University of Massachusetts Dartmouth

Department of Electrical and Computer Engineering

ECE 574/475 DISCRETE-TIME SIGNAL PROCESSING

Fall 2024

Matlab Project 2: Upsampling and Downsampling

[Project 7.3 from book Buck, Daniel & Singer]

In this exercise, you will examine how upsampling and downsampling a discrete-time signal affects its discrete-time Fourier transform (DTFT). If a discrete-time signal was originally obtained by sampling an appropriately bandlimited continuous-time signal, the upsampled or downsampled signal is the set of samples that would have been obtained by sampling the original continuous-time signal at a different sampling rate. For this reason, upsampling and downsampling are often referred to as sampling-rate conversion. Just as with sampling in continuous time, if a discrete-time signal is not sufficiently bandlimited, downsampling may introduce aliasing, which will destroy information. Individually, these operations can only increase or decrease the sampling rate by integer factors, but sampling-rate conversion by any rational factor can be achieved through a combination of upsampling and downsampling.

Learning objectives of this project:

- *Create upsampled and downsampled signals, analyzing the DTFT signals*
- *Identify aliasing*

(a) For most of this exercise, you will be working with finite segments of the two signals

$$x_1[n] = \left(\frac{\sin(0.4\pi(n-62))}{0.4\pi(n-62)}\right)^2, \qquad [1]$$

$$x_2[n] = \left(\frac{\sin(0.2\pi(n-62))}{0.2\pi(n-62)}\right)^2. \qquad [2]$$

Define x1 and x2 to be these signals for $0 \leq n \leq 124$ n using the sinc command. Plot both of these signals using stem. If you defined the signals properly, both plots should

show that the signals are symmetric about their largest sample, which has height 1. Analytically confirm your signals have their zero-crossings in the correct locations.

```
%Insert code for Section (a) to define x1 and x2 signals and plot them
```

```
%Confirm that positions of the zero-crossings
```

(b) Analytically compute the DTFTs $X_1(e^{j\omega})$ and $X_2(e^{j\omega})$ of $x_1[n]$ and $x_2[n]$ as given in Eqs. (1) and (2), ignoring the effect of truncating the signals.

```
%Insert text with the analytical solution of the DTFTs
```

Use fft to compute the samples of the DTFT of the truncated signals in x1 and x2 at $\omega_k = 2\pi \frac{k}{2048}$ for $0 \leq k \leq 2047$ and store the results in X1 and X2. Generate appropriately labeled plots of the magnitudes of X1 and X2. How do these plots compare with your analytical expressions?

```
%Insert code for Section (b)

%How the DTFT plots compare with your analytical expressions?
```

(c) Define the expansion of the signal $x[n]$ by L to be the process of inserting $L-1$ zeros between each sample of $x[n]$ to form

$$x_e[n] = \begin{cases} x\left[\frac{n}{L}\right] & n = k\,L, k \text{ integer} \\ 0 & \text{otherwise} \end{cases}$$

If x is a row vector containing $x[n]$, the following commands implement expanding by *L*

```
>> xe = zeros (1, L*length(x));
>> xe(1:L:length(xe)) = x;
```

Based on this template, define xe1 and xe2 to be x1 and x2 expanded by a factor of 3. Also, define Xe1 and Xe2 to be 2048 samples of the DTFT of these expanded signals computed using fft. Generate appropriately labeled plots of the magnitude of these DTFTs. Expanding by *L* should give a DTFT $X_e(e^{j\omega L})$. Do your plots agree with this?

Note: If you want to increase the sampling rate by L, you need to interpolate between the samples of $x_e[n]$ *with a lowpass filter with cutoff frequency* $\omega_c = \frac{\pi}{L}$ *. This filter will remove the compressed copies of* $X_1(e^{j\omega})$ *located every* $\frac{2\pi}{L}$*,except the ones centered at* $\omega_c = 2\pi k$*. The resulting spectrum is that which would have been obtained if the original bandlimited continuous-time signal had been sampled L times faster. For this reason, the combination of expansion and interpolation is often referred to as upsampling a signal.*

```
%Insert code for Section (c)


% Comment if your plots agree with the expectation
```

(d) If the row vector x contains the signal $x[n]$, the following MATLAB command will implement downsampling by an integer factor *M*

>> xd = x (1 : M : length(x));

Based on the DTFTs you found analytically in Part (b), state for both $x_1[n]$ and $x_2[n]$ if the signal can be downsampled by a factor of 2 without introducing aliasing.

If downsampling introduces aliasing, indicate which frequencies are corrupted by the aliasing and which are not affected. If the signal can be downsampled without introducing aliasing, sketch the magnitude of $X_d(e^{j\omega})$, the DTFT of downsampledsignal.

```
%Discuss if the downsampled signal of x1 and x2 presents aliasing and which
%frequencies are affected.
```

(e) Define xd1 and xd2 to be the result of downsampling x1 and x2 by 2. Define Xd1 and Xd2 to be samples of the DTFTs of the downsampled signals computed at 2048 evenly spaced samples between 0 and 2π. Generate appropriately labeled plots of the magnitudes of both DTFTs. Do the plots agree with your sketch(es) from Part (d)?

```
%Insert code for Section (e)


%Discuss if these results agree with the prediction in Part (d)
```