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Disturbance Observer-based Adaptive Fault-tolerant Dynamic Surface Control of Nonlinear System with Asymmetric Input Saturation

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Abstract: In this paper, a composite fault-tolerant control problem is studied for a class of uncertain nonlinear system with asymmetric input constraint, actuator fault and external unmatched disturbance. The radial basis function neural network (RBFNN) is employed to approximate the unknown uncertainty and asymmetric input saturation. The approximation error, external unmatched disturbance and actuator faults are integrated as the compounded disturbance. A nonlinear disturbance observer is designed to tackle the effect of the compounded disturbance which can be separated from the controller design. To handle the effect of asymmetric input saturation, a smooth continuous differentiable saturation model is explored. Adaptive NN fault-tolerant control scheme is developed to guarantee that all the signals in the closed-loop systems are semiglobally uniformly ultimately bounded (SGUUB) and the tracking errors converge to a small neighborhood of origin by choosing the appropriate design parameters. The effectiveness of the proposed control scheme is demonstrated in the simulation study.

Keywords: Disturbance observer, fault-tolerant, input saturation, nonlinear system, robust control.

1. INTRODUCTION

The stability analysis problems of nonlinear systems have received significantly increased attention [1–6]. Actuators are always subjected to limits in practical applications, which may limit the system performance severely and even lead to the system instability. In order to compensate the effect caused by actuator saturation, many research results have been carried out on this problem [5,6]. In [5], a dynamic surface control scheme was designed for an uncertain nonlinear system in the presence of input saturation. Neural adaptive control method was presented for a class of nonlinear systems with actuator saturation [6]. Adaptive control algorithm was presented for nonlinear system with input nonlinearity in [7]. It should be noted that the most of the above researches were developed for the input symmetric saturation. To a large degree, the control of a system with symmetric saturation input is easier than that of a system with asymmetric saturation input. Relatively, a fewer results are given on the control of nonlinear system with asymmetric input. Thus, the adaptive antiwidup control scheme should be further investigated for the problem of asymmetric input saturation. Adaptive neural network control was investigated for an uncertain nonlinear system with asymmetric saturation actuators [8]. In [9], an adaptive backstepping method was proposed to deal with the asymmetric input nonlinearity and constrained states. Adaptive tracking control was proposed for uncertain MIMO systems with asymmetric input constraint in [10].

On the other hand, during to the widespread existence of disturbance, the problem of anti-disturbance control has been an important topic in the control theory [11]. Neural network and fuzzy logical systems as the universal approximators have been widely employed to tackle the system uncertainty [12–14]. An adaptive tracking control was employed handle the unknown disturbances for a class of MIMO strict-feedback nonlinear systems [13]. Both input saturation and external disturbance have been further investigated in developing control scheme [15,16]. In [16], a fuzzy control scheme was presented for uncertain nonlinear systems with input saturation and external disturbance. However, the previous works only handle the problem of input nonlinearity or external disturbance in the considered system. In practical control systems, the nonlinear system may suffer the integrated effects of actuator faults, asymmetric input saturation and unknown external disturbance.

Actuators may encounter abrupt failures during operation. For the sake of safety and reliability, accommodating

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such failures/faults is important to ensure the safety of the systems, especially for life-critical systems such as aircrafts, automatic navigation, spacecrafts and nuclear power plants and so on. Many fault-tolerant control schemes have been developed various approaches: both of sliding mode observer [17], effective active adaptive control and sliding mode techniques [18], control allocation [19], and so on. Adaptive fault-tolerant control is more flexible to design, it has been extensively used as an efficient control approach for actuator failure compensation of nonlinear system [18, 20, 21]. An adaptive fault-tolerant control is presented for a class of uncertain multi-agent systems [20]. In [21] a decentralized output-feedback adaptive backstepping control is proposed for a class of interconnected nonlinear systems with unknown actuator failures.

Motivated by the above-mentioned observations, this paper developed an adaptive NN control scheme focusing on a class of nonlinear systems with asymmetric input saturation, actuator faults and unknown time-varying disturbances. Compared with the existing results, the main contributions of this brief lie in the follows:

1) Exploring a smooth hyperbolic tangent function, this is used to propose the input saturation model. It deals with the problem of input saturation, which is more meaningful in practical application compared with [6].
2) Based on the proposed smooth saturation model, the control approach presented in this paper constitutes fault-tolerant with dynamic surface control (DSC) for general uncertain nonlinear system with simultaneous unknown asymmetric saturations, actuator faults and unmatched external disturbances. It also precludes the complexity and dimension curse problems.
3) A general nonlinear system with multiple input signals is considered. There exists the asymmetric saturation and actuator failure in each input. In fact, pioneering asymmetric input saturation was handled for SISO system without considering the actuator faults [8].

The organization of this paper is as follows: The problem formulation is given in Section 2. The adaptive fault-tolerant control scheme based on disturbance observer is developed using DSC combined with backstepping method in Section 3. Stability analysis is presented in Section 4. Simulation results are provided in Section 5. Finally, Section 6 contains the conclusion.

Throughout the paper, the notations are defined as follows: for a given matrix A, A^T denotes its transpose. When A is square matrix, $trace(A)$, $A>0$ and $A<0$ denote its trace, positive-definiteness and negative-definiteness, respectively. $\|\cdot\|$ represents the Frobenius norm of matrix $(\cdot)$ or Euclidean norm of vector $(\cdot)$. $\lambda_{max}(\cdot)$ and $\lambda_{min}(\cdot)$ denotes the maximum and minimum eigenvalues of matrix $(\cdot)$, respectively.

2. PROBLEM FORMULATION AND PRELIMINARIES

To begin with, the MISO strict-feedback nonlinear system with actuator faults, asymmetric input saturation and external disturbance is considered in this paper:

$$\begin{aligned} &\dot{x}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + d_i(t), \quad i = 1,2,\ldots,n-1, \\ &\dot{x}_n = f_n(\bar{x}_n) + b^T u(v) + d_n(t), \\ &y = x_1, \end{aligned} \tag{1}$$

where $\bar{x}_i = [x_1, x_2, \ldots, x_i] \in R^i$, $i = 1,2,\ldots,n$ stands for the state vector and $y \in R$ denotes the output vector. $f_i(\bar{x}_i)$ is an unknown smooth nonlinear function and is a known smooth nonlinear function of the system. $b = [b_1, b_2, \ldots, b_m]^{\mathrm{T}} \in R^m$, $b_j(\bar{x}_i)$ is a known constant gain. $u(v) = [u_1, u_2, \ldots, u_m]^{\mathrm{T}} \in R^m$ are the actual input signals of system and the output of saturation nonlinearity. $d_i(t)$ represents the unknown time-varying external disturbance.

However, in practical engineering, the components (actuators) of input may become faulty during operation. Actuators may undergo total loss of effectiveness (TLOE) failures or partial loss of effectiveness (PLOE) faults during operation. The two kinds of actuator faults are commonly occurring in the practice. These failures may cause instability and even catastrophic accidents if they are not well handled.

When a TLOE actuator failure occurs, the output $u_j(t)$ of the faulty actuator j^{th} becomes as:

$$u_j(t) = \bar{u}_j(t), \quad t \geq t_{jf}, \quad j \in \{1,2,\ldots,m\}, \tag{2}$$

where $\bar{u}_j$ is unknown piecewise continuous bounded signal which denotes the TLOE, i.e., the case of Lock-in place, t_{jf} is the occurrence time of the fault.

When a PLOE fault occurs, the faulty model of actuator j^{th} can be described as

$$u_j(t) = \rho_j u_{c,j}(t), \quad t \geq t_{jf}, \quad j \in \{1,2,\ldots,m\}, \tag{3}$$

where $0 < \rho_j \leq 1$ is the healthy proportion of the j^{th} actuator after losing some effectiveness at the unknown failure time instant t_{jf}.

Adapted from [21], the actuator failures which cover both PLOE type of faults and TLOE failures considered in this paper can be modeled as:

$$u_j(t) = \rho_j u_{c,j}(t) + \sigma_j(\bar{u}_j(t) - \rho_j u_{c,j}(t)), \tag{4}$$

where $u_{c,j}(t)$ is the applied designed control signal. $\sigma = diag(\sigma_1, \sigma_2, \ldots, \sigma_\lambda)$, σ_i is defined as

$$\sigma_j = \begin{cases} 1, & \text{if TLOE occurs in the } j\text{th actuator,} \\ 0, & \text{otherwise.} \end{cases} \tag{5}$$

Define $u_c(t) = b_j(\bar{x}_n)u_{c,j}(t)$ and $\bar{u} = \sum_{j=1}^{m} b_j(\bar{x}_n)\sigma_j\bar{u}_j(t)$, thus

$$u_{c,j}(t) = \frac{u_c}{b_j(\bar{x}_n)}. \tag{6}$$

Let $\rho = \sum_{j=1}^{m} \rho_j(1-\sigma_j)$, from (4), (5) and (6) the system (1) can be rewritten as

$$\begin{aligned} \dot{x}_i &= f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + d_i(t), \quad i = 1,2,\ldots,n-1, \\ \dot{x}_n &= f_n(\bar{x}_n) + \rho u_c(v) + \bar{u} + d_n(t), \\ y &= x_1. \end{aligned} \tag{7}$$

On the other hand, there exists the actuator saturation nonlinearity in the uncertain nonlinear system (7). $u_c(v)$ is represented as

$$u_c(v) = sat(v(t)) = \begin{cases} u_U, & v \geq u_U, \\ v, & -u_L \leq v < u_U, \\ -u_L, & v \leq -u_L, \end{cases} \tag{8}$$

where u_L and u_U are the known lower/upper limit bounds of $u(v)$. Form [22], we know the relationship between the applied control $u_c(t)$ and the control input $v(t)$ which has a sharp corner when $v(t) = u_U$ or $v(t) = -u_L$. Backstepping technique cannot be directly applied to this case. The smooth function is a real-valued and continuous differentiable function and its Taylor expansion always converges. Thus, a novel asymmetric saturation nonlinearity model can be obtained as

$$h(v) = u_M \times \tanh(\frac{\sqrt{\pi}}{2u_M}v), \tag{9}$$

where $u_M = (u_U + u_L)/2 + sign(v) \cdot (u_U - u_L)/2$, $sign(\cdot)$ is the standard sign function.

Fig. 1 shows that saturation function (8) can be guaranteed by the novel saturation nonlinearity function (9) with smooth saturation limitation, where $u_L = -2$, $u_U = 5$ and the input signal $v(t) = 15\sin(t)$. Then $sat(v(t))$ in (8) can be expressed as

$$sat(v(t)) = u_M \times \tanh\left(\frac{\sqrt{\pi}}{2u_M}v\right) + \Delta(v). \tag{10}$$

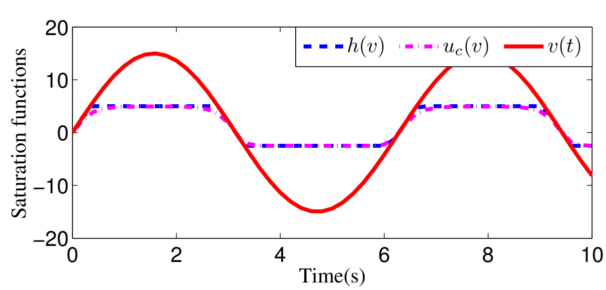


Fig. 1. Saturation function model.

Remark 1: Between the applied control $u_c(t)$ and the control input $v(t)$ has a sharp corner, when $v(t) = u_U$ or $v(t) = -u_L$. The smooth function (10) can not only obtain the asymmetric saturation nonlinearity model but also make the backstepping technique directly applied to this model.

It is straightforward to verify that

$$|\Delta(v)| = |sat(v) - h(v)| \leq \max\begin{pmatrix} u_U\left(1 - \tanh(\frac{\sqrt{\pi}}{2})\right), \\ u_L\left(1 - \tanh(\frac{\sqrt{\pi}}{2})\right) \end{pmatrix} = \bar{\Delta}. \tag{11}$$

From (11), it can be seen that $\Delta(v)$ is a bounded function. When v changes from $-u_L$ to u_U, the value of $\Delta(v)$ increase from 0 to $\bar{\Delta}$, when the value of v outside this area, the value of $\Delta(v)$ decreases from $\bar{\Delta}$ to 0. According to mean-value theorem [23], $h(v)$ in (9) can be expressed as $h(v) = h(v^*) + \frac{\partial h(\cdot)}{\partial v}\Big|_{v=v^\mu}(v - v^*)v^\mu = \mu v + (1-\mu)v^*$ with $0 < \mu < 1$.

Let $v^* = 0$, then $h(v^*) = 0$, therefore $h(v)$ can be rewritten as $h(v) = \frac{\partial h(\cdot)}{\partial v}\Big|_{v=v^\mu} v(t) = hv(t)$.

Remark 2: From the definition of $h(v)$ in (9), $tanh(\cdot)$ is an elementary function of sigmoid shape, $h(v)$ is nondecreasing. There exist positive constants $\bar{h}$ and $\bar{\bar{h}}$ such that $0 < \bar{h} \leq h(v) \leq \bar{\bar{h}}$ for every $v \in R$.

The actuator saturation nonlinearity model in (8) is rewritten as

$$u_c(t) = sat(v(t)) = hv(t) + \Delta(v). \tag{12}$$

Then, the uncertain nonlinear system (7) can be rewritten as

$$\begin{aligned} \dot{x}_i &= f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + d_i(t), \quad i = 1,2,\ldots,n-1, \\ \dot{x}_n &= f_n(\bar{x}_n) + \rho_h v + \rho\Delta(v) + \bar{u} + d_n(t), \\ y &= x_1, \end{aligned} \tag{13}$$

where $\rho_h = \rho \cdot h$ is an unknown bounded constant.

The main control objective of this paper is to design an adaptive control scheme for system (1) with the actuator faults (i.e., (2) and (3)) and actuator saturation nonlinearity (8) to obtain that the bounded properties of all signals involved in the resulting closed-loop system and output $y(t)$ can track the given reference signal $y_d(t)$ as closely as possible. To achieve the control objective, the proposed control-design needs the following assumptions.

Assumption 1: The system considered in (1) is input-to-state stable (ISS).

Assumption 2 [24]**:** The time-varying external disturbance $d_i(t)$ satisfies $\|d_i(t)\| \leq d_i^*$, where d_i^*, $i = 1,2,\ldots,n$ are unknown constants.

Assumption 3 [5, 25]**:** For a time-varying external disturbance $d_i(t)$, there exists an unknown positive constant $\bar{d}_i$ such that $\left\|\dot{d}_i(t)\right\| \leq \bar{d}_i$, $i = 1,2,\ldots,n$.

Assumption 4 [8, 11, 26]**:** The control coefficient function of the uncertain system (1) is bounded, i.e., there must exist positive constants $\bar{\bar{g}}_i$ such that with $0 < g_i(\bar{x}_i) \leq \bar{\bar{g}}_i$ and $g_i(\bar{x}_i) \neq 0$, $i = 1,2,\ldots,n-1$.

Assumption 5: The smooth function $b_i(\bar{x}_i)$ of the uncertain system (1) is not zero, i.e., $b_i(\bar{x}_i) \neq 0$.

Assumption 6: The plant (1) is so constructed that for any up to $m-1$ actuators undergoing complete failures simultaneously, the remaining actuators can still achieve the desired control objectives.

Remark 3: Assumption 3 is reasonable when the available energy of external disturbance $d_i(t)$ is considered finite. Assumption 4 is sufficient for controllability of system (1) and there are many practical systems, such as aircraft longitudinal model [11] and brush dc (BDC) motor [26], that satisfy this Assumption. Assumption 5 implies that smooth function $b_i(\bar{x}_i)$ is strictly either positive or negative. On the other hand, as similar as discussed in the existing adaptive compensation schemes, Assumption 6 is a basic assumption for the adaptive failure compensation problem.

3. DISTURBANCE OBSERVER-BASED ADAPTIVE NN CONTROL

In this section, a robust fault-tolerant dynamic surface control (DSC) is developed for the strict-feedback nonlinear system (1). A backstepping control technique and disturbance observer will be used. The process of the control is divided into n steps, at the i^{th} step $(i = 1,\ldots,n-1)$, a virtual control law and the parameter updated laws are explored, respectively. To handle the problem of "explosion of complexity" inherent, DSC is combined with the conventional backstepping method.

Define the following error variables: $z_1 = x_1 - y_d$, $z_i = x_i - \omega_i$, $\varsigma_i = \omega_i - \alpha_{i-1}, i = 1,2,\ldots,n$, where ω_i is the output of a first-order filter with α_{i-1} as the input. At each step of recursion, the quantity $\dot{\alpha}_{i-1}$ which is replaced by ω_i determined the virtual control law $\dot{\alpha}_i$, where α_{i-1} is an intermediate control. As to say, the operation of differentiation can be replaced by simpler algebraic operation. To get the main results, the control scheme comprises the following steps.

Step 1: Let $\omega_1 = y_d$, the error variables is defined as following:

$$z_1 = x_1 - \omega_1, z_2 = x_2 - \omega_2. \tag{14}$$

Considering the tracking error (14) and system (13), we have

$$\dot{z}_1 = f_1(x_1) + g_1(x_1)x_2 + d_1(t) - \dot{y}_d. \tag{15}$$

Define $F_1(X_1) = f_1(x_1) - \dot{y}_d$. Due to the prominent advantage of RBFNN in approximating any smooth nonlinear approximation with arbitrary precision [27], we employ the following RBFNN to estimate the unknown nonlinear function $F_1(X_1)$ as follows:

$$F_1(X_1) = W_1{}^T \varphi_1(X_1) + \zeta_1{}^*, \tag{16}$$

where $X_1 = [x_1, \dot{\omega}_1] \in R^2$ is the input vector, $W_1 \in R^{h_{m1}}$ is the ideal constant weight matrix between the hidden layer and the output layer, h_{m1} is the number of the hidden layer neurons. $\varphi_1(\cdot) \in R^{h_{m1}}$ is monotonically increasing activation function. $\zeta_1{}^* \in R$ is the unknown smallest approximation error of RBFNN which satisfies that $\dot{\zeta}_1^*$ is bounded.

Assumption 7: The ideal weight matrix $W_i, i = 1,2,\ldots,n$ and the activation function $\varphi_i(X_i), i = 1,2,\ldots,n$ of the NN are both bounded, i.e., $\|\varphi_i(X_i)\| \leq \mu_i$ with $\mu_i \geq 0$ and $\|W_i\| \leq \bar{W}_i$ with $\bar{W}_i > 0$.

Define the compounded disturbance $D_1(t) = d_1(t) + \zeta_1{}^*$, which combined RBFNN approximation error with external time-varying disturbance. From Assumption 3 and the approximation ability of RBFNN, it can be obtained the time derivative of the compound disturbance D_1 is bounded, i.e., $\left\|\dot{D}_1\right\| \leq \theta_1$, θ_1 is a unknown positive constant.

Substituting (16) into (15) yields

$$\dot{z}_1 = W_1{}^T \varphi_1(X_1) + g_1(x_1)x_2 + D_1(t). \tag{17}$$

Design the intermediate virtual control signal α_1 and then let it pass through an first-order filter with time constant τ_2 to generate variable ω_2 which holds a same initial value as $\alpha_1(0)$.

$$\alpha_1 = \frac{1}{g_1}\left(-k_1 z_1 - \hat{W}_1^T \varphi_1(X_1) - \hat{D}_1\right), \tag{18}$$

$$\tau_2 \dot{\omega}_2 + \omega_2 = \alpha_1,\ \omega_2(0) = \alpha_1(0), \tag{19}$$

where τ_2 is a small constant, $k_1 > 0$ is a designed parameter. $\hat{W}_1$ and $\hat{D}_1$ are the estimate of ideal weight matrix W_1 and compounded disturbance D_1, respectively.

Define an error variable $\varsigma_2 = \omega_2 - \alpha_1$, from (17), (18) and (19) we can obtain

$$\begin{aligned}\dot{\varsigma}_2 = \dot{\omega}_2 - \dot{\alpha}_1 &= -\frac{\varsigma_2}{\tau_2} + \lambda_1(z_1, \hat{W}_1, \varphi_1, \hat{D}_1, \dot{y}_d) \\ &\leq -\frac{\varsigma_2}{\tau_2} + \bar{\lambda}_1,\end{aligned} \tag{20}$$

where $\lambda_1(\cdot)$ is a continuous function with bounded. $\bar{\lambda}_1$ is a positive bounded constant of the function $\lambda_1(\cdot)$.

Lyapunov candidate function can be constructed as

$$V_1 = \frac{1}{2}z_1{}^2 + \frac{1}{2}\tilde{W}_1^T \Gamma_1{}^{-1} \tilde{W}_1 + \frac{1}{2}\tilde{D}_1^2 + \frac{1}{2}\varsigma_2{}^2, \tag{21}$$

where $\Gamma_1 = \Gamma_1{}^T > 0$ is an adaptation gain matrix. $\tilde{W}_1 = \hat{W}_1 - W_1$ and $\tilde{D}_1 = D_1 - \hat{D}_1$.

Using the time derivative of V_1, then

$$\begin{aligned}\dot{V}_1 =& z_1\dot{z}_1 + \tilde{W}_1^T\Gamma^{-1}\dot{\tilde{W}}_1 + \tilde{D}_1\dot{\tilde{D}}_1 + \varsigma_2\dot{\varsigma}_2\\ =& -k_1{z_1}^2 - z_1\tilde{W}_1^T\varphi(X_1) + g_1(x_1)z_1z_2 + z_1\tilde{D}_1\\ &+ g_1(x_1)z_1\varsigma_2 + \tilde{W}_1^T\Gamma^{-1}\dot{\hat{W}}_1 + \tilde{D}_1\dot{\tilde{D}}_1 + \varsigma_2\dot{\varsigma}_2. \quad (22)\end{aligned}$$

The adaptive parameter updated law of $\hat{W}_1$ are developed as

$$\dot{\hat{W}}_1 = \Gamma_1\left[\varphi_1(X_1)z_1 - \delta_{W1}\hat{W}_1\right], \quad (23)$$

where $\delta_{W1} > 0$ is a designed parameter.
Substituting (23) into (22) yields

$$\begin{aligned}\dot{V}_1 =& -k_1{z_1}^2 + g_1(x_1)z_1z_2 + g_1(x_1)z_1\varsigma_2\\ &+ z_1\tilde{D}_1 - \tilde{W}_1^T\delta_{W1}\hat{W}_1 + \tilde{D}_1\dot{\tilde{D}}_1 + \varsigma_2\dot{\varsigma}_2. \quad (24)\end{aligned}$$

We designed a nonlinear disturbance observer (NDO) to estimate the unknown compounded disturbance $D_1(t)$ and to avoid repeatedly computing the time derivative of virtual control laws.

$$\begin{aligned}\dot{\phi}_1 &= g_1(x_1)x_2 + \hat{W}_1^T\varphi_1(X_1) + \hat{D}_1,\\ \hat{D}_1 &= \gamma_1(z_1 - \phi_1), \quad (25)\end{aligned}$$

where ϕ_1 represents the auxiliary variable of the nonlinear disturbance observer. γ_1 is a parameter to be designed.

Based on (25), we can have $\dot{\hat{D}}_1 = \gamma_1(\dot{z}_1 - \dot{\phi}_1) = \gamma_1\left(\tilde{D}_1(t) - \tilde{W}_1^T\varphi_1(X_1)\right)$.

Considering the following Young's inequality

$$\tilde{D}_1\dot{D}_1 \le 0.5\tilde{D}_1^2 + 0.5{\theta_1}^2,, \quad (26)$$

$$\tilde{D}_1\tilde{W}_1^T\varphi_1(X_1) \le \frac{1}{2}a_1{\mu_1}^2\tilde{D}_1^2 + \frac{1}{2a_1}\left\|\tilde{W}_1\right\|^2, \quad (27)$$

$$-\delta_{W1}\tilde{W}_1^T\hat{W}_1 \le -\frac{1}{2}\delta_{W1}\left\|\tilde{W}_1\right\|^2 + \frac{1}{2}\delta_{W1}\|W_1\|^2, \quad (28)$$

where $\left\|\dot{D}_1\right\| \le \theta_1$, $a_1 > 0$ and $\delta_{w1} > 0$ are designed parameters, $\|\varphi_1(X_1)\| \le \mu_1$. Then from (25), (26) and (27), we can obtain

$$\begin{aligned}\tilde{D}_1\dot{\tilde{D}}_1 =& \tilde{D}_1\dot{D}_1 - \tilde{D}_1\dot{\hat{D}}_1\\ =& \tilde{D}_1\dot{D}_1 + \gamma_1\tilde{D}_1\tilde{W}_1^T\phi(X_1) - \gamma_1\tilde{D}^2\\ \le& \frac{1}{2}\left(\gamma_1a_1{\mu_1}^2 - 2\gamma_1 + 1\right)\tilde{D}_1^2\\ &+ 0.5\theta_1 + \frac{\gamma_1}{2a_1}\left\|\tilde{W}_1\right\|^2. \quad (29)\end{aligned}$$

From (20), we can obtain

$$\begin{aligned}\varsigma_2\dot{\varsigma}_2 =& \varsigma_2\left(\dot{\varsigma}_2 + \frac{\varsigma_2}{\tau_2}\right) - \frac{{\varsigma_2}^2}{\tau_2}\\ \le& |\varsigma_2|\bar{\lambda}_1 - \frac{{\varsigma_2}^2}{\tau_2}\\ \le& {\varsigma_2}^2\left(1 - \frac{1}{\tau_2}\right) + \frac{1}{4}\bar{\lambda}_1^2. \quad (30)\end{aligned}$$

Substituting (28), (29) and (30) into (24), we can obtain the following inequality

$$\begin{aligned}\dot{V}_1 \le& -k_1{z_1}^2 + g_1(x_1)z_1z_2 + g_1(x_1)z_1\varsigma_2 + z_1\tilde{D}_1\\ &- \frac{1}{2}\delta_{W1}\left\|\tilde{W}_1\right\|^2 + \frac{1}{2}\delta_{W1}\|W_1\|^2 + 0.5\theta_1\\ &+ \frac{1}{2}\left(\gamma_1a_1{\mu_1}^2 - 2\gamma_1 + 1\right)\tilde{D}_1^2 + \frac{\gamma_1}{2a_1}\left\|\tilde{W}_1\right\|^2\\ &+ {\varsigma_2}^2\left(1 - \frac{1}{\tau_2}\right) + \frac{1}{4}\bar{\lambda}_1^2\\ \le& -\left(k_1 - \bar{\bar{g}}_1 - \frac{1}{2}\right){z_1}^2 - \left(\frac{1}{\tau_2} - \frac{\bar{\bar{g}}_1}{4} - 1\right){\varsigma_2}^2\\ &- \frac{1}{2}\left(2\gamma_1 - \gamma_1a_1{\mu_1}^2 - 2\right)\tilde{D}_1 + g_1(x_1)z_1z_2\\ &- \frac{1}{2}\left(\delta_{W1} - \frac{\gamma_1}{a_1}\right)\left\|\tilde{W}_1\right\|^2 + \frac{1}{2}\delta_{W1}\|W_1\|^2\\ &+ 0.5\theta_1 + \frac{1}{4}\bar{\lambda}_1^2. \quad (31)\end{aligned}$$

The coupling term $g_1(x_1)z_1z_2$ in (31) will be canceled in the next step.

Step *i* $(i = 2, \ldots, n-1)$**:** The differentiation of z_i can be obtained as

$$\dot{z}_i = \dot{x}_i - \dot{\omega}_i = f_i(\bar{x}_i) + g_i(\bar{x}_i)x_{i+1} + d_i(t) - \dot{\omega}_i. \quad (32)$$

Let $F_i(X_i) = f_i(\bar{x}_i) - \dot{\omega}_i$. Like Step1, using RBFNN to approximate the unknown term $F_i(X_i)$, we have

$$F_i(X_i) = {W_i}^T\varphi_i(X_i) + {\zeta_i}^*, \quad (33)$$

where $X_i = [\bar{x}_i,\ \omega_i] \in R^{i+1}$ is input vector, $W_i \in R^{h_{mi}}$ is the ideal weight vector of the NN. $\varphi_i(\cdot) \in R^{h_{mi}}$ is monotonically increasing activation function vector. ${\zeta_i}^* \in R$ is the unknown smallest approximation error of RBFNN which satisfies that $\dot{\zeta}_i^*$ is bounded. From Assumption 7, $\varphi_i(X_i)$ is bounded with $\|\varphi_i(X_i)\| \le \mu_i$, $\mu_i \ge 0$.

Define the compounded disturbance $D_i = d_i(t) + {\zeta_i}^*$. From Assumption 3 and the approximation ability of RBFNN, it can be obtained the time derivative of D_i is bounded, i.e., $\left\|\dot{D}_i\right\| \le \theta_i$, θ_i is a positive constant. Substituting (33) into (32), we have

$$\dot{z}_i = {W_i}^T\varphi_i(X_i) + g_i(\bar{x}_i)x_{i+1} + D_i(t). \quad (34)$$

Design the virtual control law α_i, and then let it pass through an first-order filter with time constant τ_i to generate variable ω_{i+1}.

$$\alpha_i = \frac{1}{g_i(\bar{x}_i)}\left(-k_iz_i - \hat{W}_i^T\varphi_i(X_i) - \hat{D}_i - g_{i-1}(\bar{x}_{i-1})z_{i-1}\right), \quad (35)$$

$$\tau_{i+1}\dot{\omega}_{i+1} + \omega_{i+1} = \alpha_i,\ \ \omega_{i+1}(0) = \alpha_i(0), \quad (36)$$

where τ_{i+1} is a small constant, $k_i > 0$ is a designed parameter. $\hat{W}_i$ and $\hat{D}_i$ are the estimate of W_i and D_i, respectively.

Define the error variable $\varsigma_{i+1}=\omega_{i+1}-\alpha_i$, $i=1,2,\ldots,n-1$ from (34),(35) and (36) we can obtain

$$\begin{aligned}\dot{\varsigma}_{i+1}=\dot{\omega}_{i+1}-\dot{\alpha}_i&=-\frac{\varsigma_{i+1}}{\tau_{i+1}}+\bar{\lambda}_i(z_i,\hat{W}_i,\varphi_i,\hat{D}_i,\omega_i)\\&\le-\frac{\varsigma_{i+1}}{\tau_{i+1}}+\bar{\lambda}_i,\end{aligned}\quad(37)$$

where $\bar{\lambda}_i(\cdot)$ is a continuous function$\bar{\bar{\lambda}}_i$ is the positive bounded constant of the function$\bar{\lambda}_i(\cdot)$.

Consider the following Lyapunov function candidate:

$$V_i=\frac{1}{2}z_i{}^2+\frac{1}{2}\tilde{W}_i^T\Gamma_i^{-1}\tilde{W}_i+\frac{1}{2}\tilde{D}_i^2+\frac{1}{2}\varsigma_{i+1}{}^2,\quad(38)$$

where $\Gamma_i=\Gamma_i{}^T>0$ is an adaptation gain matrix. $\tilde{W}_i=\hat{W}_i-W_i$ and $\tilde{D}_i=D_i-\hat{D}_i$. Then the time derivative of V_i is written as

$$\begin{aligned}\dot{V}_i=&z_iW_i{}^T\varphi_i(X_i)+z_ig_i(\bar{x}_i)x_{i+1}+z_iD_i\\&+\tilde{W}_i^T\Gamma_i^{-1}\dot{\hat{W}}_i+\tilde{D}_i\dot{\tilde{D}}_i+\varsigma_{i+1}\dot{\varsigma}_{i+1}\\=&-k_iz_i{}^2-g_{i-1}(\bar{x}_i)z_{i-1}z_i+g_i(\bar{x}_i)z_iz_{i+1}\\&+z_ig_i(\bar{x}_i)\varsigma_{i+1}-z_i\tilde{W}_i^T\varphi_i(X_i)+z_i\tilde{D}_i\\&+\tilde{W}_i^T\Gamma_i^{-1}\dot{\hat{W}}_i+\tilde{D}_i\dot{\tilde{D}}_i+\varsigma_{i+1}\dot{\varsigma}_{i+1}.\end{aligned}\quad(39)$$

Considering (37) yields

$$\begin{aligned}\varsigma_{i+1}\dot{\varsigma}_{i+1}&=\varsigma_{i+1}\left(\dot{\varsigma}_{i+1}+\frac{\varsigma_{i+1}}{\tau_{i+1}}\right)-\frac{\varsigma_{i+1}{}^2}{\tau_{i+1}}\\&\le|\varsigma_{i+1}|\bar{\bar{\lambda}}_i-\frac{\varsigma_{i+1}{}^2}{\tau_{i+1}}\\&\le\varsigma_{i+1}{}^2\left(1-\frac{1}{\tau_{i+1}}\right)+\frac{1}{4}\bar{\bar{\lambda}}_i^2.\end{aligned}\quad(40)$$

The adaptive parameter updated law of $\hat{W}_i$ is designed as

$$\dot{\hat{W}}_i=\Gamma_i\left[\varphi_i(X_i)z_i-\delta_{Wi}\hat{W}_i\right],\quad(41)$$

where $\delta_{Wi}>0$ is a designed parameter. Substituting (40) and (41) into (39), we obtain

$$\begin{aligned}\dot{V}_i=&-k_iz_i{}^2-g_{i-1}(\bar{x}_i)z_{i-1}z_i+g_i(\bar{x}_i)z_iz_{i+1}+z_i\tilde{D}_i\\&+z_ig_i(\bar{x}_i)\varsigma_{i+1}-\delta_{W_i}\tilde{W}_i^T\hat{W}_i+\tilde{D}_i\dot{\tilde{D}}_i+\varsigma_{i+1}\dot{\varsigma}_{i+1}.\end{aligned}\quad(42)$$

Then, we can obtain

$$\begin{aligned}\dot{V}_i\le&-k_iz_i{}^2-g_{i-1}(\bar{x}_{i-1})z_{i-1}z_i+g_i(\bar{x}_i)z_iz_{i+1}\\&+g_i(\bar{x}_i)z_i\varsigma_{i+1}+z_i\tilde{D}_i-\delta_{W_i}\tilde{W}_i^T\hat{W}_i\\&+\tilde{D}_i\dot{\tilde{D}}_i+\varsigma_{i+1}{}^2\left(1-1/\tau_{i+1}\right)+\bar{\bar{\lambda}}_i^2/4.\end{aligned}\quad(43)$$

To estimate the unknown compounded disturbance $D_i(t)$, the NDO is developed as

$$\hat{D}_i=\gamma_i\left(z_i-\phi_i\right),$$

$$\dot{\phi}_i=g_i(\bar{x}_i)x_{i+1}+\hat{W}_i^T\varphi_i(X_i)+\hat{D}_i,\quad(44)$$

where ϕ_i is an auxiliary variable, γ_i is a positive designed parameter. From (44), we can obtain $\dot{\hat{D}}_i=\gamma_i\left(\tilde{D}_i(t)-\tilde{W}_i^T\varphi_i(X_i)\right)$.

Considering the following facts

$$\tilde{D}_i\dot{D}_i\le0.5\tilde{D}_i^2+0.5\theta_i{}^2,\quad(45)$$

$$\gamma_i\tilde{D}_i\tilde{W}_i^T\phi_i(X_i)\le\frac{1}{2}\gamma_ia_i\mu_i{}^2\tilde{D}_i^2+\frac{1}{2a_i}\gamma_i\left\|\tilde{W}_i\right\|^2,\quad(46)$$

$$-\delta_{Wi}\tilde{W}_i^T\hat{W}_i\le-\frac{1}{2}\delta_{Wi}\left\|\tilde{W}_i\right\|^2+\frac{1}{2}\delta_{Wi}\|W_i\|^2,\quad(47)$$

$$z_i\tilde{D}_i\le0.5\tilde{D}_i^2+0.5z_i{}^2.\quad(48)$$

Then, from (44), (45) and (46), we have

$$\begin{aligned}\tilde{D}_i\dot{\tilde{D}}_i=&\tilde{D}_i\dot{D}_i-\tilde{D}_i\gamma_i(-\tilde{W}_i^T\phi_i(X_i)\\&+g_i(\bar{x}_i)x_{i+1}+\tilde{D}_i(t)-g_i(\bar{x}_i)x_{i+1})\\=&\tilde{D}_i\dot{D}_i+\gamma_i\tilde{D}_i\tilde{W}_i^T\phi_i(X_i)-\gamma_i\tilde{D}_i^2\\\le&-\left(\gamma_i-\frac{1}{2}-\frac{1}{2}\gamma_ia_i\mu_i{}^2\right)\tilde{D}_i^2\\&+\frac{1}{2a_i}\gamma_i\left\|\tilde{W}_i\right\|^2+\frac{1}{2}\theta_i{}^2.\end{aligned}\quad(49)$$

Substituting (47), (48) and (49) into (43) yields

$$\begin{aligned}\dot{V}_i\le&-\left(k_i-\bar{\bar{g}}_i-\frac{1}{2}\right)z_i{}^2-\left(\frac{1}{2}\delta_{Wi}-\frac{1}{2a_i}\gamma_i\right)\left\|\tilde{W}_i\right\|^2\\&-\left(\gamma_i-1-\frac{1}{2}\gamma_ia_i\mu_i{}^2\right)\tilde{D}_i^2-g_{i-1}(\bar{x}_{i-1})z_{i-1}z_i\\&-\left(\frac{1}{\tau_{i+1}}-1-\frac{\bar{\bar{g}}_i}{4}\right)\varsigma_{i+1}{}^2+g_i(\bar{x}_i)z_iz_{i+1}\\&+\frac{1}{2}\delta_{Wi}\|W_i\|^2+\frac{1}{2}\theta_i{}^2+\frac{1}{4}\bar{\bar{\lambda}}_i^2.\end{aligned}\quad(50)$$

The coupling terms $g_{i-1}(\bar{x}_{i-1})z_{i-1}z_i$ and $g_i(\bar{x}_i)z_iz_{i+1}$ can be canceled in the last and next step, respectively.

Step n: In this final step, from (13), we can obtain the derivative of $z_n=x_n-\omega_n$ with time

$$\dot{z}_n=f_n(\bar{x}_n)+\rho_hv(t)+\rho\Delta(v)+\bar{u}+d_n(t)-\dot{\omega}_n.\quad(51)$$

Our control objective is to design the control input signal $v(t)$ to compensate the input saturation and actuator faults.

Let $F_n(X_n)=\rho_h{}^{-1}(f_n(\bar{x}_n)-\dot{\omega}_n)$, then using RBFNN to proximate the unknown function $F_n(X_n)$, we have

$$F_n(X_n)=W_n{}^T\varphi_n(X_n)+\zeta_n{}^*,\quad(52)$$

where $X_n=[\bar{x}_n,\ \dot{\omega}_n]\in R^{n+1}$ is input vector, $\varphi_n(X_n)$ is the basis function vector of NN. The smallest approximation error of RBFNN $\zeta_n{}^*$ satisfies $\dot{\zeta}_n^*$ is bounded.

From Assumption 7, W_n and $\varphi_n(X_n)$ are both bounded, $\|\varphi_n(X_n)\|\le\mu_n$.

Define the compounded disturbance

$$D_n = {\rho_h}^{-1}\left(d_n(t) + \rho\Delta(v) + \bar{u}\right) + {\zeta_n}^*. \tag{53}$$

From Assumption 2 and assumption 3, it can obtained that $\|D_n\| \le \eta_n$ and $\left\|\dot{D}_n\right\| \le \theta_n$, η_n and θ_n are positive constants.

Substituting (52), (53) into (51) yields

$$\dot{z}_n = \rho_h {W_n}^T \varphi_n(X_n) + \rho_h D_n(t) + \rho_h v. \tag{54}$$

Design the adaptive tolerant-fault control law $v(t)$ as following

$$v(t) = -k_n z_n - \hat{W}_n^T \varphi_n(X_n) - \hat{D}_n - g_{n-1} z_{n-1}, \tag{55}$$

where $\hat{W}_n$, $\hat{D}_n$ is the estimation of W_n and D_n, respectively. $\tilde{W}_n = \hat{W}_n - W_n$, $\tilde{D}_n = D_n - \hat{D}_n$.

Substituting (55) into (54), we obtain

$$\begin{aligned}\dot{z}_n =& -\rho_h \tilde{W}_n^T \varphi_n(X_n) + \rho_h \tilde{D}_n(t) - \rho_h k_n z_n \\ & - \rho_h g_{n-1} z_{n-1} + \rho_h {\zeta_n}^*.\end{aligned} \tag{56}$$

The parameter update law of $\hat{W}_n$ is designed as

$$\dot{\hat{W}}_n = \Gamma_n \left[\varphi_n(X_n) z_n - \delta_{Wn} \hat{W}_n\right], \tag{57}$$

where $\Gamma_n = {\Gamma_n}^T \ge 0$ is an adaptation gain matrix, $\delta_{Wn} \ge 0$ is a designed constant. Thus we can obtain

$$\begin{aligned}\tilde{W}_n^T {\Gamma_n}^{-1} \dot{\tilde{W}}_n &= \tilde{W}_n^T \Gamma^{-1} \dot{\hat{W}}_n \\ &= \tilde{W}_n^T \varphi_n(X_n) z_n - \delta_{Wn} \tilde{W}_n^T \hat{W}_n.\end{aligned} \tag{58}$$

To design the disturbance observer, another RBFNN is employed to approximate the unknown term $F_D(X_D) = f_n(\bar{x}_n) + \rho_h v - \dot{\omega}_n$ of (51), we have

$$F_D(X_D) = {W_D}^T \varphi_D(X_D) + {\xi_D}^*, \tag{59}$$

where $X_D = [\bar{x}_n,\ \dot{\omega}_n, v] \in R^{n+2}$ is input vector, $\varphi_D(X_D)$ is the basis function vector of RBFNN. The smallest approximation error ${\xi_D}^*$ satisfies $\dot{\xi_D}^*$ is bounded,i.e., $\left\|\dot{\xi}_D^*\right\| \le \bar{\xi}_D$.

Assumption 8: The ideal weight matrix and the basis function $\varphi_D(X_D)$ of the NN are both bounded, i.e., $\|\varphi_D(X_D)\| \le \mu_D$ with $\mu_D \ge 0$ and $\|W_D\| \le \bar{W}_D$ with $\bar{W}_D \ge 0$.

The (50) can be also rewritten as

$$\dot{z}_n = {W_D}^T \varphi_D(X_D) + \rho_h D_n(t) + {\xi_D}^*. \tag{60}$$

The parameter update law of $\hat{W}_D$ is designed as

$$\dot{\hat{W}}_D = -\Gamma_D \delta_{WD} \hat{W}_D, \tag{61}$$

where $\Gamma_D = {\Gamma_D}^T \ge 0$ is an adaptation gain matrix, $\delta_{WD} \ge 0$ is a designed constant. Thus we can obtain

$$\tilde{W}_D^T {\Gamma_D}^{-1} \dot{\tilde{W}}_D = \tilde{W}_D^T {\Gamma_D}^{-1} \dot{\hat{W}}_D = -\delta_{WD} \tilde{W}_D^T W_D. \tag{62}$$

To estimate the unknown compounded disturbance $D_n(t)$ in (54), a NDO is designed as

$$\begin{aligned}&\hat{D}_n = \gamma_n (z_n - \phi_n), \\ &\dot{\phi}_n = \hat{W}_D^T \varphi_D(X_D) + \hat{D}_n,\end{aligned} \tag{63}$$

where ϕ_n is an auxiliary variable, γ_n is a positive designed parameter, then we can obtain

$$\begin{aligned}\tilde{D}_n \dot{\tilde{D}}_n =& \tilde{D}_n \dot{D}_n - \tilde{D}_n \gamma_n ({W_D}^T \phi_D(X_D) \\ & + \rho_h D_n + {\xi_D}^* - \hat{W}_D^T \phi_D(X_D) - \hat{D}_n) \\ =& \tilde{D}_n \dot{D}_n + \gamma_n \tilde{D}_n \tilde{W}_D^T \phi_D(X_D) - \gamma_n \tilde{D}_n^2 \\ & + \gamma_n D_n \tilde{D}_n (1 - \rho_h) - \gamma_n \tilde{D}_n {\xi_D}^*.\end{aligned} \tag{64}$$

Consider the following Lyapunov function candidate:

$$V_n = \frac{1}{2\rho_h} {z_n}^2 + \frac{1}{2} \tilde{W}_n^T {\Gamma_n}^{-1} \tilde{W}_n + \frac{1}{2} \tilde{D}_n^2 + \frac{1}{2} \tilde{W}_D^T {\Gamma_D}^{-1} \tilde{W}_D. \tag{65}$$

From (55), (58), (62) and (64), the time derivative of V_n is

$$\begin{aligned}\dot{V}_n =& z_n \dot{z}_n / \rho_h + \tilde{W}_n^T {\Gamma_n}^{-1} \dot{\tilde{W}}_n + \tilde{D}_n \dot{\tilde{D}}_n + \tilde{W}_D^T {\Gamma_D}^{-1} \dot{\tilde{W}}_D \\ =& z_n \tilde{D}_n - k_n {z_n}^2 - g_{n-1} z_n z_{n-1} - \delta_{Wn} \tilde{W}_n^T \hat{W}_n \\ & + \tilde{D}_n \dot{D}_n + \gamma_n \tilde{D}_n \tilde{W}_D^T \varphi_D(X_D) \\ & + \gamma_n D_n \tilde{D}_n (1 - \rho_h) - \gamma_n \tilde{D}_n {\xi_D}^* \\ & - \gamma_n \tilde{D}_n^2 - \delta_{WD} \tilde{W}_D^T \hat{W}_D.\end{aligned} \tag{66}$$

Considering the following facts

$$\tilde{D}_n \dot{D}_n \le 0.5 \tilde{D}_n^2 + 0.5 {\theta_n}^2, \tag{67}$$

$$\gamma_n \tilde{D}_n \tilde{W}_D^T \phi_D(X_D) \le \frac{1}{2} \gamma_n a_n {\mu_D}^2 \tilde{D}_n^2 + \frac{1}{2a_n} \gamma_n \left\|\tilde{W}_D\right\|^2, \tag{68}$$

$$-\delta_{Wn} \tilde{W}_n^T \hat{W}_n \le -\frac{1}{2} \delta_{Wn} \left\|\tilde{W}_n\right\|^2 + \frac{1}{2} \delta_{Wn} \|W_n\|^2, \tag{69}$$

$$z_n \tilde{D}_n \le 0.5 \tilde{D}_n^2 + 0.5 {z_n}^2, \tag{70}$$

$$\begin{aligned}-\gamma_n \tilde{D}_n {\xi_D}^* &\le \gamma_n \left|\tilde{D}_n\right| \left|{\xi_D}^*\right| \\ &\le \frac{1}{4} \gamma_n \tilde{D}_n^2 + \gamma_n \bar{\xi}_D^2.\end{aligned} \tag{71}$$

Then, we have

$$\begin{aligned}\dot{V}_n \le& -k_n {z_n}^2 - g_{n-1} z_n z_{n-1} + 0.5 \tilde{D}_n^2 + 0.5 {z_n}^2 \\ & - \frac{1}{2} \delta_{Wn} \left\|\tilde{W}_n\right\|^2 + \frac{1}{2} \delta_{Wn} \|W_n\|^2 + 0.5 \tilde{D}_n^2 \\ & + 0.5 {\theta_n}^2 + \frac{1}{2} \gamma_n a_n {\mu_D}^2 \tilde{D}_n^2 + \frac{1}{2a_n} \gamma_n \left\|\tilde{W}_D\right\|^2 \\ & + \gamma_n D_n \tilde{D}_n (1 - \rho_h) + \frac{1}{4} \gamma_n \tilde{D}_n^2 + \gamma_n \bar{\xi}_D^2 \\ & - \gamma_n \tilde{D}_n^2 - \frac{1}{2} \delta_{WD} \left\|\tilde{W}_D\right\|^2 + \frac{1}{2} \delta_{WD} \|W_D\|^2 \\ \le& -\left(k_n - \frac{1}{2}\right) {z_n}^2 - g_{n-1} z_n z_{n-1}\end{aligned}$$

$$
\begin{aligned}
&-\left(\frac{1}{2}\gamma_n-\frac{1}{2}\gamma_n a_n \mu_D{}^2-1.5\right)\tilde{D}_n^2+0.5z_n{}^2\\
&-\frac{1}{2}\delta_{Wn}\|\tilde{W}_n\|^2-\left(\frac{1}{2}\delta_{WD}-\frac{1}{2a_n}\gamma_n\right)\|\tilde{W}_D\|^2\\
&+\frac{1}{2}\delta_{Wn}\|W_n\|^2+0.5\theta_n{}^2+\gamma_n|\rho_h-1|^2\eta_n{}^2\\
&+\gamma_n\bar{\xi}_D^2+\frac{1}{2}\delta_{WD}\|W_D\|^2. \qquad (72)
\end{aligned}
$$

According to the above design procedure, to further clarify the design principle in this paper, we present the proposed adaptive control scheme in Algorithm 1.

Algorithm 1:

Step 1: For $1 \le i \le n$, select the activation functions of NN $\varphi_i(\cdot)$, $\varphi_D(\cdot)$ and the sampling interval T. Initial parameters are set as $\hat{W}_i(0)$, $\hat{D}_i(0)$, $\hat{W}_D(0)$, $\hat{D}_n(0)$. Assign $l=0$, l_{end} (time to stop the algorithm), θ_W and θ_{WD} (the small positive real number) for the convergence condition.

Step 2: For $1 \le i \le n$, update $\hat{W}_i^{(l+1)}$ by (41), update $\hat{W}_D^{(l+1)}$ by (61), update $\hat{D}_i^{(l+1)}$ by (44) and (63), update $v^{(l+1)}$ by (55).

Step 3: For $1 \le i \le n$, if $\left\|\hat{W}_i^{(l+1)}-\hat{W}_i^l\right\| \le \theta_w$, $\left\|\hat{W}_D^{(l+1)}-\hat{W}_D^l\right\| \le \theta_{wD}$ and $l=l_{end}$, then stop the algorithm, else $l=l+1$, go back to Step 2.

The convergence of Algorithm 1 is given in Section 4.

4. STABILITY ANALYSIS

The main result of this paper is summarized in the following theorem.

Theorem 1: Consider a class of uncertain MISO system (1)with the unknown asymmetric input saturation, external disturbance and actuator fault. The updated laws of the RBFNN weight are chosen as (23), (41), (57) and (61). The compounded disturbance observers of each step are designed as(25), (44) and (62). The intermediate virtual control signals are designed as (18) and (35). The disturbance observer based on fault-tolerant control scheme is developed as (55). Given for all the initial conditions and the appropriate design parameters, then all the close-up signals will be uniformly ultimately bounded under the proposed adaptive NN fault-tolerant control scheme based on the nonlinear disturbance observer.

Proof: Consider the Lyapunov function candidate as

$$
\begin{aligned}
V=&\sum_{j=1}^{n}V_j=\frac{1}{2}\sum_{i=1}^{n-1}z_i{}^2+\frac{1}{2}\sum_{i=1}^{n-1}\tilde{W}_i^T\Gamma_i{}^{-1}\tilde{W}_i\\
&+\frac{1}{2}\tilde{W}_n^T\Gamma_n{}^{-1}\tilde{W}_n+\frac{1}{2\rho_h}z_n{}^2\\
&+\frac{1}{2}\sum_{i=1}^{n-1}\varsigma_{i+1}{}^2+\frac{1}{2}\sum_{i=1}^{n-1}\tilde{D}_i^2\\
&+\frac{1}{2}\tilde{D}_n^2+\frac{1}{2}\tilde{W}_D^T\Gamma_D{}^{-1}\tilde{W}_D. \qquad (73)
\end{aligned}
$$

Invoking (31), (50) and (66) in the previous recursive design procedures, we can gain

$$
\begin{aligned}
\dot{V}\le&-\sum_{i=1}^{n-1}\left(k_i-\bar{\bar{g}}_i-\frac{1}{2}\right)z_i{}^2-\left(k_n-\frac{1}{2}\right)z_n{}^2\\
&-\frac{1}{2}\sum_{i=1}^{n-1}\left(\delta_{Wi}-\frac{1}{a_i}\gamma_i\right)\|\tilde{W}_i\|^2\\
&-\left(\frac{1}{2}\delta_{WD}-\frac{1}{2a_n}\gamma_n\right)\|\tilde{W}_D\|^2\\
&-\sum_{i=1}^{n-1}\left(\frac{1}{\tau_{i+1}}-1-\frac{\bar{\bar{g}}_i}{4}\right)\varsigma_{i+1}{}^2-\frac{\delta_{Wn}\|\tilde{W}_n\|^2}{2}\\
&-\sum_{i=1}^{n-1}\left(\gamma_i-1-\frac{1}{2}\gamma_i a_i\mu_i{}^2\right)\tilde{D}_i^2\\
&-\left(\frac{1}{2}\gamma_n-\frac{1}{2}\gamma_n a_n\mu_D{}^2-1.5\right)\tilde{D}_n^2\\
&+\sum_{i=1}^{n}\frac{\delta_{Wi}\|W_i\|^2}{2}+\sum_{i=1}^{n}\frac{\theta_i{}^2}{2}+\sum_{i=1}^{n-1}\frac{\bar{\lambda}_i^2}{2}\\
&+\gamma_n\bar{\xi}_D^2+\gamma_n|\rho_h-1|^2\eta_n{}^2+\frac{1}{2}\delta_{WD}\|W_D\|^2\\
\le&-\phi V+\beta, \qquad (74)
\end{aligned}
$$

where ϕ and β are given by

$$
\phi:=\min\left\{\begin{array}{l}
2k_i-2\bar{\bar{g}}_i-1,2\bar{\rho}_h k_n-1,\\
\frac{1}{\lambda_{\max}(\Gamma_i{}^{-1})}\left(\delta_{Wi}-\frac{1}{a_i}\right),\frac{\delta_{Wn}}{\lambda_{\max}(\Gamma_n{}^{-1})},\\
2\left(\frac{1}{\tau_{i+1}}-1-\frac{\bar{\bar{g}}_i}{4}\right),2\gamma_i-2-\gamma_i a_i\mu_i{}^2,\\
\gamma_n-\gamma_n a_n\mu_D{}^2-3,\frac{1}{\lambda_{\max}(\Gamma_D{}^{-1})}\left(\delta_{WD}-\frac{\gamma_n}{a_n}\right)
\end{array}\right\}, \qquad (75)
$$

$$
\begin{aligned}
\beta:=&\sum_{i=1}^{n}\frac{\delta_{Wi}\|W_i\|^2}{2}+\sum_{i=1}^{n}\frac{\theta_i{}^2}{2}+\sum_{i=1}^{n-1}\frac{\bar{\lambda}_i^2}{2}+\gamma_n\bar{\xi}_D^2\\
&+\gamma_n|\rho_h-1|^2\eta_n{}^2+\frac{1}{2}\delta_{WD}\|W_D\|^2, \qquad (76)
\end{aligned}
$$

where $i=1,2,\ldots,n-1$.

If the following inequalities are satisfied: $2k_i-2\bar{\bar{g}}_i-1>0$, $2\gamma_i-2-\gamma_i a_i\mu_i{}^2>0$, $\frac{1}{\tau_{i+1}}-1-\frac{\bar{\bar{g}}_i}{4}>0$, $\delta_{WD}-\frac{\gamma_n}{a_n}>0$, $\delta_{Wi}-\frac{1}{a_i}>0$, $\gamma_n-\gamma_n a_n\mu_D{}^2-3>0$ and selecting the appropriate corresponding design parameter k_i, γ_i, δ_{Wi}, δ_{Wn}, δ_{WD} then $\phi>0$ will be ensured. On the other hand, the elements W_i, θ_i, $\bar{\lambda}_i$, $\bar{\xi}_D$, η_n and W_D are all bounded in β. It is clear that β is a bounded positive constant.

Finally, we have

$$
0\le V\le\beta/\phi+\vartheta e^{-\phi t}, \qquad (77)
$$

where $\vartheta=V(0)-\beta/\phi$. $V=f(z_i,z_n,\tilde{W}_i,\tilde{W}_n,\tilde{D}_i,\tilde{D}_n)$ is exponentially convergent, i.e., $\lim_{t\to\infty}V(t)\le\beta/\phi$, $\forall t>0$.

Thus, the output tracking error z_i, $\tilde{W}_i$ and the compounded disturbance approximation error $\tilde{D}_i$ are uniformly ultimately bounded. The closed-loop system under the design adaptive control scheme is stability. This proof is completed. □

Remark 4: The control scheme involves the choices of control gains $k_i > 0$ and disturbance observer gains $\gamma_i > 0$, which theoretically is no criteria. Compromise between stable performance and transient performance made for the selection of given system. Increasing the value of k_i or γ_i will result in a better tracking performance or obtain good stable performance. Otherwise, too big value will degrade the transient performance. The choices of the designed parameters should be trade-off.

Remark 5: In this note, the NDO is developed to estimate compounded disturbances which combined unknown external disturbance of nonlinear system with the RBFNN approximation error. The considered disturbances are more general and practical without satisfying the matched parametric or requiring the known boundary requirements.

Remark 6: In this paper, a robust adaptive fault-tolerant control scheme is developed, comparing with most existing methods, the proposed control scheme holds several salient features such as: 1) estimated unknown disturbance by an NDO without considering the known upper boundary requirement of unknown disturbance; 2) resolved the problem of asymmetric saturation input by proposed smooth input saturation model compared with [7]; 3) precluded the complexity and dimension curse by adaptive fault-tolerant with dynamic surface control for general nonlinear system with asymmetric input constraint, unmatched disturbances and actuator faults.

Remark 7: Our mentioned works are based an assumption that state variables are available. Moreover, many controller designs for nonlinear systems include the same assumption [7–9]. However, to apply these control strategies in wider practical engineering, the assumption can be removed. State observers can be designed first, the stability and tracking control procedure can be done based on the presented observers.

5. SIMULATION

In this section, simulation results are given to verify the effectiveness of the proposed adaptive NN DSC fault-tolerant schemes. Consider the following uncertain nonlinear system with input saturation [28]:

$$\begin{aligned} \dot{x}_1 &= x_2 + f_1(x_1) + d_1(t), \\ \dot{x}_2 &= f_2(x_1,x_2) + b_1u_1 + b_2u_2 + d_2(t), \\ y &= x_1, \end{aligned} \tag{78}$$

where $f_1(x_1) = -x_1e^{-0.5x_1}$, $f_2(\bar{x}_2) = x_1\sin(x_2{}^2)$, $b = [0.2, 0.8]$, $d_1(t) = \cos(t)$, $d_2(t) = \sin(t)$. Based on the above conditions, two cases of different simulation of the uncertain nonlinear system (78) are given to examine the effectiveness of our proposed control method.

Case 1: The desired trajectory is chosen as . The asymmetric saturation value of control input signa is chosen as $u_U = 0.8$, $u_L = -1.2$, respectively. To illustrate the effectiveness of the proposed adaptive fault-tolerant control scheme, the unknown faults are set as

$$\begin{aligned} u_1(t) &= u_{c,1}(t), \\ u_2(t) &= \begin{cases} u_{c,2}(t), & 0 < t < 10 \text{ s}, \\ 0.2, & t \geq 10 \text{ s}, \end{cases} \end{aligned} \tag{79}$$

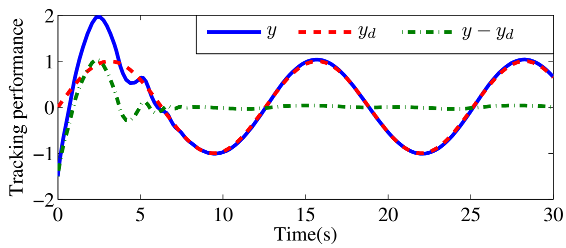
where the first actuator is normal, the second actuator is assumed to occur Lock-in-place failure at time moment $t = 10s$.

The initial state conditions are chosen as $x_1(0) = -1.5$, $x_2(0) = 0.3$. The design parameters are chosen as $k_1 = 12$, $k_2 = 4$, $\tau_2 = 0.05$, $\delta_{w1} = \delta_{w2} = 1$, $\delta_{wD} = 2$, $\gamma_1 = \gamma_2 = 1$, $\Gamma_1 = \Gamma_2 = \Gamma_D = 2I_{5\times5}$ and other initial values are all set as zero.

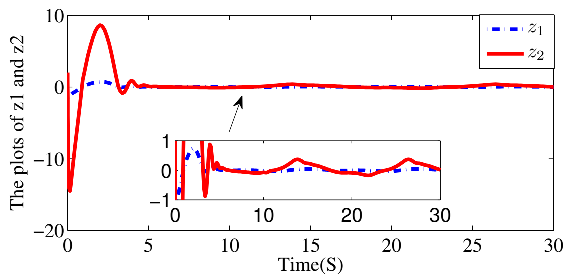
With the proposed nonlinear adaptive controller scheme, the simulation results under Case 1 are shown in Fig. 2. As shown in Fig. 2(a), it can be clearly seen that the tracking performance is satisfactory and the tracking error quickly converges to the equilibrium for the nonlinear system (78)in presence of time-varying external disturbance, input constraint and actuator failure (79). The plot of z_1 and z_2 are shown in Fig. 2(b) From Fig. 2(c), we can observe that the control input is bounded and satisfies input saturation. The trajectories of control signals u_{c1} andu_{c2} are shown in Fig. 2(d).

To illustrate the robustness of the proposed control method in this paper, the modeling uncertainties $\Delta f_1(x_1,x_2) = 0.6\cos(x_2)\sin(x_1)$ is added in the system (78) for Case 1. From Fig. 2(e), the proposed adaptive fault-tolerant control scheme is still able to achieve satisfactory tracking performance and robustness for nonlinear system with nonlinear model uncertainty. Furthermore, it should be emphasized that, comparisons are presented with recently the proposed methodology as given in [29]. The simulation results of comparisons are Fig. 3. As shown in Fig. 3(a), without actuator faults, tracking performance is still satisfactory. Then, the same input failures are added as (79), as shown in Fig. 3(b), the tracking performance is degraded largely using the proposed controller, largely because of no considering for input actuator failures in proposed control scheme in [29].

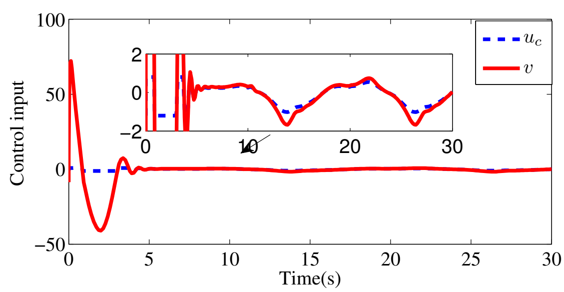
Case 2: Here, the desired trajectory is given by $y_d = 1$. In this simulation, the asymmetric saturation values of control input are chosen as $u_U = 8$ and $u_L = -10$, respec-



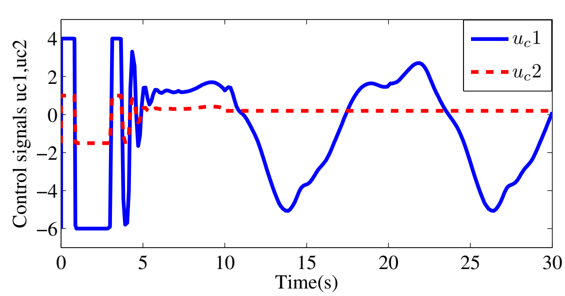
(a) System output follows desired output y_d.



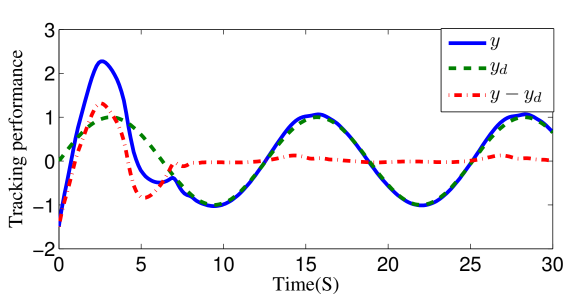
(b) The plot of z_1 and z_2.



(c) Control input.

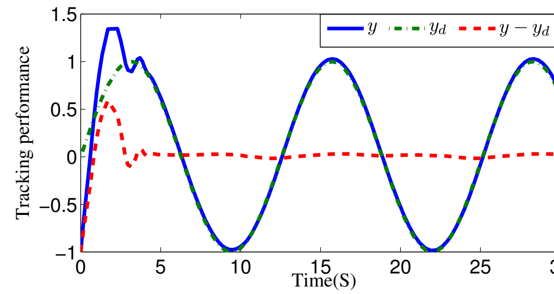


(d) Control signals of u_{c1} and u_{c2}.

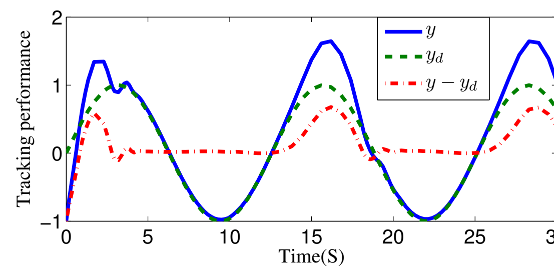


(e) Tracking performance with model uncertainty.

Fig. 2. Simulation results under Case 1.



(a) Tracking performance with no faults.



(b) Tracking performance with actuator faults.

Fig. 3. Simulaiton results under [29].

tively. The unknown failures/faults are set as

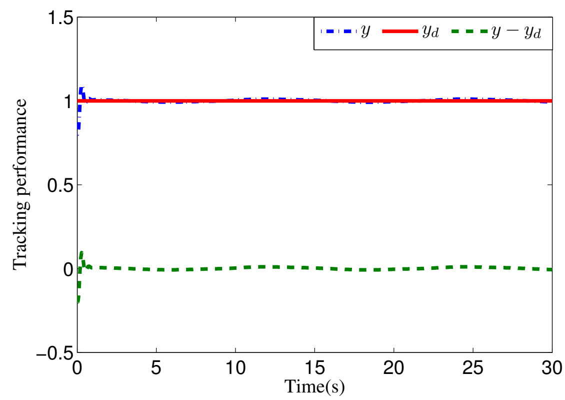
$$u_1(t) = \begin{cases} u_{c,1}(t), & 0 < t < 10 \text{ s}, \\ 0.6u_{c,1}(t), & t \geq 10 \text{ s}, \end{cases}$$
$$u_2(t) = u_{c,2}(t), \tag{80}$$

where the first actuator is assumed to occur 40% partial loss of effectiveness failure at time moment $t = 10s$, the second actuator is normal.

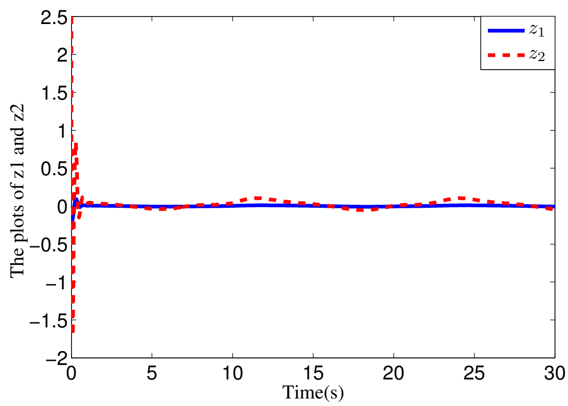
The initial state conditions are chosen as $x_1(0) = 0.8$ and $x_2(0) = 0.8$. The design parameters are chosen as $k_1 = 16$, $k_2 = 12$, $\tau_2 = 0.05$, $\delta_{w1} = \delta_{w2} = 1$, $\delta_{wD} = 2$, $r_1 = 10$, $r_2 = 8$, $\Gamma_1 = \Gamma_2 = \Gamma_D = 2I_{5\times 5}$ and other initial values are all set as zero.

With the proposed nonlinear adaptive controller scheme, the simulation results under Case 2 are shown in Fig. 4. As shown in Fig. 4(a), it can be clearly seen that the tracking performance is satisfactory and the tracking error quickly converges to the equilibrium for the nonlinear system (78) in presence of time-varying external disturbance, input constraint and actuator failure (80). The plot of z_1 and z_2 are shown in Fig. 4(b). It is shown in Fig. 4(c) that the control input is bounded and satisfies input saturation, while the tracking performance is satisfactory for Case 2. The trajectories of control signals u_{c1} and u_{c2} are shown in Fig. 4(d).

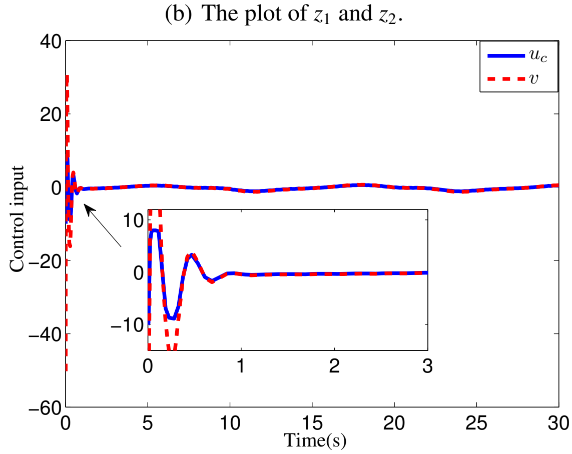
From all two groups of simulations, the results obviously show that the closed-loop system signals are bounded and converge in a short time. Even the considered system is added extra unknown model uncertainty, the tracking performances are still satisfactory.



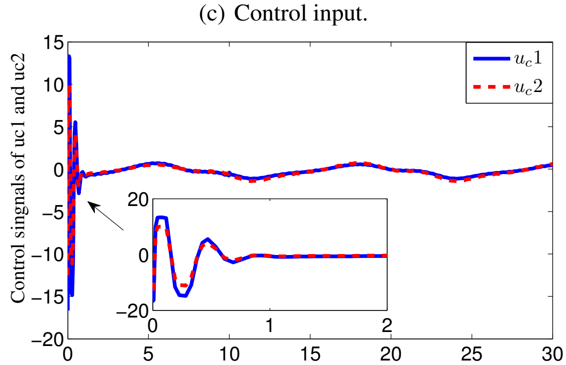
(a) System output follows desired output y_d.



(b) The plot of z_1 and z_2.



(c) Control input.



(d) Control signals of u_{c1} and u_{c2}.

Fig. 4. Simulation results under Case 2.

According to the above simulation results, it is clearly that the presented control algorithm in this note is valid for the nonlinear system with unknown disturbances and input constraints and actuator faults.

6. CONCLUSION

In this paper, an adaptive neural fault-tolerant control scheme is proposed for uncertainty nonlinear system with asymmetric input constraint, unknown external disturbance and actuator failures. A smooth hyperbolic tangent function is introduced to present the symmetric input saturation. The disturbance observer is employed to estimate the compound unknown disturbance consisting of the NN approximation error and the external disturbance and actuator faults. By the developed adaptive control algorithm, the uniformly ultimately bounded convergence of all the signals of the closed-loop system are guaranteed via Lyapunov approach. Simulation results demonstrate the effectiveness of the proposed approach. The primary goal of our future work is to devise novel adaptive neuro-fuzzy backstepping control scheme to handle large MIMO nonlinear system subject to time delays and full states saturation.

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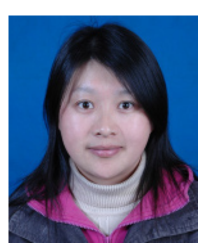

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