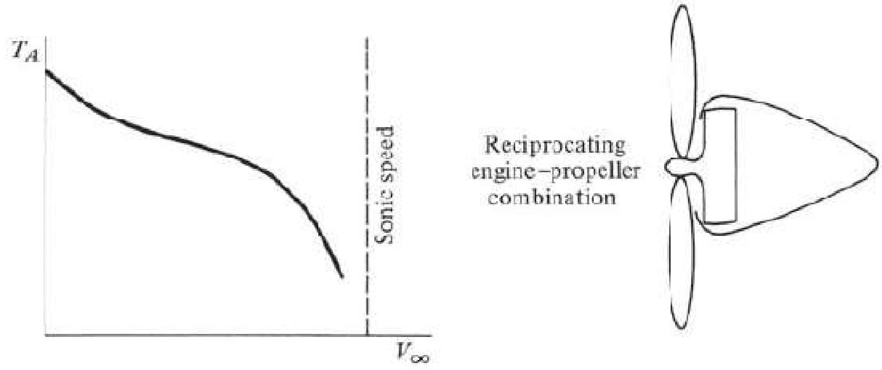
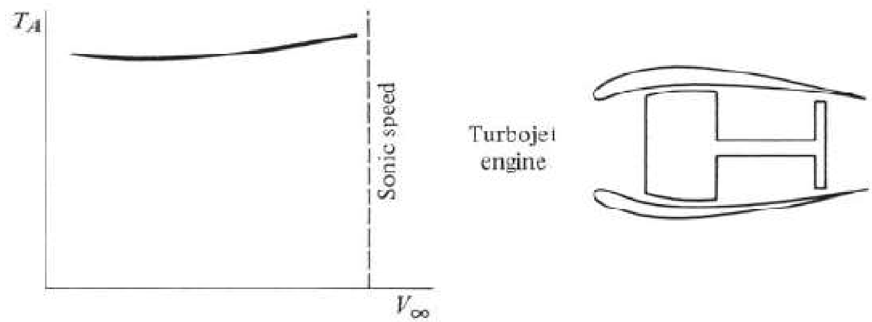
THRUST AVAILABLE AND MAXIMUM VELOCITY



(a)



(b)

POWER REQUIRED FOR LEVEL, UNACCELERATED FLIGHT

$$\text{Power} = \frac{\text{energy}}{\text{time}} = \frac{\text{force} \times \text{distance}}{\text{time}} = \text{force} \times \frac{\text{distance}}{\text{time}}$$

$$\text{Power} = F\left(\frac{d}{t_2 - t_1}\right) = FV$$

$$\boxed{P_R = T_R V_\infty}$$

$$P_R = T_R V_\infty = \frac{W}{C_L/C_D} V_\infty$$

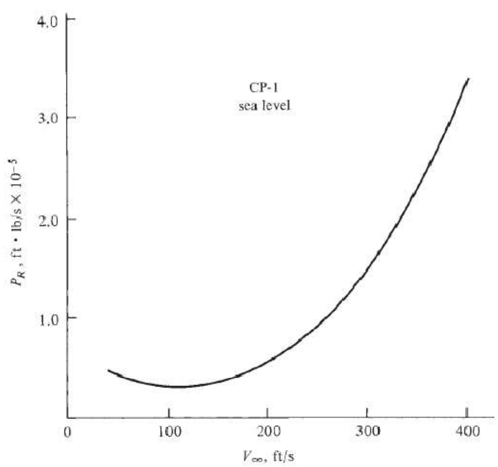
$$L = W = q_\infty S C_L = \tfrac{1}{2}\rho_\infty V_\infty^2 S C_L$$

$$V_\infty = \sqrt{\frac{2W}{\rho_\infty S C_L}}$$

$$P_R = \frac{W}{C_L/C_D}\sqrt{\frac{2W}{\rho_\infty S C_L}}$$

$$\boxed{P_R = \sqrt{\frac{2W^3 C_D^2}{\rho_\infty S C_L^3}} \propto \frac{1}{C_L^{3/2}/C_D}}$$

قابل توجه آنکه رانش مورد نیاز متناسب با عکس نسبت C_L/C_D بود اما قدرت مورد نیاز متناسب با عکس نسبت $C_L^{3/2}/C_D$ متناسب است



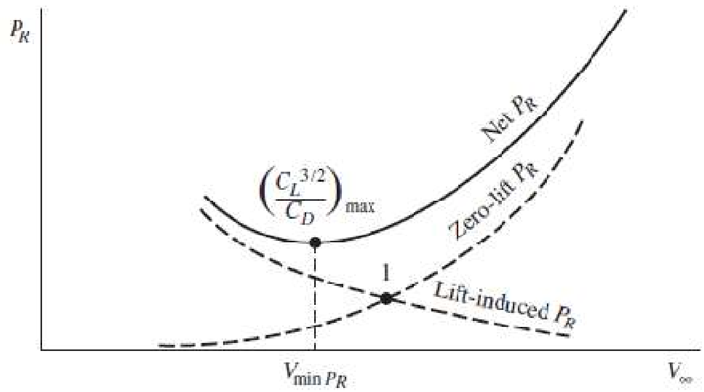
$$P_R = T_R V_\infty = D V_\infty = q_\infty S\left(C_{D,0} + \frac{C_L^2}{\pi e AR}\right)V_\infty$$

$$P_R = \underbrace{q_\infty S C_{D,0} V_\infty}_{\text{Zero-lift power required}} + \underbrace{q_\infty S V_\infty \frac{C_L^2}{\pi e AR}}_{\text{Lift-induced power required}}$$

می دانیم $C_L = W/(\frac{1}{2}\rho_\infty V_\infty^2 S)$ پس

$$P_R = \frac{1}{2}\rho_\infty V_\infty^3 S C_{D,0} + \frac{1}{2}\rho_\infty V_\infty^3 S \frac{[W/(\frac{1}{2}\rho_\infty V_\infty^2 S)]^2}{\pi e \mathrm{AR}}$$

$$P_R = \frac{1}{2}\rho_\infty V_\infty^3 S C_{D,0} + \frac{W^2/(\frac{1}{2}\rho_\infty V_\infty S)}{\pi e \mathrm{AR}}$$



For minimum power required, $dP_R/dV_\infty = 0$.

$$\frac{dP_R}{dV_\infty} = \frac{3}{2}\rho_\infty V_\infty^2 S C_{D,0} - \frac{W^2/(\frac{1}{2}\rho_\infty V_\infty^2 S)}{\pi e \mathrm{AR}}$$

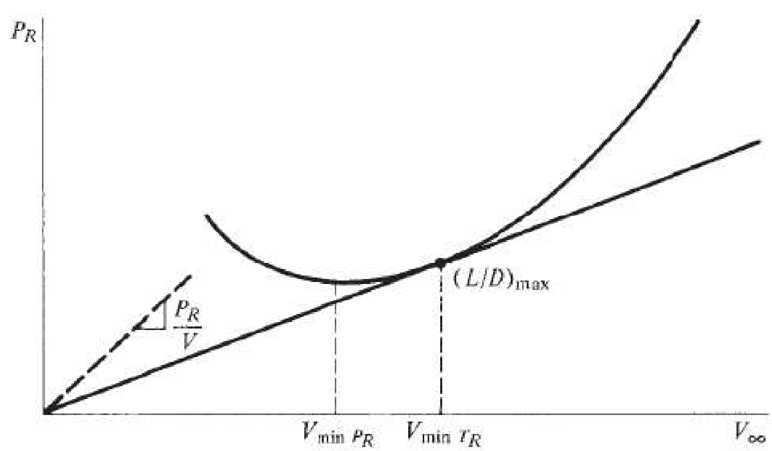
$$= \frac{3}{2}\rho_\infty V_\infty^2 S\left[C_{D,0} - \frac{W^2/(\frac{3}{4}\rho_\infty^2 S^2 V_\infty^4)}{\pi e \mathrm{AR}}\right]$$

$$= \frac{3}{2}\rho_\infty V_\infty^2 S\left(C_{D,0} - \frac{\frac{1}{3}C_L^2}{\pi e \mathrm{AR}}\right)$$

$$= \frac{3}{2}\rho_\infty V_\infty^2 S\left(C_{D,0} - \frac{1}{3}C_{D,i}\right) = 0 \qquad \text{for minimum } P_R$$

$$\boxed{C_{D,0} = \tfrac{1}{3}C_{D,i}}$$

$C_{D,0} = C_{D,i}$ (that is, minimum T_R)



$$\frac{d(P_R/V_\infty)}{dV_\infty} = \frac{d(T_R V_\infty/V_\infty)}{dV_\infty} = \frac{dT_R}{dV_\infty} = 0$$

EXAMPLE

Calculate the power-required curves for (*a*) the **CP-1** at sea level and (*b*) the **CJ-1** at an altitude of 22,000 ft.

■ **Solution**

a. For the CP-1, the values of T_R at sea level have already been tabulated and graphed in Example 6.1. Hence, from Eq. (6.24),

$$P_R = T_R V_\infty$$

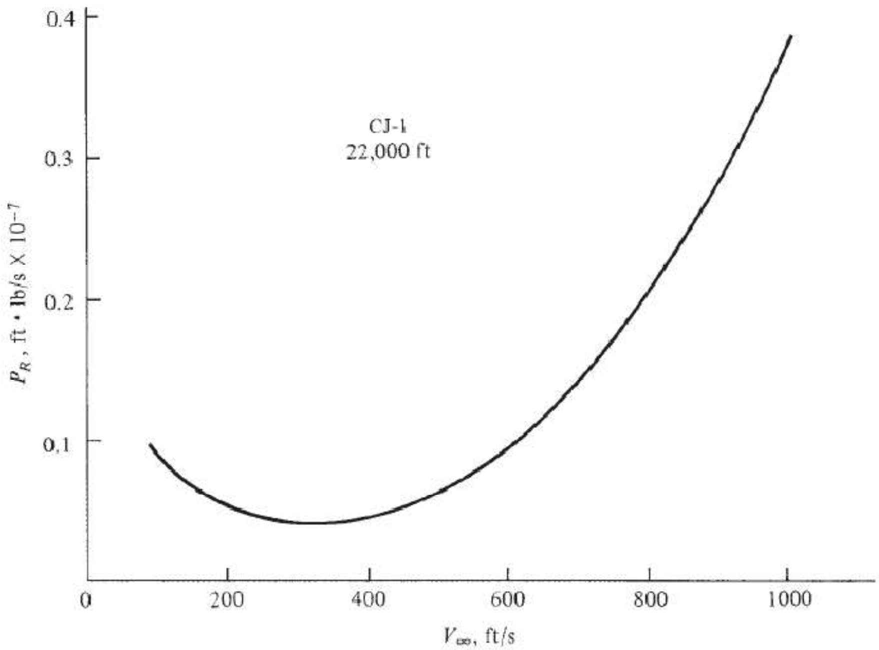
we obtain the following tabulation:

V, ft/s	T_R, lb	P_R, ft · lb/s
100	279	27,860
150	217	32,580
250	359	89,860
300	491	147,200
350	652	228,100

b. For the CJ-1 at 22,000 ft, $\rho_\infty = 0.001184$ slug/ft^3. The calculation of T_R is done with the same method as given in Example 6.1, and P_R is obtained from Eq. (6.24). Some results are tabulated here:

V_∞, ft/s	C_L	C_D	L/D	T_R, lb	P_R, ft · lb/s
300	1.17	0.081	14.6	1358	0.041×10^7
500	0.421	0.028	15.2	1308	0.065×10^7
600	0.292	0.024	12.3	1610	0.097×10^7
800	0.165	0.021	7.76	2553	0.204×10^7
1000	0.105	0.020	5.14	3857	0.386×10^7

The reader should attempt to reproduce these results.



POWER AVAILABLE AND MAXIMUM VELOCITY

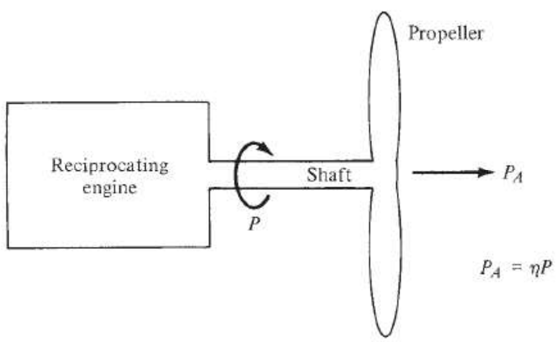
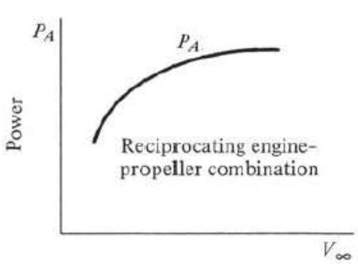
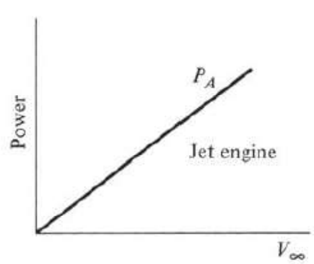


Figure 6.19 Relation between shaft brake power and power available.





$$\boxed{1\ \text{hp} = 550\ \text{ft} \cdot \text{lb/s} = 746\ \text{W}}$$

$$P_A = T_A V_\infty$$

$$\text{hp}_A = (\eta)(\text{bhp})$$

EXAMPLE 6.4

Calculate the maximum velocity for (*a*) the CP-1 at sea level and (*b*) the CJ-1 at 22,000 ft.

■ **Solution**

a. For the CP-1, the information in Example 6.1 gave the horsepower rating of the power plant at sea level as 230 hp. Hence

$$\mathrm{hp}_A = (\eta)(\mathrm{bhp}) = 0.80(230) = 184\ \mathrm{hp}$$

The results of Example 6.3 for power required are replotted in Fig. 6.21*a* in terms of horsepower. The horsepower available is also shown, and $V_{\max}$ is determined by the intersection of the curves as

$$\boxed{V_{\max} = 265\ \mathrm{ft/s} = 181\ \mathrm{mi/h}}$$

b. For the CJ-1, again from the information given in Example 6.1, the sea-level static thrust for each engine is 3650 lb. There are two engines; hence $T_A = 2(3650) = 7300$ lb. From Eq. (6.33), $P_A = T_A V_\infty$; and in terms of horsepower, where T_A is in pounds and V_∞ is in feet per second,

$$\mathrm{hp}_A = \frac{T_A V_\infty}{550}$$

Let $\mathrm{hp}_{A,0}$ be the horsepower at sea level. As we will see in Ch. 9, the thrust of a jet engine is, to a first approximation, proportional to air density. If we make this approximation here, the thrust at altitude becomes

$$T_{A,\mathrm{alt}} = \frac{\rho}{\rho_0} T_{A,0}$$

Hence

$$\mathrm{hp}_{A,\mathrm{alt}} = \frac{\rho}{\rho_0} \mathrm{hp}_{A,0}$$

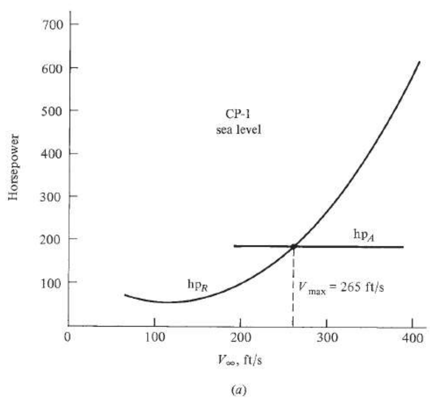
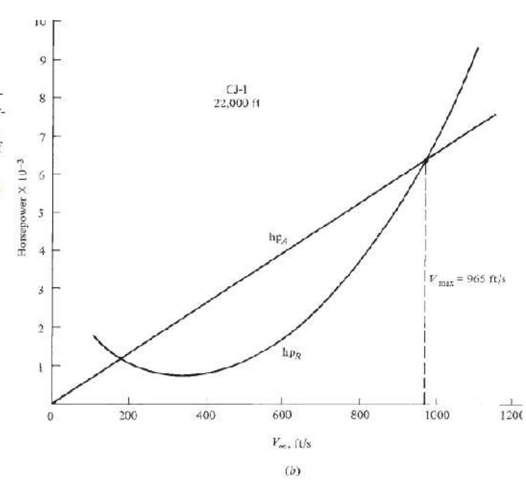


Figure 6.21 Power-available and power-required curves and the determination of maximum velocity. (*a*) Propeller-driven airplane. (*b*) Jet-propelled airplane.



For the CJ-1 at 22,000 ft, where $\rho = 0.001184$ slug/ft^3,

$$\mathrm{hp}_{A,\mathrm{alt}} = \frac{(\rho/\rho_0)\, T_A V_\infty}{550} = \frac{(0.001184/0.002377)(7300)V_\infty}{550} = 6.61 V_\infty$$

The results of Example 6.3 for power required are replotted in Fig. 6.21*b* in terms of horsepower. The horsepower available, obtained from the preceding equation, is also shown, and $V_{\max}$ is determined by the intersection of the curves as

$$\boxed{V_{\max} = 965\ \mathrm{ft/s} = 658\ \mathrm{mi/h}}$$

ALTITUDE EFFECTS ON POWER REQUIRED AND AVAILABLE

$$V_0 = \sqrt{\frac{2W}{\rho_0 S C_L}}$$

$$P_{R,0} = \sqrt{\frac{2W^3 C_D^2}{\rho_0 S C_L^3}}$$

$$V_{\text{alt}} = \sqrt{\frac{2W}{\rho S C_L}}$$

$$P_{R,\text{alt}} = \sqrt{\frac{2W^3 C_D^2}{\rho S C_L^3}}$$

می دانیم ضریب برا در تغییرات ارتفاع ثابت می ماند و همچنین می دانیم $C_D = C_{D,0} + C_L^2/(\pi e \mathrm{AR})$ پس تغییرات ضریب پسا هم در تغییرات ارتفاع نداریم بنابراین با توجه به معادلات بالا می توان نوشت:

$$V_{\text{alt}} = V_0 \left(\frac{\rho_0}{\rho}\right)^{1/2}$$

$$P_{R,\text{alt}} = P_{R,0} \left(\frac{\rho_0}{\rho}\right)^{1/2}$$

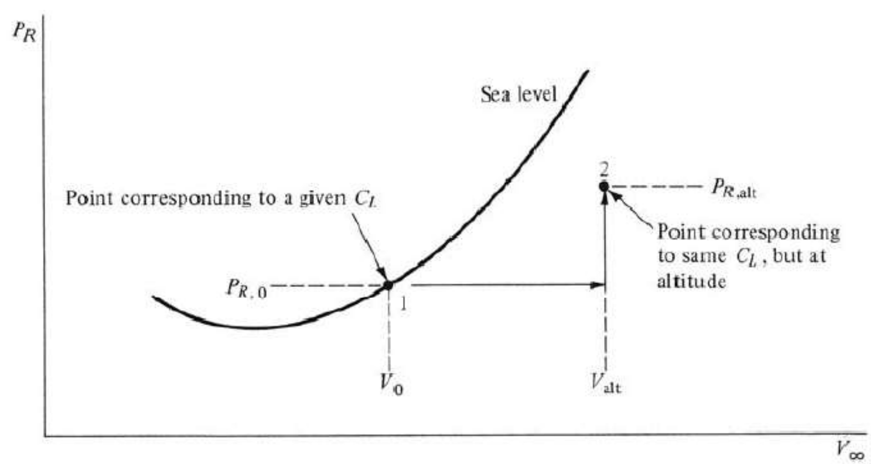


Figure 6.22 Correspondence of points on sea-level and altitude power-required curves.

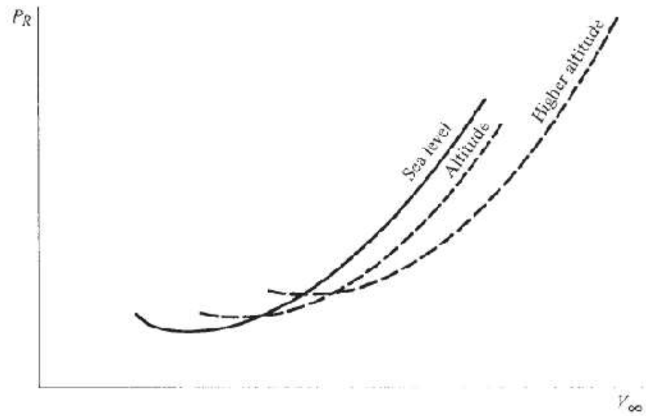


Figure 6.23 Effect of altitude on power required.

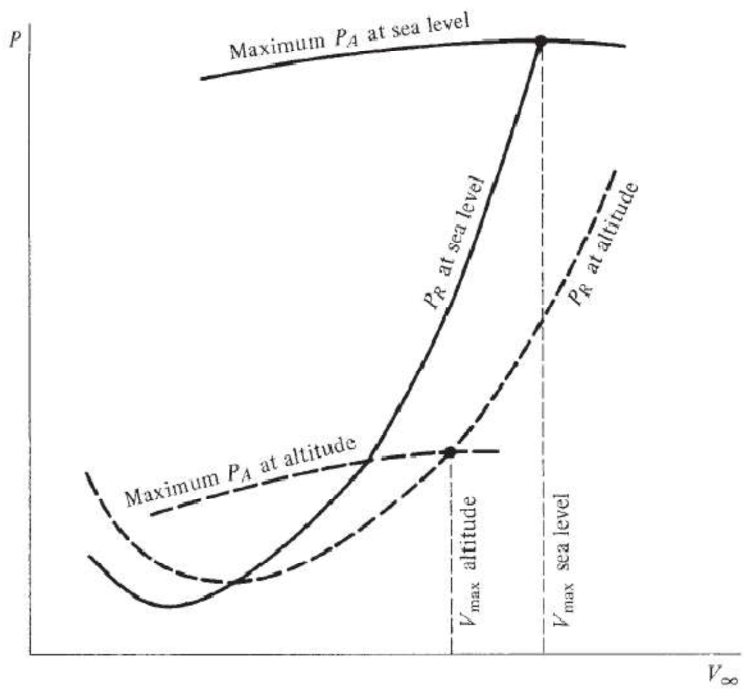


Figure 6.24 Effect of altitude on maximum velocity. (*a*) Propeller-driven airplane. (*continued*)

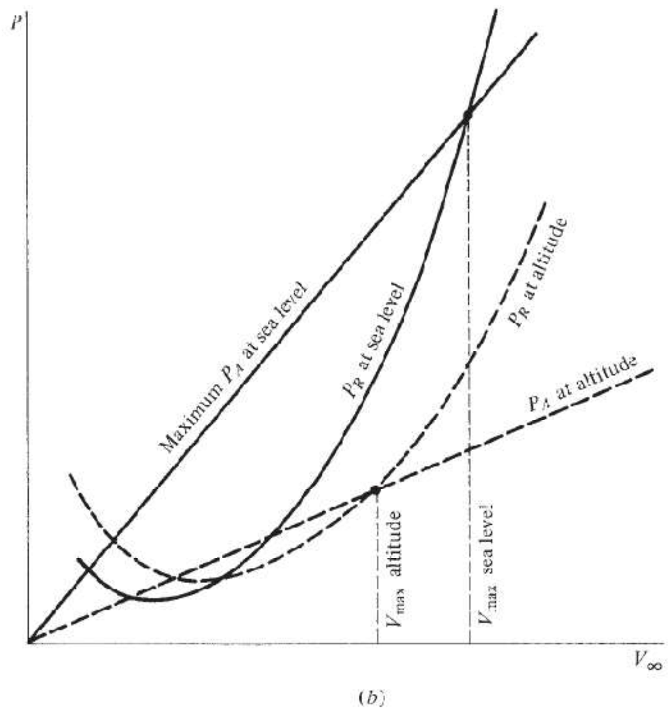


Figure 6.24 (*concluded*) (*b*) Jet-propelled airplane.

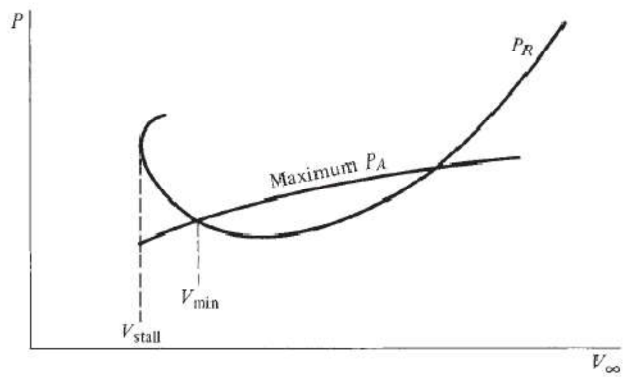


Figure 6.25 Situation when minimum velocity at altitude is greater than stalling velocity.

EXAMPLE 6.5

Using the method of this section, from the CJ-1 power-required curve at 22,000 ft in Example 6.4, obtain the CJ-1 power-required curve at sea level. Compare the maximum velocities at both altitudes.

■ Solution

From Eqs. (6.38) and (6.39), corresponding points on the power-required curves for sea level and altitude are, respectively,

$$V_0 = V_{alt}\left(\frac{\rho}{\rho_0}\right)^{1/2}$$

and

$$\text{hp}_{R,0} = \text{hp}_{R,alt}\left(\frac{\rho}{\rho_0}\right)^{1/2}$$

We are given V_{alt} and $hp_{R,alt}$ for 22,000 ft from the CJ-1 curve in Example 6.4. Using the formulas here, we can generate V_0 and $hp_{R,0}$ as in the following table, noting that

$$\left(\frac{\rho}{\rho_0}\right)^{1/2} = \left(\frac{0.001184}{0.002377}\right)^{1/2} = 0.706$$

Given Points		$\left(\frac{\rho}{\rho_0}\right)^{1/2}$	**Generated Points**	
V_{alt}, **ft/s**	hp_R,**alt**		V_0, **ft/s**	$hp_{R,0}$
200	889	0.706	141	628
300	741	↓	212	523
500	1190	↓	353	840
800	3713	↓	565	2621
1000	7012	↓	706	4950

These results, along with the hp_A curves for sea level and 22,000 ft, are plotted in Fig. 6.26. Looking closely at Fig. 6.26, note that point 1 on the hp_R curve at 22,000 ft is used to generate point 2 on the hp_R curve at sea level. This illustrates the idea of this section. Also note that V_{max} at sea level is 975 ft/s = 665 mi/h. This is slightly larger than V_{max} at 22,000 ft, which is 965 ft/s = 658 mi/h.

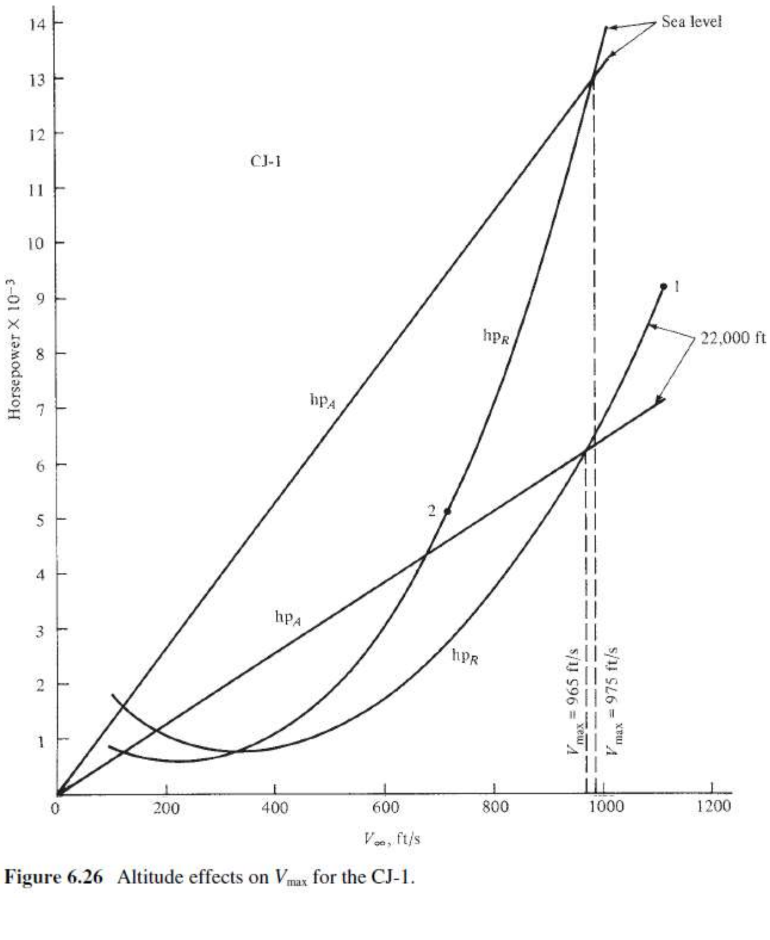


Figure 6.26 Altitude effects on V_{max} for the CJ-1.

EXAMPLE

For a given airplane in steady, level flight, prove that Eq. (6.39) relates the minimum power required at altitude, $(P_{R,\text{alt}})_{\min}$, to the minimum power required at sea level, $(P_{R,0})_{\min}$. In other words, prove that

$$\frac{(P_{R,\text{alt}})_{\min}}{(P_{R,0})_{\min}} = \left(\frac{\rho_0}{\rho}\right)^{1/2} \qquad \text{(E 6.6.1)}$$

■ **Solution**

Equation (6.39) relates a point on the power-required curve at altitude (point 2 in Fig 6.22) to the corresponding point on the power-required curve at sea level (point 1 in Fig. 6.22)

where C_L is the same value at both points. For the special case where point 1 in Fig. 6.22 pertains to the *minimum* P_R at sea level, we wish to prove that point 2 in Fig. 6.22 then pertains to the *minimum* P_R at altitude. This is not immediately obvious from the derivation given for Eq. (6.39), which depends just on the assumption of the same C_L at points 1 and 2.

From Eq. (6.27), the general formula for P_R is

$$P_R = \sqrt{\frac{2\,W^3 C_D^2}{\rho_s\, S\, C_L^3}} = \left(\frac{C_D}{C_L^{3/2}}\right)\sqrt{\frac{2\,W^3}{\rho\, S}} \qquad \text{(E 6.6.2)}$$

$$P_R = \frac{C_D}{C_L^{3/2}}\sqrt{\frac{2W^3}{\rho S}} = \frac{1}{(C_L^{3/2}/C_D)}\sqrt{\frac{2W^3}{\rho S}}$$

Thus

$$\frac{P_{R,\text{alt}}}{P_{R,0}} = \frac{(C_L^{3/2}/C_D)_0\sqrt{\dfrac{2W^3}{\rho\, S}}}{(C_L^{3/2}/C_D)_{\text{alt}}\sqrt{\dfrac{2W^3}{\rho_0\, S}}} = \left(\frac{\rho_0}{\rho}\right)^{1/2}\frac{(C_L^{3/2}/C_D)_0}{(C_L^{3/2}/C_D)_{\text{alt}}}$$

$$\frac{(P_{R,\text{alt}})_{\min}}{(P_{R,0})_{\min}} = \left(\frac{\rho_0}{\rho}\right)^{1/2}\frac{\left[(C_L^{3/2}/C_D)_0\right]_{\max}}{\left[(C_L^{3/2}/C_D)_{\text{alt}}\right]_{\max}}$$

می دانیم : $\left[(C_L^{3/2}/C_D)_0\right]_{\max} = \left[(C_L^{3/2}/C_D)_{\text{alt}}\right]_{\max}$ در نتیجه

$$\frac{(P_{R,\text{alt}})_{\min}}{(P_{R,0})_{\min}} = \left(\frac{\rho_0}{\rho}\right)^{1/2}$$

RATE OF CLIMB

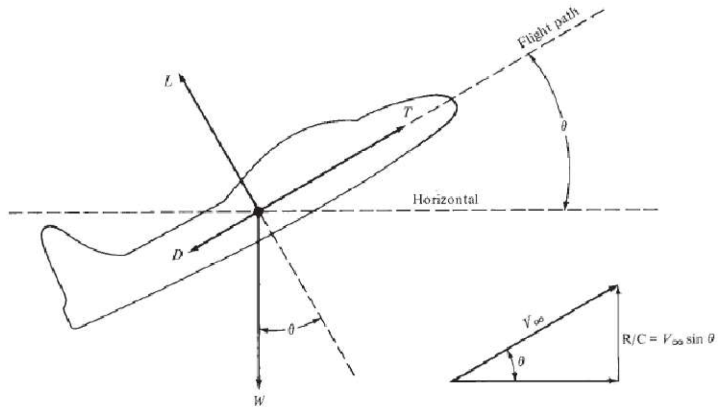


Figure 6.28 Airplane in climbing flight.

$$T = D + W\sin\theta$$

$$L = W\cos\theta$$

$$TV_\infty = DV_\infty + WV_\infty\sin\theta$$

$$\frac{TV_\infty - DV_\infty}{W} = V_\infty\sin\theta$$

از آنجا که $V_\infty \sin\theta$ سرعت در راستای قائم تعریف می شود به آن نرخ صعود می گویند

$$\boxed{\text{R/C} \equiv V_\infty\sin\theta}$$

$$\boxed{TV_\infty - DV_\infty = \text{excess power}}$$

$$\boxed{\text{R/C} = \frac{\text{excess power}}{W}}$$

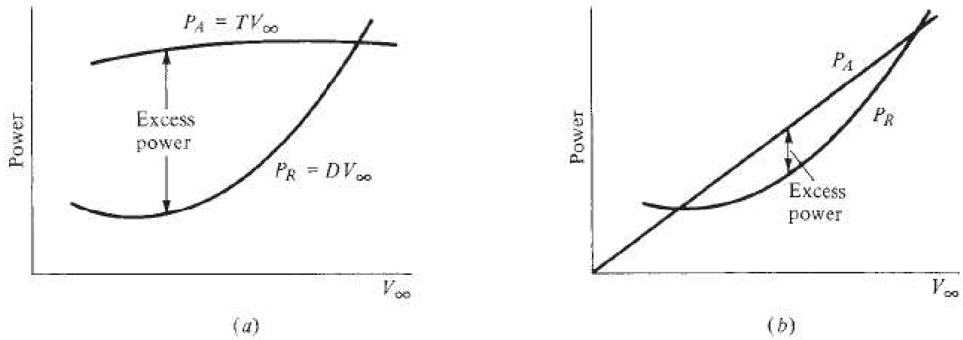


Figure 6.29 Illustration of excess power. (*a*) Propeller-driven airplane. (*b*) Jet-propelled airplane.

$$\max \text{ R/C} = \frac{\text{maximum excess power}}{W}$$

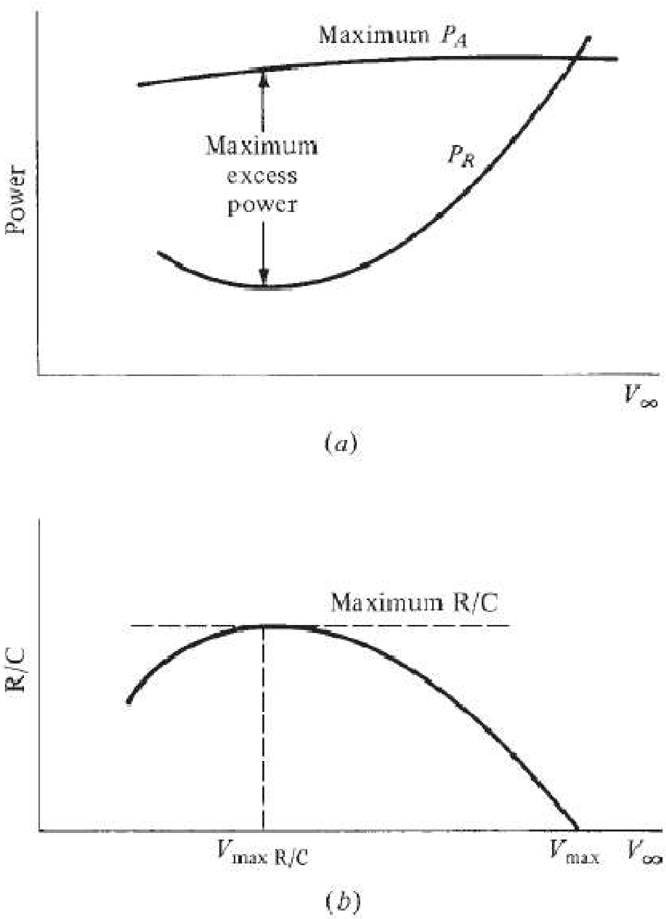


Figure 6.30 Determination of maximum rate of climb for a given altitude.

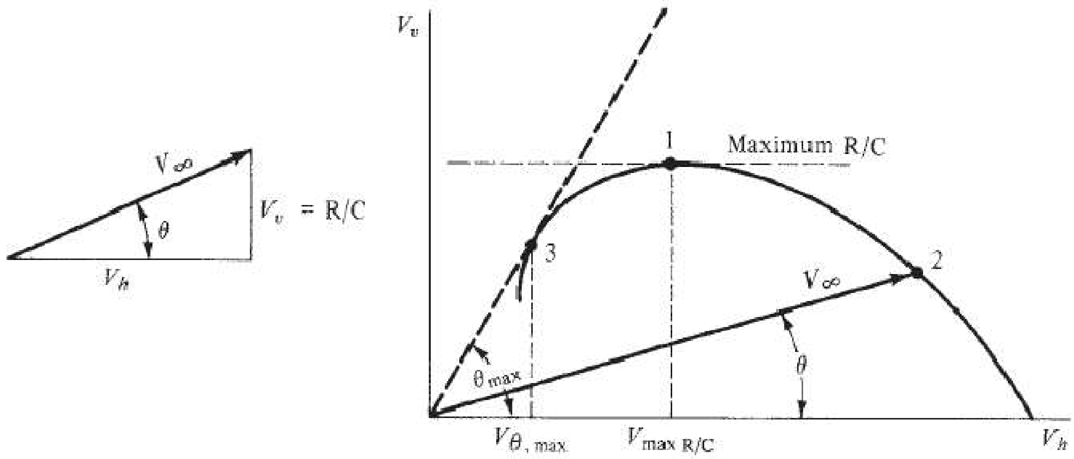


Figure 6.31 Hodograph for climb performance at a given altitude.

EXAMPLE

Calculate the rate of climb versus velocity at sea level for (*a*) the CP-1 and (*b*) the CJ-1.

■ **Solution**

a. For the CP-1, from Eq. (6.50),

$$\text{R/C} = \frac{\text{expcess power}}{W} = \frac{P_A - P_R}{W}$$

With power in foot-pounds per second and W in pounds, for the CP-1, this equation becomes

$$\text{R/C} = \frac{P_A - P_R}{2950}$$

From Example 6.3, at V_∞ = 150 ft/s, $P_R = 0.326 \times 10^5$ ft · lb/s. From Example 6.4, P_A = 550(hp$_A$) = 550(184) = 1.012×10^5 ft · lb/s. Hence

$$\text{R/C} = \frac{(1.012 - 0.326)\times 10^5}{2950} = 23.3 \text{ ft/s}$$

In terms of feet per minute,

$$\text{R/C} = 23.3(60) = \boxed{1398 \text{ ft/min}} \qquad \text{at } V_\infty = 150 \text{ ft/s}$$

This calculation can be repeated at different velocities:

V, ft/s	R/C, ft/min
100	1492
130	1472
180	1189
220	729
260	32.6

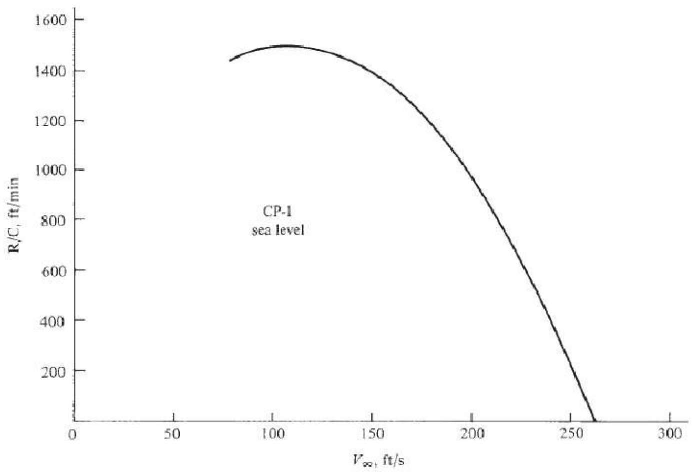


Figure 6.32 Sea-level rate of climb for the CP-1.

These results are plotted in Fig. 6.32.

b. For the CJ-1, from Eq. (6.50),

$$\text{R/C} = \frac{P_A - P_R}{W} = \frac{550\,(\text{hp}_A - \text{hp}_R)}{19{,}815}$$

From the results and curves of Example 6.5, at $V_\infty = 500$ ft/s, $\text{hp}_R = 1884$ and $\text{hp}_A = 6636$.

Hence
$$\text{R/C} = 550\left(\frac{6636 \quad 1884}{19{,}815}\right) = 132 \text{ ft/s}$$

or
$$\text{R/C} = 132(60) = \boxed{7914 \text{ ft/min}} \qquad \text{at } V_\infty = 500 \text{ ft/s}$$

Again, here is a short tabulation for other velocities for the reader to check:

V, ft/s	R/C, ft/min
200	3546
400	7031
600	8088
800	5792
950	1230

These results are plotted in Fig. 6.33.

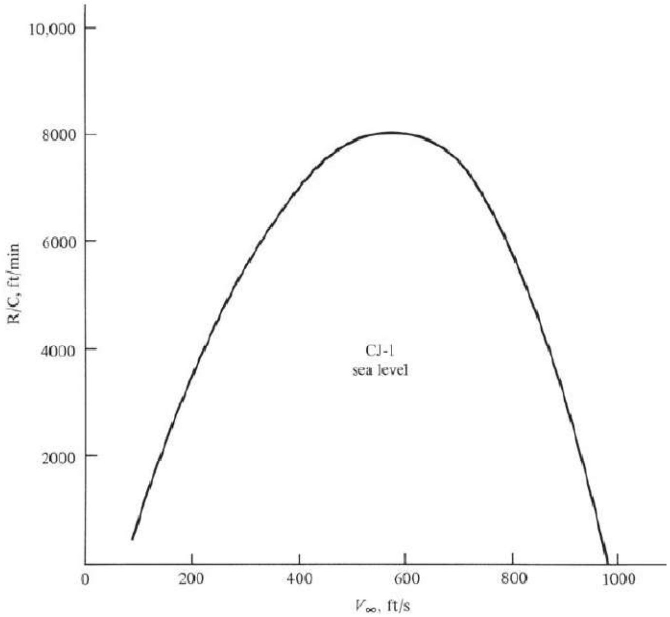


Figure 6.33 Sea-level rate of climb for the CJ-1.