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Viscous dissipation effects of power-law fluid flow within parallel plates with constant heat fluxes

C.P. Tso [a], J. Sheela-Francisca [b,*], Yew-Mun Hung [c]

[a] School of Mechanical & Aerospace Engineering, Nanyang Technological University, Post Code 639798, Singapore, Singapore
[b] Faculty of Engineering, Multimedia University, 63100 Cyberjaya, Malaysia
[c] School of Engineering, Monash University, 46150 Bandar Sunway, Malaysia

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ABSTRACT

Both hydro-dynamically and thermally fully developed laminar heat transfer of non-Newtonian fluids between fixed parallel plates has been analyzed taking into account the effect of viscous dissipation of the flowing fluid. Thermal boundary condition considered is that both the plates kept at different constant heat fluxes. The energy equation, and in turn the Nusselt number, were solved analytically in terms of Brinkman number and power-law index. The findings show that the heat transfer depends on the power-law index of the flowing fluid. Pseudo-plastic and dilatant fluids manifest themselves differently in the heat transfer characteristics under the influence of viscous dissipation. Under certain conditions, the viscous dissipation effects on heat transfer between parallel plates are significant and should not be neglected.

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1. Introduction

Heat transfer with effect of viscous dissipation has been studied intensively [1–5], and this effect is frequently encountered in many applications such as in material processing and in high speed flows. However, the effect of viscous dissipation in non-Newtonian flow of power-law fluids is comparatively less well-known.

Considering different thermal boundary conditions, simultaneously developing steady laminar flow of viscous non-Newtonian fluid flowing between parallel plates was investigated numerically [6]. When the fluid is having high viscosity, the flow is generally assumed to be dynamically fully developed. This happens in polymer processing where the high elastic fluids flow under non-isothermal conditions. The issue of thermal entry problem of pipe and channel flow was modeled and solved semi-analytically considering either an imposed wall temperature or a specified wall heat flux as thermal boundary conditions [7].

For the parallel plates, one plate kept at constant heat flux and the other plate being insulated, but moving with constant velocity, the analytical solution was obtained for visco-elastic fluids [8]. Giving importance to viscous dissipation, a numerical investigation is done for the Poiseuille–Couette flow of non-Newtonian fluids when one wall is at constant heat flux and the other insulated [9]. Another study [10] was done on heat transfer with effect of viscous dissipation for the flow of non-Newtonian fluids through parallel plates and circular tubes with thermal boundary conditions of uniform wall temperature. From the point of view of the second law of thermodynamics, the effect of viscous dissipation of single-phase non-Newtonian fluids on entropy generation in a circular microchannel was studied [11]. By considering non-Newtonian fluid flowing through a channel of heated parallel plates, and taking into account the effect of viscous dissipation, the second law was analysed and the temperature and entropy generation were reported [12]. Effect of viscous dissipation and convective heat transfer within a non-Newtonian thin liquid film on an unsteady stretching sheet was discussed [13].

Although numerous studies have been carried out on flow of power-law fluids with viscous dissipation in parallel plates, the analytical results for the case of both plates kept at different constant heat fluxes have not been reported in the literature. Hence, the present analytical study is motivated for scrutinizing the changes entailed in the convection heat transfer characteristics for power-law fluids due to the incorporation of the effect of viscous dissipation.

2. Statement of problem and mathematical formulation

Consider a steady laminar flow of a non-Newtonian fluid with constant properties between fixed infinitely long parallel plates distanced W or $2w$ apart, to be fully developed both thermally and hydro-dynamically. For the thermal boundary conditions, the case where the upper plate at constant heat flux q_1 while the lower plate

* Corresponding author. Tel.: +60 3 83125367; fax: +60 3 83183029.
E-mail address: franciscasheela@hotmail.com (J. Sheela-Francisca).

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Nomenclature

A_1–A_6	coefficients defined in Eqs. (18), (19), and (26)–(29)
A_c	cross-sectional area of channel (m^2)
Br_{q_1}	modified Brinkman number defined in Eq. (7)
C_1–C_4	coefficients defined in Eqs. (12)–(15)
c_p	specific heat at constant pressure (J/kg K)
h	convective heat transfer coefficient (W/m^2 K)
k	thermal conductivity (W/m K)
L	width of plate (m)
n	power-law index
Nu	Nusselt number, defined in Eq. (24)
P	pressure (Pa)
q_1	upper wall heat flux (W/m^2)
q_2	lower wall heat flux (W/m^2)
T	temperature (K)
T_0	wall temperature when both walls are kept at the same constant heat flux (K)
T_1	upper wall temperature (K)
T_2	lower wall temperature (K)
ΔT	general temperature difference (K)
u	velocity (m/s)
U	dimensionless velocity
w	half-channel height (m)
W	channel height (= $2w$) (m)
x	coordinate in the axial direction (m)
y	coordinate in the vertical direction (m)
Y	dimensionless vertical coordinate

Greek symbols

α	thermal diffusivity (m^2/s)
β	parameter defined in Eq. (7)
γ	parameter defined in Eq. (9)
θ	dimensionless temperature
θ_m	mean dimensionless temperature
η	consistency factor (pa s^n)
ρ	density (kg/m^3)
τ	shear-stress (Pa)

Subscripts

c	center-line
e	fluids entering
m	mean

at different constant heat flux q_2, as shown in Fig. 1, is considered.

For non-Newtonian fluids, the rheological behavior of a power-law fluid with constant fluid properties in between fixed parallel plates is described by the shear–stress relationship

$$\tau = -\eta\left|\frac{du}{dy}\right|^{n-1}\frac{du}{dy}, \tag{1}$$

where τ is the shear–stress, η is the consistency factor and du/dy is the velocity gradient perpendicular to the flow direction. The n is the power-law index, where the fluid is shear thinning or pseudo-plastic for $0<n<1$, Newtonian for $n=1$ and shear thickening or dilatant for $n>1$.

When the velocity boundary conditions are $u=0$ when $y=w$ and $y=-w$, the maximum velocity, u_c, occurs midway ($y=0$) between the two parallel plates. Following this, the well-known velocity distribution is given by

$$u = u_c\left[1-\left(\frac{y}{w}\right)^{(n+1)/n}\right] \tag{2}$$

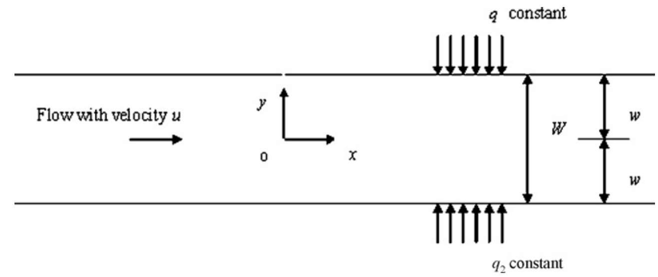

Fig. 1. Notation to the problem.

The energy equation, including the effect of viscous dissipation, is given by

$$\rho c_p u\frac{\partial T}{\partial x} = k\frac{\partial^2 T}{\partial y^2} + \eta\left|\frac{du}{dy}\right|^{n-1}\left(\frac{du}{dy}\right)^2, \tag{3}$$

where the second term on the right-hand side is the viscous-dissipative term. In accordance to the assumption of a thermally fully developed flow with uniformly heated boundary walls, the longitudinal conduction term is neglected in the energy equation [14]. Following this, the temperature gradient along the axial direction is independent of the transverse direction and given as

$$\frac{\partial T}{\partial x} = \frac{dT_1}{dx} = \frac{dT_2}{dx}, \tag{4}$$

where T_1 and T_2 are the upper and lower wall temperatures, respectively.

By taking $\alpha = k/\rho c_p$ and substituting Eqs. (2) and (4) into Eq. (3), it becomes

$$\frac{\partial^2 T}{\partial y^2} = \frac{u_c}{\alpha}\left[1-\frac{y^{(n+1)/n}}{w^{(n+1)/n}}\right]\frac{dT_1}{dx} - \frac{\eta}{\alpha\rho c_p}u_c^{n+1}\left(\frac{n+1}{n}\right)^{n+1}\frac{y^{(n+1)/n}}{w^{(n+1)^2/n}} \tag{5}$$

By introducing the non-dimensional quantities

$$Y = \frac{y}{W}, \quad \text{and} \quad \theta = \frac{T-T_1}{q_1 W/k}, \tag{6}$$

and by letting β, which is simply a dimensionless constant, and modified Brinkman number Br_{q_1}, respectively, be

$$\beta = \frac{u_c k W}{\alpha q_1}\frac{dT_1}{dx} \quad \text{and} \quad Br_{q_1} = \frac{\eta u_c^{n+1}}{q_1 W^n}, \tag{7}$$

Eq. (5) can be written as

$$\frac{d^2\theta}{dY^2} = \beta - \gamma Y^{(n+1)/n}, \tag{8}$$

where

$$\gamma = 2^{(n+1)/n}\left\{\beta + \left[\frac{2(n+1)}{n}\right]^{n+1} Br_{q_1}\right\}. \tag{9}$$

The thermal boundary conditions are

$$k\frac{\partial T}{\partial y} = q_1 \text{ at } y=w, \text{ or } \frac{\partial\theta}{\partial Y} = 1 \text{ at } Y=\frac{1}{2}, \quad T=T_1 \text{ at } y=w, \quad \text{or } \theta = 0 \text{ at } Y=\frac{1}{2}. \tag{10}$$

The solution of Eq. (8) under the above thermal boundary conditions can be obtained as

$$\theta(Y) = C_1 Y^{(3n+1)/n} + C_2 Y^2 + C_3 Y + C_4, \tag{11}$$

where

$$C_1 = -\frac{\gamma n^2}{(2n+1)(3n+1)}, \tag{12}$$

$$C_2 = \frac{\beta}{2}, \tag{13}$$

$$C_3 = \frac{\gamma n(1/2)^{(n+1)/n} + (2n+1)(2-\beta)}{2(2n+1)}, \tag{14}$$

$$C_4 = -\frac{\gamma n(1/2)^{1/n} + (3n+1)(4-\beta)}{8(3n+1)}, \tag{15}$$

To evaluate β in the above equation, a third boundary condition is required:

$$-k\frac{\partial T}{\partial y} = q_2 \quad \text{at} \quad y=-w, \quad \text{or} \quad \frac{\partial \theta}{\partial Y} = -\frac{q_2}{q_1} \quad \text{at} \quad Y=-\frac{1}{2}. \tag{16}$$

By substituting Eq. (16) into Eq. (11), β can be expressed as

$$\beta = A_1\left(1+\frac{q_2}{q_1}\right) + A_2 \mathrm{Br}_{q_1}, \tag{17}$$

with the coefficients A_1 and A_2, in terms of n, defined as

$$A_1 = \frac{2n+1}{(2n+1) - 2^{(2n+1)/n} n[(1/2)^{(3n+1)/n} + (-1/2)^{(3n+1)/n}]}, \tag{18}$$

$$A_2 = \frac{2^{(n^2+3n+1)/n} n[(n+1)/n]^{n+1}[(1/2)^{(3n+1)/n} + (-1/2)^{(3n+1)/n}]}{(2n+1) - 2^{(2n+1)/n} n[(1/2)^{(3n+1)/n} + (-1/2)^{(3n+1)/n}]} \tag{19}$$

In fully developed flow, it is usual to utilize the mean fluid-temperature, T_m, rather than the center-line temperature, when defining the Nusselt number. This mean or bulk temperature is given by

$$T_m = \frac{\int_{A_c} \rho u T dA_c}{\int_{A_c} \rho u dA_c}, \tag{20}$$

with A_c the cross-sectional area of the channel and the denominator on the right-hand side of Eq. (20) can be written as

$$\rho\int_{-w}^{w} u_c\left[1-\left(\frac{y}{w}\right)^{(n+1)/n}\right] dA_c = \rho u_c LW\left(\frac{3n+2+n(-1)^{1/n}}{4n+2}\right) \tag{21}$$

Using Eqs. (2) and (11), the numerator of Eq. (20) can be found. Therefore the dimensionless mean temperature is given by

$$\theta_m = \frac{k}{q_1 W}(T_m - T_1) \tag{22}$$

At this point, the convective heat transfer coefficient can be evaluated by the equation

$$q_1 = h(T_1 - T_m) \tag{23}$$

Defining Nusselt number to be

$$\mathrm{Nu} = \frac{hW}{k} = \frac{q_1 W}{k(T_1 - T_m)} = -\frac{1}{\theta_m}, \tag{24}$$

the Nusselt number can be evaluated and its explicit expression is given as

$$\mathrm{Nu} = \frac{A_3}{A_1A_5(1+q_2/q_1) + (A_4 + A_2A_5)\mathrm{Br}_{q_1} + A_6}, \tag{25}$$

where

$$A_3 = -12(5n+2)(4n+1)(3n+1)[3n+2+(-1)^{1/n}], \tag{26}$$

$$A_4 = 2^n\left(\frac{n+1}{n}\right)^n 6[n^2(4n+1)(n+1)(-1)^{2/n} + n(35n^3+59n^2+28n+4)(-1)^{(n+1)/n} - (81n^4+174n^3+127n^2+38n+4)] \tag{27}$$

$$A_5 = 6n(25n^3+35n^2+15n+2)(-1)^{1/n} + 3n^3(4n+1)(-1)^{2/n} + 222n^4+427n^3+286n^2+80n+8, \tag{28}$$

$$A_6 = 6n(100n^3+105n^2+36n+4)(-1)^{(n+1)/n} - 6(220n^4+343n^3+192n^2+46n+4). \tag{29}$$

3. Results and discussion

As the general result is too complex, various particular cases will next be presented in order to reveal the heat transfer characteristics. The values of n selected for discussion are 1/4, 1/2, 1, and 2.

3.1. Cases of unequal heat fluxes

3.1.1. Newtonian fluids

Nusselt number expressed in Eq. (24) characterizes the heat transfer between the fluid and the upper wall, with the inclusion of the effect of viscous dissipation. For a Newtonian fluid ($n = 1$), we have the established result,

$$\mathrm{Nu} = \frac{70}{26 - 9(q_2/q_1) + 24\mathrm{Br}_{q_1}}, \tag{30}$$

agreeing with Ref. [5].

3.1.2. Shear thinning fluids

For the pseudo-plastic fluids ($n < 1$), when $n = 1/4$,

$$\mathrm{Nu} = \frac{1}{16/39 - 44(q_2/q_1)/273 + 24711\mathrm{Br}_{q_1}/33721}, \tag{31}$$

and when $n = 1/2$,

$$\mathrm{Nu} = \frac{1}{4/9 - 7(q_2/q_1)/45 + 56638\mathrm{Br}_{q_1}/63061} \tag{32}$$

As expected, from Eq. (31), at a given ratio of (q_2/q_1), the graph (Fig. 2) Nu at $n = 1/4$ versus Br_{q_1} will form a rectangular hyperbola on both sides of an asymptote of

$$\mathrm{Br}_{q_1} = -\left(\frac{539536}{963729}\right) + \frac{1483724(q_2/q_1)}{6746103} \tag{33}$$

Five sets of curves are shown in Fig. 2, for the heat flux ratios of 0, 1, 5, 10 and 28/11.

The ratio 0 corresponds to the case of insulated lower plate, and the ratio of unity depicts the case of equal constant heat flux on both plates. The ratio 28/11 is of interest because the asymptote lies on the vertical axis.

3.1.3. Shear thickening fluids

For dilatant fluids ($n > 1$), when $n = 2$, the real part of Nu is

$$\mathrm{Nu} = \frac{126[-320(q_2/q_1) + 3753\mathrm{Br}_{q_1} + 976]}{6025(q_2/q_1)^2 - 36820(q_2/q_1) - 142560\mathrm{Br}_{q_1}(q_2/q_1) + 463968\mathrm{Br}_{q_1} + 1102977\mathrm{Br}_{q_1}^2 + 57028} \tag{34}$$

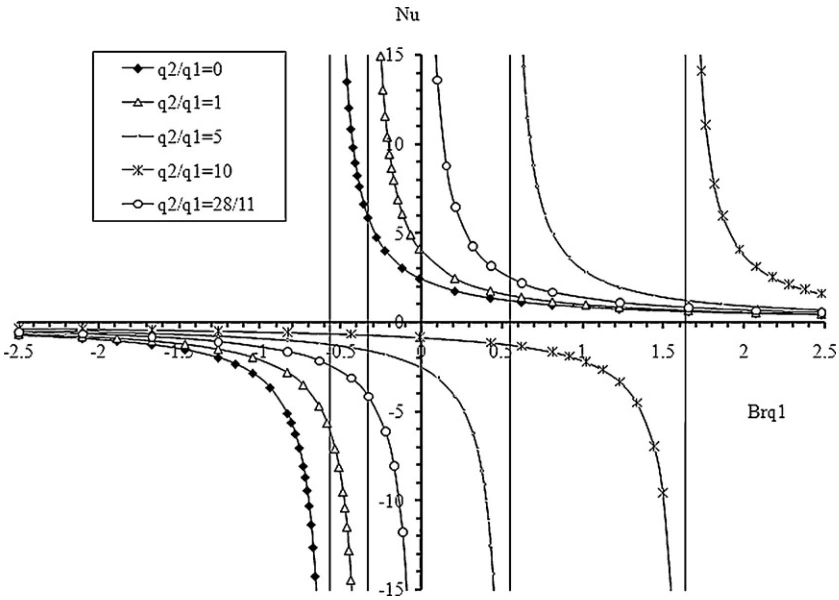

Fig. 2. Graph of Nu versus Br_{q_1} for $n = 1/4$.

The Nu–Br_{q_1} curves for dilatant fluids feature differently from those for Newtonian and pseudo-plastic fluids. Instead of manifesting as rectangular hyperbola with asymptotic values of Nu and Br_{q_1}, the Nu–Br_{q_1} curves for dilatant fluids appear as non-asymptotic forms, showing turning points in the variation of Br_{q_1} against Nu.

Table 1 shows the values of the turning points in the variation of Br_{q_1} against Nusselt number for $n = 2$ for the specified heat flux ratios. Moreover, when q_2/q_1 increases from 0 to 2.45 the Nusselt number decreases as Br_{q_1} increases. When $q_2/q_1 = 2.45$, the minimum occurs at (−0.0528, −266.55) and the maximum occurs at (−0.0495, 86.235). When q_2/q_1 increases from 2.451 to 10, there is an increase in Nusselt number as Br_{q_1} increases. When $q_2/q_1 = 2.451$, the non-asymptotic curve has the minimum at (−0.0528, −250.25) and the maximum at (−0.0494, 83.5). Therefore when q_2/q_1 increases from 2.45 to 2.451, the Nusselt number changes from decreasing to increasing. It is clear that the value of the heat flux ratio and the Brinkman number play significant roles in the heat transfer characteristics for a dilatant fluid with a power-law index.

Based on Eq. (34), four sets of curves are shown in Fig. 3, for the heat flux ratios of 0, 1, 5, and 10, for $n = 2$. It is observed again that the curves are not asymptotic and they have the maximum and minimum values for Nu. When $q_2/q_1 = 0, 1, 5, 10$, the minimum value that Nu takes is −1.4346, −2.4522, −3.9904, −1.3619, respectively, whereas the maximum value that Nu takes is 4.29, 7.3339, 1.3344, 0.2554 respectively.

It is noted that for pseudo-plastic fluids, when $n = 1/4$ and for Newtonian fluids when $n = 1$ the Nusselt number profiles against Br_{q_1} are asymptotic, and for dilatant fluids, when $n = 2$ the Nusselt number profiles against Br_{q_1} are not asymptotic, but they have turning points as explained in Table 1.

Table 1
Minimum and maximum points when Nu versus Br_{q_1} for various ratios of (q_2/q_1) at $n = 2$.

q_2/q_1	Minimum points		Maximum points	
	Br_{q_1}	Nu	Br_{q_1}	Nu
0	−0.3598	−1.4346	−0.1604	4.2900
1	−0.2331	−2.4522	−0.1165	7.3339
2	−0.1065	−8.4395	−0.0726	25.240
2.25	−0.0748	−21.655	−0.0616	64.994
2.35	−0.0621	−57.933	−0.0572	172.72
2.45	−0.0528	−266.55	−0.0495	86.235
2.451	−0.0528	−250.25	−0.0494	83.500
2.455	−0.0526	−235.29	−0.0489	74.886
2.46	−0.0524	−215.12	−0.0482	67.759
2.48	−0.0515	−145.91	−0.0457	49.115
3	−0.0287	−17.500	0.0202	5.8529
4	0.0152	−6.4994	0.1468	2.1731
5	0.0591	−3.9904	0.2735	1.3344
10	0.2785	−1.3619	0.9067	0.4554

3.2. Special case of lower plate insulated

For the case of lower plate insulated, $q_2 = 0$, and for Newtonian fluid, we obtain the established result

$$\mathrm{Nu} = \frac{35}{13 + 12\mathrm{Br}_{q_1}}, \tag{35}$$

agreeing with Ref. [5].

For the pseudo-plastic fluids, from Fig. 4, for $n = 1/4$, it is observed that when $\mathrm{Br}_{q_1} = -3, -2, -1$, the temperature distribution assumes positive values and it becomes 0 at $Y = 0.5$. When $\mathrm{Br}_{q_1} = 0, 1, 2$ and 3, the temperature distribution assumes negative values and it becomes 0 at $Y = 0.5$.

For the dilatant fluids, from Fig. 5, for $n = 2$, the real part of theta is plotted. It is observed that when $\mathrm{Br}_{q_1} = -3, -2, -1$, the temperature distribution assumes positive values and it becomes 0 at $Y = 0.5$. When $\mathrm{Br}_{q_1} = 0, 1, 2$ and 3, the temperature distribution assumes negative values and it becomes 0 at $Y = 0.5$. For $n = 1/4$,

$$\mathrm{Nu} = \frac{1}{16/39 + (24711/33721)\mathrm{Br}_{q_1}} \tag{36}$$

From Fig. 2, it is observed that when $q_2 = 0$ at $n = 1/4$, Nu versus Br_{q_1} is asymptotic and the asymptote appears at $\mathrm{Br}_{q_1} = -0.55984$.

For $n = 1/2$,

$$\mathrm{Nu} = \frac{1}{4/9 + (56638/63061)\mathrm{Br}_{q_1}} \tag{37}$$

For dilatants, at $n = 2$, the real part of Nu is

$$\mathrm{Nu} = \frac{126(976 + 3753\mathrm{Br}_{q_1})}{1102977\mathrm{Br}_{q_1}^2 + 463968\mathrm{Br}_{q_1} + 57028}, \tag{38}$$

verifying the findings in Table 1 and Fig. 3.

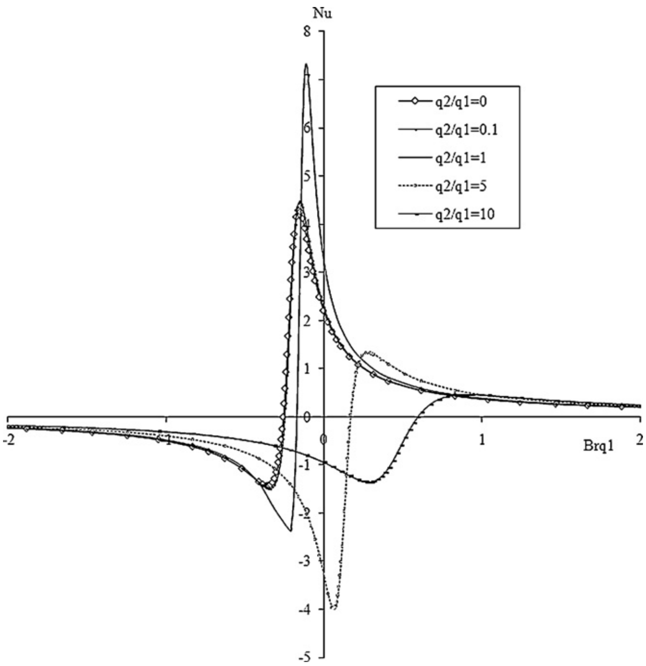

Fig. 3. Graph of Nu versus Br_{q_1} for $n = 2$.

3.3. *Case of equal heat fluxes*

Of particular interest here is the case when both the upper and lower plates are of equal heat flux, i.e., $q_1 = q_2$. An implicit expression was given in [15], but our explicit form, in Eq. (25) with $q_2/q_1 = 1$ is ready to be used.

3.3.1. *Newtonian fluids*

For the Newtonian fluid, the Nusselt number is reduced to

$$\mathrm{Nu} = \frac{70}{17 + 24\mathrm{Br}_{q_1}} = \frac{70}{17 + 27\mathrm{Br}'_{q_1}}, \tag{39}$$

where

$$\mathrm{Br}'_{q1} = \frac{\eta \bar{u}^{n+1}}{q_1 w^n}, \tag{40}$$

with $\bar{u}$ the mean velocity of the fluid. The expression of Nu in Eq. (39) corresponds to the classical problem of Poiseuille viscous-dissipative Newtonian flow in parallel plate channel. For verification of the present model, we observe that the $\mathrm{Nu} - \mathrm{Br}_{q_1}$ correlations in Eq. (39) are identical to those in [4] and [1], respectively, for fully developed flow of Newtonian fluid with isoflux boundary condition. For the case of no viscous dissipation, $\mathrm{Br}_{q_1} = 0$, the Nusselt number becomes Nu = 70/17.

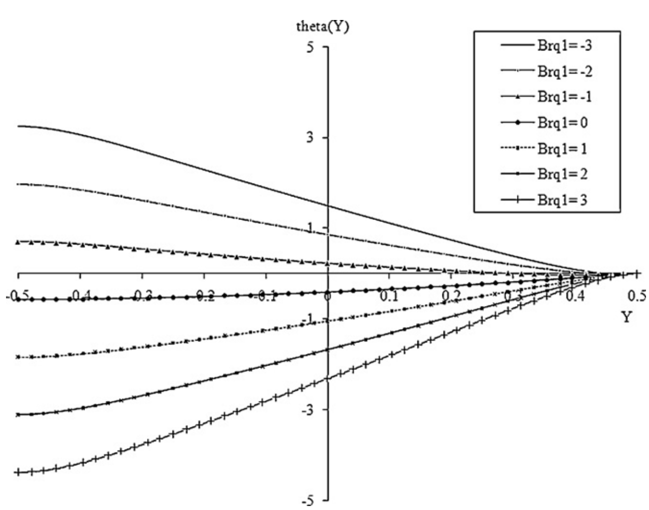

Fig. 4. Graph of $\theta(Y)$ versus Y for the case of insulated lower plate at $n = 1/4$.

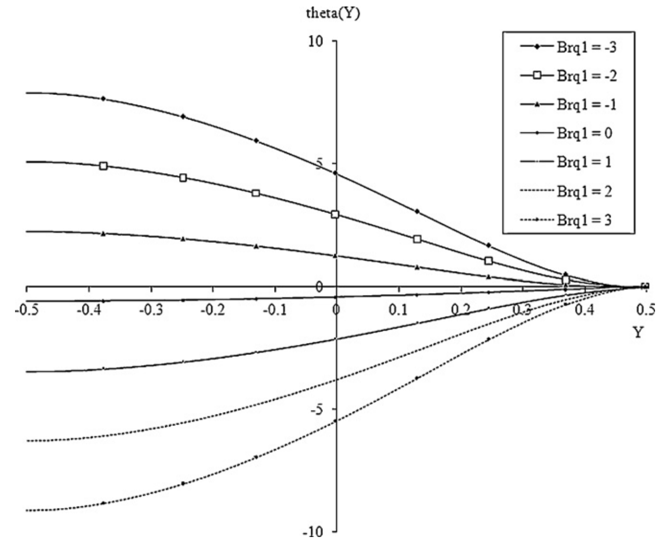

Fig. 5. Graph of $\theta(Y)$ versus Y for the case of insulated lower plate at $n = 2$.

3.3.2. *Shear thinning fluids*

For the pseudo-plastic fluids, when $n = 1/4$,

$$\mathrm{Nu} = \frac{1}{68/273 + (24711/33721)\mathrm{Br}_{q_1}}, \tag{41}$$

and when $\mathrm{Br}_{q_1} = 0$, Nu = 273/68.When $n = 1/2$,

$$\mathrm{Nu} = \frac{1}{13/45 + (56638/63061)\mathrm{Br}_{q_1}}, \tag{42}$$

and when $\mathrm{Br}_{q_1} = 0$, Nu = 45/13.

3.3.3. *Shear thickening fluids*

For dilatants, when $n = 2$, the real part of Nu is,

$$\mathrm{Nu} = \frac{126(656 + 3753\mathrm{Br}_{q_1})}{1102977\mathrm{Br}_{q_1}^2 + 321408\mathrm{Br}_{q_1} + 26233}, \tag{43}$$

and when $\mathrm{Br}_{q_1} = 0$, Nu = 82656/26233.

4. Conclusions

An explicit expression for Nusselt number has been obtained for fully developed power-law fluid flow between fixed parallel plates. The effect of viscous dissipation is found to be of essential importance in the heat transfer analysis. When both plates are kept at different constant heat fluxes, the dimensionless temperature distribution is given by Eq. (11), and the Nusselt number by Eq. (25), for all $n > 0$ and they are in terms of Br_{q_1}. When the upper plate is at constant heat flux and the lower plate insulated, the Nusselt number is obtained by substituting $q_2 = 0$ in Eq. (25), and selected results are Eqs. (35)–(38). For the case of equal constant heat fluxes at both the plates, the Nusselt number is obtained by substituting $q_1 = q_2$ in Eq. (25), and selected results are Eqs. (39), (41)–(43). For $n = 1/4$, the Nusselt number distribution against Br_{q_1} is asymptotic, whereas, for $n = 2$, Nusselt number distribution against Br_{q_1} is not asymptotic and maximum and minimum values occur at various points depending upon the ratio (q_2/q_1).

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