# Secure Relay Beamforming for SWIPT in Amplify-and-Forward Two-Way Relay Networks 

Quanzhong Li, Qi Zhang, Member, IEEE, and Jiayin Qin


#### Abstract

In this paper, we investigate secure relay beamforming problem for simultaneous wireless information and power transfer in an amplify-and-forward two-way relay network. We consider scenarios that the eavesdropper's channel state information (CSI) is and is not available. When the eavesdropper's CSI is available, our objective is to maximize achievable secrecy sum rate under transmit power constraint and energy harvesting constraint. Since the optimization problem is nonconvex, we derive its performance upper bound which requires two-dimensional search where in each step a semidefinite programming is solved. We also propose an upper bound based rankone solution by employing Gaussian randomization method. To reduce computational complexity, we transform the optimization problem into a difference of convex programming and propose a sequential parametric convex approximation (SPCA) based iterative algorithm to find a locally optimal solution. Furthermore, we also propose a zero-forcing (ZF) based suboptimal solution. Simulation results demonstrate that the upper bound based rank-one solution archives performance almost the same as upper bound while has high computational complexity. The low-complexity SPCA based locally optimal solution performs close to upper bound. The ZF based suboptimal solution has the lowest computational complexity among proposed solutions. When the eavesdropper's CSI is not available, we propose an artificial noise-aided secure relay beamforming scheme.


Index Terms-Energy harvesting (EH), secure relay beamforming, simultaneous wireless information and power transfer (SWIPT), two-way relay network.

## I. Introduction

SIMULTANEOUS wireless information and power transfer (SWIPT) is a promising energy harvesting (EH) technique to solve the energy scarcity problem in energy constrained wireless communications [1]-[8]. The SWIPT schemes for multiple-input-multiple-output (MIMO) broadcast channel and multiple-input-single-output (MISO) broadcast channel with a single information-decoding (ID) receiver were investigated in [1]-[3]. For multiple ID receivers, the transmit beamforming

[^0]design was studied for SWIPT in MISO broadcast channel [5], [6]. In [7], the SWIPT in a two-user MISO interference channel was considered. For the SWIPT scheme in amplify-and-forward (AF) MIMO relay networks, rate-energy tradeoff for one-way relaying was investigated in [8] and sum-rate maximization for two-way relaying was studied in [9].

Because of openness of wireless transmission medium, wireless information is susceptible to eavesdropping. Thus, secure communication is a critical issue for SWIPT. Without considering SWIPT, secure beamforming schemes in MIMO channel were derived in [10], [11]. The secure beamforming schemes for SWIPT in MISO broadcast channel were studied in [12], [13]. For SWIPT in AF one-way relay networks, secure relay beamforming design was investigated in [14]. However, to the best of our knowledge, the research on secure relay beamforming design for SWIPT in AF two-way relay networks is missing.

In this paper, we study the secure relay beamforming design problem which maximizes secrecy sum rate of AF two-way relay networks under transmit power constraint at the relay and EH constraint at the EH receiver. We consider scenarios that the eavesdropper's CSI is and is not available at the sources and the relay. When the eavesdropper's CSI is available, for the non-convex EH-constrained secure relay beamforming design problem, we derive its performance upper bound by employing the rank-one relaxation and Charnes-Cooper transformation. To obtain the performance upper bound requires twodimensional (2-D) search where in each step a semidefinite programming (SDP) is solved. We also propose an upper bound based rank-one solution by employing the Gaussian randomization method. To reduce computational complexity, we transform the original non-convex optimization problem into a difference of convex (DC) programming [10], [11], [18] and propose a sequential parametric convex approximation (SPCA) [19] based iterative algorithm to find a local optimum of the DC programming. To further reduce complexity, we also propose a zero-forcing (ZF) based suboptimal solution.

When the eavesdropper's CSI is not available, we propose an artificial noise (AN)-aided secure relay beamforming scheme, where the relay allocates its power for both AN and information bearing signals. The secure relay beamforming problem is formulated as to maximize allocated power for AN under transmit power constraint at relay, EH constraint at EH receiver and the additional constraint that achievable sum rate of two sources is larger than or equal to a predefined threshold. We propose to find the optimal solution by one-dimensional (1-D) search where in each step an SDP is solved.

Compared with conventional secure relay beamforming de-


Fig. 1. The system model of secure relay beamforming with SWIPT in the AF two-way multi-antenna relay network.
sign problem in AF two-way relay networks without SWIPT [15], the problem with SWIPT is more difficult. In [15], without EH constraint, ZF is employed to convert the achievable secrecy sum rate maximization problem into a generalised Rayleigh quotient. With EH constraint, the achievable secrecy sum rate maximization problem cannot be converted into a generalised Rayleigh quotient. Furthermore, besides ZF based suboptimal solution, we also propose an upper bound based solution and an SPCA based locally optimal solution in this paper.

Our main contributions, compared with secure relay beamforming design problem in AF one-way relay networks with SWIPT [14], are summarized as follows. We theoretically derive the system model and performance upper bound of secure relay beamforming in AF two-way relay networks with SWIPT. We propose an SPCA based iterative algorithm to find a local optimum of the DC programming. Furthermore, considering that the eavesdropper's CSI is not available, we propose an AN-aided secure relay beamforming scheme.

The rest of this paper is organized as follows. In Section II, we describe the system model and problem formulation. In Sections III, we propose three algorithms for secure relay beamforming design problem when the eavesdropper's CSI is available. In Sections IV, we propose the AN-aided secure relay beamforming scheme when the eavesdropper's CSI is not available. In Section V, we analyze complexities of the proposed algorithms. Simulation results are provided in Section VI. We conclude our paper in Section VII.

Notations: Boldface lowercase and uppercase letters denote vectors and matrices, respectively. The conjugate, transpose, conjugate transpose, Frobenius norm and trace of the matrix $\mathbf{A}$ are denoted as $\mathbf{A}^{*}, \mathbf{A}^{T}, \mathbf{A}^{\dagger},\|\mathbf{A}\|$, and $\operatorname{tr}(\mathbf{A})$, respectively. The $\otimes$ denotes Kronecker product. vec $(\mathbf{A})$ denotes to stack the columns of a matrix $\mathbf{A}$ into a single vector $\mathbf{a}$. $\operatorname{Re}\{\mathbf{a}\}$ denotes the real part of $\mathbf{a} .\|\mathbf{a}\|$ denotes the Euclidean norm of vector a. $\lambda_{\max }(\mathbf{A})$ and $\lambda_{\min }(\mathbf{A})$ denote the maximum and minimum eigenvalues of $\mathbf{A}$, respectively. By $\mathbf{A} \succeq \mathbf{0}$ or $\mathbf{A} \succ \mathbf{0}$, we mean that the matrix $\mathbf{A}$ is positive semidefinite or positive definite, respectively.

## II. System Model

Consider an AF two-way relay network which consists of two sources, one relay, one eavesdropper and one EH receiver, as shown in Fig. 1. We assume that the direct link between two sources is sufficiently weak to be ignored. This occurs when the direct link is blocked due to long-distance path loss or obstacles. The relay, equipped with $N$ antennas, is responsible to establish reliable communications between two sources, to transfer power to the EH receiver and to guarantee the secure communications. Each of the other nodes is equipped with a single antenna. As in [8], [15], we assume that all the channels remain unchanged during the period for obtaining CSI at the relay and exchanging information between two sources via the relay. Thus, the quasi-static channels are considered in this paper.

The exchange of information symbols between two sources, i.e., sources 1 and 2, is divided into two phases. In the first phase, sources 1 and 2 simultaneously transmit the symbols $x_{1} \in \mathbb{C}^{1 \times 1}$ and $x_{2} \in \mathbb{C}^{1 \times 1}$, respectively, to the relay. The received signals at relay and eavesdropper are expressed as

$$
\begin{align*}
\mathbf{r} & =\mathbf{h}_{1, f} x_{1}+\mathbf{h}_{2, f} x_{2}+\mathbf{n}_{r}  \tag{1}\\
y_{e, 1} & =f_{1} x_{1}+f_{2} x_{2}+n_{e, 1} \tag{2}
\end{align*}
$$

where $\mathbf{h}_{i, f} \in \mathbb{C}^{N \times 1}$ and $f_{i}, i \in\{1,2\}$, denote the forward channel response from source $i$ to relay and eavesdropper, respectively; $\mathbf{n}_{r} \in \mathbb{C}^{N \times 1}$ is the additive Gaussian noise vector at relay which has zero mean and covariance matrix $\sigma^{2} \mathbf{I} ; n_{e, 1}$ is the additive Gaussian noise at eavesdropper which has zero mean and variance $\sigma^{2}$. The harvested energy at EH receiver is given by [1]

$$
\begin{equation*}
E_{1}=\rho\left(P_{1}\left\|g_{1}\right\|^{2}+P_{2}\left\|g_{2}\right\|^{2}\right) \tag{3}
\end{equation*}
$$

where $g_{i}, i \in\{1,2\}$, denotes the channel response from source $i$ to EH receiver, $P_{i}=\mathbb{E}\left[\left|x_{i}\right|^{2}\right], i \in\{1,2\}$, denotes the average transmit power of source $i$, the factor $\rho$ denotes the EH efficiency that accounts for the loss in energy transducer. Without loss of generality, we assume that the EH efficiency $\rho=1$ in this paper as in [1].

In the second phase, the relay multiplies the received signal by a beamforming matrix, denoted as $\mathbf{W} \in \mathbb{C}^{N \times N}$, and forwards the product to two sources. The transmitted signal from relay is

$$
\begin{equation*}
\tilde{\mathbf{r}}=\mathbf{W r} \tag{4}
\end{equation*}
$$

By using the equality $\operatorname{vec}\left(\mathbf{A}_{1} \mathbf{A}_{2} \mathbf{A}_{3}\right)=\left(\mathbf{A}_{3}^{T} \otimes \mathbf{A}_{1}\right) \operatorname{vec}\left(\mathbf{A}_{2}\right)$, the transmit power at relay is expressed as

$$
\begin{equation*}
\mathbb{E}\left[\|\tilde{\mathbf{r}}\|^{2}\right]=\mathbf{w}^{\dagger} \mathbf{C w} \tag{5}
\end{equation*}
$$

where

$$
\begin{align*}
& \mathbf{w}=\operatorname{vec}(\mathbf{W})  \tag{6}\\
& \mathbf{C}=\left(P_{1} \mathbf{h}_{1, f} \mathbf{h}_{1, f}^{\dagger}+P_{2} \mathbf{h}_{2, f} \mathbf{h}_{2, f}^{\dagger}+\sigma^{2} \mathbf{I}\right)^{T} \otimes \mathbf{I} \tag{7}
\end{align*}
$$

The received signals at source 1 , source 2, and the eavesdropper are given by

$$
\begin{align*}
y_{d, 1} & =\mathbf{h}_{1, b}^{T} \mathbf{W} \mathbf{h}_{2, f} x_{2}+\mathbf{h}_{1, b}^{T} \mathbf{W} \mathbf{h}_{1, f} x_{1}+\mathbf{h}_{1, b}^{T} \mathbf{W} \mathbf{n}_{r}+n_{d, 1}  \tag{8}\\
y_{d, 2} & =\mathbf{h}_{2, b}^{T} \mathbf{W} \mathbf{h}_{1, f} x_{1}+\mathbf{h}_{2, b}^{T} \mathbf{W} \mathbf{h}_{2, f} x_{2}+\mathbf{h}_{2, b}^{T} \mathbf{W} \mathbf{n}_{r}+n_{d, 2}  \tag{9}\\
y_{e, 2} & =\mathbf{f}_{r}^{T} \mathbf{W} \mathbf{h}_{2, f} x_{2}+\mathbf{f}_{r}^{T} \mathbf{W} \mathbf{h}_{1, f} x_{1}+\mathbf{f}_{r}^{T} \mathbf{W} \mathbf{n}_{r}+n_{e, 2} \tag{10}
\end{align*}
$$

respectively, where $\mathbf{h}_{i, b}, \mathbf{f}_{r} \in \mathbb{C}^{N \times 1}$ denote the channel response vector from relay to source $i$ and eavesdropper, respectively; $n_{d, i}, i \in\{1,2\}$, and $n_{e, 2}$ are the additive Gaussian noises at source $i$ and eavesdropper, respectively, which have zero mean and variance $\sigma^{2}$. Since source $i$ knows its own transmitted signal, it can subtract the self-interference term $\mathbf{h}_{i, b}^{T} \mathbf{W h}_{i, f} x_{i}$ from the received signal. Thus, the remaining received signals at source 1 and source 2, denoted as $\tilde{y}_{d, 1}$ and $\tilde{y}_{d, 2}$, respectively, are

$$
\begin{align*}
& \tilde{y}_{d, 1}=\mathbf{h}_{1, b}^{T} \mathbf{W} \mathbf{h}_{2, f} x_{2}+\mathbf{h}_{1, b}^{T} \mathbf{W} \mathbf{n}_{r}+n_{d, 1}  \tag{11}\\
& \tilde{y}_{d, 2}=\mathbf{h}_{2, b}^{T} \mathbf{W} \mathbf{h}_{1, f} x_{1}+\mathbf{h}_{2, b}^{T} \mathbf{W} \mathbf{n}_{r}+n_{d, 2} \tag{12}
\end{align*}
$$

From (11) and (12), the received signal-to-noise ratio (SNR) at source 1 and source 2 are

$$
\begin{align*}
\gamma_{1} & =\frac{\mathbf{w}^{\dagger} \mathbf{Q}_{2,1} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{R}_{1} \mathbf{w}+\sigma^{2}}  \tag{13}\\
\gamma_{2} & =\frac{\mathbf{w}^{\dagger} \mathbf{Q}_{1,2} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{R}_{2} \mathbf{w}+\sigma^{2}} \tag{14}
\end{align*}
$$

respectively, where

$$
\begin{align*}
\mathbf{Q}_{2,1} & =P_{2}\left[\left(\mathbf{h}_{2, f} \mathbf{h}_{2, f}^{\dagger}\right) \otimes\left(\mathbf{h}_{1, b} \mathbf{h}_{1, b}^{\dagger}\right)\right]^{T}  \tag{15}\\
\mathbf{Q}_{1,2} & =P_{1}\left[\left(\mathbf{h}_{1, f} \mathbf{h}_{1, f}^{\dagger}\right) \otimes\left(\mathbf{h}_{2, b} \mathbf{h}_{2, b}^{\dagger}\right)\right]^{T}  \tag{16}\\
\mathbf{R}_{i} & =\left[\left(\sigma^{2} \mathbf{I}\right) \otimes\left(\mathbf{h}_{i, b} \mathbf{h}_{i, b}^{\dagger}\right)\right]^{T}, i \in\{1,2\} \tag{17}
\end{align*}
$$

Here, the equalities $\operatorname{vec}\left(\mathbf{A}_{1} \mathbf{A}_{2} \mathbf{A}_{3}\right)=\left(\mathbf{A}_{3}^{T} \otimes \mathbf{A}_{1}\right) \operatorname{vec}\left(\mathbf{A}_{2}\right)$ and $\left(\mathbf{A}_{1} \otimes \mathbf{A}_{2}\right)\left(\mathbf{A}_{3} \otimes \mathbf{A}_{4}\right)=\left(\mathbf{A}_{1} \mathbf{A}_{3}\right) \otimes\left(\mathbf{A}_{2} \mathbf{A}_{4}\right)$ have been employed. In the second phase, the harvested energy at EH receiver is [8]

$$
\begin{equation*}
E_{2}=\mathbf{w}^{\dagger} \mathbf{D} \mathbf{w} \tag{18}
\end{equation*}
$$

where

$$
\begin{equation*}
\mathbf{D}=\left(P_{1} \mathbf{h}_{1, f} \mathbf{h}_{1, f}^{\dagger}+P_{2} \mathbf{h}_{2, f} \mathbf{h}_{2, f}^{\dagger}+\sigma^{2} \mathbf{I}\right)^{T} \otimes\left(\mathbf{g}_{r}^{*} \mathbf{g}_{r}^{T}\right) \tag{19}
\end{equation*}
$$

in which $\mathbf{g}_{r} \in \mathbb{C}^{N \times 1}$ denotes the channel response vector from relay to EH receiver.

For the eavesdropper, in each transmission phase it has the opportunity to decode the information symbols from two sources. The received signals at the eavesdropper in the first and second phases can be expressed as [15]

$$
\begin{equation*}
\mathbf{y}_{e}=\mathbf{H}_{e} \mathbf{x}+\mathbf{n}_{e} \tag{20}
\end{equation*}
$$

where $\mathbf{y}_{e}=\left[y_{e, 1}, y_{e, 2}\right]^{T}, \mathbf{x}=\left[x_{1}, x_{2}\right]^{T}$,

$$
\begin{align*}
\mathbf{H}_{e} & =\left[\begin{array}{cc}
f_{1} & f_{2} \\
\mathbf{f}_{r}^{T} \mathbf{W} \mathbf{h}_{1, f} & \mathbf{f}_{r}^{T} \mathbf{W} \mathbf{h}_{2, f}
\end{array}\right] \\
\mathbf{n}_{e} & =\left[\begin{array}{cc}
n_{e, 1} \\
\mathbf{f}_{r}^{T} \mathbf{W} \mathbf{n}_{r}+n_{e, 2}
\end{array}\right] \tag{21}
\end{align*}
$$

In (20), the received signals at eavesdropper in two phases are equivalent to those of an MIMO system. Thus, from [10], [11], the information rate leaked to the eavesdropper is

$$
\begin{align*}
R_{e} & =\frac{1}{2} \log _{2} \operatorname{det}\left(\mathbf{I}+\mathbf{H}_{e} \mathbf{P} \mathbf{H}_{e}^{\dagger} \mathbf{R}_{e}^{-1}\right) \\
& =\frac{1}{2} \log _{2} \frac{\mathbf{w}^{\dagger} \mathbf{Q}_{e} \mathbf{w}+\alpha}{\sigma^{4}\left(1+\mathbf{w}^{\dagger} \mathbf{R}_{c c} \mathbf{w}\right)} \tag{22}
\end{align*}
$$

where the factor $\frac{1}{2}$ is included because the signals are transmitted in two consecutive phases [15], and

$$
\begin{align*}
\mathbf{R}_{e} & =\operatorname{diag}\left(\sigma^{2}, \sigma^{2}\left(1+\mathbf{w}^{\dagger} \mathbf{R}_{c c} \mathbf{w}\right)\right)  \tag{23}\\
\mathbf{P} & =\operatorname{diag}\left(P_{1}, P_{2}\right)  \tag{24}\\
\mathbf{Q}_{e} & =\left(P_{1} P_{2}\left|f_{2}\right|^{2}+P_{1} \sigma^{2}\right) \mathbf{R}_{c f}+\left(P_{1} P_{2}\left|f_{1}\right|^{2}+P_{2} \sigma^{2}\right) \mathbf{R}_{c g} \\
& +\alpha \mathbf{R}_{c c}-P_{1} P_{2} f_{1} f_{2}^{*} \mathbf{a}_{c f}^{*} \mathbf{a}_{c g}^{T}-P_{1} P_{2} f_{1}^{*} f_{2} \mathbf{a}_{c g}^{*} \mathbf{a}_{c f}^{T},  \tag{25}\\
\mathbf{R}_{c c} & =\left[\mathbf{I} \otimes\left(\mathbf{f}_{r} \mathbf{f}_{r}^{\dagger}\right)\right]^{T}, \mathbf{R}_{c f}=\mathbf{a}_{c f} \mathbf{a}_{c f}^{\dagger}, \mathbf{R}_{c g}=\mathbf{a}_{c g} \mathbf{a}_{c g}^{\dagger},  \tag{26}\\
\mathbf{a}_{c f} & =\operatorname{vec}\left(\mathbf{f}_{r} \mathbf{h}_{1, f}^{T}\right), \mathbf{a}_{c g}=\operatorname{vec}\left(\mathbf{f}_{r} \mathbf{h}_{2, f}^{T}\right),  \tag{27}\\
\alpha & =P_{1} \sigma^{2}\left|f_{1}\right|^{2}+P_{2} \sigma^{2}\left|f_{2}\right|^{2}+\sigma^{4} . \tag{28}
\end{align*}
$$

Note that to obtain the information rate leaked to the eavesdropper, $R_{e}$, the eavesdropper may employ minimum-mean-square-error equalizer and successive interference cancelation scheme.

From (13), (14), and (22), the achievable secrecy sum rate of two-way relay network is [10], [11]

$$
\begin{gather*}
R_{s}=\frac{1}{2}\left[\log _{2}\left(1+\frac{\mathbf{w}^{\dagger} \mathbf{Q}_{2,1} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{R}_{1} \mathbf{w}+\sigma^{2}}\right)\left(1+\frac{\mathbf{w}^{\dagger} \mathbf{Q}_{1,2} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{R}_{2} \mathbf{w}+\sigma^{2}}\right)\right. \\
\left.\cdot\left(\frac{\sigma^{4}\left(1+\mathbf{w}^{\dagger} \mathbf{R}_{c c} \mathbf{w}\right)}{\mathbf{w}^{\dagger} \mathbf{Q}_{e} \mathbf{w}+\alpha}\right)\right]^{+} \tag{29}
\end{gather*}
$$

The transmit power constraint at relay and the EH constraint at EH receiver are

$$
\begin{align*}
& \mathbf{w}^{\dagger} \mathbf{C w} \leq P_{r}  \tag{30}\\
& E_{1}+E_{2} \geq Q \tag{31}
\end{align*}
$$

where $P_{r}$ is the transmit power constraint at relay and $Q$ is the EH constraint at EH receiver. According to [1], [2], the EH constraint $Q$ should be chosen such that $0 \leq Q \leq Q_{\max }$ where

$$
\begin{align*}
Q_{\max } & =E_{1}+\max _{\mathbf{w} \mid \mathbf{w}^{\dagger} \mathbf{C w} \leq P_{r}} E_{2} \\
& =E_{1}+\lambda_{\max }\left(\mathbf{C}^{-1} \mathbf{D}\right) P_{r} \tag{32}
\end{align*}
$$

## III. Secure Relay Beamforming with the EAVESDROPPER's CSI

In this section, we assume that the eavesdropper's CSI is available at two sources and relay. This assumption is valid when the eavesdropper is active [20]. The active eavesdropper may register in the network as a subscribed user [21]. The active eavesdropper may also act as either (both) a jammer or (and) a classical eavesdropper [22]. The aforementioned scenarios are typical in future device-to-device communications [23] where a mobile phone in the network may be remotely intercepted and used as an active eavesdropper. Furthermore, even for a passive eavesdropper, there is a possibility for
one to estimate the CSI through the local oscillator power inadvertently leaked from the eavesdropper's receiver radio frequency frontend [24].

When the eavesdropper's CSI is available, our objective is to maximize achievable secrecy sum rate of the two-way relay network under transmit power constraint at relay and EH constraint at EH receiver by designing the beamforming vector $\mathbf{w}$, which is formulated as follows

$$
\begin{array}{ll}
\max _{\mathbf{w}} & \left(1+\frac{\mathbf{w}^{\dagger} \mathbf{Q}_{2,1} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{R}_{1} \mathbf{w}+\sigma^{2}}\right)\left(1+\frac{\mathbf{w}^{\dagger} \mathbf{Q}_{1,2} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{R}_{2} \mathbf{w}+\sigma^{2}}\right) \\
& \cdot\left(\frac{\sigma^{4}\left(1+\mathbf{w}^{\dagger} \mathbf{R}_{c c} \mathbf{w}\right)}{\mathbf{w}^{\dagger} \mathbf{Q}_{e} \mathbf{w}+\alpha}\right)  \tag{33}\\
\text { s.t. } & \mathbf{w}^{\dagger} \mathbf{C w} \leq P_{r}, \\
& \mathbf{w}^{\dagger} \mathbf{D} \mathbf{w} \geq \bar{Q}
\end{array}
$$

where $\frac{1}{2} \log _{2}(\cdot)$ is omitted in (33) due to it is monotonically increasing function and $\bar{Q}=Q-E_{1}$. It is noted that the achievable secrecy sum rate optimization problem for AF twoway relay networks without EH constraint has been investigated in [15]. Without EH constraint, ZF is employed in [15] to convert the achievable secrecy sum rate maximization problem into a generalised Rayleigh quotient. With EH constraint, the achievable secrecy sum rate maximization problem cannot be converted into a generalised Rayleigh quotient. Furthermore, besides ZF based suboptimal solution, we also propose an upper bound based solution and an SPCA based locally optimal solution in this paper.

Remark 1: It is noted that in [15], [17], the sum transmit power constraint at sources and relay is considered, which is suitable for the scenario of energy-constraint relay networks. In the aforementioned scenario, the limited energy of whole relay networks should be consumed efficiently. In this paper, we consider the relay is responsible for not only relaying the transmitted signals from two sources but also transferring energy to an EH receiver. Thus, the relay should have sufficient power supply. Under this condition, the individual transmit power constraints at sources and relay may be proper. If the sum transmit power constraint at sources and relay is considered, the solution to the achievable secrecy sum rate maximization problem with EH constraint can be obtained by the combination of 2-D search over $\left(P_{1}, P_{2}\right)$ and our proposed algorithms to obtain $\mathbf{W}$. The more efficient solution to aforementioned optimization problem is an interesting future work.

For proceeding, we have the following lemma.
Lemma 1: The optimal beamforming vector $\mathbf{w}$ in (33) satisfies

$$
\begin{equation*}
\mathbf{w}^{\dagger} \mathbf{C w}=P_{r} . \tag{34}
\end{equation*}
$$

Proof: See Appendix A.
From Lemma 1, optimal transmit power of relay is equal to the transmit power constraint at relay. In the following, assuming that individual transmit power constraints of source 1 and source 2 are $\bar{P}_{1}$ and $\bar{P}_{2}$, respectively, we will find the optimal transmit powers of two sources. When optimal beamforming vector $\mathbf{w}$ is known, the optimization problem (33) with respect to $\left(P_{1}, P_{2}\right)$ is still non-convex and the global optimal solution is difficult to obtain. However, the local
optimum of $\left(P_{1}, P_{2}\right)$ can be found by alternating optimization of $P_{1}$ and $P_{2}$. We have the following lemma.

Lemma 2: With a fixed $P_{2}$, the objective of (33) is monotonically increasing or decreasing with respect to $P_{1}$.

Proof: See Appendix B.
According to Lemma 2, for a given $P_{2}$, the optimal $P_{1}$ is either 0 or the maximum allowable transmit power of source 1. Similar result holds for the objective of (33) with respect to $P_{2}$. These results are reasonable because if the transmit power constraint at relay, $P_{r}$, is relatively small compared with those of two source or the eavesdropper is close to one source, two-way relay network may be proactively degraded to one-way relay network. In this paper, our focus is on secure relay beamforming. Therefore, we assume that $P_{r}$ is sufficiently large and the eavesdropper is close to the relay such that aforementioned degradation will not happen. Under this condition, the optimal transmit powers of two sources are $P_{i}=\bar{P}_{i}, i \in\{1,2\}$.

Substituting (34) into problem (33), we obtain

$$
\begin{array}{cl}
\max _{\mathbf{w}} & \frac{\mathbf{w}^{\dagger} \mathbf{A}_{1} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{B}_{1} \mathbf{w}} \cdot \frac{\mathbf{w}^{\dagger} \mathbf{A}_{2} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{B}_{2} \mathbf{w}} \cdot \frac{\mathbf{w}^{\dagger} \mathbf{A}_{3} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{B}_{3} \mathbf{w}}  \tag{35}\\
\text { s.t. } & \mathbf{w}^{\dagger} \mathbf{B}_{4} \mathbf{w}-\mathbf{w}^{\dagger} \mathbf{A}_{4} \mathbf{w} \leq 0
\end{array}
$$

where

$$
\begin{align*}
\mathbf{A}_{1} & =\mathbf{Q}_{2,1}+\mathbf{R}_{1}+\sigma^{2} \mathbf{C} / P_{r}  \tag{36}\\
\mathbf{A}_{2} & =\mathbf{Q}_{1,2}+\mathbf{R}_{2}+\sigma^{2} \mathbf{C} / P_{r}  \tag{37}\\
\mathbf{A}_{3} & =\mathbf{R}_{c c}+\sigma^{4} \mathbf{C} / P_{r}, \mathbf{A}_{4}=P_{r} \mathbf{D}  \tag{38}\\
\mathbf{B}_{i} & =\mathbf{R}_{i}+\sigma^{2} \mathbf{C} / P_{r}, i \in\{1,2\}  \tag{39}\\
\mathbf{B}_{3} & =\mathbf{Q}_{e}+\alpha \mathbf{C} / P_{r}, \mathbf{B}_{4}=\bar{Q} \mathbf{C} \tag{40}
\end{align*}
$$

Note that in (35), we have removed the equality constraint of relay transmit power (34). This is because the objective function and the constraint in (35) are homogeneous in w. An arbitrary positive scaling of $w$ has no effect on the value of the objective function.

It is observed that the objective function and the constraint in (35) are non-convex which causes problem (35) a nonconvex optimization problem. In general, it is difficult or even intractable to obtain the global optimal solution to a nonconvex problem. In the following, we will propose several suboptimal solutions for (35).

## A. Upper Bound Based Solution

In this subsection, we derive an upper bound for problem (35) and then propose an upper bound based suboptimal solution by employing Gaussian randomization method.

By introducing two slack variables $t_{1}$ and $t_{2}$ such that $\frac{\mathbf{w}^{\dagger} \mathbf{A}_{1} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{B}_{1} \mathbf{w}} \geq t_{1}$ and $\frac{\mathbf{w}^{\dagger} \mathbf{A}_{2} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{B}_{2} \mathbf{w}} \geq t_{2}$, we equivalently rewrite problem (35) as

$$
\begin{align*}
\max _{\mathbf{w}, t_{1}, t_{2}} & t_{1} \cdot t_{2} \cdot \frac{\mathbf{w}^{\dagger} \mathbf{A}_{3} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{B}_{3} \mathbf{w}} \\
\text { s.t. } & \mathbf{w}^{\dagger}\left(\mathbf{A}_{1}-t_{1} \mathbf{B}_{1}\right) \mathbf{w} \geq 0  \tag{41}\\
& \mathbf{w}^{\dagger}\left(\mathbf{A}_{2}-t_{2} \mathbf{B}_{2}\right) \mathbf{w} \geq 0 \\
& \mathbf{w}^{\dagger}\left(\mathbf{B}_{4}-\mathbf{A}_{4}\right) \mathbf{w} \leq 0
\end{align*}
$$

Consider the rank-one relaxation of (41) as follows

$$
\begin{align*}
\max _{\mathbf{X} \succeq \mathbf{0}, t_{1}, t_{2}} & t_{1} \cdot t_{2} \cdot \frac{\operatorname{tr}\left(\mathbf{A}_{3} \mathbf{X}\right)}{\operatorname{tr}\left(\mathbf{B}_{3} \mathbf{X}\right)} \\
\text { s.t. } & \operatorname{tr}\left(\left(\mathbf{A}_{1}-t_{1} \mathbf{B}_{1}\right) \mathbf{X}\right) \geq 0  \tag{42}\\
& \operatorname{tr}\left(\left(\mathbf{A}_{2}-t_{2} \mathbf{B}_{2}\right) \mathbf{X}\right) \geq 0 \\
& \operatorname{tr}\left(\left(\mathbf{B}_{4}-\mathbf{A}_{4}\right) \mathbf{X}\right) \leq 0
\end{align*}
$$

If problem (42) has an optimal rank-one solution $\mathbf{X}$, (42) is equivalent to problem (35). Given $t_{1}$ and $t_{2}$, problem (42) is a linear fractional programming, which can be solved by employing Charnes-Cooper transformation [25]. Let $1 / \operatorname{tr}\left(\mathbf{B}_{3} \mathbf{X}\right)=\nu$ and $\nu \mathbf{X}=\mathbf{Y}$. Assuming that $t_{1}$ and $t_{2}$ are given, we express problem (42) as

$$
\begin{array}{cl}
\max _{\mathbf{Y} \succeq \mathbf{0}} & t_{1} \cdot t_{2} \cdot \operatorname{tr}\left(\mathbf{A}_{3} \mathbf{Y}\right) \\
\text { s.t. } & \operatorname{tr}\left(\mathbf{B}_{3} \mathbf{Y}\right)=1 \\
& \operatorname{tr}\left(\left(\mathbf{A}_{1}-t_{1} \mathbf{B}_{1}\right) \mathbf{Y}\right) \geq 0  \tag{43}\\
& \operatorname{tr}\left(\left(\mathbf{A}_{2}-t_{2} \mathbf{B}_{2}\right) \mathbf{Y}\right) \geq 0 \\
& \operatorname{tr}\left(\left(\mathbf{B}_{4}-\mathbf{A}_{4}\right) \mathbf{Y}\right) \leq 0
\end{array}
$$

which is an SDP. We have the following lemma.
Lemma 3: Given $t_{1}$ and $t_{2}$, problems (42) and (43) have the same optimal value.

Proof: See Appendix C.
The SDP (43) can be solved effectively using the interiorpoint method [26]. It is noted that given $t_{1}$ and $t_{2}$, the obtained optimal solution to (43), denoted as $\mathbf{Y}^{o}$, is also the optimal solution to (42). The upper and lower bounds of $t_{1}$ and $t_{2}$ can be chosen as

$$
\begin{align*}
t_{1, u} & =\max _{\mathbf{w}} \frac{\mathbf{w}^{\dagger} \mathbf{A}_{1} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{B}_{1} \mathbf{w}}=\lambda_{\max }\left(\mathbf{B}_{1}^{-1} \mathbf{A}_{1}\right)  \tag{44}\\
t_{1, l} & =\min _{\mathbf{w}} \frac{\mathbf{w}^{\dagger} \mathbf{A}_{1} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{B}_{1} \mathbf{w}}=\lambda_{\min }\left(\mathbf{B}_{1}^{-1} \mathbf{A}_{1}\right) \tag{45}
\end{align*}
$$

respectively. Similarly,

$$
\begin{equation*}
t_{2, u}=\lambda_{\max }\left(\mathbf{B}_{2}^{-1} \mathbf{A}_{2}\right), t_{2, l}=\lambda_{\min }\left(\mathbf{B}_{2}^{-1} \mathbf{A}_{2}\right) \tag{46}
\end{equation*}
$$

Thus, the optimal solution to problem (42) can be found by 2-D search over $\left(t_{1}, t_{2}\right)$, which is summarized in Algorithm 1.

```
Algorithm 1 Find the optimal solution to problem (42)
    : Choose some large \(L_{1}\) and \(L_{2}\). Define \(\Delta t_{1}=\frac{t_{1, u}-t_{1, l}}{L_{1}}\),
    \(\Delta t_{2}=\frac{t_{2, u}-t_{2, l}}{L_{2}}\). Initialize \(\lambda^{\star}=0 ;\)
    For \(i=0: L_{1}\)
        Set \(t_{1}=i \Delta t_{1}+t_{1, l}\);
            For \(j=0: L_{2}\)
                Set \(t_{2}=j \Delta t_{2}+t_{2, l}\);
                Solve problem (43);
                If the optimal value of (43) is \(\lambda^{o}>\lambda^{\star}\)
                    Update \(\lambda^{\star}=\lambda^{o}\);
                    Save the optimal solution as \(\mathbf{Y}^{o}\).
        End
            End
    End
```

The optimal value obtained by Algorithm 1 is an upper bound for problem (35) because of the rank-one relaxation. If the optimal solution to (42) found by Algorithm 1 is rank-one, i.e., $\operatorname{rank}\left(\mathbf{Y}^{o}\right)=1$, the optimal solution of (35) is $\mathbf{w}^{o}=\mathbf{y}^{o}$ where $\mathbf{Y}^{o}=\mathbf{y}^{o} \mathbf{y}^{o \dagger}$. Otherwise, we can employ the Gaussian randomization method proposed in [27] to find a suboptimal solution.

According to Lemma 3.1 in [28], there exists an optimal solution $\mathbf{Y}^{o}$ to problem (42) such that $\operatorname{rank}\left(\mathbf{Y}^{o}\right) \leq 2$. Under some special conditions, we can construct an optimal rank-one solution for (35) even when $\mathbf{Y}^{o}$ is not rank-one. We have the following lemma.

Lemma 4: In Algorithm 1, if $\left(\mathbf{A}_{4}-\mathbf{B}_{4}\right) \succ \mathbf{0}$ or $\left(\mathbf{A}_{1}-\right.$ $\left.t_{1}^{o} \mathbf{B}_{1}\right) \succ \mathbf{0}$ or $\left(\mathbf{A}_{2}-t_{2}^{o} \mathbf{B}_{2}\right) \succ \mathbf{0}$ where $t_{1}^{o}$ and $t_{2}^{o}$ are the optimal values of $t_{1}$ and $t_{2}$, respectively, an optimal rank-one solution to problem (35) can be constructed when $\mathbf{Y}^{o}$ is not rank-one.

Proof: See Appendix D.
Remark 2: From Lemma 4, when $\bar{Q}$ is much lower than the transmit power constraint at relay, $P_{r}$, the probability of obtaining an optimal rank-one solution is high. The probability of obtaining an optimal rank-one solution is related to the channel responses, including $\mathbf{h}_{1, f}, \mathbf{h}_{2, f}, f_{1}, f_{2}, g_{1}, g_{2}, \mathbf{h}_{1, b}$, $\mathbf{h}_{2, b}, \mathbf{f}_{r}$, and $\mathbf{g}_{r}$, the transmit power constraint at relay, $P_{r}$, and the EH constraint at EH receiver, $Q$. Theoretical derivation of aforementioned probability, which is difficult if not impossible, may be an interesting future work. In Section VI, we present the probability of obtaining an optimal rank-one solution in our simulations.

Remark 3: When $\left(\mathbf{A}_{4}-\mathbf{B}_{4}\right) \succ \mathbf{0}$ or $\left(\mathbf{A}_{1}-t_{1}^{o} \mathbf{B}_{1}\right) \succ \mathbf{0}$ or $\left(\mathbf{A}_{2}-t_{2}^{o} \mathbf{B}_{2}\right) \succ \mathbf{0}$, the construction of optimal rank-one solution to problem (35) when $\mathbf{Y}^{o}$ is not rank-one is following the steps of Algorithm 3 in [29].

## B. SPCA Based Locally Optimal Solution

In Algorithm 1, the 2-D search over $\left(t_{1}, t_{2}\right)$ has high computational complexity. In practice, developing a lower complexity algorithm to find a local optimum of problem (35) is meaningful. In this section, we transform problem (35) into an equivalent DC programming whose objective can be written as a DC function [18]. To solve this DC programming, we propose an SPCA based iterative algorithm to achieve its local optimum.

Because of monotonicity of logarithm, we equivalently rewrite problem (35) as

$$
\begin{align*}
& \max _{\mathbf{w}} \sum_{i=1}^{3} \ln \left(\mathbf{w}^{\dagger} \mathbf{A}_{i} \mathbf{w}\right)-\sum_{i=1}^{3} \ln \left(\mathbf{w}^{\dagger} \mathbf{B}_{i} \mathbf{w}\right) \\
& \text { s.t. } \mathbf{w}^{\dagger} \mathbf{B}_{4} \mathbf{w}-\mathbf{w}^{\dagger} \mathbf{A}_{4} \mathbf{w} \leq 0 \tag{47}
\end{align*}
$$

Assume that $\mathbf{X}=\mathbf{w} \mathbf{w}^{\dagger}$, problem (47) is further expressed as

$$
\begin{align*}
\min _{\mathbf{X} \succeq \mathbf{0}} & \sum_{i=1}^{3} \ln \left(\operatorname{tr}\left(\mathbf{B}_{i} \mathbf{X}\right)\right)-\sum_{i=1}^{3} \ln \left(\operatorname{tr}\left(\mathbf{A}_{i} \mathbf{X}\right)\right) \\
\text { s.t. } & \operatorname{tr}\left(\mathbf{B}_{4} \mathbf{X}\right)-\operatorname{tr}\left(\mathbf{A}_{4} \mathbf{X}\right) \leq 0  \tag{48}\\
& \operatorname{rank}(\mathbf{X})=1
\end{align*}
$$

Since the functions $-\ln \left(\operatorname{tr}\left(\mathbf{B}_{i} \mathbf{X}\right)\right)$ and $-\ln \left(\operatorname{tr}\left(\mathbf{A}_{i} \mathbf{X}\right)\right), i \in$ $\{1,2,3\}$, are convex, problem (48) is a DC programming without considering the rank-one constraint. To proceed, we have the following lemma.

Lemma 5: The constraint $\operatorname{rank}(\mathbf{X})=1$ with $\mathbf{X} \succeq \mathbf{0}$ is equivalent to

$$
\begin{equation*}
\mathbf{X} \succeq \mathbf{0}, \operatorname{tr}(\mathbf{X})-\lambda_{\max }(\mathbf{X}) \leq 0 \tag{49}
\end{equation*}
$$

Proof: Since $\mathbf{X} \succeq \mathbf{0}$, we have $\operatorname{tr}(\mathbf{X})-\lambda_{\max }(\mathbf{X}) \geq 0$, which combines $\operatorname{tr}(\mathbf{X})-\lambda_{\max }(\mathbf{X}) \leq 0$ such that $\operatorname{tr}(\mathbf{X})-\lambda_{\max }(\mathbf{X})=$ 0 . Thus, $\mathbf{X}$ has only one nonzero eigenvalue.

Using Lemma 5, we equivalently transform problem (48) into

$$
\begin{align*}
\min _{\mathbf{X} \succeq \mathbf{0}} & \sum_{i=1}^{3} \ln \left(\operatorname{tr}\left(\mathbf{B}_{i} \mathbf{X}\right)\right)-\sum_{i=1}^{3} \ln \left(\operatorname{tr}\left(\mathbf{A}_{i} \mathbf{X}\right)\right) \\
\text { s.t. } & \operatorname{tr}\left(\mathbf{B}_{4} \mathbf{X}\right)-\operatorname{tr}\left(\mathbf{A}_{4} \mathbf{X}\right) \leq 0  \tag{50}\\
& \operatorname{tr}(\mathbf{X})-\lambda_{\max }(\mathbf{X}) \leq 0
\end{align*}
$$

To solve the above problem, we employ the exact penalty method [26] and rewrite the above problem as

$$
\begin{align*}
\min _{\mathbf{X} \succeq \mathbf{0}} & \sum_{i=1}^{3} \ln \left(\operatorname{tr}\left(\mathbf{B}_{i} \mathbf{X}\right)\right)-\sum_{i=1}^{3} \ln \left(\operatorname{tr}\left(\mathbf{A}_{i} \mathbf{X}\right)\right) \\
& +\kappa\left(\operatorname{tr}(\mathbf{X})-\lambda_{\max }(\mathbf{X})\right)  \tag{51}\\
\text { s.t. } & \operatorname{tr}\left(\mathbf{B}_{4} \mathbf{X}\right)-\operatorname{tr}\left(\mathbf{A}_{4} \mathbf{X}\right) \leq 0
\end{align*}
$$

where $\kappa>0$ is a sufficient large penalty factor. For problems (50) and (51), we have the following lemma.

Lemma 6: There exists $0<\kappa_{0}<+\infty$ such that problems (50) and (51) are equivalent when $\kappa>\kappa_{0}$.

Proof: See Appendix E.
According to Lemma 6, we can obtain the optimal solution to (50) by solving problem (51). Since the spectral function $\lambda_{\max }(\mathbf{X})$ is convex, problem (51) is a DC programming. To deal with the nonconvex terms in the objective, i.e., $\ln \left(\operatorname{tr}\left(\mathbf{B}_{i} \mathbf{X}\right)\right), i \in\{1,2,3\}$, and $-\lambda_{\max }(\mathbf{X})$, we employ the result of [19] which shows that if we replace the nonconvex terms with their convex upper bounds and iteratively solve the resulting problem by judiciously updating the variables until convergence, we can obtain a local optimum of (51).

Suppose that $\tilde{\mathbf{X}}$ is a feasible point to problem (50). Using the property of concave functions [26], we have

$$
\begin{align*}
\ln \left(\operatorname{tr}\left(\mathbf{B}_{i} \mathbf{X}\right)\right) & \leq \ln \left(\operatorname{tr}\left(\mathbf{B}_{i} \tilde{\mathbf{X}}\right)\right)+\frac{\operatorname{tr}\left(\mathbf{B}_{i}(\mathbf{X}-\tilde{\mathbf{X}})\right)}{\operatorname{tr}\left(\mathbf{B}_{i} \tilde{\mathbf{X}}\right)}  \tag{52}\\
-\lambda_{\max }(\mathbf{X}) & \leq-\lambda_{\max }(\tilde{\mathbf{X}})-\operatorname{tr}\left(\tilde{\mathbf{x}} \tilde{\mathbf{x}}^{\dagger}(\mathbf{X}-\tilde{\mathbf{X}})\right) \tag{53}
\end{align*}
$$

where $\tilde{\mathbf{x}}$ is the unit-norm eigenvector corresponding to the maximal eigenvalue $\lambda_{\max }(\tilde{\mathbf{X}})$. Since the righthand sides of (52) and (53) are linear, they are convex upper bounds of $\ln \left(\operatorname{tr}\left(\mathbf{B}_{i} \mathbf{X}\right)\right)$ and $-\lambda_{\max }(\mathbf{X})$, respectively.

We propose an SPCA based iterative algorithm which iteratively optimizes $\mathbf{X}$. In the $(n+1)$ th iteration, given $\mathbf{X}^{(n)}$ which is optimal in the $n$th iteration and $\mathbf{x}^{(n)}$ which is the unitnorm eigenvector corresponding to the maximal eigenvalue
$\lambda_{\max }\left(\mathbf{X}^{(n)}\right)$, we solve the following convex optimization problem,

$$
\begin{align*}
\min _{\mathbf{X} \succeq \mathbf{0}} & \sum_{i=1}^{3} \frac{\operatorname{tr}\left(\mathbf{B}_{i}(\mathbf{X})\right)}{\operatorname{tr}\left(\mathbf{B}_{i} \mathbf{X}^{(n)}\right)}-\sum_{i=1}^{3} \ln \left(\operatorname{tr}\left(\mathbf{A}_{i} \mathbf{X}\right)\right) \\
& +\kappa\left(\operatorname{tr}(\mathbf{X})-\operatorname{tr}\left(\mathbf{x}^{(n)} \mathbf{x}^{(n) \dagger} \mathbf{X}\right)\right)  \tag{54}\\
\text { s.t. } & \operatorname{tr}\left(\mathbf{B}_{4} \mathbf{X}\right)-\operatorname{tr}\left(\mathbf{A}_{4} \mathbf{X}\right) \leq 0
\end{align*}
$$

to obtain $\mathbf{X}^{(n+1)}$ which is optimal in the $(n+1)$ th iteration. Thus, the proposed SPCA based iterative algorithm to obtain the locally optimal solution to problem (50) is summarized in Algorithm 2.

```
Algorithm 2 Find the locally optimal solution to problem (50)
    Initialization: \(n=0, \mathbf{X}^{(0)}=\mathbf{x}^{(0)} \mathbf{x}^{(0) \dagger}\), and a small
    positive number, \(\varepsilon\).
    Repeat:
    Solve the convex problem (54) to obtain \(\mathbf{X}^{(n+1)}\);
    \(n:=n+1\);
    Until: \(\eta^{(n)}-\eta^{(n-1)}<\varepsilon\), where \(\eta^{(n)}\) denotes the obtained
    objective value of (54) in the \(n\)th iteration.
```

Remark 4: It is noted that problem (54) is not a linear SDP. However, it is still convex and can be solved effectively using the interior-point method [26].

## C. ZF Based Suboptimal Solution

In this subsection, we propose a ZF based suboptimal solution for secure relay beamforming. We force the information leakage to the eavesdropper in the second phase to be zero, i.e., $\mathbf{a}_{c f}^{T} \mathbf{w}=0$ and $\mathbf{a}_{c g}^{T} \mathbf{w}=0$, which can also be expressed as

$$
\begin{equation*}
\left[\mathbf{a}_{c f}, \mathbf{a}_{c g}\right]^{T} \mathbf{w}=\mathbf{0} \tag{55}
\end{equation*}
$$

From (55), the secure relay beamforming vector $\mathbf{w}$ is

$$
\begin{equation*}
\mathbf{w}=\mathbf{V} \mathbf{x} \tag{56}
\end{equation*}
$$

where $\mathbf{V} \in \mathbb{C}^{N^{2} \times\left(N^{2}-2\right)}$ consists of $N^{2}-2$ singular vectors of the matrix $\left[\mathbf{a}_{c f}, \mathbf{a}_{c g}\right]^{T}$ associating with zero singular values and $\mathrm{x} \in \mathbb{C}^{\left(N^{2}-2\right) \times 1}$ is an arbitrary vector. Substituting (56) into problem (35), we obtain

$$
\begin{array}{cl}
\max _{\mathbf{x}} & \frac{\mathbf{x}^{\dagger} \overline{\mathbf{A}}_{1} \mathbf{x}}{\mathbf{x}^{\dagger} \overline{\mathbf{B}}_{1} \mathbf{x}} \cdot \frac{\mathbf{x}^{\dagger} \overline{\mathbf{A}}_{2} \mathbf{x}}{\mathbf{x}^{\dagger} \overline{\mathbf{B}}_{2} \mathbf{x}}  \tag{57}\\
\text { s.t. } & \mathbf{x}^{\dagger} \overline{\mathbf{B}}_{4} \mathbf{x}-\mathbf{x}^{\dagger} \overline{\mathbf{A}}_{4} \mathbf{x} \leq 0
\end{array}
$$

where $\overline{\mathbf{A}}_{i}=\mathbf{V}^{\dagger} \mathbf{A}_{i} \mathbf{V}$ and $\overline{\mathbf{B}}_{i}=\mathbf{V}^{\dagger} \mathbf{B}_{i} \mathbf{V}$.
Let

$$
\begin{equation*}
\mathbf{x}=\mathbf{U} \tilde{\mathbf{x}} \tag{58}
\end{equation*}
$$

where $\tilde{\mathbf{x}} \in \mathbb{C}^{M \times 1}$ is an arbitrary vector and $\mathbf{U} \in \mathbb{C}^{\left(N^{2}-2\right) \times M}$ consists of the $M$ eigenvectors of the matrix $\overline{\mathbf{B}}_{4}-\overline{\mathbf{A}}_{4}$ associating with the eigenvalues being no greater than zero where $1 \leq M \leq N^{2}-2$. Since $\tilde{\mathbf{x}}^{\dagger} \mathbf{U}^{\dagger}\left(\overline{\mathbf{B}}_{4}-\overline{\mathbf{A}}_{4}\right) \mathbf{U} \tilde{\mathbf{x}} \leq 0$, substituting (58) into (57), we have

$$
\begin{equation*}
\max _{\tilde{\mathbf{x}}} \frac{\tilde{\mathbf{x}}^{\dagger} \tilde{\mathbf{A}}_{1} \tilde{\mathbf{x}}}{\tilde{\mathbf{x}}^{\dagger} \tilde{\mathbf{B}}_{1} \tilde{\mathbf{x}}} \cdot \frac{\tilde{\mathbf{x}}^{\dagger} \tilde{\mathbf{A}}_{2} \tilde{\mathbf{x}}}{\tilde{\mathbf{x}}^{\dagger} \tilde{\mathbf{B}}_{2} \tilde{\mathbf{x}}} \tag{59}
\end{equation*}
$$

where $\tilde{\mathbf{A}}_{i}=\mathbf{U}^{\dagger} \overline{\mathbf{A}}_{i} \mathbf{U}$ and $\tilde{\mathbf{B}}_{i}=\mathbf{U}^{\dagger} \overline{\mathbf{B}}_{i} \mathbf{U}$.
According to [32], the global optimum to problem (59) must be one of the vectors $\tilde{\mathbf{x}}$ for which the gradient of objective function is zero. Using the condition of zero gradient, we obtain

$$
\begin{equation*}
\left(\tilde{\mathbf{A}}_{1}+\zeta_{A} \tilde{\mathbf{A}}_{2}\right) \tilde{\mathbf{x}}=\frac{P_{A, 1}}{P_{B, 1}}\left(\tilde{\mathbf{B}}_{1}+\zeta_{B} \tilde{\mathbf{B}}_{2}\right) \tilde{\mathbf{x}} \tag{60}
\end{equation*}
$$

where $P_{A, i}=\tilde{\mathbf{x}}^{\dagger} \tilde{\mathbf{A}}_{i} \tilde{\mathbf{x}}, P_{B, i}=\tilde{\mathbf{x}}^{\dagger} \tilde{\mathbf{B}}_{i} \tilde{\mathbf{x}}, \zeta_{A}=P_{A, 1} / P_{A, 2}, \zeta_{B}=$ $P_{B, 1} / P_{B, 2}$.

From (60), the optimal $\tilde{\mathbf{x}}$ is a generalized eigenvector of the matrix pair $\left(\tilde{\mathbf{A}}_{1}+\zeta_{A} \tilde{\mathbf{A}}_{2}\right)$ and $\left(\tilde{\mathbf{B}}_{1}+\zeta_{B} \tilde{\mathbf{B}}_{2}\right)$. However, we cannot compute $\tilde{\mathbf{x}}$ directly from the generalized eigenvector because the unknown parameters $\zeta_{A}$ and $\zeta_{B}$ are correlated with $\tilde{\mathbf{x}}$. Therefore, 2-D search over $\left(\zeta_{A}, \zeta_{B}\right)$ is required to find the optimal $\tilde{\mathbf{x}}$. It has been shown in [32] that for every value of $\zeta_{B}$, the corresponding maximization over $\zeta_{A}$ yields the maximal value which depends on $\zeta_{B}$ only very weakly. Thus, the 2-D search over $\left(\zeta_{A}, \zeta_{B}\right)$ can be replaced essentially without any loss by a 1-D bisection search over $\zeta_{A}$ only for one fixed value of $\zeta_{B}$, e.g., the geometric mean of upper and lower bounds [32].

For 1-D search over $\zeta_{A}$, the required upper and lower bounds of $\zeta_{A}$, denoted as $\zeta_{A, u}$ and $\zeta_{A, l}$, respectively, are

$$
\begin{equation*}
\zeta_{A, u}=\lambda_{\max }\left(\tilde{\mathbf{A}}_{2}^{-1} \tilde{\mathbf{A}}_{1}\right), \zeta_{A, l}=\lambda_{\min }\left(\tilde{\mathbf{A}}_{2}^{-1} \tilde{\mathbf{A}}_{1}\right) \tag{61}
\end{equation*}
$$

Similarly,

$$
\begin{equation*}
\zeta_{B, u}=\lambda_{\max }\left(\tilde{\mathbf{B}}_{2}^{-1} \tilde{\mathbf{B}}_{1}\right), \zeta_{B, l}=\lambda_{\min }\left(\tilde{\mathbf{B}}_{2}^{-1} \tilde{\mathbf{B}}_{1}\right) \tag{62}
\end{equation*}
$$

Defining the function

$$
\begin{equation*}
f\left(\zeta_{A}\right)=\left.\frac{\tilde{\mathbf{x}}^{\dagger} \tilde{\mathbf{A}}_{1} \tilde{\mathbf{x}}}{\tilde{\mathbf{x}}^{\dagger} \tilde{\mathbf{B}}_{1} \tilde{\mathbf{x}}} \cdot \frac{\tilde{\mathbf{x}}^{\dagger} \tilde{\mathbf{A}}_{2} \tilde{\mathbf{x}}}{\tilde{\mathbf{x}}^{\dagger} \tilde{\mathbf{B}}_{2} \tilde{\mathbf{x}}}\right|_{\tilde{\mathbf{x}}=\phi\left(\tilde{\mathbf{A}}_{1}+\zeta_{A} \tilde{\mathbf{A}}_{2}, \tilde{\mathbf{B}}_{1}+\zeta_{B} \tilde{\mathbf{B}}_{2}\right)} \tag{63}
\end{equation*}
$$

where $\phi(\mathbf{A}, \mathbf{B})$ denotes the generalized eigenvector of the matrix pair $(\mathbf{A}, \mathbf{B})$ with respect to the largest eigenvalue, the proposed 1-D bisection search based algorithm to obtain the suboptimal solution to problem (35) is summarized in Algorithm 3.

```
Algorithm 3 Find the low-complexity suboptimal solution to
problem (35) by 1-D bisection search
    Initialization: \(\zeta_{B}=\sqrt{\zeta_{B, u} \zeta_{B, l}}\), and a small positive
    number, \(\varepsilon\).
2: Repeat:
    Compute \(\zeta_{A}=\frac{\zeta_{A, u}+\zeta_{A, l}}{2}, \zeta_{1}=\zeta_{A}-\varepsilon, \zeta_{2}=\zeta_{A}+\varepsilon\).
    Compute \(f\left(\zeta_{1}\right)\) and \(\stackrel{2}{f}\left(\zeta_{2}\right)\) according to (63);
    If \(f\left(\zeta_{2}\right)<f\left(\zeta_{1}\right)\)
        \(\zeta_{A, u}=\zeta_{2} ;\)
    Else
        \(\zeta_{A, l}=\zeta_{1} ;\)
    End
    Until: \(\zeta_{A, u}-\zeta_{A, l}<\varepsilon\).
```


## IV. Secure Relay Beamforming without the EAVESDROPPER's CSI

When the eavesdropper is passive, the eavesdropper's CSI is difficult to obtain. Thus, the proposed solutions in Section III may not be applied. In this section, we consider that the eavesdropper's CSI is not available at sources and relay. Under this condition, we propose effective secure relay beamforming using the artificial noise (AN)-aided scheme [30]. In the ANaided scheme, the relay transmits AN to mask the concurrent transmission of information bearing signal to the two sources. Thus, the transmitted signal from the relay in the second phase is expressed as

$$
\begin{equation*}
\tilde{\tilde{\mathbf{r}}}=\mathbf{W r}+\mathbf{z} \tag{64}
\end{equation*}
$$

where $\mathbf{z}$ is the AN. After subtracting the self-interference, the remaining received signals at two sources are given by

$$
\begin{align*}
& \tilde{\tilde{y}}_{d, 1}=\mathbf{h}_{1, b}^{T} \mathbf{W} \mathbf{h}_{2, f} x_{2}+\mathbf{h}_{1, b}^{T} \mathbf{z}+\mathbf{h}_{1, b}^{T} \mathbf{W} \mathbf{n}_{r}+n_{d, 1}  \tag{65}\\
& \tilde{\tilde{y}}_{d, 2}=\mathbf{h}_{2, b}^{T} \mathbf{W} \mathbf{h}_{1, f} x_{1}+\mathbf{h}_{2, b}^{T} \mathbf{z}+\mathbf{h}_{2, b}^{T} \mathbf{W} \mathbf{n}_{r}+n_{d, 2} \tag{66}
\end{align*}
$$

and the received signal at the eavesdropper is
$\tilde{\tilde{y}}_{e, 2}=\mathbf{f}_{r}^{T} \mathbf{W} \mathbf{h}_{2, f} x_{2}+\mathbf{f}_{r}^{T} \mathbf{W} \mathbf{h}_{1, f} x_{1}+\mathbf{f}_{r}^{T} \mathbf{z}+\mathbf{f}_{r}^{T} \mathbf{W} \mathbf{n}_{r}+n_{e, 2}$.

To eliminate the interference to two sources, AN is injected into the null space of the channels of $\mathbf{h}_{1, b}$ and $\mathbf{h}_{2, b}$, i.e., $\mathbf{h}_{1, b}^{T} \mathbf{z}=\mathbf{h}_{2, b}^{T} \mathbf{z}=0$. It is noted that to ensure that AN lies on the null space of $\mathbf{h}_{1, b}$ and $\mathbf{h}_{2, b}$, an additional assumption that $N>2$ should be included. Since the relay has no information on the eavesdropper's CSI, i.e., the relay doesn't know $\mathbf{f}_{r}$, it has to transmit AN isotropically instead of concentrating the AN power toward some directions. Therefore, the AN z is expressed as

$$
\begin{equation*}
\mathbf{z}=\mathbf{V}^{\perp} \mathbf{n} \tag{68}
\end{equation*}
$$

where $\mathbf{V}^{\perp}$ is the projection matrix onto the null space of $\mathbf{V}=\left[\mathbf{h}_{1, b}, \mathbf{h}_{2, b}\right]$, i.e., $\mathbf{V}^{\perp}=\mathbf{I}-\mathbf{V}\left(\mathbf{V}^{T} \mathbf{V}\right)^{-1} \mathbf{V}^{T}$, and $\mathbf{n}$ is a Gaussian random vector with zero mean and covariance $\sigma_{n}^{2} \mathbf{I}$.

Using (68), the transmit power at the relay is expressed as

$$
\begin{equation*}
p_{r}=\mathbf{w}^{\dagger} \mathbf{C w}+\sigma_{n}^{2}(N-2) \tag{69}
\end{equation*}
$$

where the term $\sigma_{n}^{2}(N-2)=\mathbb{E}\left[\|\mathbf{z}\|^{2}\right]$ is the allocated power for AN. The harvested energy at the EH receiver in the second phase is given by

$$
\begin{equation*}
\tilde{E}_{2}=\mathbf{w}^{\dagger} \mathbf{D} \mathbf{w}+\sigma_{n}^{2}\left\|\mathbf{g}_{r}^{T} \mathbf{V}^{\perp}\right\|^{2} \tag{70}
\end{equation*}
$$

The achievable secrecy sum rate $R_{s}$ is maximized only when $R_{e}$ is minimized. In order to minimize $R_{e}$, we should maximize allocated power for AN under the transmit power constraint at relay, the EH constraint at EH receiver and the additional constraint that achievable sum rate of two sources is larger than or equal to a predefined threshold. Thus, assuming that $N>2$, the secure relay beamforming problem
is formulated as

$$
\begin{align*}
\max _{\mathbf{w}, \sigma_{n}^{2} \geq 0} & \sigma_{n}^{2}(N-2) \\
\text { s.t. } & \frac{1}{2} \log _{2}\left(1+\frac{\mathbf{w}^{\dagger} \mathbf{Q}_{2,1} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{R}_{1} \mathbf{w}+\sigma^{2}}\right) \\
& \cdot\left(1+\frac{\mathbf{w}^{\dagger} \mathbf{Q}_{1,2} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{R}_{2} \mathbf{w}+\sigma^{2}}\right) \geq r_{d},  \tag{71}\\
& \mathbf{w}^{\dagger} \mathbf{C w}+\sigma_{n}^{2}(N-2) \leq P_{r}, \\
& \mathbf{w}^{\dagger} \mathbf{D} \mathbf{w}+\sigma_{n}^{2}\left\|\mathbf{g}_{r}^{T} \mathbf{V}^{\perp}\right\|^{2} \geq \bar{Q}
\end{align*}
$$

where $r_{d}$ denotes the predefined threshold of achievable sum rate of two sources. The optimization problem (71) is nonconvex. It can be proved that the optimal solution to this problem satisfies that achievable sum rate constraint is active,

$$
\begin{equation*}
\frac{1}{2} \log _{2}\left(1+\frac{\mathbf{w}^{\dagger} \mathbf{Q}_{2,1} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{R}_{1} \mathbf{w}+\sigma^{2}}\right) \cdot\left(1+\frac{\mathbf{w}^{\dagger} \mathbf{Q}_{1,2} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{R}_{2} \mathbf{w}+\sigma^{2}}\right)=r_{d} \tag{72}
\end{equation*}
$$

Replacing inequality sign with equality sign in achievable sum rate constraint and introducing a slack variable $\tau$, problem (71) is rewritten as

$$
\begin{align*}
\max _{\mathbf{w}, \sigma_{n}^{2} \geq 0,0 \leq \tau \leq r_{d}} & \sigma_{n}^{2}(N-2) \\
\text { s.t. } & 1+\frac{\mathbf{w}^{\dagger} \mathbf{Q}_{2,1} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{R}_{1} \mathbf{w}+\sigma^{2}}=2^{2 \tau} \\
& 1+\frac{\mathbf{w}^{\dagger} \mathbf{Q}_{1,2} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{R}_{2} \mathbf{w}+\sigma^{2}}=2^{2\left(r_{d}-\tau\right)}  \tag{73}\\
& \mathbf{w}^{\dagger} \mathbf{C w}+\sigma_{n}^{2}(N-2) \leq P_{r} \\
& \mathbf{w}^{\dagger} \mathbf{D} \mathbf{w}+\sigma_{n}^{2}\left\|\mathbf{g}_{r}^{T} \mathbf{V}^{\perp}\right\|^{2} \geq \bar{Q}
\end{align*}
$$

Consider the rank-one relaxation of (73) as follows

$$
\begin{array}{cl}
\max _{\mathbf{X} \succeq \mathbf{0}, \sigma_{n}^{2} \geq 0,0 \leq \tau \leq r_{d}} & \sigma_{n}^{2}(N-2) \\
\text { s.t. } & \operatorname{tr}\left(\left(\left(2^{2 \tau}-1\right) \mathbf{R}_{1}-\mathbf{Q}_{2,1}\right) \mathbf{X}\right) \\
& +\sigma^{2}\left(2^{2 \tau}-1\right)=0 \\
& \operatorname{tr}\left(\left(\left(2^{2\left(r_{d}-\tau\right)}-1\right) \mathbf{R}_{2}-\mathbf{Q}_{1,2}\right) \mathbf{X}\right)  \tag{74}\\
& +\sigma^{2}\left(2^{2\left(r_{d}-\tau\right)}-1\right)=0 \\
& \operatorname{tr}(\mathbf{C X})+\sigma_{n}^{2}(N-2) \leq P_{r} \\
& \operatorname{tr}(\mathbf{D X})+\sigma_{n}^{2}\left\|\mathbf{g}_{r}^{T} \mathbf{V}^{\perp}\right\|^{2} \geq \bar{Q}
\end{array}
$$

If problem (74) has an optimal rank-one solution $\mathbf{X}$, (74) is equivalent to problem (73). Given $\tau$, problem (74) is an SDP, which can be solved effectively using the interior-point method [26]. Furthermore, given $\tau$, an optimal rank-one solution to (74) can be always found according to Theorem 2.3 in [29]. Thus, the optimal solution to problem (73) is obtained by 1-D search over $\tau$.

Obtaining the solution to problem (73), denoted as $\left(\mathbf{w}^{\star}, \sigma_{n}^{2 \star}\right)$, the received signals at the eavesdropper in the first and second phases can be expressed as in (20) where $\mathbf{n}_{e}$ is replaced by $\tilde{\mathbf{n}}_{e}$,

$$
\tilde{\mathbf{n}}_{e}=\left[\begin{array}{c}
n_{e, 1}  \tag{75}\\
\mathbf{f}_{r}^{T} \mathbf{z}+\mathbf{f}_{r}^{T} \mathbf{W}^{\star} \mathbf{n}_{r}+n_{e, 2}
\end{array}\right]
$$

in which $\mathbf{w}^{\star}=\operatorname{vec}\left(\mathbf{W}^{\star}\right)$. The covariance matrix of $\tilde{\mathbf{n}}_{e}$ is

$$
\mathbf{R}_{e}=\left[\begin{array}{cc}
\sigma^{2} & 0  \tag{76}\\
0 & \sigma_{n}^{2 \star}\left\|\mathbf{f}_{r}^{T} \mathbf{V}^{\perp}\right\|^{2}+\sigma^{2}\left(1+\mathbf{w}^{\star \dagger} \mathbf{R}_{c c} \mathbf{w}^{\star}\right)
\end{array}\right]
$$

Substituting (76) into (22), the information rate leaked to the eavesdropper can be computed as

$$
\begin{equation*}
\bar{R}_{e}=\frac{1}{2} \log _{2} \frac{\mathbf{f}_{r}^{T} \overline{\mathbf{Q}}_{e} \mathbf{f}_{r}^{*}+\alpha}{\sigma^{4}+\sigma^{2} \mathbf{f}_{r}^{T} \overline{\mathbf{R}}_{c c} \mathbf{f}_{r}^{*}} \tag{77}
\end{equation*}
$$

where

$$
\begin{align*}
\overline{\mathbf{Q}}_{e}= & \left(P_{1} P_{2}\left|f_{2}\right|^{2}+P_{1} \sigma^{2}\right) \overline{\mathbf{R}}_{c f}+\left(P_{1} P_{2}\left|f_{1}\right|^{2}+P_{2} \sigma^{2}\right) \overline{\mathbf{R}}_{c g} \\
& +\left(P_{1} \sigma^{2}\left|f_{1}\right|^{2}+P_{2} \sigma^{2}\left|f_{2}\right|^{2}\right) \overline{\mathbf{R}}_{c c}-P_{1} P_{2} f_{1} f_{2}^{*} \mathbf{\Theta}_{1} \\
& -P_{1} P_{2} f_{1}^{*} f_{2} \boldsymbol{\Theta}_{2},  \tag{78}\\
\overline{\mathbf{R}}_{c c}= & \sigma_{n}^{2 \star} \mathbf{V}^{\perp} \mathbf{V}^{\perp \dagger}+\sigma^{2} \mathbf{W}^{\star} \mathbf{W}^{\star \dagger},  \tag{79}\\
\overline{\mathbf{R}}_{c f}= & \mathbf{W}^{\star} \mathbf{h}_{1, f} \mathbf{h}_{1, f}^{\dagger} \mathbf{W}^{\star \dagger}, \overline{\mathbf{R}}_{c g}=\mathbf{W}^{\star} \mathbf{h}_{2, f} \mathbf{h}_{2, f}^{\dagger} \mathbf{W}^{\star \dagger},  \tag{80}\\
\boldsymbol{\Theta}_{1}= & \mathbf{W}^{\star} \mathbf{h}_{2, f} \mathbf{h}_{1, f}^{\dagger} \mathbf{W}^{\star \dagger}, \quad \boldsymbol{\Theta}_{2}=\mathbf{W}^{\star} \mathbf{h}_{1, f} \mathbf{h}_{2, f}^{\dagger} \mathbf{W}^{\star \dagger} . \tag{81}
\end{align*}
$$

According to the definition in [31], the secrecy outage probability is given by

$$
\begin{equation*}
\epsilon=\operatorname{Pr}\left(r_{d}-\bar{R}_{e}<R_{s}^{\epsilon}\right) \tag{82}
\end{equation*}
$$

where $R_{s}^{\epsilon}$ is a target secrecy rate. Substituting (77) into (82), we have

$$
\begin{align*}
\epsilon & =\operatorname{Pr}\left(\bar{R}_{e}>r_{d}-R_{s}^{\epsilon}\right) \\
& =\operatorname{Pr}\left(\frac{\mathbf{f}_{r}^{T} \overline{\mathbf{Q}}_{e} \mathbf{f}_{r}^{*}+\alpha}{\sigma^{4}+\sigma^{2} \mathbf{f}_{r}^{T} \overline{\mathbf{R}}_{c c} \mathbf{f}_{r}^{*}}>2^{2\left(r_{d}-R_{s}^{\epsilon}\right)}\right) . \tag{83}
\end{align*}
$$

In (83), when $f_{1}, f_{2}$, and the entries of $\mathbf{f}_{r}$ are considered as zero-mean independent and identically distributed (i.i.d.) Gaussian random variables, the probability density function of $\bar{R}_{e}$ is not known. Therefore, it is difficult, if not impossible, to obtain the closed-form expression of secrecy outage probability. In this paper, we employ Monte Carlo simulations to evaluate the secrecy outage probability.

## V. Complexity Analysis

In this section, we evaluate the computational complexity of our proposed algorithms.

For Algorithm 1, the computation burden is solving the SDP (43). To analyze the complexity of solving the SDP (43), we transform it into a standard form as in Section 6.6.3 of [33]. By introducing slack variables, the SDP (43) is rewritten with only equality constraints,

$$
\begin{array}{cl}
\max _{\mathbf{Y} \succeq \mathbf{0}, \lambda_{1} \geq 0, \lambda_{2} \geq 0, \lambda_{3} \geq 0} t_{1} \cdot t_{2} \cdot \operatorname{tr}\left(\mathbf{A}_{3} \mathbf{Y}\right) \\
\text { s.t. } & \operatorname{tr}\left(\mathbf{B}_{3} \mathbf{Y}\right)=1, \\
& \operatorname{tr}\left(\left(-\mathbf{A}_{1}+t_{1} \mathbf{B}_{1}\right) \mathbf{Y}\right)+\lambda_{1}=0,  \tag{84}\\
& \operatorname{tr}\left(\left(-\mathbf{A}_{2}+t_{2} \mathbf{B}_{2}\right) \mathbf{Y}\right)+\lambda_{2}=0, \\
& \operatorname{tr}\left(\left(\mathbf{B}_{4}-\mathbf{A}_{4}\right) \mathbf{Y}\right)+\lambda_{3}=0 .
\end{array}
$$

Let

$$
\begin{align*}
\tilde{\mathbf{Y}} & =\left[\begin{array}{cc}
\operatorname{diag}\{\boldsymbol{\lambda}\} & \mathbf{0}^{T} \\
\mathbf{0} & \mathbf{Y}
\end{array}\right], \\
\tilde{\mathbf{A}}_{1} & =\left[\begin{array}{cc}
\mathbf{0} & \mathbf{0}^{T} \\
\mathbf{0} & t_{1} t_{2} \mathbf{A}_{3}
\end{array}\right], \tilde{\mathbf{A}}_{2}=\left[\begin{array}{ll}
\mathbf{0} & \mathbf{0}^{T} \\
\mathbf{0} & \mathbf{B}_{3}
\end{array}\right], \\
\tilde{\mathbf{A}}_{3} & =\left[\begin{array}{cc}
\operatorname{diag}\left\{\mathbf{e}_{1}\right\} & \mathbf{0}^{T} \\
\mathbf{0} & -\mathbf{A}_{1}+t_{1} \mathbf{B}_{1}
\end{array}\right],  \tag{85}\\
\tilde{\mathbf{A}}_{4} & =\left[\begin{array}{cc}
\operatorname{diag}\left\{\mathbf{e}_{2}\right\} & \mathbf{0}^{T} \\
\mathbf{0} & -\mathbf{A}_{2}+t_{2} \mathbf{B}_{2}
\end{array}\right], \\
\tilde{\mathbf{A}}_{5} & =\left[\begin{array}{cc}
\operatorname{diag}\left\{\mathbf{e}_{3}\right\} & \mathbf{0}^{T} \\
\mathbf{0} & \mathbf{B}_{4}-\mathbf{A}_{4}
\end{array}\right]
\end{align*}
$$

where $\tilde{\mathbf{Y}}, \tilde{\mathbf{A}}_{i} \in \mathbb{C}^{\left(N^{2}+3\right) \times\left(N^{2}+3\right)}, \boldsymbol{\lambda}=\left[\lambda_{1}, \lambda_{2}, \lambda_{3}\right]^{T}$, and $\mathbf{e}_{i}$, $i \in\{1,2,3\}$, denotes a column vector with the $i$ th element being 1 and the others being 0 . The SDP (84) is expressed as

$$
\begin{align*}
\max _{\mathbf{Y}} \geq & \operatorname{tr}\left(\tilde{\mathbf{A}}_{1} \tilde{\mathbf{Y}}\right) \\
\text { s.t. } & \operatorname{tr}\left(\tilde{\mathbf{A}}_{2} \tilde{\mathbf{Y}}\right)=1, \operatorname{tr}\left(\tilde{\mathbf{A}}_{3} \tilde{\mathbf{Y}}\right)=0,  \tag{86}\\
& \operatorname{tr}\left(\tilde{\mathbf{A}}_{4} \tilde{\mathbf{Y}}\right)=0, \operatorname{tr}\left(\tilde{\mathbf{A}}_{5} \tilde{\mathbf{Y}}\right)=0
\end{align*}
$$

which is the dual problem to the following primal standard SDP problem [33],

$$
\begin{equation*}
\min _{\mu} \mu_{1} \text { s.t. }-\tilde{\mathbf{A}}_{1}+\sum_{m=1}^{4} \mu_{m} \tilde{\mathbf{A}}_{m+1} \succeq \mathbf{0} \tag{87}
\end{equation*}
$$

where $\boldsymbol{\mu}=\left[\mu_{1}, \mu_{2}, \mu_{3}, \mu_{4}\right]^{T}$. From Section 6.6.3 of [33], the SDP (87) with block-diagonal matrices is solved efficiently using the primal-dual interior-point method whose complexity is

$$
\begin{align*}
& \mathcal{O}\left(\left(1+\sum_{i=1}^{m_{\text {sdp }}} k_{i, \text { sdp }}\right)^{1 / 2}\right.  \tag{88}\\
& \left.\cdot\left(n_{\text {sdp }}^{3}+n_{\text {sdp }}^{2} \sum_{i=1}^{m_{\text {sdp }}} k_{i, \text { sdp }}^{2}+n_{\text {sdp }} \sum_{i=1}^{m_{\text {sdp }}} k_{i, \text { sdp }}^{3}\right) \cdot \log (1 / \epsilon)\right)
\end{align*}
$$

where $n_{\text {sdp }}$ is the number of equality constraints in (86), $m_{\text {sdp }}$ is the number of diagonal blocks in semidefinite cone, $k_{i, \text { sdp }}$ is the dimension of the $i$ th diagonal block in semidefinite cone, and $\epsilon$ is the computation accuracy. Comparing the SDP (87) in our paper with the standard form in Section 6.6 .3 of [33], we have $n_{\text {sdp }}=4, m_{\text {sdp }}=4, k_{1, \text { sdp }}=k_{2, \text { sdp }}=k_{3, \text { sdp }}=1$, and $k_{4}=N^{2}$. Therefore, the complexity of Algorithm 1 is

$$
\begin{equation*}
\mathcal{O}\left(L_{1} L_{2} \sqrt{N^{2}+4}\left(4 N^{6}+16 N^{4}+128\right) \cdot \log (1 / \epsilon)\right) . \tag{89}
\end{equation*}
$$

Similarly, the complexity of the proposed AN-aided secure relay beamforming scheme in Section IV is

$$
\begin{equation*}
\mathcal{O}\left(\tilde{L} \sqrt{N^{2}+2}\left(4 N^{6}+16 N^{4}+84\right) \cdot \log (1 / \epsilon)\right) \tag{90}
\end{equation*}
$$

where $\tilde{L}$ is the number of 1-D search over $\tau$.
For Algorithm 2, the computation burden is solving the convex problem (54). According to [34], problem (54) is a nonlinear SDP (NLSDP), which is an extension of the linear SDP and can be solved by the nonsmooth Newton's method [34]. During each iteration of the nonsmooth Newton's method, a nonsmooth equation for (54) is solved with the complexity
$\mathcal{O}\left(4 N^{7}+8 N^{5}+32 N+64\right)$ [34]. Since the algorithm in [34] to solve NLSDP has the same convergence rate with that to solve linear SDP, it needs at most $\mathcal{O}\left(\sqrt{N^{2}+4} \cdot \log (1 / \epsilon)\right)$ iterations. Thus, the complexity of Algorithm 2 is

$$
\begin{equation*}
\mathcal{O}\left(\hat{L} \sqrt{N^{2}+4}\left(4 N^{7}+8 N^{5}+32 N+64\right) \cdot \log (1 / \epsilon)\right) . \tag{91}
\end{equation*}
$$

where $\hat{L}$ is the number of iterations of the SPCA based iterative algorithm.

For Algorithm 3, the main computation burden is to compute the function (63). The computation of the generalized eigenvector of matrix pair $\left(\tilde{\mathbf{A}}_{1}+\zeta_{A} \tilde{\mathbf{A}}_{2}\right)$ and $\left(\tilde{\mathbf{B}}_{1}+\zeta_{B} \tilde{\mathbf{B}}_{2}\right)$ requires $\mathcal{O}\left(M^{3}\right)$ arithmetic operations. Thus, the complexity of Algorithm 3 is about

$$
\begin{equation*}
\mathcal{O}\left(2 M^{3} \cdot \log _{2} \frac{\rho_{A, u}-\rho_{A, l}}{\varepsilon}\right) . \tag{92}
\end{equation*}
$$

## VI. Simulation Results

In this section, we present the simulation results of our proposed algorithms for secure relay beamforming problems. We assume that in the two-way relay network, all the entries in the channel response vectors are i.i.d. complex Gaussian random variables with zero mean and unit variance. To solve the SDPs, we use the Matlab-based CVX optimization software [35]. In all simulations, the transmit power to noise power ratios of two sources, if not specified, are $P_{i} / \sigma^{2}=5 \mathrm{~dB}, i \in\{1,2\}$.

## A. Convergence Performance of SPCA Based Locally Optimal Solution

In Fig. 2, we present the average secrecy sum rate achieved by the SPCA based locally optimal solution versus the number of iterations for different transmit power constraints at the relay. The number of antennas at relay is $N=4$. The EH constraint is $Q=0.5 Q_{\max }$. The penalty factor $\kappa$ for Algorithm 2 is set as 0.01 and the initial point $\mathbf{X}^{(0)}$ is chosen by using $\hat{\mathbf{V}} \hat{\mathbf{x}}^{\dagger} \hat{\mathbf{V}}^{\dagger}$ where $\hat{\mathbf{V}}$ consists of the eigenvectors of matrix $\mathbf{B}_{4}-\mathbf{A}_{4}$ which are associated with the eigenvalues being no greater than zero and $\hat{\mathbf{x}}$ is a randomly generated vector. It is observed from Fig. 2 that the SPCA based locally optimal solution converges after about $3 \sim 4$ iterations under different transmit power constraints at the relay, $P_{r} / \sigma^{2}$.

In Fig. 3, we present the average secrecy sum rate achieved by the SPCA based locally optimal solution versus the number of iterations for different numbers of antennas at relay. The transmit power to noise power ratio of relay is $P_{r} / \sigma^{2}=20$ dB. The EH constraint is $Q=0.5 Q_{\max }$. It is found from Fig. 3 that after about 4 iterations, the stable average secrecy sum rate is achieved.

## B. Program Execution Time Comparison of Proposed Solutions When the Eavesdropper's CSI Is Available

In Table I, we present the program execution time comparison to obtain the proposed solutions for different number of antennas at relay, where the transmit power to noise power ratio of relay is $P_{r} / \sigma^{2}=20 \mathrm{~dB}$ and the EH constraint is $Q=0.5 Q_{\text {max }}$. The Central Processing Unit (CPU) is Intel


Fig. 2. Average secrecy sum rate versus the number of iterations; performance of proposed SPCA based locally optimal solution where the number of antennas at relay is $N=4$ and the EH constraint is $Q=0.5 Q_{\max }$.


Fig. 3. Average secrecy sum rate versus the number of iterations; performance of proposed SPCA based locally optimal solution where the transmit power to noise power ratio of relay is $P_{r} / \sigma^{2}=20 \mathrm{~dB}$ and the EH constraint is $Q=0.5 Q_{\max }$.

Core i7-4790K 4.0 GHz. The size of Random Access Memory (RAM) is 8 GB . The version of Matlab is R2014b and the version of employed SDPT3 solver in CVX is 4.0. From Table I , it is observed that to obtain the upper bound based solution requires the running time about $14 \sim 32$ times more than to obtain the SPCA based local optimal solution and about $3.5 \times 10^{3} \sim 3.3 \times 10^{4}$ times more than to obtain the ZF based suboptimal solution.

TABLE I
Program Execution Time Comparison to Obtain the Proposed Solutions

| Solutions | $N=2$ | $N=4$ | $N=6$ |
| :---: | :---: | :---: | :---: |
| Upper Bound Based Solution | 158.4 s | 191.7 s | 342.6 s |
| SPCA Based Local Optimal Solution | 4.812 s | 8.363 s | 23.49 s |
| ZF Based Suboptimal Solution | 0.0048 s | 0.015 s | 0.096 s |



Fig. 4. Average secrecy sum rate versus $P_{r} / \sigma^{2}$; performance comparison of different schemes when the eavesdropper's CSI is available where the EH constraint is $Q=0.5 Q_{\max }$ and the number of antennas at relay is $N=4$.

## C. Average Secrecy Sum Rate Comparison When the Eavesdropper's CSI Is Available

In Fig. 4, we present the average secrecy sum rate comparison of different secure relay beamforming schemes including the proposed upper bound based solution without Gaussian randomization (denoted as "Upper Bound" in the legend), the proposed upper bound based solution with Gaussian randomization (denoted as "UB-GR"), the proposed SPCA based locally optimal solution (denoted as "SPCA"), the ZF based suboptimal solution (denoted as "ZF"). The EH constraint is $Q=0.5 Q_{\max }$. The number of antennas at relay is $N=4$. In Fig. 4, the average secrecy sum rate without considering EH constraint (denoted as "No EH" in the legend) is also presented, which serves as the benchmark for our proposed solutions.

It is observed from Fig. 4 that the "UB-GR" scheme archives almost the same performance as the "Upper Bound" scheme. This is because the obtained upper bound based solution without Gaussian randomization is rank-one with probability close to one. In Table II, we present the probability of obtaining the rank-one solution for different values of $P_{r} / \sigma^{2}$. From Fig. 4, it is also found that the "SPCA" scheme performs close to the "UB-GR" scheme and has substantial performance improvement over the "ZF" scheme.

TABLE II
Probability of Obtaining Rank-One Solution

| $P_{r} / \sigma^{2}(\mathrm{~dB})$ | 5 | 10 | 15 | 20 | 25 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Probability | 0.997 | 0.998 | 0.999 | 0.999 | 1 |

Let $Q=\tau Q_{\max }$. In Fig. 5, we present the average secrecy sum rate comparison of different schemes for various EH constraints, i.e. $\tau$, where the transmit power to noise power ratio of relay is $P_{r} / \sigma^{2}=20 \mathrm{~dB}$ and the number of antennas at relay is $N=4$. From Fig. 5, it is found that with the increase of $\tau$, the average secrecy sum rates achieved by all the schemes decrease. When $\tau$ is small, the average secrecy


Fig. 5. Average secrecy sum rate versus $\tau$; performance comparison of different schemes when the eavesdropper's CSI is available where the transmit power to noise power ratio of relay is $P_{r} / \sigma^{2}=20 \mathrm{~dB}$ and the number of antennas at relay is $N=4$.
sum rates decrease slowly and as $\tau$ grows the average secrecy sum rates decrease fast. It is also found that the performances achieved by "Upper Bound", "UB-GR" and "SPCA" schemes are close, and the performance gap between "SPCA" scheme and "ZF" scheme becomes large with the increase of $\tau$.

In Fig. 6, we present the average secrecy sum rate comparison of different schemes for different number of antennas at relay where the transmit power to noise power ratio of relay is $P_{r} / \sigma^{2}=20 \mathrm{~dB}$ and the EH constraint is $Q=0.5 Q_{\max }$. It is observed from Fig. 6 that with the increase of $N$, the average secrecy sum rates of our proposed schemes increase. It is also found the "SPCA" scheme performs close to the "Upper Bound" and "UB-GR" schemes and outperforms the "ZF" scheme.

## D. Secrecy Outage Probability When the Eavesdropper's CSI Is Not Available

In Fig. 7, when the eavesdropper's CSI is not available at sources and relay, we present the secrecy outage probability of proposed AN-aided secure relay beamforming versus the transmit powers to noise power ratio of sources and relay, $P_{i} / \sigma^{2}=P_{r} / \sigma^{2}=P / \sigma^{2}, i \in\{1,2\}$, for different predefined threshold of achievable sum rate, $r_{d}$, where the target secrecy rate is $R_{s}^{\epsilon}=1.0 \mathrm{bps} / \mathrm{Hz}$, the number of antennas at relay is $N=4$, and the EH constraint is $Q=0.5 Q_{\max }$. The secrecy outage probabilities with and without considering the EH constraint are denoted as "EH" and "No EH" in the legend, respectively. From Fig. 7, it is observed that when $P / \sigma^{2}$ is higher than 20 dB , the secrecy outage probability increases with the increase of $P / \sigma^{2}$. This is because the eavesdropper is able to decode signals transmitted from two sources in the first phase when the transmit powers are large. From Fig. 7, it is also found that when $r_{d}$ is equal to $4.5 \mathrm{bps} / \mathrm{Hz}$ and 5.5 $\mathrm{bps} / \mathrm{Hz}$, respectively, the lowest secrecy outage probability is achieved when $P / \sigma^{2}$ is equal to 16 dB and 20 dB , respectively.


Fig. 6. Average secrecy sum rate versus $N$; performance comparison of different schemes when the eavesdropper's CSI is available where the transmit power to noise power ratio of relay is $P_{r} / \sigma^{2}=20 \mathrm{~dB}$ and the EH constraint is $Q=0.5 Q_{\text {max }}$.

In Fig. 8, we present the percentage of consumed AN power, $\eta$, of proposed AN-aided secure relay beamforming, where $\eta$ is defined as $\eta=(N-2) \sigma_{n}^{2} / P_{r}$, for different predefined threshold of achievable sum rate, $r_{d}$, where the target secrecy rate is $R_{s}^{\epsilon}=1.0 \mathrm{bps} / \mathrm{Hz}$, the number of antennas at relay is $N=4$, and the EH constraint is $Q=0.5 Q_{\max }$. From Fig. 8, it is observed that when $P / \sigma^{2}$ is lower than 10 dB , the percentage of consumed AN power is 0 . Combined with Fig. 7, it is also found that when $r_{d}$ is equal to $4.5 \mathrm{bps} / \mathrm{Hz}$ and $5.5 \mathrm{bps} / \mathrm{Hz}$, respectively, the percentage of consumed AN power is about $40 \%$ and $50 \%$ when the lowest secrecy outage probability is achieved.

## VII. Conclusions

In this paper, we have proposed upper bound base rank-one solution, SPCA based locally optimal solution, and ZF based suboptimal solution for secure relay beamforming with SWIPT in the AF two-way relay network when the eavesdropper's CSI is not available. Simulation results have shown that the upper bound based rank-one solution archives the performance almost the same as upper bound while has high computational complexity. The low-complexity SPCA based locally optimal solution performs close to upper bound. The ZF based suboptimal solution has the lowest computational complexity among proposed solutions. When the eavesdropper's CSI is not available, we propose the AN-aided secure relay beamforming scheme.

## Appendix A <br> Proof of Lemma 1

We prove Lemma 1 by reductio ad absurdum. Assume that $\mathbf{w}_{o}$ is the optimal solution to (33) such that $\mathbf{w}_{o}^{\dagger} \mathbf{C w} \mathbf{w}_{o}<$ $P_{r}$. Define $\overline{\mathbf{U}} \in \mathbb{C}^{N^{2} \times\left(N^{2}-3 N-2\right)}$ as a matrix which consists of $N^{2}-3 N-2$ singular vectors of the matrix $\left[\mathbf{R}_{1}^{T}, \mathbf{R}_{2}^{T}, \mathbf{R}_{c f}^{T}, \mathbf{R}_{c g}^{T}, \mathbf{R}_{c c}^{T}\right]^{T}$ which are associated with zero


Fig. 7. Secrecy outage probability versus $P / \sigma^{2}$; performance of proposed AN-aided secure relay beamforming when the eavesdropper's CSI is not available where the target secrecy rate is $R_{s}^{\epsilon}=1.0 \mathrm{bps} / \mathrm{Hz}$, the number of antennas at relay is $N=4$, and the EH constraint is $Q=0.5 Q_{\text {max }}$.


Fig. 8. Percentage of consumed AN power versus $P / \sigma^{2}$; performance of proposed AN-aided secure relay beamforming when the eavesdropper's CSI is not available where the target secrecy rate is $R_{s}^{\epsilon}=1.0 \mathrm{bps} / \mathrm{Hz}$, the number of antennas at relay is $N=4$, and the EH constraint is $Q=0.5 Q_{\text {max }}$.
singular values. Let $\Delta \mathbf{w}=\beta \overline{\mathbf{U}} \overline{\mathbf{U}}^{\dagger} \mathbf{w}_{o}$ where $\beta>0$. We have $\hat{\mathbf{w}}^{\dagger} \mathbf{C} \hat{\mathbf{w}}>\mathbf{w}_{o}^{\dagger} \mathbf{C w}{ }_{o}, \hat{\mathbf{w}}^{\dagger} \mathbf{D} \hat{\mathbf{w}}>\mathbf{w}_{o}^{\dagger} \mathbf{D} \mathbf{w}_{o}$, and

$$
\begin{align*}
1+\frac{\hat{\mathbf{w}}^{\dagger} \mathbf{Q}_{2,1} \hat{\mathbf{w}}}{\hat{\mathbf{w}}^{\dagger} \mathbf{R}_{1} \hat{\mathbf{w}}+\sigma^{2}} & >1+\frac{\mathbf{w}_{o}^{\dagger} \mathbf{Q}_{2,1} \mathbf{w}_{o}}{\mathbf{w}_{o}^{\dagger} \mathbf{R}_{1} \mathbf{w}_{o}+\sigma^{2}}  \tag{93}\\
1+\frac{\hat{\mathbf{w}}^{\dagger} \mathbf{Q}_{1,2} \hat{\mathbf{w}}}{\hat{\mathbf{w}}^{\dagger} \mathbf{R}_{2} \hat{\mathbf{w}}+\sigma^{2}} & >1+\frac{\mathbf{w}_{o}^{\dagger} \mathbf{Q}_{1,2} \mathbf{w}_{o}}{\mathbf{w}_{o}^{\dagger} \mathbf{R}_{2} \mathbf{w}_{o}+\sigma^{2}}  \tag{94}\\
\frac{\sigma^{4}\left(1+\hat{\mathbf{w}}^{\dagger} \mathbf{R}_{c c} \hat{\mathbf{w}}\right)}{\hat{\mathbf{w}}^{\dagger} \mathbf{Q}_{e} \hat{\mathbf{w}}+\alpha} & >\frac{\sigma^{4}\left(1+\mathbf{w}_{o}^{\dagger} \mathbf{R}_{c c} \mathbf{w}_{o}\right)}{\mathbf{w}_{o}^{\dagger} \mathbf{Q}_{e} \mathbf{w}_{o}+\alpha} \tag{95}
\end{align*}
$$

where $\hat{\mathbf{w}}=\mathbf{w}_{o}+\Delta \mathbf{w}$. Therefore, we can choose $\beta$ appropriately such that $\hat{\mathbf{w}}^{\dagger} \mathbf{C} \hat{\mathbf{w}}=P_{r}$. It is noted that $\hat{\mathbf{w}}$ which is feasible has larger objective value than $\mathbf{w}_{o}$. It is contradictory with the assumption that $\mathbf{w}_{o}$ is the optimal solution to (33).

## Appendix B Proof of Lemma 2

Define

$$
\begin{align*}
a_{1} & =\frac{\mathbf{w}^{\dagger}\left[\left(\mathbf{h}_{2, f} \mathbf{h}_{2, f}^{\dagger}\right) \otimes\left(\mathbf{h}_{1, b} \mathbf{h}_{1, b}^{\dagger}\right)\right]^{T} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{R}_{1} \mathbf{w}+\sigma^{2}}  \tag{96}\\
a_{2} & =\frac{\mathbf{w}^{\dagger}\left[\left(\mathbf{h}_{1, f} \mathbf{h}_{1, f}^{\dagger}\right) \otimes\left(\mathbf{h}_{2, b} \mathbf{h}_{2, b}^{\dagger}\right)\right]^{T} \mathbf{w}}{\mathbf{w}^{\dagger} \mathbf{R}_{2} \mathbf{w}+\sigma^{2}}  \tag{97}\\
b_{1} & =\sigma^{4}\left(1+\mathbf{w}^{\dagger} \mathbf{R}_{c c} \mathbf{w}\right)  \tag{98}\\
b_{2} & =\mathbf{w}^{\dagger} \mathbf{R}_{c f} \mathbf{w}, b_{3}=\mathbf{w}^{\dagger} \mathbf{R}_{c g} \mathbf{w}  \tag{99}\\
b_{4} & =\mathbf{w}^{\dagger}\left(f_{1} f_{2}^{*} \mathbf{a}_{c f}^{*} \mathbf{a}_{c g}^{T}\right) \mathbf{w}, b_{5}=\mathbf{w}^{\dagger}\left(f_{1}^{*} f_{2} \mathbf{a}_{c g}^{*} \mathbf{a}_{c f}^{T}\right) \mathbf{w} \tag{100}
\end{align*}
$$

With a fixed $P_{2}$, the objective in (33) can be rewritten as

$$
\begin{equation*}
\psi=\frac{c_{1}+c_{2} P_{1}}{c_{3}+c_{4} P_{1}} \tag{101}
\end{equation*}
$$

where

$$
\begin{align*}
c_{1}= & b_{1}+a_{1} b_{1}, c_{2}=a_{1} a_{2} b_{1} P_{2}+a_{2}  \tag{102}\\
c_{3}= & \sigma^{2} b_{3} P_{2}+\sigma^{2} b_{1}\left|f_{2}\right|^{2} P_{2}+1  \tag{103}\\
c_{4}= & \sigma^{2} b_{2}+b_{1}\left|f_{1}\right|^{2} / \sigma^{2} \\
& +\left(b_{2}\left|f_{1}\right|^{2}+b_{3}\left|f_{2}\right|^{2}-b_{4}-b_{5}\right) P_{2} \tag{104}
\end{align*}
$$

Taking the partial derivative of $\psi$ with respect to $P_{1}$, we obtain

$$
\begin{equation*}
\frac{\partial \psi}{\partial P_{1}}=\frac{c_{1} c_{4}-c_{3} c_{2}}{\left(c_{3}+c_{4} P_{1}\right)^{2}} \tag{105}
\end{equation*}
$$

When $c_{1} c_{4}-c_{3} c_{2}>0$, the function $\psi$ is monotonically increasing. Otherwise, $\psi$ is monotonically decreasing.

## Appendix C <br> Proof of Lemma 3

Suppose the optimal objective function values and optimal solutions of (42) and (43) are $\varphi_{i}, i \in\{1,2\}, \mathbf{X}^{o}$ and $\mathbf{Y}^{o}$, respectively. It is noted that $\mathbf{Y}^{o}$ is feasible for (42) and the objective value of (42) at $\mathbf{Y}^{o}$ is $\varphi_{2}$. Thus, $\varphi_{1} \geq \varphi_{2}$.

On the other hand, it can be verified that $\frac{\mathbf{X}^{o}}{\operatorname{tr}\left(\mathbf{B}_{3} \mathbf{X}^{o}\right)}$ is feasible for (43) and the objective value of (43) at $\frac{\mathbf{X}^{\circ}}{\operatorname{tr}\left(\mathbf{B}_{3} \mathbf{X}^{\circ}\right)}$ is $\varphi_{1}$. Therefore, $\varphi_{2} \geq \varphi_{1}$ which results in $\varphi_{1}=\varphi_{2}$.

## Appendix D <br> Proof of Lemma 4

Consider the case that $\left(\mathbf{A}_{4}-\mathbf{B}_{4}\right) \succ \mathbf{0}$. Suppose the optimal objective function value and optimal solution of (42) are $\varphi^{\circ}$ and $\left(\mathbf{Y}^{o}, t_{1}^{o}, t_{2}^{o}\right)$. We have

$$
\begin{equation*}
\operatorname{tr}\left(t_{1}^{o} t_{2}^{o} \mathbf{A}_{3} \mathbf{Y}^{o}\right)-\varphi^{o} \operatorname{tr}\left(\mathbf{B}_{3} \mathbf{Y}^{o}\right)=0 \tag{106}
\end{equation*}
$$

Since $\left(\mathbf{A}_{4}-\mathbf{B}_{4}\right) \succ \mathbf{0}$, we have

$$
\begin{align*}
& {\left[\operatorname{tr}\left(\left(t_{1}^{o} t_{2}^{o} \mathbf{A}_{3}-\varphi^{o} \mathbf{B}_{3}\right) \mathbf{Y}\right), \operatorname{tr}\left(\left(\mathbf{A}_{1}-t_{1}^{o} \mathbf{B}_{1}\right) \mathbf{Y}\right)\right.} \\
& \quad \operatorname{tr}\left(\left(\mathbf{A}_{2}-t_{2}^{o} \mathbf{B}_{2}\right) \mathbf{Y}\right), \operatorname{tr}\left(\left(\mathbf{B}_{4}-\mathbf{A}_{4}\right) \mathbf{Y}\right] \neq[0,0,0,0] \tag{107}
\end{align*}
$$

for any nonzero $\mathbf{Y} \succeq \mathbf{0}$. If $\operatorname{rank}\left(\mathbf{Y}^{o}\right) \geq 2$, according to Theorem 2.3 in [29], we can find a rank-one matrix $\mathbf{y y}^{\dagger}$ such that

$$
\begin{align*}
\operatorname{tr}\left(\left(t_{1}^{o} t_{2}^{o} \mathbf{A}_{3}-\varphi^{o} \mathbf{B}_{3}\right) \mathbf{y} \mathbf{y}^{\dagger}\right) & =\operatorname{tr}\left(\left(t_{1}^{o} t_{2}^{o} \mathbf{A}_{3}-\varphi^{o} \mathbf{B}_{3}\right) \mathbf{Y}^{o}\right)  \tag{108}\\
\operatorname{tr}\left(\left(\mathbf{A}_{1}-t_{1}^{o} \mathbf{B}_{1}\right) \mathbf{y} \mathbf{y}^{\dagger}\right) & =\operatorname{tr}\left(\left(\mathbf{A}_{1}-t_{1}^{o} \mathbf{B}_{1}\right) \mathbf{Y}^{o}\right),  \tag{109}\\
\operatorname{tr}\left(\left(\mathbf{A}_{2}-t_{2}^{o} \mathbf{B}_{2}\right) \mathbf{y} \mathbf{y}^{\dagger}\right) & =\operatorname{tr}\left(\left(\mathbf{A}_{2}-t_{2}^{o} \mathbf{B}_{2}\right) \mathbf{Y}^{o}\right)  \tag{110}\\
\operatorname{tr}\left(\left(\mathbf{B}_{4}-\mathbf{A}_{4}\right) \mathbf{y} \mathbf{y}^{\dagger}\right) & =\operatorname{tr}\left(\left(\mathbf{B}_{4}-\mathbf{A}_{4}\right) \mathbf{Y}^{o}\right) \tag{111}
\end{align*}
$$

Thus, problem (35) has an optimal rank-one solution $\mathbf{y y}^{\dagger}$.
For the cases that $\left(\mathbf{A}_{1}-t_{1}^{o} \mathbf{B}_{1}\right) \succ \mathbf{0}$ and $\left(\mathbf{A}_{2}-t_{2}^{o} \mathbf{B}_{2}\right) \succ \mathbf{0}$, using the similar method, we can prove that an optimal rankone solution for (35) can be constructed.

## Appendix E <br> Proof of Lemma 6

Assume that the optimal values of problems (50) and (51) are $\tau_{1}$ and $\tau_{2}$, respectively. Since each feasible point to problem (50) is also feasible to problem (51), we have $\tau_{1} \geq \tau_{2}$ and the optimal value of problem (51) is upper bounded by the optimal value of problem (50).

Next, we show that $\tau_{1} \leq \tau_{2}$. In fact, we just need to show there exists $0<\kappa_{0}<+\infty$ such that for $\kappa>\kappa_{0}$, any optimal solution given $\kappa$ to problem (51), denoted as $\mathbf{X}(\kappa)$ are also feasible to problem (50), i.e., $\mathbf{X}(\kappa)$ satisfies $\operatorname{tr}(\mathbf{X}(\kappa))-\lambda_{\max }(\mathbf{X}(\kappa))=0$. We prove this by reductio ad absurdum. Assume that there exists no $0<\kappa_{0}<+\infty$ such that for $\kappa>\kappa_{0}, \operatorname{tr}(\mathbf{X}(\kappa))-\lambda_{\max }(\mathbf{X}(\kappa))=0$. Since the feasible set of problem (51) is convex and compact, by taking a subsequence if necessary we can assume that $\mathbf{X}(\kappa) \rightarrow$ $\mathbf{X}(+\infty)$ as $\kappa \rightarrow+\infty$ with $\operatorname{tr}(\mathbf{X}(+\infty))-\lambda_{\max }(\mathbf{X}(+\infty))>0$. This means that $\kappa\left(\operatorname{tr}(\mathbf{X}(\kappa))-\lambda_{\max }(\mathbf{X}(\kappa))\right) \rightarrow+\infty$, which contradict that the optimal value of (51) is upper bounded by the optimal value of problem (50).

## References

[1] R. Zhang and C. K. Ho, "MIMO broadcasting for simultaneous wireless information and power transfer," IEEE Trans. Wireless Commun., vol. 12, no. 5, pp. 1989-2001, May 2013.
[2] C. Xing, N. Wang, J. Ni, Z. Fei, and J. Kuang, "MIMO beamforming designs with partial CSI under energy harvesting constraints," IEEE Signal Process. Lett., vol. 20, no. 4, pp. 363-366, Apr. 2013.
[3] Z. Xiang and M. Tao, "Robust beamforming for wireless information and power transmission," IEEE Wireless Commun. Lett., vol. 1, no. 4, pp. 372-375, Aug. 2012.
[4] Z. Ding, I. Krikidis, B. Sharif, and H. V. Poor, "Wireless information, and power transfer in cooperative networks with spatially random relays," IEEE Trans. Wireless Commun., vol. 13, no. 8, pp. 4440-4453, Aug. 2014.
[5] J. Xu, L. Liu and R. Zhang, "Multiuser MISO beamforming for simultaneous wireless information and power transfer," IEEE Trans. Signal Process., vol. 62, no. 18, pp. 4798-4810, Sept. 2014.
[6] Q. Shi, W. Xu, and D. Li, "Joint beamforming and power splitting for multi-user MISO SWIPT system," IEEE Trans. Wireless Commun., vol. 13, no. 6, pp. 3269-3280, June 2014.
[7] J. Park and B. Clerckx, "Joint wireless information and energy transfer in a two-user MIMO interference channel," IEEE Trans. Wireless Commun., vol. 12, no. 8, pp. 4210-4221, Aug. 2013.
[8] B. K. Chalise, W.-K. Ma, Y. D. Zhang, H. A. Suraweera, and M. G. Amin, "Optimum performance boundaries of OSTBC based AF-MIMO relay system with energy harvesting receiver," IEEE Trans. Signal Process., vol. 61, no. 17, pp. 4199-4213, Sept. 2013.
[9] Q. Li, Q. Zhang, and J. Qin, "Beamforming in non-regenerative twoway multi-antenna relay networks for simultaneous wireless information and power transfer," IEEE Trans. Wireless Commun., vol. 13, no. 10, pp. 5509-5520, Oct. 2014.
[10] K. Cumanan, Z. Ding, B. Sharif, G. Y. Tian, and K. K. Leung, "Secrecy rate optimizations for a MIMO secrecy channel with a multiple-antenna eavesdropper," IEEE Trans. Veh. Technol., vol. 63, no. 4, pp. 1678-1690, May 2014.
[11] Z. Chu, K. Cumanan, Z. Ding, M. Johnston, and S. Y. Le Goff, "Secrecy rate optimizations for a MIMO secrecy channel with a cooperative jammer," IEEE Trans. Veh. Technol., vol. 64, no. 5, pp. 1833-1847, May 2015.
[12] D. W. K. Ng, E. S. Lo, and R. Schober, "Robust beamforming for secure communication in systems with wireless information and power transfer," IEEE Trans. Wireless Commun., vol. 13, no. 8, pp. 4599-4615, Aug. 2014.
[13] L. Liu, R. Zhang, and K. C. Chua, "Secrecy wireless information and power transfer with MISO beamforming," IEEE Trans. Signal Process., vol. 62, no. 7, pp. 1850-1863, Apr. 2014.
[14] Q. Li, Q. Zhang, and J. Qin, "Secure relay beamforming for simultaneous wireless information and power transfer in non-regenerative relay networks," IEEE Trans. Veh. Technol., vol. 63, no. 5, pp. 2462-2467, Jun. 2014.
[15] H.-M. Wang, Q. Yin, and X.-G. Xia, "Distributed beamforming for physical-layer security of two-way relay networks," IEEE Trans. Signal Process., vol. 60, no. 7, pp. 3532-3545, Jul. 2012.
[16] Z. Ding, K. K. Leung, D. L. Goeckel, and D. Towsley, "On the application of cooperative transmission to secrecy communications," IEEE J. Select. Areas Commun., vol. 30, no. 2, pp. 359-368, Feb. 2012.
[17] Y. Yang, H Zhao, C Sun, and W. Weng, "Iterative algorithm for secrecy guarantee with null space beamforming in two-way relay networks," in Proc. IEEE WCNC, 2013, pp. 3420-3425.
[18] R. Horst and N. V. Thoai, "Dc programming: Overview", J. Optim. Theory Appl., vol. 103, no. 1, pp. 1-43, Oct. 1999.
[19] A. Beck, A. Ben-Tal, and L. Tetruashvili, "A sequential parametric convex approximation method with applications to nonconvex truss topology design problems," J. Global Optim., vol. 47, no. 1, pp. 29-51, 2010.
[20] P. K. Gopala, L. Lai, and H. El Gamal, "On the secrecy capacity of fading channels," IEEE Trans. Inf. Theory, vol. 54, no. 10, pp. 46864698, Oct. 2008.
[21] A. Chorti, S. M. Perlaza, Z. Han, and H. V. Poor, "On the resilience of wireless multiuser networks to passive and active eavesdroppers," IEEE J. Sel. Areas Commun., vol. 31, no. 9, pp. 1850-1863, Aug. 2012.
[22] X. Zhou, B. Maham, and A. Hjorungnes, "Pilot contamination for active eavesdropping," IEEE Trans. Wireless Commun., vol. 11, no. 3, pp. 903907, Mar. 2012.
[23] P. Janis, C.-H. Yu, K. Doppler, C. Ribeiro, C. Wijting, K. Hugl, O. Tirkkonen, and V. Koivunen, "Device-to-device communication underlaying cellular communications systems," Int. J. Communications, Network and System Sciences, doi:10.4236/ijens.2009.23019, pp. 169-247, 2009.
[24] A. Mukherjee and A. L. Swindlehurst, "Detecting passive eavesdroppers in the MIMO wiretap channel," in Proc. IEEE ICASSP, 2012, pp. 28092812.
[25] A. Charnes and W. W. Cooper, "Programming with linear fractional functionals," Naval Res. Logist. Quarter., vol. 9, pp. 181-186, 1962.
[26] S. Boyd and L. Vandenberghe, Convex Optimization. Cambridge, U.K.: Cambridge Univ. Press, 2004.
[27] Z.-Q. Luo, W.-K.Ma, A. M.-C. So, Y. Ye, and S. Zhang, "Semidefinite relaxation of quadratic optimization problems," IEEE Signal Process. Mag., vol. 27, no. 3, pp. 20-34, May 2010.
[28] Y. Huang and D. P. Palomar, "Rank-constrained separable semidefinite programming with applications to optimal beamforming," IEEE Trans. Signal Process., vol. 58, no. 2, pp. 664-678, Feb. 2010.
[29] W. Ai, Y. Huang, and S. Zhang, "New results on Hermitian matrix rankone decompostion," Mathematical Programming, vol. 128, pp. 253-283, 2011.
[30] S. Goel and R. Negi, "Guaranteeing secrecy using artificial noise," IEEE Trans. Wireless Commun., vol. 7, no. 6, pp. 2180-2189, Jun. 2008.
[31] S. Gerbracht, C. Scheunert, and E. A. Jorswieck, "Secrecy outage in MISO systems with patial channel state information," IEEE Trans. Inf. Forensics Security, vol. 7, no. 2, pp. 704-716, Apr. 2012.
[32] A. Khabbazibasmenj, F. Roemer, S. A. Vorobyov, and M. Haardt, " Sum-rate maximization in two-way AF MIMO relaying: polynomial time solutions to a class of DC programming problems," IEEE Trans. Signal Process., vol. 60, no. 10, pp. 5478-5493, Oct. 2012.
[33] A. Ben-Tal and A. Nemirovski, Lectures on Modern Convex Optimization: Analysis, Algorithms, and Engineering Applications, ser. MPSSIAM Series on Optimization, 2001.
[34] B. Benahmed and A. Alloun, "Semismooth reformulation and nonsmooth newton's method for solving nonlinear semidefinite programming," in Modelling, Computation and Optimization in Information Systems and Management Sciences, H.A. Le Thi et al. editors. Springer, 2015.
[35] M. Grant and S. Boyd, CVX: Matlab Software for Disciplined Convex Programming [Online]. Available: http://cvxr.com/cvx.


Quanzhong Li received the B.S. and Ph.D. degrees from Sun Yat-Sen University (SYSU), Guangzhou, China, in 2009 and 2014, respectively, both in information and communications engineering.

He is currently a Lecturer with the School of Data and Computer Science, SYSU, China. His research interests are in wireless communications powered by energy harvesting, cognitive radio, cooperative communications, and multiple-input-multiple-output (MIMO) communications.


Qi Zhang (S'04-M'11) received the B.Eng. (Hons.) and M.S. degrees from the University of Electronic Science and Technology of China (UESTC), Chengdu, Sichuan, China, in 1999 and 2002, respectively. He received the Ph.D. degree in Electrical and Computer Engineering from the National University of Singapore (NUS), Singapore, in 2007.

He is currently an Associate Professor with the School of Electronics and Information Technology, Sun Yat-Sen University, China. From 2007 to 2008, he was a Research Fellow in the Communications Lab, Department of Electrical and Computer Engineering, NUS. From 2008 to 2011, he was at the Center for Integrated Electronics, Shenzhen Institutes of Advanced Technology, Chinese Academy of Sciences and The Chinese University of Hong Kong. His research interests are in wireless communications powered by energy harvesting, cooperative communications, ultra-wideband (UWB) communications.


Jiayin Qin received the M.S. degree in radio physics from Huazhong Normal University, China, in 1992 and the Ph.D. degree in Electronics from Sun Yat-Sen University (SYSU), Guangzhou, China, in 1997.

He is currently a Professor with the School of Electronics and Information Technology, Sun YatSen University, China. From 2002 to 2004, he was the Head of the Department of Electronics and Communication Engineering, SYSU, China. From 2003 to 2008, he was the Vice Dean of the School of Information Science and Technology, SYSU, China. His research areas include wireless communication and submillimeter wave technology.

Dr. Qin is the recipient of the IEEE Communications Society Heinrich Hertz Award for Best Communications Letter in 2014, the Second Young Teacher Award of Higher Education Institutions, Ministry of Education (MOE), China in 2001, the Seventh Science and Technology Award for Chinese Youth in 2001, the New Century Excellent Talent, MOE, China in 1999.


[^0]:    Copyright (c) 2015 IEEE. Personal use of this material is permitted. However, permission to use this material for any other purposes must be obtained from the IEEE by sending a request to pubs-permissions @ieee.org.

    This work was supported in part by the National Natural Science Foundation of China under Grant 61472458, Grant 61202498, and Grant 61173148, in part by the Guangdong Natural Science Foundation under Grant 2014A030311032, Grant 2014A030313111, and Grant 2014A030310374, in part by the Fundamental Research Funds for the Central Universities under Grant $15 \operatorname{lgzd1} 10$ and Grant $15 \lg p y 15$.
    Q. Li is with the School of Data and Computer Science, Sun YatSen University, Guangzhou 510006, Guangdong, China, and also with Collaborative Innovation Center of High Performance Computing, National University of Defense Technology, Changsha, Hunan, China (email: liquanzhong2009@gmail.com). Q. Zhang and J. Qin are with the School of Electronics and Information Technology, Sun Yat-Sen University, Guangzhou 510006, Guangdong, China (zhqi26@ mail.sysu.edu.cn; issqjy@mail.sysu.edu.cn).

