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# The *p*-center flow-refueling facility location problem

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ABSTRACT

The *p*-center flow-refueling facility location problem locates *p* refueling facilities to minimize the maximum percentage deviation of all drivers. It is a social equity resource allocation model as opposed to a social efficiency *p*-maximum coverage flow-refueling model. We propose a nonlinear integer program based on link formulation and analyze its relationship with *p*-maximum coverage and set covering location flow-refueling problems. We develop a link-based implicit enumeration algorithm with an embedded vehicle range-constrained shortest path subproblem to optimally solve the problem. The computational results show that multiple optimal solutions may exist but that they are associated with different total trip distances. In addition, the maximum deviation may not decrease as the vehicle range or the number of refueling facilities marginally increases.



## 1. Introduction

The transportation sector is one of the largest contributors to greenhouse gas (GHG) emissions, accounting for 27% of all GHG emissions in the US in 2011. Within the sector, cars are by far the greatest contributors, generating 43% of all emissions (US EPA, 2013). Due to the negative environmental impacts of gasoline-powered vehicles, the development of alternative-fuel vehicles (AFVs) powered by liquid biofuels, hydrogen and/or electricity has become a priority for governments around the world to help ensure a healthy environment. However, some AFV drivers must fuel their vehicles at alternative-fuel refueling facilities (AFFs) to complete trips. Thus, the development of an equitable and/or cost-effective refueling infrastructure is essential for establishing consumer confidence in the deployment of AFVs.

AFFs provide services required by drivers (the terms 'user,' 'driver' and 'traveler' are used interchangeably herein) to complete trips (the terms 'trip' and 'journey' are used interchangeably). Flow-refueling facility location models design an infrastructure to successfully refuel drivers' AFVs and ensure the completion of covered trips, and they are *flow-based* refueling facility location models, which differ from *point-based* models. Point-based refueling facility location models design an infrastructure that guarantees that drivers can reach a refueling station within a minimum travel time (Stephens-Romero et al., 2010). These *flow-refueling* problems are analyzed in three primary research streams: *p*-maximum coverage, set (complete) covering and tour-based refueling facility location models. In the design of the flow-refueling facility infrastructure, *two* opposing social objectives may be considered: social efficiency and social equity. Models of *p-maximum coverage flow-refueling with a maximum allowable deviation* locate *p* refueling facilities to maximize the total number of trips completed within a *predetermined* maximum allowable deviation. Of course, when zero deviation is imposed, all drivers travel along their shortest paths (Kuby and Lim, 2005). The *p*-maximum coverage flow-refueling models are *system optimization* (e.g., *social*

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*efficiency*) models. These models maximize the travel flows of a system as a whole, though some vehicles may not be able to complete their trips when feasible refueling paths exceed the maximum allowable deviation.

In contrast, *social equity* optimization models explicitly consider levels of fairness among users. The goal of these models is to allocate resources such that *all* drivers are indiscriminately served by the system and the *worst* deviation allowable is equally applied to each driver. *Set covering* flow-refueling models minimize the total investment cost of refueling facilities while addressing the refueling needs of *all* drivers so that they can complete their trips. Certain set covering flow-refueling models prohibit deviations, thus requiring that all drivers travel along their *shortest* paths (MirHassani and Ebrazi, 2013; Wang and Lin, 2009; 2013; Wang and Wang, 2010). These models are the most desirable social equity models but are applied only in extreme cases, as they potentially require considerable governmental/industrial investment. In this paper, we propose an alternative means of addressing such social equity issues regarding flow-refueling infrastructure design by examining the *p-center flow-refueling facility location* problem. We locate *p* refueling facilities so that the occurrence of the maximum allowable deviation (i.e., the *worst* detour for all drivers in the system) is minimized. The number of *p* refueling facilities can be treated as a surrogate for budget investment limitations.

The remainder of this paper is organized as follows. In Section 2, we review *three* primary streams of relevant research on flow-refueling facility location models, which include *p*-maximum coverage, set covering and tour-based facility location models. Section 3 presents mathematical models for the *p*-center flow-refueling facility location problem applied to an uncapacitated directed network. Two exclusive but related *p*-center flow-refueling models—symmetric trips in a symmetric network and single-direction trips in symmetric or asymmetric networks—are presented. Both models are developed as nonlinear integer programs by link formulation. We analyze the relationship among *p*-center, *p*-maximum coverage and set covering flow-refueling problems. Section 4 presents the algorithmic design. We propose an exact solution for optimally solving the proposed problem without the explicit enumeration of paths. This solution method involves a *link-based* approach and determines travel paths as needed. The problem employs a bi-level structure, *p*-infrastructure design and path determination. Through the determination of the locations of refueling facilities, the problem is simplified to a *vehicle-range-constrained routing* subproblem (VRCRP). We develop a nondominated labeling shortest path algorithm for this subproblem. This algorithm is embedded in the implicit enumeration algorithm branched on the zero-one facility decision variable in a search tree. To improve computational efficiency, we develop a *proliferate* algorithm to determine the associated lower bound. In Section 5, we present numerical experiments on a 25-node network (Hodgson, 1990) and its expanded 35-node network to examine the computational efficiency of the proposed solution method. We analyze optimal solutions and compare them to those of set-covering flow-refueling modeling. A sensitivity analysis of the impact of vehicle range on the number of refueling facilities required is also conducted. A summary of our study is provided in Section 6.

## 2. Literature review

In the literature, conventional location-allocation models simultaneously locate and allocate demand to a set of service facilities in a manner that minimizes total cost. Studies on these models assume that all journeys involve direct travel between demand nodes and facilities. Accordingly, individuals who require services travel directly from their locations to service facilities, or conversely, a service provider will directly deliver its services from its location to the customer's location. These models assign customer locations and fulfill customer demands through the delivery of service facilities. The models have been used to plan public facilities, warehouses, emergency facilities, etc.

However, certain forms of demand that emerge during travel are dependent on the journey. These demands are addressed by facilities positioned along travel paths. Such facilities include vehicle refueling facilities, which have two unique characteristics (Kelly and Kuby, 2013; Kuby et al., 2013) that cannot be addressed by conventional location models based on an assumption of *node-based* demand. First, individuals do not typically make special trips from their homes to refueling stations and then return home immediately afterward. Vehicle refueling demands thus generally take the form of traffic flows that pass refueling facilities on the way to other destinations. Second, AFVs with a limited driving range must be refueled multiple times to complete longer trips and/or must deviate from typical paths for refueling purposes when necessary.

Flow-refueling facility location models study *flow-based* facility location problems. Three primary streams of research, *p*-maximum coverage, set covering and tour-based flow-refueling models, have focused on this subject. Notably, we present only a subset of the related research and their classifications, which are tabulated in Table 1.

The *first* stream examines *p-maximum coverage* flow-refueling models that design an infrastructure that is confined to a specific number of *p* refueling facilities to maximize the total travel flow. These models examine social efficiency problems. This modeling approach was inspired by Hodgson (1990), who developed the *uncapacitated flow-capturing location model*. The objective of such models is to maximize the total flow volume passing by any *one* facility positioned on a node along a driver's *shortest* paths in the network. However, AFVs with limited range may need to make *multiple stops* to complete journeys. Thus, Kuby and Lim (2005) and Lim and Kuby (2010) developed the *uncapacitated flow-refueling location model* (u-FRLM) designed specifically for siting AFFs. This model optimally locates a set of *p* refueling facilities in a network and considers opportunities to refuel AFVs multiple times to maximize the total flow volume of *fuel-feasible* paths.

Other researchers have developed reformulating mathematical models. Capar and Kuby (2012) proposed the 'node cover-path cover (NC-PC)' formulation, and Capar et al. (2013) developed the 'arc cover-path cover (AC-PC)' formulation. The NC-PC formulation identifies a set of candidate facility locations that renders a node passible, while the AC-PC formulation identifies a set of candidate facility locations that renders a directed arc travelable. Thus, when each node/directed arc along

**Table 1**
A subset of selected research and their classifications.

| Flow-refueling models | Aim | Time period | Key characteristics: Path/tour | Key characteristics: Refueling station | Research |
|---|---|---|---|---|---|
| *p*-maximum coverage | Social efficiency/system optimization | Single | Shortest/predetermined single path | Uncapacitated | Kuby and Lim (2005) |
| | | | | | Lim and Kuby (2010) |
| | | | | | Capar and Kuby (2012) |
| | | | | | Capar et al. (2013) |
| | | | | | Arslan and Karaşan (2016) |
| | | | | Capacitated | Upchurch et al. (2009) |
| | | Multiple | | | Zhang et al. (2017) |
| | | Single | Predetermined $\varepsilon$ deviation | Uncapacitated | Kim and Kuby (2012) |
| | | | | | Kim and Kuby (2013) |
| | | | | | Tran et al. (2018) |
| Set covering | Social equity | Single | Shortest path | Uncapacitated | Wang and Lin (2009) MirHassani and Ebrazi (2013) |
| | | | | | Wang and Wang (2010) |
| | | | | | Wang and Lin (2013) |
| *p*-center | | | Determine the maximum deviation $\varepsilon$ | | *This research* |
| Tour interception (Activity-based) | Fulfilling daily activity | Single | Actual/resequencing | Uncapacitated | Kang and Recker (2009) |
| | | | | | Kang and Recker (2014) |
| | | | | | Dong et al. (2014) |
| | | | Predetermined $\varepsilon$ deviation | | Kang and Recker (2015) |

a path is passible/travelable, a trip is refuel-feasible. Although both formulations assume that paths for all OD pairs must be known in advance, they do not require the computationally intensive enumeration of facility combinations along each path, thus resulting in a reduced number of decision variables and constraints and potentially solving broader problems than previous formulations.

Arslan and Karaşan (2016) extended the flow-refueling location model by considering both electric and plug-in hybrid electric vehicles. They proposed a generalization model that maximizes vehicle miles traveled using electricity (and thereby minimizes total transportation cost) and presented an exact solution methodology, called the Benders decomposition algorithm. The computation showed that the approach may effectively handle instances of realistic size. Upchurch et al. (2009), in contrast, studied the *capacitated flow-refueling location model* (c-FRLM) to determine an infrastructure of *p* refueling facilities such that the total flow volume is maximized while meeting multiple refueling needs and refueling facility capacity requirements. Zhang et al. (2017) extended a single-period c-FRLM to a multiple-period c-FRLM for electric vehicles. In addition, they also studied an alternative objective function for maximizing electric vehicle demand.

Berman et al. (1995) extended the flow-capturing location model to consider the fact that customers may deviate from their shortest paths to visit a service facility. One of the objectives of this new model is to minimize expected inconveniences as measured by the deviation distance. Similarly, Kim and Kuby (2012) provided a deviated version of the FRLM to consider the fact that drivers may make necessary deviations from their shortest paths to travel to fueling stations when a refueling station network is sparse. The authors used the *k*-shortest path algorithm to generate potential deviation paths for all OD pairs until the deviation distance of these paths reaches an upper limit. The quality of this solution is closely correlated with the upper limit or the number of predetermined refuel-feasible paths. Furthermore, it is fairly burdensome and computationally time consuming to enumerate all potential deviation paths and potential combinations of refueling facilities that cover each path for large-scale problems. Kim and Kuby (2013) proposed a heuristic algorithm for managing large-scale networks. Their bi-level method applies a greedy approach to the determination of facility infrastructures and an embedded shortest refuel-feasible path subproblem. The heuristic algorithm is a linked-based approach that implicitly determines paths when needed without the predetermination of refuel-feasible paths. Tran et al. (2018) developed an efficient heuristic algorithm based on solving a sequence of subproblems restricted to a set of promising station candidates, some of which are fixed as the most promising station locations to improve computation time. The experimental results show that the algorithm outperforms the genetic and greedy algorithms. However, the solution to those heuristic algorithms is a local optimum.

The *second* stream of flow-refueling station location models is the *set covering* models, which minimize the total investment cost of refueling facilities and determine their locations to ensure that all AFVs have enough fuel to travel along their *shortest* paths from origins to destinations in a network. These models examine social equity problems to ensure that no deviations occur in any AFV. Wang and Lin (2009) used such variables to track AFV fuel levels to ensure that AFVs are supplied with enough fuel at each intermediate node to complete journeys on their shortest paths. To more efficiently address this issue, MirHassani and Ebrazi (2013) expanded the network by adding virtual nodes and refuel-feasible-directed links to the shortest paths. They formulated a mixed integer linear programming problem for this expanded network. The computational results for a 192-node network with 3160 origin–destination (OD) pairs show that this approach can be used to obtain an optimal solution efficiently. Wang and Wang (2010) extended the set covering problem to simultaneously consider nodal population demands such that the total coverage of population demand is maximized while path demands are satisfied. Later, Wang and Lin (2013) extended the set covering problem to study the problem of the deployment of multiple types of recharging stations (i.e., slow-, fast-, and battery-exchange stations) for refueling electric vehicles.

The *third* stream of flow-based models includes the *tour interception location models*. These models are also referred to *activity-based* models (Kang and Recker, 2009; Dong et al., 2014) and determine an infrastructure of refueling facilities that captures travelers' *tours/itineraries* while allowing for itinerary rescheduling or activity resequencing during trip completion. Kang and Recker (2009) used 1-day travel diaries drawn from 2000 to 2001 California state household travel survey to study the potential impacts of electric vehicles on energy profiles. Dong et al. (2014) used GPS-based travel survey data to estimate drivers' travel activities, origins, destinations, travel distances, dwell times and other travel patterns. Under the assumption that travelers' daily travel scheduling/sequencing remains *unchanged*, this research determines the locations and types of chargers used within maximum allowable budgets to minimize the number of missed trips. Based on a limited refueling infrastructure, Kang and Recker (2014) measured the inconveniences suffered by individuals who switch from using internal combustion engine vehicles to using AFVs. The authors assumed that individuals may rearrange their travel sequencing habits to perform identified activities. Issues of travel with allowable deviations were considered by Kang and Recker (2015). They studied the hydrogen refueling facility location model, which couples a location strategy for the set covering problem with a routing and scheduling strategy for the household activity pattern problem. The model determines a refueling infrastructure such that each household's travel disutility is bounded by a *predetermined* maximum tolerable inconvenience value for trips involving refueling only once. Jung et al. (2014) considered stochastic demand that occur dynamically over time. Their model determines the locations and the number of servers at each location for a maximum of *p* servers to minimize the total travel times and average queue delays at each server for all travelers.

The *p-maximum coverage* flow-refueling location models are *system optimization* (i.e., *social efficiency*) models. These models identify the maximum efficiency of a system regardless of the impacts on drivers who are unable to refuel to complete journeys in a system. In contrast, *set covering* flow-refueling models are social equity models that minimize total investments made while allowing all travelers to travel along *zero* or *predetermined* deviation-allowable paths. Such models may set an unrealistic maximum deviation for travelers to tolerate or potentially require considerable governmental/industrial investment. In the present study, we examine *p-center flow-refueling facility location* problems that locate *p* refueling facilities such that, for the maximum percentage deviation of all drivers, the occurrence of the worst detour for any driver in a system is minimized. We develop a social equity optimization model that explicitly considers levels of fairness among AFVs. The proposed approach may illuminate how limited budgets may impact travel equity or the maximum allowable deviation in driver paths.

## 3. Mathematical model

In this study, we define the network examined as an *uncapacitated directed* network $G = (N, A)$, where $N$ is a set of nodes including the origins and destinations of all OD pairs with element $i, j \in N$ and $A$ is the set of *directed* links with element $ij \in A$ with the direction running from (tail) node $i$ to (head) node $j$. Some/all origins and destinations may be designated candidate refueling facility locations with element $s \in S$. In addition, we assume that the service capacity of each facility is unlimited to form an uncapacitated network. Key definitions and assumptions are as follows.

(1) *Two types of networks*.
  a. *Symmetric network*. A network that, for any pair of nodes, includes a pair of links in opposing directions. In addition, their attributes (distance and fuel consumption) are identical.
  b. *Asymmetric network*. A network that consists of a single link for a pair or for some/all pairs of nodes, or a pair of links in opposing directions for a pair or for some/all pairs of nodes wherein their attributes are different, or a combination of the above.

(2) *Two types of trips*.
  a. *Single-direction trips*. For such trips, drivers depart from their origins and arrive at their destinations. AFVs have an initial fuel level and a nonzero fuel level at any node in their journals. Vehicles may refuel along the way as needed.
  b. *Symmetric round trips*. For such trips, drivers travel along their *forward* (directed) paths from their origins to their destinations and return along their *reverse* (directed) paths, which are identical to their forward paths but are oriented in the opposite direction, and arrive at the locations in the reverse sequence. To ensure the completion of round trips, drivers require half the fuel capacity upon departing from their origins (Kuby and Lim, 2005) and the same amount of fuel upon arriving at their destinations (MirHassani and Ebrazi, 2013).

(3) *Vehicle range feasible paths* (VRFPs) are directed paths from origins to destinations in which drivers have enough fuel or can refuel to complete their journeys. However, we assume that all drivers are rational and thus always choose to travel along the shortest paths, namely, via *vehicle range shortest feasible paths* (VRSPs).

(4) We define the *absolute shortest path* (ASP) of an OD pair as the *shortest distance* from an origin to a destination assuming that the driver has an *unlimited* fuel capacity.
(5) The percentage of deviation for a driver is the percentage increase in distance from the distance to the ASP to the distance to the VRSP in a $p$-infrastructure.

### 3.1. Notations

The notations used throughout this study are defined as follows.

A. Sets and parameters

$O$: The set of origin nodes with generic $o \in O$.
$D$: The set of destination nodes with generic $d \in D$.
$S$: The set of candidate refueling facility nodes with generic $s \in S$.
$N$: The set of nodes $N = \{O \cup D \cup S\} \backslash \{\{O \cap S\}, \{D \cap S\}\}$ with generic $i/j \in N$.
$A$: The set of directed arcs with generic element $ij \in A$.
$V$: Vehicle full fuel tank capacity.
$d_{ij}$: The distance from node $i$ to node $j$.
$\hat{e}_o^{od}$: The initial fuel level at trip origin $o$ for O-D pair $od \in OD$
$f_{ij}$: Fuel consumed after traveling from node $i$ to node $j$.
$\bar{z}^{od}$: The absolute shortest distance for O-D pair$od \in OD$
$B$: A large value.

B. Decision variables

$e_i^{od,f}$: Fuel remaining with arrival at node $i$ on a forward path of O-D pair $od$. Note that the *superscript f* is used to denote the forward trip.
$y_i$: 1 when a refueling facility is located at node $i, i \in S$and$=0$ otherwise.
$x_{ij}^{od}$: 1 when a driver travels on directed link $ij$ for OD pair $od$ and$=0$ otherwise.
$r^{od}$: The percentage of deviation between VRSP and ASP for OD pair $od$.

### 3.2. The p-center flow-refueling problem in an uncapacitated directed network

The $p$-center flow-refueling facility location problem in an uncapacitated directed network involves selecting a set of $p$ facilities from the set of candidate refueling sites such that the maximum of all drivers' percentage deviations is minimized. We study the $p$-center problem for the following two cases: symmetric round trips taken in a symmetric network and single-direction trips taken in symmetric or asymmetric networks.

#### 3.2.1. Symmetric round trips occurring in a symmetric network

The mathematical formulation for symmetric round trips in a symmetric network is as follows:

$$Minimize\ \varepsilon = \max\left\{ r^{od} = \frac{\left(\sum_{ij} d_{ij} x_{ij}^{od} - \bar{z}^{od}\right)}{\bar{z}^{od}}, \forall od \in M \right\} \tag{1}$$

Subject to:

$$\sum_{i \in S} y_i = p \tag{2}$$

$$\sum_{j \in N} x_{ij}^{od} - \sum_{j \in N} x_{ji}^{od} = \begin{cases} 1, i = o; \\ -1, i = d; \\ 0, o/w; \end{cases} \quad \forall i \in N, od \in M \tag{3}$$

$$e_j^{od,f} \leq \left(e_i^{od,f}(1 - y_i) + V y_i - f_{ij}\right)\left(x_{ij}^{od}\right) + B\left(1 - x_{ij}^{od}\right) \quad \forall ij \in A, od \in M \tag{4}$$

$$\begin{cases} e_o^{od,f} = \hat{e}_o^{od} \geq 0.5V \\ 0 \leq e_j^{od,f} \leq V \forall j \in N \backslash \{o, d\} \quad \forall od \in M \\ e_d^{od,f} \geq 0.5V \end{cases} \tag{5}$$

$$\left(x_{ij}^{od}, y_i\right) \in \{0, 1\}; \quad \forall i \in N, ij \in A, od \in M \tag{6}$$

The objective function minimizes the maximum percentage deviation of all drivers. Constraint (2) states that there is an exact number of $p$ refueling facilities to be allocated within the network. Constraints (3), the set of flow conservation constraints, state that each driver travels on his/her *forward* VRSP. This path includes links valued at 1. Constraints (4) track the fuel level at each node on a forward path. Note that these constraints are a set of *either-or* formulations. Let us observe the following cases for Constraints (4). (1) For $y_i = 1, x_{ij}^{od} = 1$, the constraint becomes $e_j^{od,f} \le V - f_{ij}$, or the amount of fuel remaining upon arrival at node *j*. (2) For $y_i = 0, x_{ij}^{od} = 1$, the constraint becomes $e_j^{od,f} \le e_i^{od,f} - f_{ij}$. Finally, (3) for cases $y_i = 1, x_{ij}^{od} = 0$ and $y_i = 0, x_{ij}^{od} = 0$, as link *ij* is not located on the path, the constraint becomes $e_j^{od,f} \le B$ and is thus *lifted*. Thus, Constraints (4) ensure that enough fuel is available for the completion of a journey along link *ij*. In addition, fuel remaining upon arrival at node *j* cannot exceed the level of consumption required to travel along link *ij*. Of course, the vehicle is fully refueled to its capacity when it departs a refueling facility at node $i, y_i = 1$. Constraints (5) state that the fuel level at any intermediate node of all directed forward paths is a non-negative decision variable and is bounded by the full capacity. In addition, the initial fuel level of AFVs of drivers at their respective origin nodes and destinations must be at least half capacity. All decision variables, links on VRSPs and facility location decision variables are binary, as stated in Constraints (6).

#### 3.2.2. Single-direction trips occurring in asymmetric or symmetric networks

Mathematical formulations (1)–(6) apply to symmetric round trips. These symmetric assumptions improve computational efficiency, as the model needs to determine only a single forward path for all OD trips. Suppose that all trips are single-direction trips in *asymmetric* or *symmetric* networks. No half-capacity fuel levels are required for origins or destinations. Constraints (5) can simply be replaced with the following:

$$\begin{cases} e_o^{od,f} = \hat{e}_o^{od} \ge 0 \\ 0 \le e_j^{od,f} \le V \forall j \in N \backslash \{o\} \end{cases} \quad \forall od \in M \tag{5a}$$

The amount of fuel required depends on the path traveled and is sufficient as long as AFVs have enough fuel to complete their journeys.

### 3.3. The relationship with flow-based models

In this section, we analyze the relationship among the *p*-center, *p*-maximum coverage and set covering flow-refueling facility location models.

#### 3.3.1. *p*-maximum coverage flow-refueling facility problems

The *p*-center flow-refueling facility location problem determines the *lowest* percentage deviation under which *all* flows in a system can be served by a *p*-infrastructure. Thus, we have the following:

**Lemma 1.** *Denote $\varepsilon^*$as the optimal solution to the p-center flow-refueling facility location problem. In turn, $\varepsilon^*$ is the lowest predetermined percentage deviation that guarantees that all flows will be served under the p-maximum coverage flow-refueling facility location problem.*

**Proof.** Denote $\{(r^{od})^*, od \in OD\}$ as the optimal percentage deviation of OD pairs of the *p*-center flow-refueling facility location problem. When $\varepsilon^*$ is the optimal solution to the *p*-center flow-refueling facility location problem, $\{y_s^* \in \{0,1\}, \forall s \in S : \sum_s y_s^* = p\}$; this condition implies the following: $\varepsilon^* \ge \max\{(r^{od})^*, \forall od \in OD\}$. Thus, $\{(r^{od})^*, od \in OD\}$ is a feasible solution to the *p*-maximum coverage flow-refueling facility location problem with a predetermined maximum allowable deviation of $\varepsilon^*$. Now, assume the opposite, where there is another *p*-infrastructure $\{y_{\bar{s}} \in \{0,1\}, \forall \bar{s} \in S : \sum_{\bar{s}} y_{\bar{s}} = p\}$ with a set of refuel-feasible paths that satisfies condition $\varepsilon^* > \max\{\bar{r}^{od}, \forall od \in OD\}$ for the *p*-maximum coverage flow-refueling facility location problem. Because $\{\bar{r}^{od}, \forall od \in OD\}$ is a feasible solution to the *p*-center flow-refueling facility location problem, $\varepsilon^*$ cannot be the optimal solution to the *p*-center flow-refueling facility location problem. A contradiction is thus established. □

Of course, pursuing any ambitious goal to predetermine a lower (i.e., smaller than $\varepsilon^*$) maximum percentage deviation may indicate only that there is a trip flow that cannot be served by any such *p*-infrastructure.

**Corollary 2.** *If $\varepsilon^*$ is the optimal solution to the p-center flow-refueling facility location problem with a predetermined number of refueling facilities p, no p-infrastructures of the p-maximum coverage* flow-refueling facility location *problem can serve all OD flows by a predetermined maximum deviation of* $\{\varepsilon : 0 \le \varepsilon < \varepsilon^*\}$.

**Proof.** Again, there is a set of refuel-feasible paths in the *p*-maximum coverage flow-refueling facility location problem that satisfies condition $\varepsilon^* > \varepsilon = \max\{\bar{r}^{od}, \forall od \in OD\}$. $\{\bar{r}^{od}, \forall od \in OD\}$ is a feasible solution to the *p*-center flow-refueling facility location problem, implying that $\varepsilon^*$ cannot be the optimal solution to the *p*-center flow-refueling facility location problem. A contradiction is thus established. □

#### 3.3.2. Set covering flow-refueling facility problems

The $p$-center flow-refueling facility location problem determines the maximum percentage deviation for a predetermined $p$ number of refueling facilities. In contrast, the set covering flow-refueling facility location problem determines the minimum number of refueling facilities for a predetermined maximum percentage deviation. Thus, there is a strong relationship between the two problems, which can be stated as the following lemma:

**Lemma 3.** *Suppose $\varepsilon^*$ is the optimal solution to the $p^*$-center flow-refueling facility location problem with a predetermined number of refueling facilities $p^*$ and that $s^*$ is the optimal number of refueling facilities for the set covering flow-refueling facility location problem with a predetermined maximum deviation of $\varepsilon^*$. We in turn have the following relationship: $s^* \leq p^*$.*

**Proof.** Because $\varepsilon^*$ is the optimal solution to the $p^*$-center flow-refueling facility location problem, this implies that $\varepsilon^* \geq \max\{(r^{od})^*, \forall od \in OD\}$. Thus, $p^*$-infrastructure serves as a feasible solution to the set covering flow-refueling facility location problem with a predetermined maximum deviation of $\varepsilon^*$. □

This proof shows that the center flow-refueling facility location problem determines the maximum percentage deviation $\varepsilon^*$ for a predetermined size of refueling facilities $p$. The set covering flow-refueling facility location problem will determine the smallest size $s^*$ when $p$-infrastructures $p \in \{1, ...|S|\}$ all achieve the maximum percentage deviation of $\varepsilon^*$. Thus, we have the following corollary:

**Corollary 4.** $s^* = \min\{p\colon \varepsilon(p) = \varepsilon^*, p \in \{1, ...|S|\}\}$.

**Proof.** From Lemma 3, we have $s^* \leq p^*$ where $\varepsilon^* = \varepsilon(p^*)$. Assume that there is a $p'$-infrastructure such that $\{p'|\varepsilon(p') = \varepsilon^*\} < s^*$. Then, $p'$-infrastructure must be the optimal solution to the set covering flow-refueling facility location problem with a maximum deviation of $\varepsilon^*$. This condition contradicts the conclusion that $s^*$ is the optimal solution to the set covering flow-refueling facility location problem. □

Finally, as $p$ continuously increases, all drivers may travel along their shortest paths, thus leading to the following:

**Lemma 5.** *The optimal solution to the p-center flow-refueling facility location problem when p is sufficiently large is the optimal solution to the set covering flow-refueling facility location problem that involves the shortest paths.*

**Proof.** As all origins and destinations are candidate locations for refueling facilities, all drivers may travel along their shortest paths when all locations are designated a refueling facility. □

Collectively, the $p$-center flow-refueling facility location problem determines the *lowest* percentage deviation at which all flows in a system can be served by a predetermined $p$. Any ambitious goal to establish a lower deviation may imply only that a trip flow cannot be addressed by the $p$-maximum coverage flow-refueling facility location problem. In addition, although the center flow-refueling facility location problem determines the maximum percentage deviation for infrastructure of a predetermined size, the set covering flow-refueling facility location problem will determine the smallest size associated with that maximum percentage deviation. Finally, there is a large $p$ value such that $p$-infrastructure may allow all drivers to travel along their shortest paths.

## 4. Design of the algorithm

One solution scheme is a *path-based* approach. That is, when reformulating the model as a set covering problem that selects a set of $p$ refueling facilities to completely *cover* at least one VRFP for all OD pairs. This approach requires access to a *reasonable* set of VRFPs. This set may be determined by a predetermined number of paths or from all paths based on the *best estimated* maximum deviation for all OD pairs. One of the drawbacks associated with this approach is that it is time consuming to simply generate a set of VRFPs. In addition, a set may need to be expanded when an insufficient number of paths are generated to provide feasible results. In this study, we design an *exact* optimal solution without explicitly generating any sets of deviation paths using a *link-based* approach.

Observe that the $p$-center flow-refueling problem has a bi-level structure. The upper level forms a $p$-infrastructure (Constraint 2), while the lower level forms VRSPs for all OD pairs in the *vehicle range-constrained shortest path* (VRCSP) subproblem through Constraints (3)–(5). The VRCSP determines the shortest path constrained by a vehicle's driving range even when a vehicle may be refueled along its path when necessary. We embed the VRCSP into an implicit enumeration scheme to determine the maximum percentage deviation.

### 4.1. The VRCSP subproblem

We propose a labeling shortest path (SP) algorithm with dequeue implementation to determine the VRSP of all OD pairs. The algorithm may solve either directional or symmetric trips. Two main steps are involved: *node selection* and *distance/fuel level update*. The algorithm starts at origin $o$ and proceeds with a forward search. This algorithm is characterized by node labeling. For each node $i$, we create a set of labels $\omega_i^n = (e_i^{od,f,n}, d_i^{od,f,n})$ to represent the fuel level and distance of the $n$th

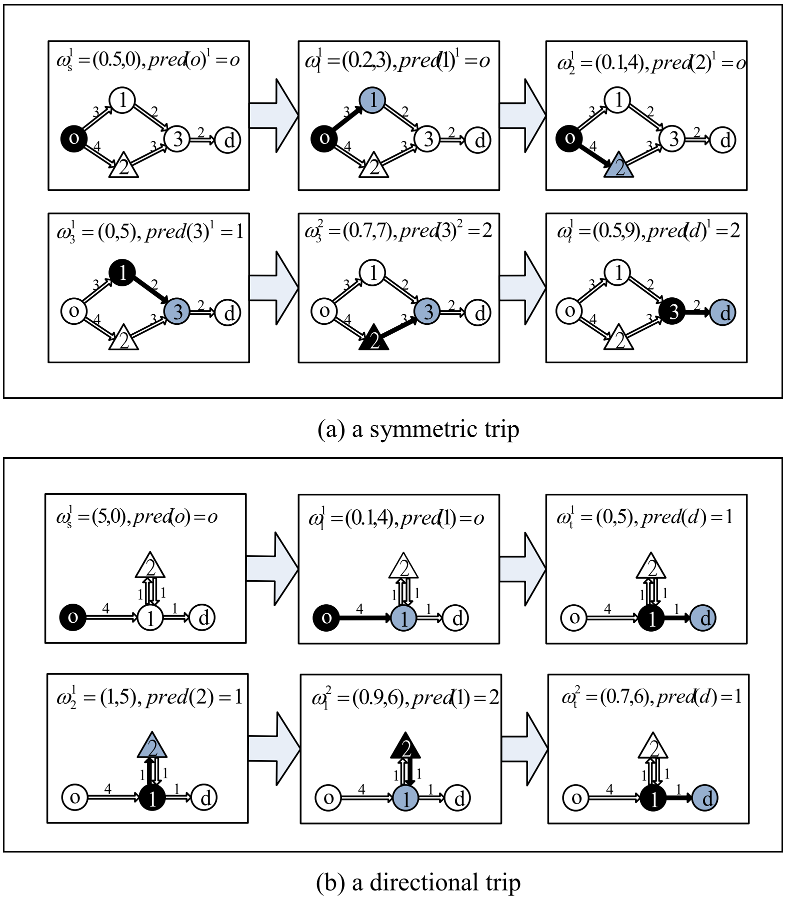
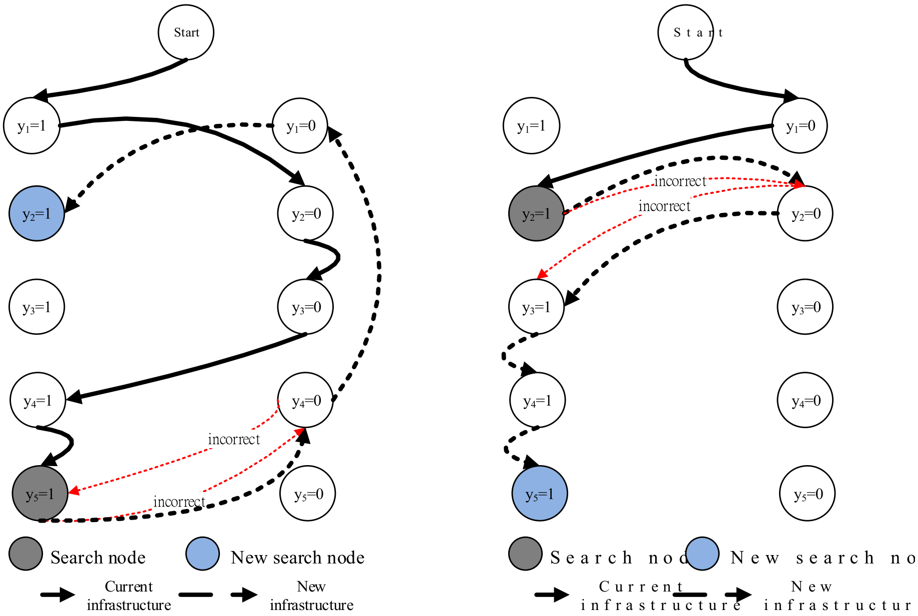



**Fig. 1.** Two illustrative examples of the VRCSP algorithm.

forward path$n \in M(i)$ at node $i$ from origin $o$. At any node $i$ on path $n$, we scan all associated *leaving* links with a common *tail* node $i$. A vehicle may reach head node $j$ upon leaving link $ij$ if it is supplied with enough fuel to travel from tail node $i$ to head node $j$. Whenever head node $j$ is reached, we calculate two *temporary* attributes, fuel level and distance $(\tilde{e}_j^{od,f}, \tilde{d}_j^{od,f})$, as $\tilde{e}_j^{od,f} = e_i^{od,f,n} - f_{ij}$ and $\tilde{d}_j^{od,f} = d_i^{od,f,n} + d_{ij}$, respectively. Of course, the fuel level will be reset to its full level $\tilde{e}_j^{od,f} = V$ when node $j$ keeps a refueling facility open. We maintain only *nondominated* labels. Thus, $\exists n \in M(j)$:$e_j^{od,f,n} \geq \tilde{e}_j^{od,f}$ and $d_j^{od,f,n} \leq \tilde{d}_j^{od,f}$, and *no* labels shall be recorded, which means that a rational driver cannot travel along this path segment, as another path segment runs from origin $o$ to node $j$ over a distance that is not longer, and the driver may travel with the same amount of remaining fuel. Otherwise, we increment the nondominated set of node $j$ as $e_j^{od,f,m} = \tilde{e}_j^{od,f}$ and $d_j^{od,f,m} = \tilde{d}_j^{od,f}$ for $m = n + 1$. Whenever a nondominated label is recorded for a *head* node, that node will be inserted into the *search node sequence*. Dequeue implementation will place this node in the first position of the sequence if it was previously positioned in the sequence. Otherwise, it will be placed at the end of the sequence. The algorithm will not end until the search node sequence is empty. Note that this algorithm does not require any additional preprocessing of network configurations for computation purposes as is necessary with, for example, an artificial feasible network constructed to represent all feasible paths, as proposed by Ichimori et al. (1981) and Kim and Kuby (2013) and Adler et al. (2016).

We now illustrate two examples, shown in Fig. 1(a) and (b), in a directed network, where $o$ is the origin node, $d$ is the destination node and the number shown on each link is its associated distance. We assume that a full-capacity vehicle can travel a distance of 10. For a symmetric round trip for an OD pair $od$ of the $p = 1$ center problem, at least half a tank of fuel is required at the origin and destination nodes to guarantee that enough fuel is available for journey completion. In Fig. 1(a), node 2 is assigned a refueling facility. In Fig. 1(a), a vehicle with a range of 10 cannot travel along a forward path $o$-1-3-$d$ or complete its round trip even though it is the ASP between $o$ and $d$. The vehicle cannot complete its associated backward path $d$-3-1-$o$ due to fuel limitations. Observe the first label at node 3 $\omega_3^1 = (0, 5)$ on path segment $o$-1-3. After traveling along another path segment $o$-2 and 3 and reaching node 3, its associated attributes are $\omega_3^2 = (0.7, 7)$. This label can be recorded as the second label at node 3, as it is a nondominated solution. Ultimately, at the destination, the SP with sufficient fuel is the VRSP.




*(a) finding an incomplete infrastructure; (b) finding a complete infrastructure*

**Fig. 2.** An illustrative search tree and backward search scenarios for $p = 3$.

Note that the VRSP may not necessarily be a path with no duplicated nodes. In other words, the path may include a *cycle*. Observe the second example illustrated in Fig. 1(b). The VRSP is *o*-1-2-1-*d* for OD pair *od*, which includes a cycle of 1-2-1.

### 4.2. Implicit enumeration algorithm

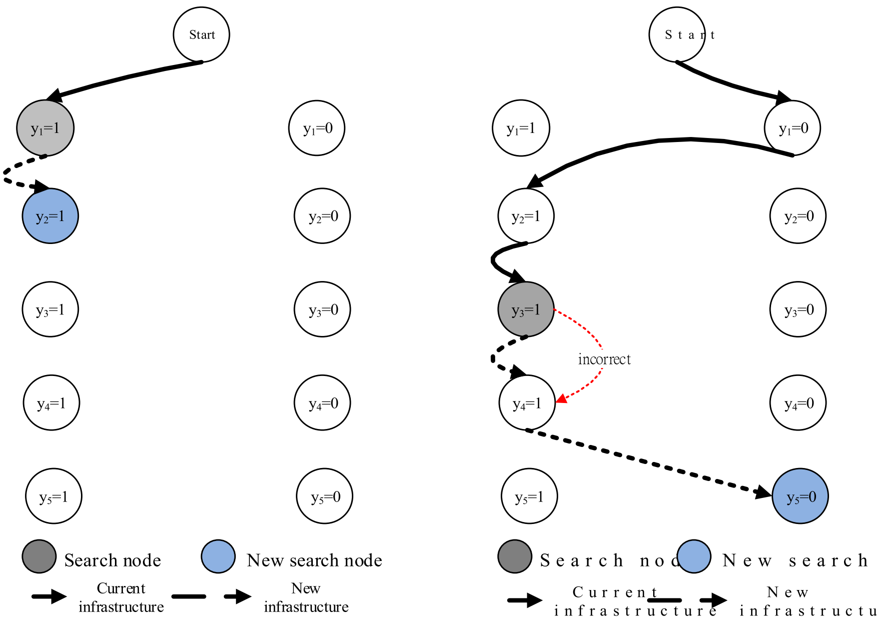
The VRCSP algorithm is embedded in an implicit enumeration on binary decision variable *y*'s open or closed refueling facilities at candidate locations. These decisions collectively form the search tree illustrated in Fig. 2.

A node in the search tree shown in Fig. 2 is denoted by *search* node. A search node is associated with either *complete* or *incomplete p*-infrastructure. When the open/closed status of *all* candidate refueling facilities is assigned and the total number of facilities open is exactly equal to *p*, the infrastructure is a *complete p*-infrastructure. We may evaluate the objective value and the maximum deviation and update the incumbent solution when a superior solution is obtained. We search *backward* (toward lower tiers) to determine the next search node. The infrastructure is *incomplete*, implying that the status of *certain* refueling facilities has yet to be assigned. We can determine the feasibility level of the *branch* rooted at the search node and then determine its associated lower bound when feasible. When the lower bound is *lower* than the incumbent, we perform a *depth-first* search by moving *downward* (toward higher tiers) to determine the next search node. Otherwise, we search backward.

#### 4.2.1. Backward search

For the *backward* search applied at search node *s*, we iteratively move backward toward lower tiers until reaching node $\bar{s}$ with $y_{\bar{s}} = 1$, and while applying $y_{\bar{s}} = 0$, condition $\sum_{s=1}^{s=\bar{s}} y_s + |S| - |\bar{s}| \geq p$ must hold. This condition implies the presence of *p*-infrastructure on the branch rooted at $\bar{s}$. Otherwise, while $\sum_{s=1}^{s=\bar{s}} y_s + |S| - |\bar{s}| < p$, no *p*-infrastructure may exist even though all unassigned refueling facilities $y_s = 1, \forall s \in \{\bar{s}+1, \ldots |S|\}$ remain open. Of course, when none exist, the program terminates. With the determination of $\bar{s}$, *two* branching scenarios occur as follows.

(a) Suppose that $\sum_{s=1}^{s=\bar{s}} y_s + |S| - |\bar{s}| > p$ and that there are more than enough refueling facilities available to form a p-infrastructure. Thus, node $\bar{s}$ becomes the next search node. An example for $p = 3$ is shown in Fig. 2(a). The search node applies $y_5 = 1$ in a $p = 3$ infrastructure where $y_1 = y_4 = 1$ and $y_2 = y_3 = 0$. Now, when performing the backward search, node 5 cannot simply be closed, i.e., $y_5 = 0$, and assigned as the next search node. This situation leaves only 2 refueling facilities for a $p = 3$ problem. Thus, we continuously trace backward to node 4 and close its refueling facility $y_4 = 0$. Unfortunately, $y_1 = y_5 = 1$ with $y_2 = y_3 = y_4 = 0$ cannot form the $p = 3$ infrastructure. Thus, we continuously trace back to node 1 and close its refueling facility $y_1 = 0$, and we move to node 2, where $y_2 = 1$. This node becomes the search node, as in addition to node 2, there are 3 more candidate nodes (3, 4, and 5) of the $p = 3$ problem.




*(a) an incomplete infrastructure; (b) a complete infrastructure*

**Fig. 3.** An illustrative forward search scenario for $p = 3$.

(b) Suppose that $\sum_{s=1}^{s=\bar{s}} y_s + |S| - |\bar{s}| = p$ and there are exactly enough refueling facilities to form a $p$-infrastructure. Thus, we apply $y_s = 1, \forall s \in \{\bar{s}+1, \ldots |S|\}$ and designate node $|S|$ as the next search node with $y_{|S|} = 1$, as shown in Fig. 2(b) as an example of $p = 3$.

*4.2.2. Forward search*

For the *forward* search at search node $s$, we perform a depth-first search by moving downward by one tier higher in value relative to node $s + 1$. Of course, *two* branching scenarios occur. These involve branching to node $s + 1$ with $y_{s+1} = 1$, as shown in Fig. 3(a), or to node $|S|$ with $y_{|S|} = 0$, as shown in Fig. 3(b) as an example of the $p = 3$ problem.

*4.2.3. The computational procedure*

A flowchart of the implicit enumeration algorithm is shown in Fig. 4, and the steps involved are discussed below.

**Step 0.** *Feasibility and initialization*. To determine feasibility levels, we solve the *set covering* flow-refueling facility location problem, which is discussed in Section 4.3. This solution determines the lowest number of refueling facilities feasible. When this number is greater than the predetermined $p$, the solution space of the $p$-center flow-refueling problem is *empty*, and we terminate the program due to issues of infeasibility. Otherwise, we construct a search tree, as shown in Fig. 2, and we set the initial incumbent solution as $\varepsilon^* = \infty$. We open the first $p$ refueling facilities with $y_s = 1$, $s = \{1, \ldots, p\}$ and close the remaining refueling facilities $y_s = 0$, $s = \{p + 1, \ldots, |S|\}$ and denote $y_{|S|} = 0$ as the first search node $s \leftarrow y_{|S|}$.

**Step 1.** *Is the infrastructure complete?* If search node $s$ represents complete $p$-infrastructure as shown in Fig. 2 (a) and (b), then we proceed to Step 2. Otherwise, we proceed to Step 3.

**Step 2**. *Is there a better feasible solution?* We determine VRSP and its associated deviation $(\bar{r}^{od})$, $od \in M$for each OD pair by solving the VRCSP algorithm. If $\varepsilon^* > \max\{(\bar{r}^{od}), od \in M\}$, then we update the incumbent solution $\varepsilon^* = \max\{(\bar{r}^{od}), od \in M\}$ together with respective minimum deviations $(r^{od})^* = \bar{r}^{od}$, $od \in M$. However, this new incumbent solution does not imply that the lowest deviation of all OD pairs has been improved such that $(r^{od})^* < \bar{r}^{od}$, $od \in M$. We proceed to Step 5 for branching.

**Step 3**. *Does a better solution exist on the branch*? Prior to determining the lower bound associated with search node $s$ in an *incomplete* $p$-infrastructure, we determine whether there is a superior solution on the *branch* rooted at the search node. We *open* all *unassigned* refueling facilities even though the total number of open facilities may exceed $p$. We determine the VRSP and its associated $\{\bar{r}^{od}, od \in M\}$ for all OD pairs using the VRCSP algorithm. If the maximum deviation of VRSPs is *larger* than the incumbent, then $\varepsilon^* < \max\{(\bar{r}^{od}), od \in M\}$, we may conclude that any smaller subsets of refueling facilities in the branch must represent an *inferior* solution to the incumbent. We proceed to Step 5 for branching.

**Step 4.** *The lower bound*. We apply a sequence of *three* steps to each OD, $od \in M$, to determine the lower bound. These steps are described as follows.

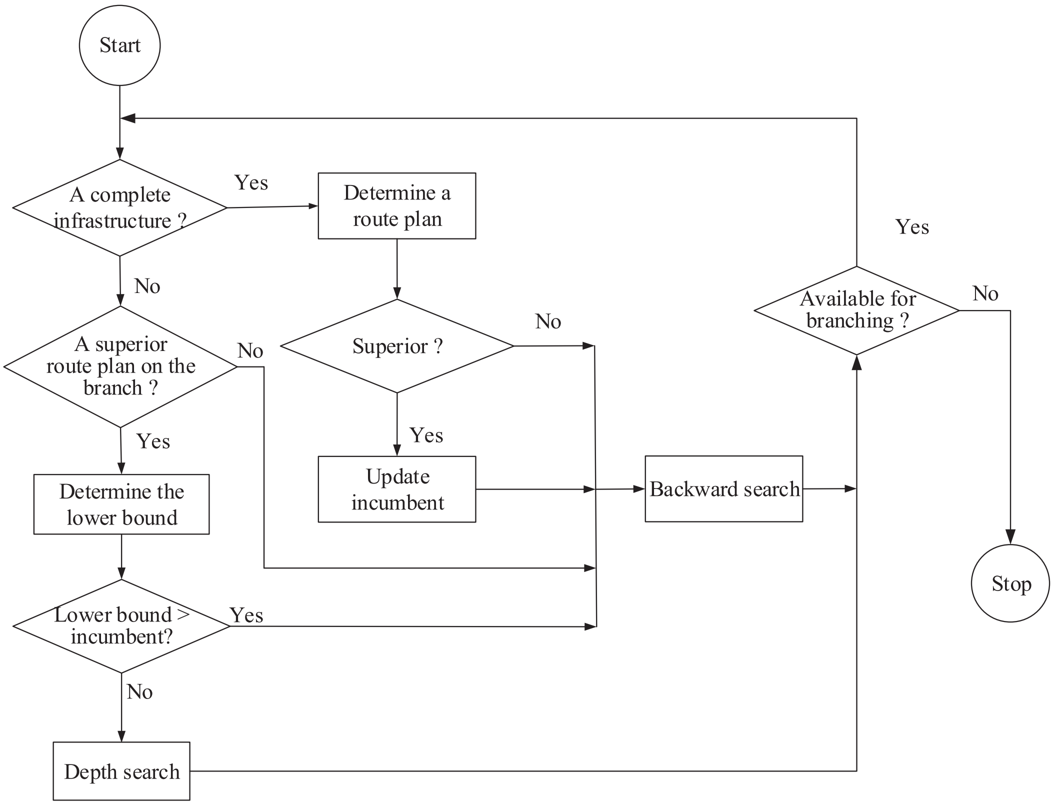



**Fig. 4.** Flowchart of the implicit enumeration algorithm.

(a) We first determine the additional number of refueling facilities that must open on the ASP to form a VRFP for OD pair od. When the number is no greater than the available number of refueling facilities to be assigned, the minimum deviation of OD pair od is zero.
(b) We next determine the lowest number of additional refueling facilities that must open such that there is at least a VRFP for OD pair od. We apply the proliferate algorithm to the set covering flow-refueling facility location problem to construct a reachable directed out-tree. Further detail on this step is provided in Section 4.3. When this value is greater than the available value to be assigned, no feasible VRFP can exist. The minimum deviation of OD pair od is infinite.
(c) We finally assign the available number of refueling facilities as driver reserve fuel. We then apply the VRCSP algorithm to determine the VRSP. If the AFV's tank depletes (empty) during travel, then the reserve will be used. Of course, the driver may refuel to the full tank level $\tilde{e}_j^{od,f} = V$ whenever an open refueling facility is reached. The ratio between the VRSP and ASP is the minimum percentage of deviation of OD pair od.

With the completion of all OD pairs, we can determine the maximum minimum deviations of the OD pairs, which form the *lower bound* associated with the search node *s*.

**Step 5.** *Branching*: Three branching schemes are used to determine the *next* search node: a complete *p*-infrastructure and superior and inferior values in an incomplete *p*-infrastructure. We discuss each scheme as follows.

(1) *A complete infrastructure*. Suppose that $y_{|S|} = 1$ at node $|S|$ is the last refueling facility in the sequence with $\sum_s y_s = p$. We cannot simply complete this stage by applying $y_{|S|} = 0$, as the result of $\sum_s y_s = p - 1 < p$ is an infeasible solution. Thus, for $y_{|S|} = 1$ or $y_{|S|} = 0$, we must perform a backward search, as discussed in Section 4.2.1. Once a new search node is identified, we can proceed to Step 2. When a new search node is not found, the incumbent $\varepsilon^*$ is the optimal solution and the program terminates.
(2) *Fathom in an incomplete infrastructure*. The occurrence of a fathom at search node *s* implies that an inferior solution for the rooted branch or the lower bound associated with search node *s* is no better than the incumbent. We apply the backward search method discussed in Section 4.2.1. Either a new search node is identified and Step 2 is repeated or the program terminates with incumbent $\varepsilon^*$ identified as the optimal solution.
(3) *Depth search in an incomplete infrastructure*. When the lower bound is *lower* than the incumbent, we must perform a forward search, as discussed in Section 4.2.2, and return to Step 2.

### 4.3. The lowest number of facilities

We use a *proliferate* algorithm, which is a link-based approach, to determine the *lowest* number of refueling facilities available so that all OD pairs have at least a VRFP in an incomplete $p$-infrastructure. For an unreachable OD pair, the algorithm iteratively expands a *reachable directed out-tree* (RDOT) originating at its origin node until it reaches its destination node. Upon completion, we may determine the *lowest* number of *additional* refueling facilities necessary. Upon completion, the *largest* of the *smallest* additional facilities of all OD pairs is the lower bound associated with this incomplete $p$-infrastructure.

We define an RDOT as a tree wherein all nodes can be reached from the original node by using the dequeue implementation, which is a type of *label-correcting* algorithm (Ahuja et al., 1993), and preserve the path with the *highest* residual fuel level at any node. When a destination has not yet been reached, we *open* a refueling facility from all *undetermined* facility candidate nodes in the directed out-tree. Note that we cannot alternate the statuses of candidate nodes that have already been assigned, open or closed, in the incomplete $p$-infrastructure. Therefore, we must open at least 1 additional refueling facility at one of the undetermined candidate nodes in the incomplete $p$-infrastructure. The result is the 1st outward proliferate wave. We then expand the RDOT by including all nodes that can be reached. We now either reach the destination or repeat the process to expand another directed out-tree. In the latter case, we open a refueling facility at all undetermined candidate nodes. We now open a second additional refueling facility, thus forming a second outward proliferate wave. This process continues until we eventually reach the destination. Upon reaching the destination, the number of additional facilities is the lower bound of this OD pair.

The Hodgson network is depicted in Fig. 5(a). As an example, suppose that the origin is node 1 and that the destination is node 12 with a vehicle range of 8. Node 6 is the search node. The destination is unreachable within the incomplete infrastructure that contains the first 6 nodes. There are refueling facilities at nodes 1, 4 and 6. However, nodes 2, 3 and 5 do not include a refueling facility, as shown in Fig. 5(a). First, we determine the directed out-tree rooted at the origin (node 1), which is depicted in Fig. 5(b). Because no feasible paths reach the destination, we assign all undetermined nodes in the directed out-tree as refueling facilities, as shown in Fig. 5(c). At this stage, we add the 1st additional refueling facility at nodes 7 to 9 for OD pair (1, 12). We expand our directed out-tree with all reachable nodes, as shown in Fig. 5(d). Unfortunately, we still cannot reach the destination. Again, we assign undetermined nodes with a refueling station from the result of 5(e). We now add a second additional refueling facility to the incomplete infrastructure. Finally, we reach the destination, i.e., node 12. As a result, the lowest number of refueling facilities is 2 for OD pair (1, 12).

One may observe that the directed out-tree includes links that may reach any node with the highest remaining fuel level found at any node, and it contains a VRFP for all OD pairs.

**Lemma 6.** *There is a VRFP of an OD pair in the RDOT.*

**Proof.** The origin node may reach all nodes in the first directed out-tree. *All* nodes of the second directed out-tree may be reached by at least one node in the first directed out-tree. The directed out-tree continuously expands until the destination is reached. Thus, there is at least a VRFP in the directed out-tree. □

## 5. Computational results

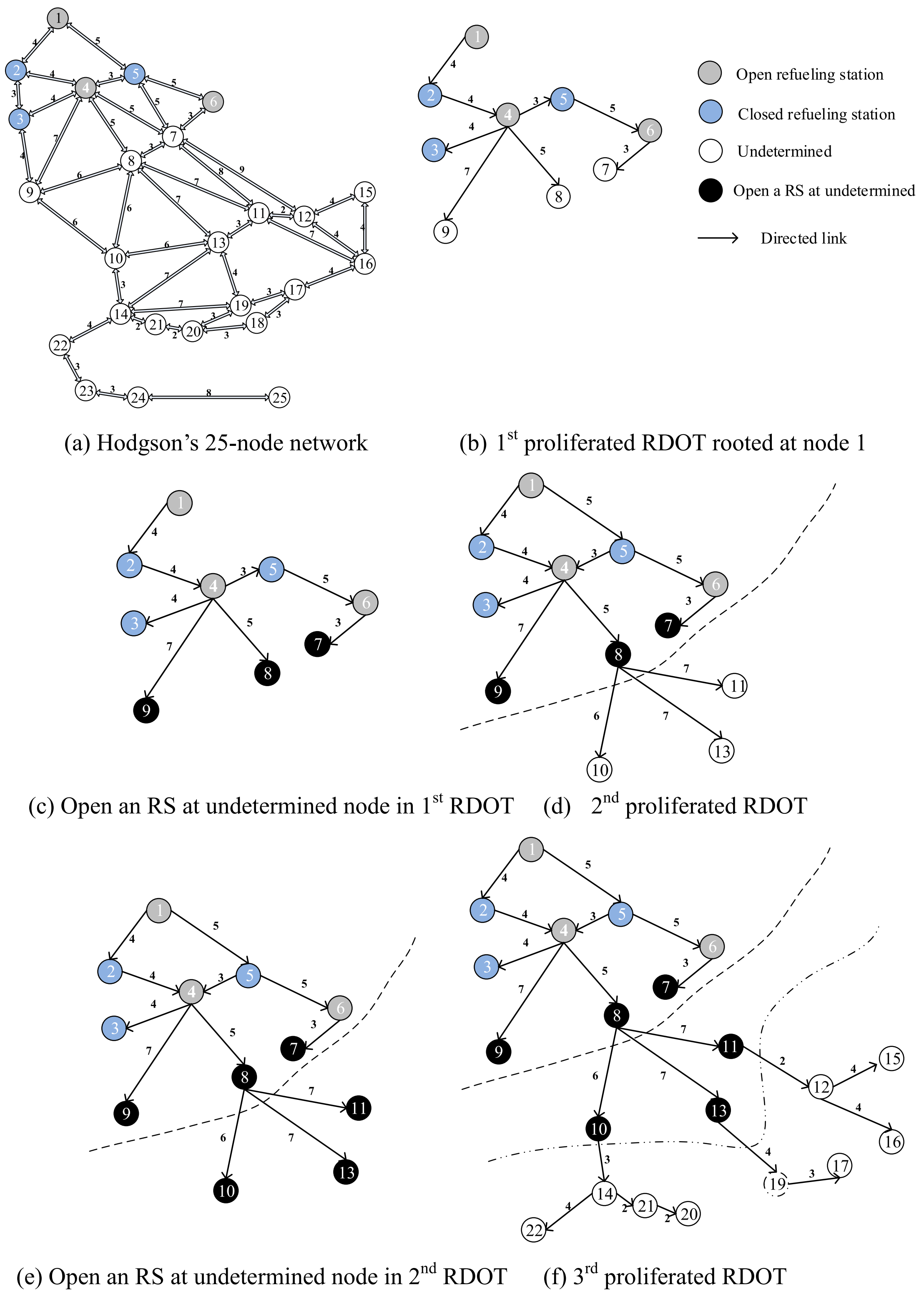
We conduct our numerical test using the 25-node network studied by Hodgson (1990), as shown in Fig. 5(a). The network applied is an *undirected* network. We can expand the network by any pair of nodes with 2 directed links to form a *directed* network. We assume that all 25 nodes are origins, destinations, and candidate locations of refueling facilities. Thus, there are 600 OD pairs. All trips are symmetric round trips. Our *base* experiment assumes that a vehicle may travel a distance of 9 with full capacity. All vehicles are running on half a tank of fuel at their origins and must be running on half a tank upon arriving at their destinations for symmetric round trips. In addition, we experimented with levels of sensitivity in the number of refueling facilities involved, $p$, and with vehicle ranges. Our algorithm is coded in C and is tested using Linux O/S equipped with a Core$^{TM}$ i7 CPU 7700 processor of 3.6 GHz.

### 5.1. The base experiment

For our computational results for a vehicle range of 9, base experiments are collectively tabulated in Table 2.

As the size of infrastructure $p$ increases, the number of combinations for both complete and incomplete infrastructures increases, thus resulting in longer computational times, as shown in Table 2. However, one of our main observations derived from the base experiment is as follows.

There are several optimal solutions for different total travel distances. Multiple optimal solutions identified for the $p$-center problem result from the fact that, while the maximum deviation at optimality remains unchanged, the paths of some ODs can vary with different total system distances. For example, when 18 fuel stations must be positioned, 14 alternative optimal solutions are available with an identical maximum deviation of 42.8%. However, the highest and lowest total system distances are 17,344 and 17,184, which are presented in Fig. 6(a) and (b), respectively. The distance gap is 160, which is 0.93% of the lowest total distance.




(a) Hodgson's 25-node network (b) 1st proliferated RDOT rooted at node 1

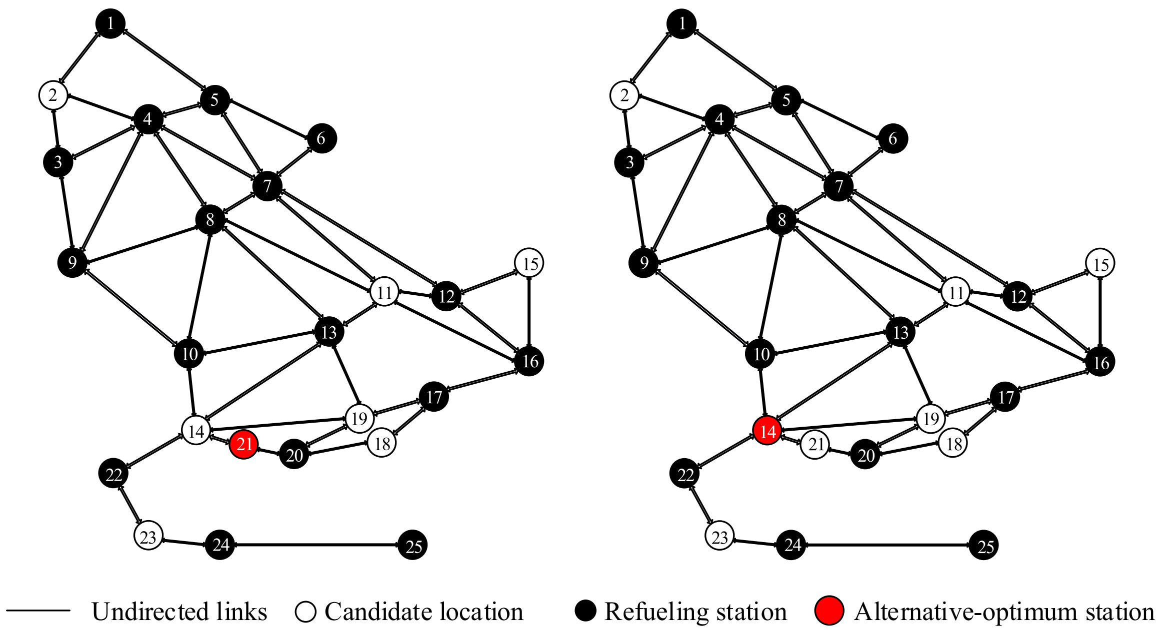
(c) Open an RS at undetermined node in 1st RDOT (d) 2nd proliferated RDOT

(e) Open an RS at undetermined node in 2nd RDOT (f) 3rd proliferated RDOT

**Fig. 5.** Illustration of the lower bounding procedure of Hodgson's 25-node network (1990).

**Table 2**
Computational results for a vehicle range of 9.

| $p$ | Max deviation (%) | Iteration | Comp. time (m/s) | Number of alternate optima | Total system distance | | Solution |
|---|---|---|---|---|---|---|---|
| | | | | | Highest | Lowest | |
| 11 | 400.0 | 4035 | 14.181 s | 25 | 21,932 | 20,156 | 2,5,7,9,12,14,19, 20,23,24,25 |
| 12 | 200.0 | 6934 | 24.973 s | 6 | 19,368 | 18,752 | 2,5,7,9,10,12,13,17, 20,22,24,25 |
| 13 | 120.0 | 10,381 | 38.271 s | 10 | 18,868 | 17,736 | 2,5,7,9,10,11,16,18,19, 21,22,24,25 |
| 14 | 120.0 | 17,708 | 1 m 6.121 s | 185 | 18,868 | 17,440 | 2,5,7,9,10,11,16,18,19, 21,22,23,24,25 |
| 15 | 100.0 | 6261 | 23.639 s | 10 | 18,336 | 17,576 | 1,3,4,5,7,9,10,11,16,18,19, 21,22,24,25 |
| 16 | 100.0 | 7322 | 27.106 s | 155 | 18,344 | 17,280 | 1,3,4,5,7,9,10,11,16,18, 19,21,22,23,24,25 |
| 17 | 60.0 | 6293 | 22.943 s | 14 | 17,368 | 17,208 | 1,3,4,5,7,8,9,10,12,13,16, 18,19,21,22,24,25 |
| 18 | 42.8 | 3580 | 12.890 s | 14 | 17,344 | 17,184 | 1,3,4,5,6,7,8,9,10,12,13, 16,18,19,21,22,24,25 |
| 19 | 0 | 1249 | 4.368 s | 8 | 17,080 | 17,080 | 1,3,4,5,6,7,8,9,10,11,12,13, 14,16,17,20,23,24,25 |




(a) The highest total distance (17,344) (b) The lowest total distance (17,184)

**Fig. 6.** An illustration of multiple optimal solutions for *18*-infrastructure.

One of the optimal *18*-infrastructures, as shown in Fig. 6(a), includes a refueling station at location 21. Suppose that we substitute the location at 21 with that at 14, as shown in Fig. 6(b). The maximum deviation remains at 42.8%. This problem has multiple optimal solutions. Observe OD pair (8, 11) with a VRSP of 8-13-11. Its distance is 10 against its ASP of 7, thus resulting in a deviation of 42.8% for both *18*-infrastructures. The maximum deviation remains unchanged. While the alternation of refueling stations from location 21 to location 14 does not improve the travel distance of OD pair (8, 11), some OD pairs, such as (11, 14) and (11, 22), alternate their VRSPs to ASPs as 11-13-14 and 11-13-10-14-22, respectively.

### *5.2. Sensitivity analysis*

We also conducted a sensitivity analysis on the number of refueling facilities based on various vehicle ranges. The vehicle range is increased to 10 and is then increased by an increment of 2 to 16. Corresponding computational results are graphically shown in Fig. 7.

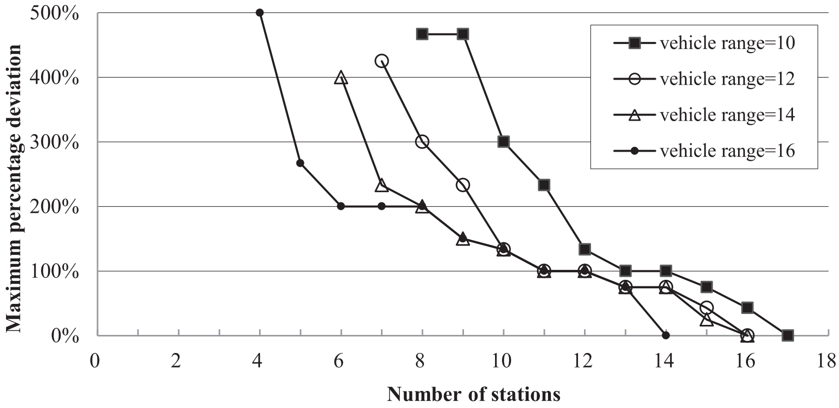



**Fig. 7.** Sensitivity analysis of vehicle ranges.

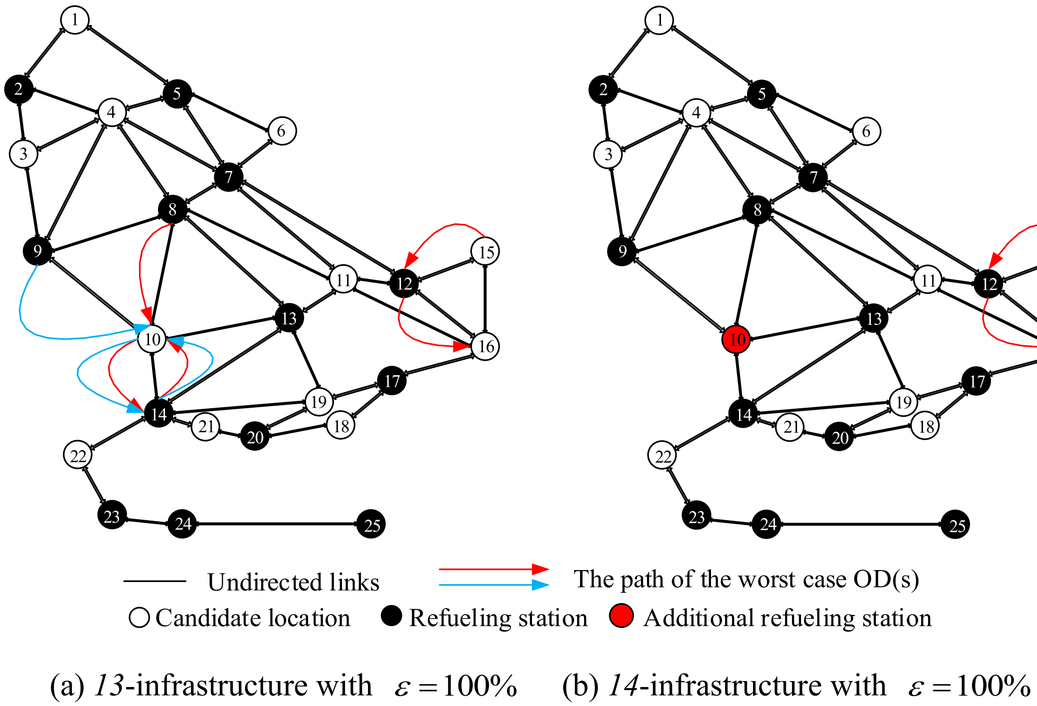



**Fig. 8.** Sensitivity analysis of the number of refueling facilities for a vehicle range of 10.

From our sensitivity analysis and base experiment, we make a few key observations that are discussed as follows.

(1) *The maximum deviation may not always decrease as the number of refueling facilities increases by one*. When more refueling facilities are present, drivers are more likely to identify shorter VRSPs. Curves follow a *monotonically* decreasing function. However, this does not directly imply that an increase in the number of refueling stations by 1 can guarantee a decrease in the maximum deviation. For the vehicle range of 9 shown in Table 2, an increase in the number of refueling stations from 13 to 14 or from 15 to 16 does not always reduce the maximum deviation. Similarly, for vehicle ranges of 10 and 16 shown in Fig. 7, an increase in the number of refueling stations from 13 to 14 or from 6 to 7 does not change the maximum deviation. In other words, some drivers' deviations do not improve when the size of the infrastructure increases by 1. As an example, for a vehicle range of 10, the maximum deviation is 100% for the 13- and 14-infrastructures shown in Fig. 8. In a 13-infrastructures setup, there are OD pairs of (8, 10), (9, 10) and (15, 16) with respective VRSPs of 8-10-14-10, 9-10-14-10, and 15-12-16 with a maximum deviation of 100%. Increasing the number of refueling facilities by 1 at location 10 may reduce the deviation of (8, 10) and (9, 10) to 0%, but it will

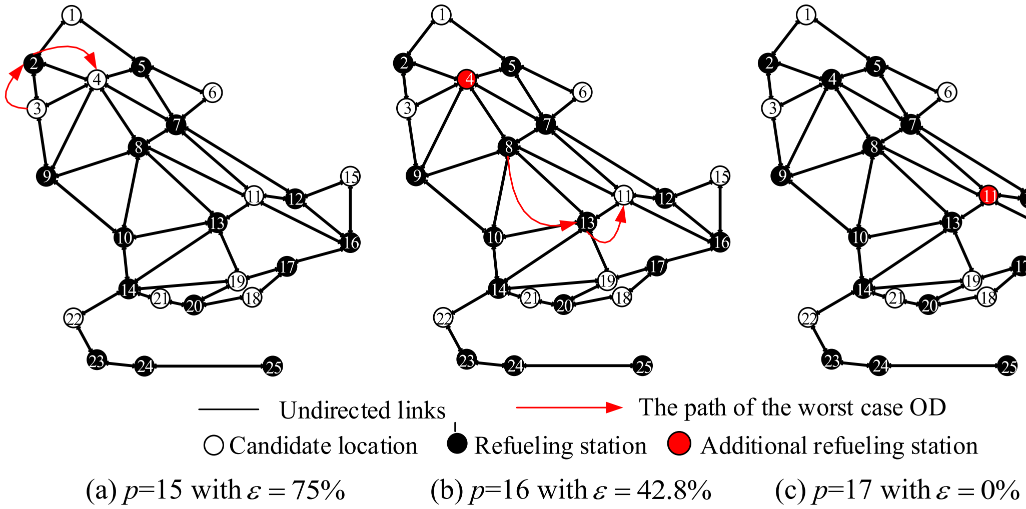



**Fig. 9.** The optimal infrastructure for $p = 15$, 16, 17 for a vehicle range of 10.

have no effect on OD pair (15, 16). Thus, the maximum deviation fails to improve with an increase in the number of refueling stations by 1.

(2) *Different marginal deviations in the number of refueling facilities with improvements*. Observe the infrastructures of 15, 16, and 17 with a vehicle range of 10 shown in Fig. 9. When $p = 15$, no refueling facilities are positioned at locations 3 and 4, and thus, for OD pair (3, 4), drivers must travel on a VRSP of 3-2-4 rather than on an ASP of 3–4. The respective travel distances are 7 and 4, thus producing a minimum deviation of 75%. When $p = 16$, 1 additional refueling station at location 4 is added to the infrastructure, and OD pair (3, 4) can now travel along its ASP. OD pair (8, 11) in turn adheres to the maximum deviation for all OD pairs. Rather, the VRSP is 8-13-11 with a distance of 10, and the ASP of 8–11 applies a distance of 7, thus forming a maximum deviation of 42.8%. When a 16th refueling station is located at location 4, the *marginal* maximum deviation is improved by 32.2%. Now, observe infrastructure $p = 17$, where 1 new refueling station at location 11 is added to the infrastructure. All drivers may travel along their respective ASPs with no deviations. The *marginal* maximum deviation of the seventeenth refueling station is improved by 42.8%, thus representing a stronger improvement than that achieved by the sixteenth refueling station.

(3) *The maximum deviation may not always decrease as the vehicle range increases.* It is clear that, for longer vehicle ranges, fewer stations are needed to achieve the same maximum deviation, as drivers may have more opportunities to refuel, which alleviates the need for drivers to make detours to refuel their vehicles and shortens VRFPs. In Fig. 7, the maximum percentage deviation is 300% for an infrastructure of 10 refueling facilities when the vehicle range is 10. However, for a vehicle range of 12, only 8 stations are required, and for a vehicle range of 14, 7 stations are required. For a range of 16, 5 stations are required to surpass a value of 300%.

However, an increase in the vehicle range may not guarantee a decrease in the maximum deviation. As shown in Fig. 10, for the infrastructure of 14 refueling facilities, the maximum percentage deviation is 75% for vehicle ranges of 12 and 14. An increase in the vehicle range by an increment of 2 does not lead to a decrease in maximum percentage deviations. For OD pair (24, 25), a vehicle cannot directly travel over a distance of 8 with half a tank of fuel for vehicle ranges 12 and 14. Thus, a vehicle must travel to a refueling station at 23 to refuel. The vehicle may then be able to travel to its destination at 25, where it may also refuel to a full tank and then complete its return trip to its original location at 24.

These findings suggest that an adequate deployment of stations can limit the number of deviations drivers must make without an increase in vehicle ranges. This practice may be central to promoting AFVs, especially when the improvement in AFV ranges is expensive and challenging for manufacturers to achieve.

### 5.3. Computational analysis of the size of the network

We propose an exact *link-based* optimal solution for optimally solving the $p$-center flow-refueling problem without the explicit enumeration of paths. In this section, we use the number of iterations and the computational times to analyze the computational performance of the algorithm on the network scale. We gradually expand our base network, Hodgson's 25-node network, to a 35-node network, as depicted in Fig. 11, with an increment of 5 nodes. For all network sizes, we again assume all nodes as origins, destinations and candidate sites for refueling facilities. The network size of 35 nodes contains 1,190 OD pairs, which is comparable to a CA network of 1,167 OD pairs (Arslan and Karaşan, 2016).

The computational results are tabulated in Table 3. The number of OD pairs and tree nodes (complete/incomplete infrastructures) on the search tree increased at a much faster rate than the network size. However, there is a profound impact

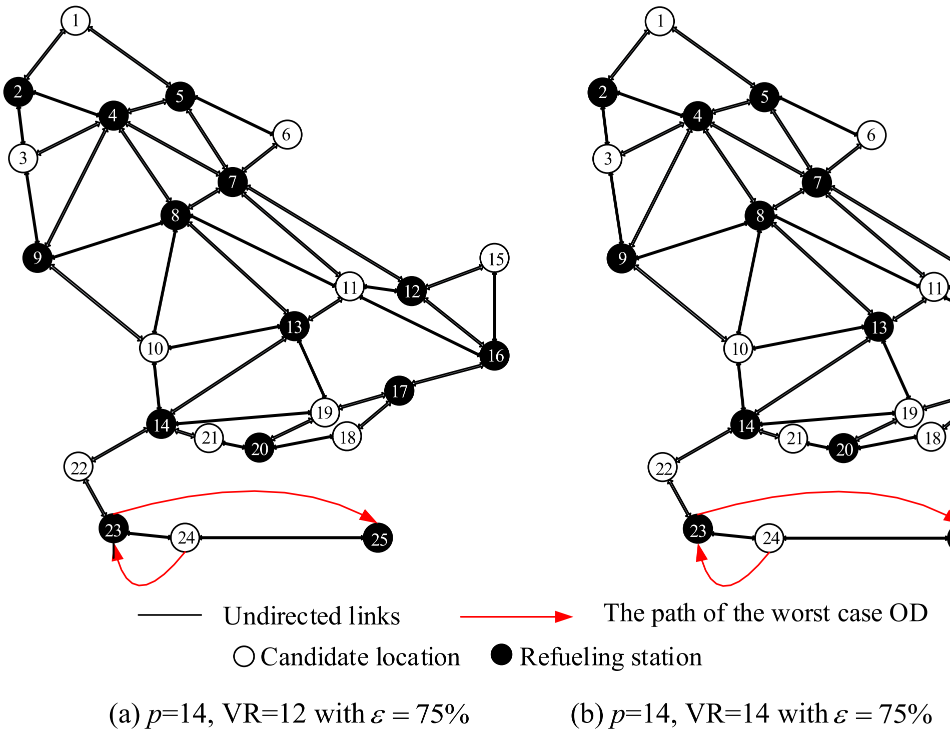



**Fig. 10.** The optimal infrastructure for vehicle ranges of 12 and 14 for $p = 14$.

**Table 3**
Computational performances of networks of various size for a vehicle range of 9.

| | 25 | | | 30 | | | 35 | | |
|---|---|---|---|---|---|---|---|---|---|
| | Max deviation (%) | Iteration | Comp. time (m/s) | Max deviation (%) | Iteration | Comp. time (m/s) | Max deviation (%) | Iteration | Comp. time (m/s) |
| 10 | Infeasible | 1153 | 3.713 s | Infeasible | 349 | 1.118 s | Infeasible | 305 | 1.707 s |
| 15 | 100.0 | 6261 | 23.639 s | 120.0 | 41,397 | 3 m 31.299 s | 883.0 | 324,371 | 43 m 5.804 s |
| 20 | 0 | 672 | 2.044 s | 75.0 | 19,350 | 1 m 43.567 s | 120.0 | 899,794 | 127 m 25.744 s |
| 25 | 0 | 25 | 0.008 s | 0 | 5097 | 24.760 s | 62.5 | 279,819 | 41 m 5.942 s |
| 30 | – | | – | 0 | 30 | 0.006 s | 0 | 11,938 | 1 m 30.563 s |
| 35 | – | | – | – | | – | 0 | 35 | 0.009 s |

on the number of complete/incomplete infrastructures. With a network size of 35, the infrastructure size of $p = 20$ has the largest set of complete infrastructures at 3.247 billion, while the largest sets of network sizes of 30 and 25 are more than 30 and 3.27 million. As seen in Table 3, increasing the network size obviously increases the computational time. However, all instances except one required more than 2 h of computational time, and the link-based exact algorithm may solve all instances optimally in less than 45 min. The link-based algorithm enables us to efficiently handle problems of manageable sizes.

One of the key observations is that the complete infrastructures may increase exponentially as the network size increases; for example, they increase 47.4- and 20.9-fold with an increment of 5 nodes from a 25-node network if all nodes are candidate refueling locations. However, the number of OD pairs are increased by much smaller magnitudes (1.45- and 1.37-fold, respectively). When all candidate locations are designated refueling stations, the VRCSP subproblem will consume most of the computational time. Observing the computational times in Table 3, when comparing the largest combination of complete infrastructures between 30- and 35-node networks, the time increased 36.2-fold, but when designating all locations as refueling stations, the times increased by relatively smaller magnitudes of 1.5-fold, from 0.006 s to 0.009 s. Thus, our exact link-based optimal solution without the explicit enumeration of paths is more sensitive to the number of candidate locations in the network and thus has a profound impact on the combinations and the computational efficiency.

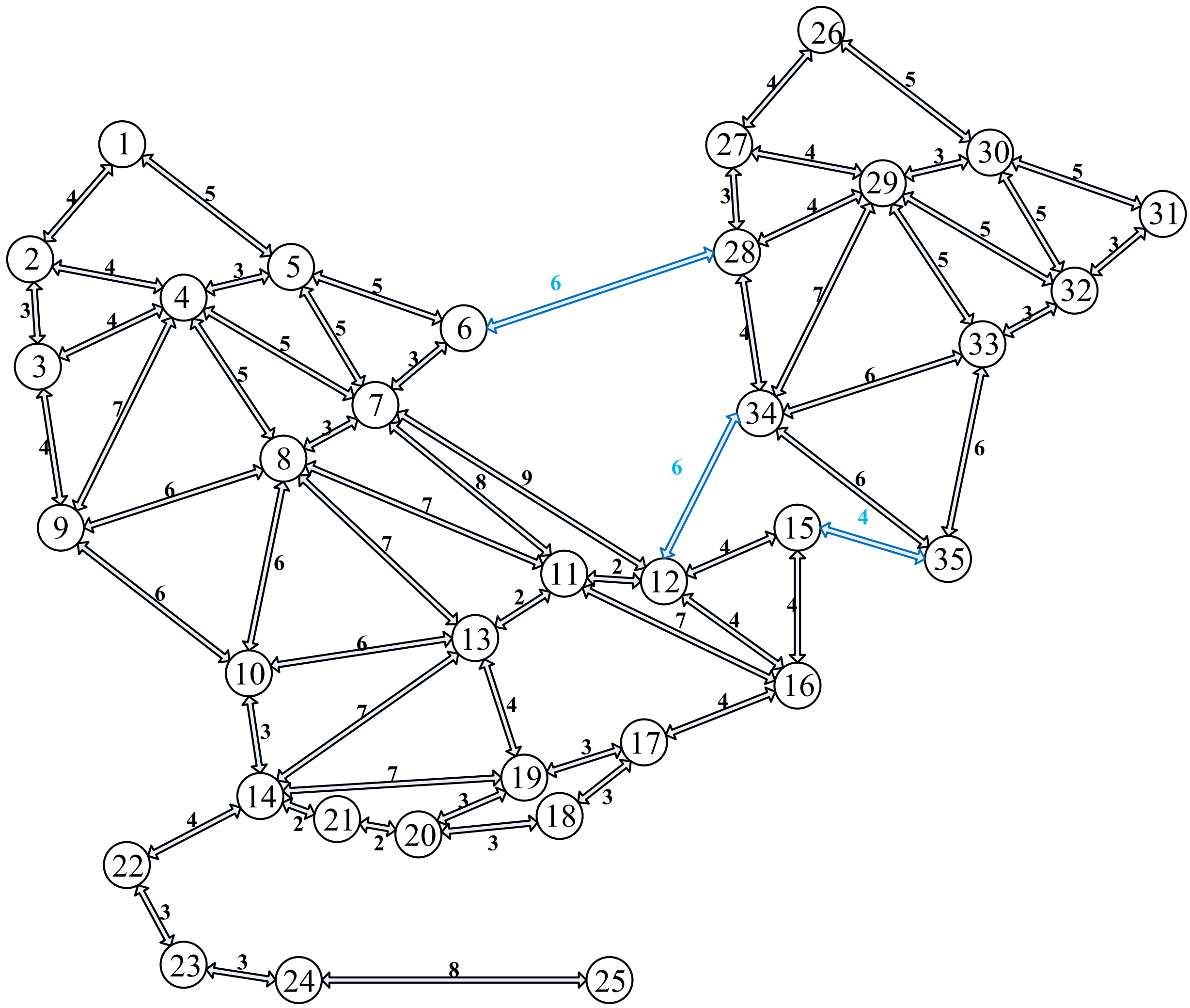



**Fig. 11.** The expanded 35-node network.

## 6. Conclusions

Alternative-fuel vehicles (AFVs) are on pace to replace petroleum-based vehicles to reduce global $CO_2$ emissions and contribute to a healthy living environment. However, fuel storage technologies limit the AFV driving range. This limitation requires drivers to deviate from their normal/ideal trips to replenish their AFVs to complete their journeys. Despite high levels of capital investment, the development of a suitable refueling infrastructure is essential for establishing consumer confidence in AFVs.

Most of refueling facility location models are classified as flow-based facility location problems. In the design of a flow-refueling facility infrastructure, two opposing social issues must be considered: social efficiency and social equity. *System optimization* (i.e., *social efficiency*) models design an infrastructure to maximize total travel flow. However, in the present study, we propose the *social equity* optimization model, which is a *p*-center flow-refueling facility location problem that explicitly considers the levels of fairness among users. The goal of this method is to allocate resources such that *all* AFV users are indiscriminately served by the system while the occurrence of *worst* deviations is minimized and applied equally across individuals.

Through the study of *p*-center, *p*-maximum coverage and set covering flow-refueling facility location problems, we show that the *p*-center flow-refueling facility location problem determines the *lowest* percentage deviation from which all flows in a system can be served by a predetermined *p*. The *p*-maximum coverage flow-refueling facility location problem cannot serve all drivers wishing to reach any deviation that is lower than the maximum percentage deviation, thus supporting a higher level of social equity. In addition, through problem formulation, the set covering flow-refueling facility location problem determines the smallest size associated with the optimal percentage deviation determined from the *p*-center flow-refueling facility location problem. Finally, a large $p$ is used such that the $p$-infrastructure can allow all drivers to travel along their SPs.

We formulate the $p$-center flow-refueling facility location problem as a link-based nonlinear integer program. The model uses a bi-level structure. The upper level determines the facility design master problem, while the lower level determines the VRCSP subproblem. We propose a link-based approach, in which we develop a nondominated labeling SP using the dequeue implementation algorithm to solve the VRCSP subproblem. This algorithm is embedded in the implicit enumeration algorithm to optimality solve the model. We use Hodgson's 25-node and expand it to a 35-node undirected network for our numerical experiments. The computational results show that our exact link-based optimal solution is more sensitive to the number of candidate locations than the number of OD pairs. In addition, we conduct a sensitivity analysis of the vehicle range, the number of refueling facilities and the network scale. Our computations show the following: (1) there are multiple optimal solutions for different total travel distances; (2) the maximum deviation may not always decrease as the vehicle range increases or as the number of refueling facilities increases by one; and (3) different marginal deviations occur in the number of refueling facilities when improvements are made.

Because there are multiple optimal solutions for different total travel distances, we may incorporate the total travel distance as the *secondary* objective term. It can be implemented in the link-based exact algorithm with a fairly minor modification. The first modification is needed in Step 5(2). The condition was as follows: when the lower bound associated with search node is no better than the incumbent, we perform a backward search. It shall be modified as we perform the backward search if the lower bound is *worse* than the incumbent. The second modification is needed in Step 2. The original condition was that, whenever the minimum of the maximum percentage deviation of all drivers is improved (i.e., smaller), we update the incumbent solution. The new condition is that, although the minimum of the maximum deviations remained the same, the total travel distance improves (i.e., decreased); thus, we also need to update the incumbent.

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