No ....

Date

سوال ۳

الف:

$$f(x)=\begin{cases} 0 & -\pi<x<0 \\ \frac{x}{2} & 0<x<\pi \end{cases}$$

تابع ما در بازه $(\pi,-\pi)$

$$f(x)\sim \frac{a_0}{2}+\sum_{1}^{\infty} a_n\cos(nx)+b_n\sin(nx))$$

$$\rightarrow a_0=\frac{1}{\pi}\int f(x)\,dx \qquad a_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx$$

$$\rightarrow b_n=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx$$

چون تابع ما در نیم‌بازه منفی صفر است پس عبارت ما فقط از ۰ تا π انتگرال‌گیری می‌شود.

$a_0=?$

$$a_0=\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\,dx=\frac{1}{\pi}\int_0^{\pi}\frac{x}{2}\,dx=\frac{1}{2\pi}\int_0^{\pi}x\,dx=\frac{1}{2\pi}\left[\frac{x^2}{2}\right]_0^{\pi}=$$

$$\boxed{\frac{1}{2\pi}}$$

$$a_n\Rightarrow\frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\cos(nx)\,dx=\frac{1}{\pi}\int_0 \frac{x}{2}\cos(nx)=\frac{1}{2\pi}\int_0^{\pi}$$

انتگرال را به روش جزء به جزء

$$u=x \quad dv=\cos(nx)\,dx \xrightarrow{\text{پس}} du=dx$$

$$\frac{\sin(nx)}{n}=v$$

$$\int x\cos(nx)\,dx=x\cdot\frac{\sin(nx)}{n}-\int\frac{\sin(nx)}{n}\,dx=\frac{x\sin(nx)}{n}+\frac{\cos(nx)}{n^2}$$

$$\Rightarrow\int_0^{\pi}x\cos(nx)\,dx=\left[\frac{x\sin(nx)}{n}+\frac{\cos(nx)}{n^2}\right]_0^{\pi}=$$

$$=\left(\frac{\pi\sin(n\pi)}{n}+\frac{\cos(n\pi)}{n^2}\right)-\left(0+\frac{1}{n^2}\right)=\frac{(-1)^n-1}{n^2}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)\sin(nx)\,dx = \frac{1}{\pi}\int_0^{\pi} \frac{x}{2}\sin(nx)\,dx =$$ ادامه سوال ۳

از طریق جزء به جزء $\longrightarrow u = x,\ du = \sin(nx)\,dx \Rightarrow du = dx,$

$$-\frac{\cos(nx)}{n} = v$$

$$\int x\sin(nx)\,dx = -x\cdot\frac{\cos(nx)}{n} - \int\left(-\frac{\cos(nx)}{n}\right)dx = -\frac{x\cos(nx)}{n} + \frac{\sin(nx)}{n^2}$$

$$\to \int_0^{\pi} x\sin(nx)\,dx = \left[-\frac{x\cos(nx)}{n} + \frac{\sin(nx)}{n^2}\right]_0^{\pi} = \left(-\frac{\pi(-1)^n}{n} + 0\right) - 0 = -\frac{\pi(-1)^n}{n}$$

پس $b_n = \frac{1}{2\pi}\cdot\left(-\frac{\pi(-1)^n}{n}\right) = \frac{(-1)^n}{2n}$

$$a_0 = \frac{\pi}{4},\quad a_n = \frac{(-1)^n - 1}{2\pi n^2} = \begin{cases} 0 \\ -\frac{1}{\pi n^2} \end{cases},\quad b_n = -\frac{(-1)^n}{2n}$$

$$\xrightarrow{\text{سری فوریه}} \frac{\pi}{8} + \sum_1^{\infty}\left[a_n\cos(nx) + b_n\sin(nx)\right]$$

ب.

$$f(0) = \frac{\pi}{8} - \frac{1}{\pi}\sum_1^{\infty}\frac{\cos(0)}{(2k-1)^2} + \sum_1^{\infty}\frac{\sin 0}{2(2k-1)} - \sum_1^{\infty}\frac{\sin(0)}{4k}$$

داریم $\Leftarrow \sin(0) = 0,\ \cos(0) = 1$

$$\frac{1}{\pi}\sum_1^{\infty}\frac{1}{(2k-1)^2} = \frac{\pi}{8} \qquad \sum_1^{\infty}\frac{1}{(2k-1)^2} = \frac{\pi^2}{8}$$