Notes for Anand Tumpati

Lorenzo Vangelista

May 25, 2002 first revision; this revision October 12, 2022

1 Introduction - May 25, 2002

This notes are following a conversation with Anand Tumpati where he outlined he could not understand the paper [3]. In the following we will analyse the paper step by step.

2 System model - May 25, 2002

Equation (5) in [3] is slightly different from equation (13) in [1], which is correct. Th reason is that the paper [1] is centering the spectrum at 0. Note that the paper discuss about phase continuity wrongly since, as demonstrated in [1] LoRa is a phase continuous modulation by definition.

Tackling equation (9)

$$y_1(q) = \langle r(p, kT), c(q, kT) \rangle \tag{1}$$

$$= \sum_{k=0}^{2^{SF}-1} \sum_{i=0}^{L-1} h(i) c\left((k-i)T, p\right) \cdot c^*(q, kT)) \tag{2}$$

$$= \frac{1}{2^{SF}} \sum_{k=0}^{2^{SF}-1} \sum_{i=0}^{L-1} h(i) e^{\jmath\pi \frac{1}{2^{SF}} \left((k-i)^2 + 2p(k-i)\right)} e^{-\jmath\pi \frac{1}{2^{SF}} \left(k^2 + 2qk\right)} \tag{3}$$

$$= \frac{1}{2^{SF}} \sum_{k=0}^{2^{SF}-1} \sum_{i=0}^{L-1} h(i) e^{\jmath\pi \frac{1}{2^{SF}} \left(k^2 - 2ki + i^2 + 2pk - 2pi - k^2 - 2qk\right)} \tag{4}$$

$$= \frac{1}{2^{SF}} \sum_{k=0}^{2^{SF}-1} \sum_{i=0}^{L-1} h(i) e^{\jmath\pi \frac{1}{2^{SF}} \left(i^2 - 2ki - 2pi + 2pk - 2qk\right)} \tag{5}$$

$$= \frac{1}{2^{SF}} \sum_{i=0}^{L-1} h(i) e^{\jmath\pi \frac{1}{2^{SF}} \left(i^2 - 2pi\right)} \sum_{k=0}^{2^{SF}-1} e^{-\jmath \frac{2\pi}{2^{SF}} k(i-p+q)} \tag{6}$$

Now remembering that $\sum_{k=0}^{n} x^n = \frac{1-x^{n+1}}{1-x}$ we have

$$y_1(q) = \langle r(p,kT), c(q,kT) \rangle \tag{7}$$

$$= \begin{cases} \frac{1}{2^{SF}} \sum_{i=0}^{L-1} h(i) e^{\jmath\pi \frac{1}{2^{SF}}\left(i^2-2pi\right)} \frac{1-e^{-\jmath\frac{2\pi}{2^{SF}}(i-p+q)2^{SF}}}{1-e^{-\jmath\frac{2\pi}{2^{SF}}(i-p+q)}} & \text{if } i-p+q \neq 0, mSF\, m \in \mathbf{Z} \\ \frac{1}{2^{SF}} \sum_{i=0}^{L-1} h(i) e^{\jmath\pi \frac{1}{2^{SF}}\left(i^2-2pi\right)} \cdot SF & \text{if } i-p+q = 0, mSF\, m \in \mathbf{Z} \end{cases} \tag{8}$$

$$= \begin{cases} 0 & \text{if } i-p+q \neq 0, , mSF\, m \in \mathbf{Z} \\ \frac{1}{2^{SF-1}} \sum_{i=0}^{L-1} h(i) e^{\jmath\pi \frac{1}{2^{SF}}\left(i^2-2pi\right)} & \text{if } i-p+q = 0, mSF\, m \in \mathbf{Z} \end{cases} \tag{9}$$

Ignoring the case $i-p+q = mSF \neq 0$, $m \in \mathbf{Z}$ we have

$$y_1(q) = \langle r(p,kT), c(q,kT) \rangle = \frac{1}{2^{SF-1}} \sum_{i=0}^{L-1} h(i) e^{\jmath\pi \frac{1}{2^{SF}}\left(i^2-2pi\right)} \delta\left(i-p+q\right) \tag{10}$$

3 Open Issues - May 25, 2002

There are still several open issues:

1. π multiplicative coefficient in equation (9)
2. the $\mod 2^{SF}$ in equation (9)

Furthermore the inner product $\langle r(p,kT), c(q,kT) \rangle$ is ill–defined since the support of the function $r(p,kT)$ is larger than $0 \ldots 2^{SF}-1$ because of the convolution $c(p,kT) \otimes h(kT)$.

4 What is to be done to get to a suitable thesis - October 12, 2022

Please note that the following is not a comprehensive and fully detailed list of what should be done to have a suitable thesis; this is what is needed.

This is a preliminary provisional list which must be discussed in a face to face meeting. Bi-weekly progress reports must be provided. The meeting must be in person during office hours. I am not assuming the risk of misunderstandings, delays etc. if the meeting are not in person. Anyway the meetings, if online, should be during my office hours.

First of all you must use a style less "marketing" oriented and actually technical / scientific. There must be mathematical definitions of what you discuss, mathematical derivations, suitable graphs, tables etc.

- **introduction**n the introduction you should give a brief overview of the Low Power Wide Area Networks (i.e. SigFox, LoRaWAN, NB-IoT, Mioty), explain why LoRaWAN is the focus of your thesis, explain that in the current state of art the channel estimation and equalization is not widespread but it is useful, explain that you have found a way to improve the paper [3].
- **System model** an overall introduction of the LoRaWAN system should be given; then the LoRa signal mathematical description, e.g., according to [1], is to be given; the structure of the LoRaWAN packet should finally be given (preamble, etc.)
- **Channel estimation** Explain in detail the algorithms in [3] and explain in detail the "corrections" to be made to [3].
- **Performance evaluation** Showing that your channel estimation algorithm is improving the performance in terms of BER w.r.t. a system without the channel estimation; ideally you need to show also the improvement w.r.t. to [3]; for sure you need to describe the channels you use for testing the algorithms; you must be specific and not generic (e.g., saying "Rayleigh").
- **Conclusions** - *obvious*
- **Appendix 1** simulator validation: you should compare the performance in terms of BER in AWGN of the simulator against the thepretical results in [2]
- **Appendix 2 - totally optional and almost useless** printout of the MATLAB code

References

[1] Marco Chiani and Ahmed Elzanaty. "On the LoRa Modulation for IoT: Waveform Properties and Spectral Analysis". In: *IEEE Internet of Things Journal* 6.5 (2019), pp. 8463–8470. DOI: 10.1109/JIOT.2019.2919151.

[2] Tallal Elshabrawy and Joerg Robert. "Closed-Form Approximation of LoRa Modulation BER Performance". In: *IEEE Communications Letters* 22.9 (2018), pp. 1778–1781. DOI: 10.1109/LCOMM.2018.2849718.

[3] Yurong Guo and Zujun Liu. "Time-Delay-Estimation-Liked Detection Algorithm for LoRa Signals Over Multipath Channels". In: *IEEE Wireless Communications Letters* 9.7 (2020), pp. 1093–1096. DOI: 10.1109/LWC.2020.2981597.