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OPTIMIZATION AND SENSITIVITY OF PRESTRESSED CONCRETE BEAMS

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Abstract—This paper is concerned with the optimum design of prestressed concrete beams. Both minimum weight and minimum cost optimization formulations are given for simply supported beams having three different sections. Sensitivity of the optimum designs, with respect to various design parameters, are also discussed. The formulation is programmed for interactive use on micro-computers. An example is given and results are discussed.

1. INTRODUCTION

This paper deals with the optimum design of prestressed concrete beams using linear programming. Mathematical programming techniques, and in particular linear programming, have been successfully used in several optimum designs of fully prestressed concrete [1–10], and recently, in partially prestressed concrete beams [11].

Optimal design of a prestressed concrete member is interesting, as well as being complex. The problem covers a variety of design considerations and parameters, including investigations into service and ultimate strengths, material characteristics, loading conditions, prestressing force, tendon configuration and cross-sectional dimensions. The objective function is either the weight or the cost; cost minimization addressing a more general problem.

The available studies on optimization of prestressed concrete beams differ from each other in (i) the objective function and what it includes, (ii) the fixed problem parameters, (iii) design variables, (iv) constraints and (v) solution algorithms. A comparatively general problem would be posed as a nonlinear program [1, 6, 9, 11]. Various linear programming formulations are suggested in [2–5, 7, 10].

In this paper, both minimum weight and minimum cost optimization formulations and solutions are given for simply supported beams having three different sections. In the minimum cost problem, the costs of concrete, steel and forming are included. The minimum weight problem considers weights of concrete and steel. The constraints cover working stresses, deflections, ultimate strength, buckling and section adequacy requirements. The prestressing force and the width of the cross-section (rectangular sections), or the width of the web (flanged sections) are chosen to be the design variables. This addresses not only a practical problem in prestressed concrete construction, but also allows for a linear programming formulation which has well-known advantages [3].

The paper presents sample results and also discusses the sensitivity of the optimum designs with respect to some unit cost, geometric, material and loading design parameters. The numerical calculations are carried out using a specially written interactive computer program operational on IBM personal computers, their compatibles and Apple Macintosh computers.

2. PROBLEM DEFINITION AND FORMULATION

We consider the total cost and the total weight minimization of simply supported prestressed concrete beams, having the cross-sections shown in Fig. 1.

For all cross-sections the depth, h, is shown, and the objective is to find the optimum width, b (rectangular cross-sections), or width of the web, b (flanged sections), and the prestressing force, F_i. The cross-sectional properties of the sections in Fig. 1 are taken as follows:

$$A_c = a_A b, \quad Z_t = a_{zt} b, \quad Z_b = a_{zb} b, \quad I = a_I b, \quad p = a_p b, \quad (1)$$

where A_c = area of concrete, Z_t = section modulus with respect to extreme top fibre, Z_b = section modulus with respect to extreme bottom fibre, I = moment of inertia, p = perimeter of concrete section, and a_A, a_{zt}, a_{zb}, a_I and a_p are constant parameters.

The objective function is either the total cost, i.e.

$$Z_c = C_s(L_s A_p) + C_c(LA_c) + C_f(pL) \quad (2)$$

or the total weight, i.e.

$$Z_w = A_p L_s \gamma_s + A_c L \gamma_c. \quad (3)$$

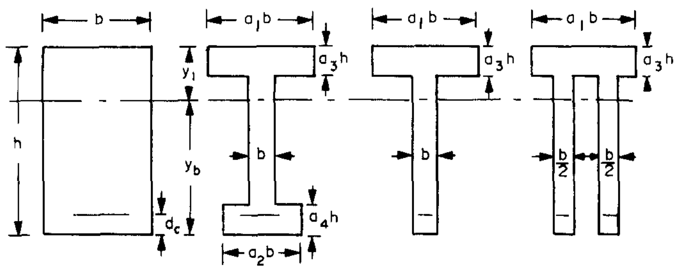

Fig. 1. Beam sections.

In eqns (2) and (3), C_s = cost of prestressing steel per unit weight, L_s = length of prestressing steel, A_p = area of prestressed reinforcement in tension zone, γ_s = unit weight of steel, C_c = unit cost of concrete, L = beam span, C_f = unit cost of forming and γ_c = unit weight of concrete.

We consider two categories of constraints, namely behavioural and side. The behavioural constraints are the working stresses, deflections, and the buckling load. The side constraints are the section adequacy constraints. The ultimate and cracking moments are not included as constraints, however, they are checked at the end of the optimization process.

The stresses under service load conditions must stay within permissible limits stated by design specifications. In this study the design requirements of ACI [12] are used. For deflections we impose three conditions representing (i) the initial stage of loading which normally starts at one-week-old concrete, (ii) a stage of loading which normally starts at 28-day-old concrete and (iii) the final loading stage taking care of sustained loads. These are represented as

$$\delta_p + \delta_G \geqslant \bar{\Delta}_c, \quad \delta_{ll} \leqslant \Delta_{ll}, \quad \delta_{sus} + \delta_{dl} + \delta_{ll} \leqslant \bar{\Delta}_u, \tag{4}$$

where δ_p = deflection due to prestressing force, δ_G = deflection due to self-weight, δ_{ll} = deflection due to superimposed live load, δ_{dl} = deflection due to superimposed dead load, δ_{sus} = deflection due to sustained loading = $\delta_{psus} + \delta_{Gsus}$, $\bar{\Delta}_c$ = allowable camber, $\bar{\Delta}_{ll}$ = allowable live load deflection and $\bar{\Delta}_u$ = maximum allowable deflection.

Since, prestressed concrete simply supported beams are members under compression, and since in many cases very long and slender beams are used, buckling may occur. To avoid this, we restrict F_i to be less than the critical axial load defined by Euler's equation.

ACI [12] requirements for the ultimate strength of the section and the cracking moment are

$$M_u \leqslant \phi M_n \tag{5}$$

and

$$\phi M_n \geqslant 1.2 M_{cr},$$

where M_u = strength design moment, M_n = nominal moment resistance, M_{cr} = cracking moment and $\phi = 0.9$. As was stated before, these two conditions have not been used as constraints, but were checked for the optimal solution.

Finally, two more constraints imposed are the section adequacy constraints for the minimum required section moduli

$$Z_t \geqslant \frac{\Delta M + (1+\eta) M_{\min}}{\bar{\sigma}_{cs} - \eta \bar{\sigma}_{ti}}, \quad Z_b \geqslant \frac{\Delta M + (1-\eta) M_{\min}}{\eta \bar{\sigma}_{ci} - \bar{\sigma}_{ts}}, \tag{6}$$

where ΔM = bending moment due to superimposed dead and live load, $M_{\min}$ = minimum bending moment at section considered under service load conditions, η = ratio of final prestressing force to initial prestressing force, $\bar{\sigma}_{cs}$ = allowable service (final) compressive stress in concrete, $\bar{\sigma}_{ti}$ = allowable initial tensile stress in concrete, $\bar{\sigma}_{ci}$ = allowable initial compressive stress in concrete and, $\bar{\sigma}_{ts}$ = allowable service (final) tensile stress in concrete.

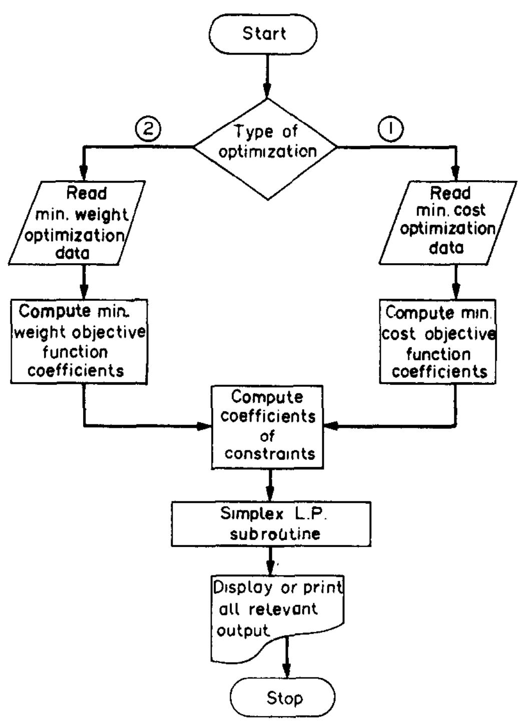

Fig. 2. Flowchart of PRECO.

Table 1. Data for the examples

Design variables		Value
Sections, Rect. (1), I(2), T(3)		1, 2, 3
Height	h	40
Concrete cover (in)	d_c	3
Beam span (ft)	L	70
Unit weight of concrete (lb/ft^3)	γ_c	150
Unit weight of steel (lb/ft^3)	γ_d	490
Ratio of final to initial PF	η	0.83
Concrete compressive strength (psi)	f'_c	5000
Tensile strength of PS steel (ksi)	f_{pu}	250
Superimposed dead load (lb/ft)	DL	40
Superimposed live load (lb/ft)	LL	400
Allowable camber (in)	Δ_{cam}	−2.3333
Allowable live load deflection (in)	Δ_{ll}	2.333
Upper limit of displacement (in)	Δ_u	3
Reinforcement edge eccentricity (in)	e_1	0
Reinforcement trajectory		1
Age of beam at initial load (Days)	t_A	7
Age of beam at final loading (Days)		30
Total life of structure (Years)	t	40
Type of curing, moist (1) or steam (2)		1
Type of cement, Type I (1) or Type III (2)		1

3. COMPUTER IMPLEMENTATION

The ten constraints indicated above and the objective functions given in eqns (2) and (3) are then used to define a linear program using two design variables, namely, b and F_i, and the solution is carried out using the Simplex algorithm. The computer program PRECO (Fig. 2) is written in Microsoft Quick Basic version 4.0, and is operational on any IBM computer or its compatibles. The program can also be introduced to Apple Macintosh computers without any changes if Macintosh Microsoft Basic is used. PRECO is an interactive program, therefore the required data for any problem is fed to the computer by answering questions appearing on the computers monitor.

4. NUMERICAL APPLICATIONS

For illustrative purposes we report the results obtained for a 70 ft long beam with a parabolic prestressing reinforcement trajectory. The beam carries 400 lb/ft as dead load, in addition to its own weight. Rest of the relevant data on materials, geometry and loading are given in Table 1. Using this data three examples are considered for cost optimization. The results are shown in Table 2.

Since we have only two design variables the solution can also be obtained using the direct method. The results are shown in Figs 3–5. The check for the ultimate strength and cracking moment of the sections showed that they do not govern the solutions.

Table 2. Results of examples

Example	Section	b (in)	F_i (lb)	Z_c (SR)†
1	□	6.66	230,156.7	1798.02
2	I	2.83	122,620.3	1435.48
3	T	3.81	195,621.2	1204.38

† SR = Saudi Riyals.

5. SENSITIVITY OF THE OPTIMAL DESIGNS

In any optimization problem a change in the design parameters may affect the optimal solution, i.e. the solution may be sensitive with respect to such parameters. For the optimal design problem reported in this paper an extensive sensitivity analysis in cost and weight minimization have been carried out to test the effects of (i) C_s/C_c ratio, (ii) beam span, L, (iii) beam height, h, and (iv) compressive strength of concrete, f'_c, on the optimal solution. Some of the results are shown in Table 3 and Figs 6–13. The effect of C_s/C_c ratio on the cost minimization problem is shown in Table 3. The rectangular section b and F_i values are

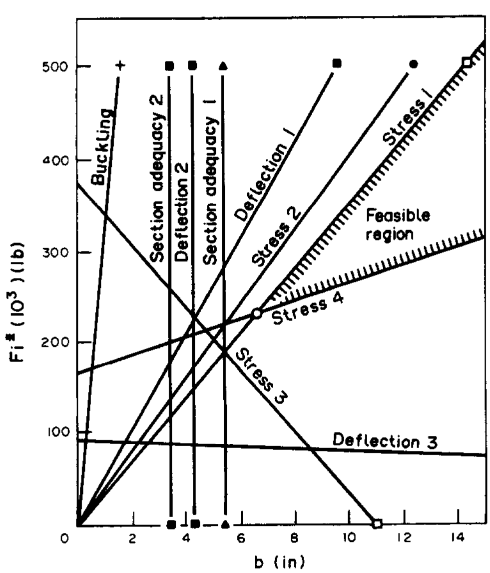

Fig. 3. Graphical solution of Example 1.

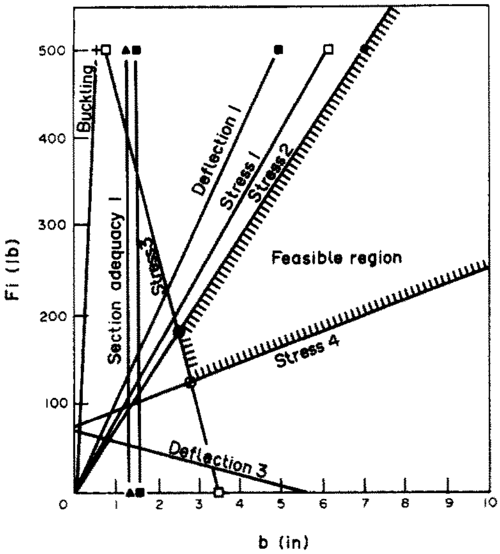

Fig. 4. Graphical solution of Example 2.

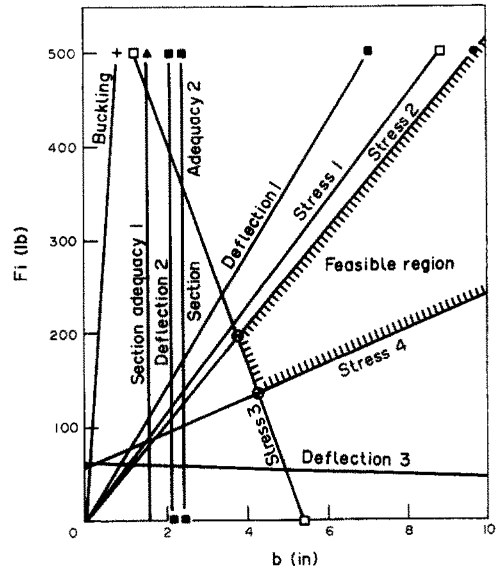

Fig. 5. Graphical solution of Example 3.

Table 3. Sensitivity of optimum design vs C_s/C_c ratio

	React. section beam			I-section beam			T-section beam		
$\frac{C_s}{C_c}$	b (in)	F_i (lb)	O.F. (SR)	b (in)	F_i (lb)	O.F. (SR)	b (in)	F_i (lb)	O.F. (SR)
45	6.66	230156.70	1230.25	2.83	122620.30	971.12	4.32	134915.10	1256.45
40	6.66	230156.70	1210.61	2.83	122620.30	939.31	4.32	134915.10	1230.44
35	6.66	230156.70	1150.97	2.83	122620.30	907.31	3.81	195621.20	1194.69
30	6.66	230156.70	1091.33	2.83	122620.30	875.70	3.81	195621.20	1143.92
25	6.66	230156.70	1031.69	2.83	122620.30	843.90	3.81	195621.20	1093.16
20	6.66	230156.70	972.05	2.53	177631	797.99	3.81	195621.20	1042.39

not sensitive to the change in C_s/C_c ratio. The I-section b and F_i values are sensitive when the C_s/C_c ratio is less than 25 and the T-section b and F_i values show sensitivity for $C_a/C_c > 35$.

The effect of changing the beam span, L, on the design variables and the total weight are shown in Figs 6–8. It is seen that the shape of the b versus L, curve changes when $L > 80$. The same applies to the total weight versus l curve. The prestressing force increases with the increase in l, as expected.

Figures 9–11 show the effect of the cross-sectional height, h, on the design variables and the total weight of the beam. b, F_i and the total weight all decrease with increase in h. For 20 in $< h <$ 35 in, b and F_i are

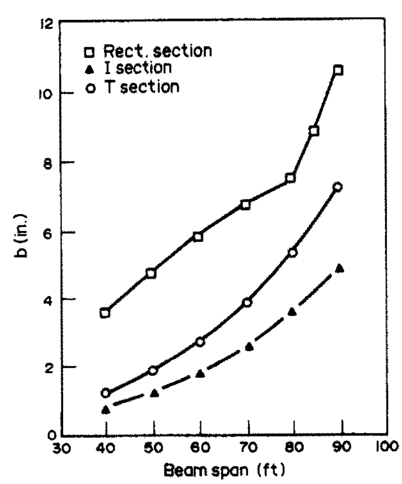

Fig. 6. Span effect on the widths of the beams.

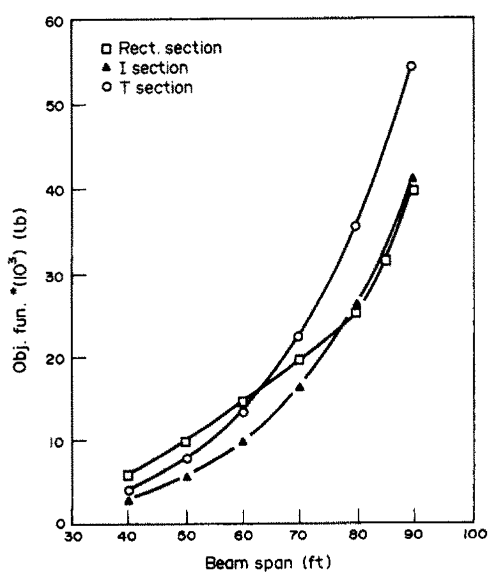

Fig. 7. Span effect on the weight of the beams.

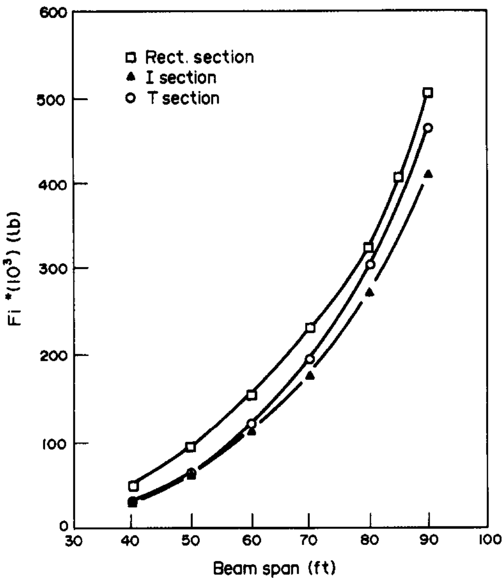

Fig. 8. Span effect on the prestressing force.

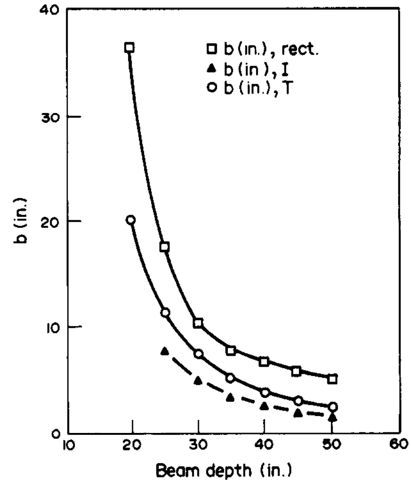

Fig. 10. Beam depth effect on the widths of the beams.

quite sensitive to a change in h. The rectangular section beam does not show much sensitivity in changes for h for $h > 35$ in.

The effect of f_c' on b, F_i and the total weight of the rectangular section beam are shown in Figs 12 and 13, showing that some savings in b and F_i can be made but not at a significant degree.

6. CONCLUSIONS

The paper considers optimum design of simply supported prestressed concrete beams using linear programming. Both minimum cost and minimum weight designs are studied. The formulation is programmed for interactive use on micro-computers. Sensitivity of the optimum solutions with respect to several design parameters is also discussed. Some of the main points and results of the study may be summarized as follows:

1. A linear programming problem is obtained using transformation of variables.
2. The design variables have been chosen to be the prestressing force, F_i and the widths, b, of the considered sections. This allows the use of the direct search method to be used graphically. The constraints are defined on stresses, deflections, section adequacy and buckling.
3. The working stress inequality constraints are expressed in terms of F_i and b. The eccentricity is taken at its maximum practical value, leading to a more efficient use of the prestressing force. This seems to be more realistic than the common practice of assigning a known section and determining the optimum F_i and the eccentricity.

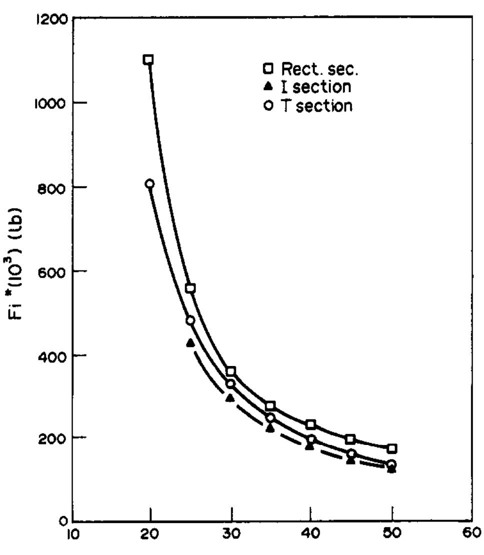

Fig. 9. Beam depth effect on the widths of the beams.

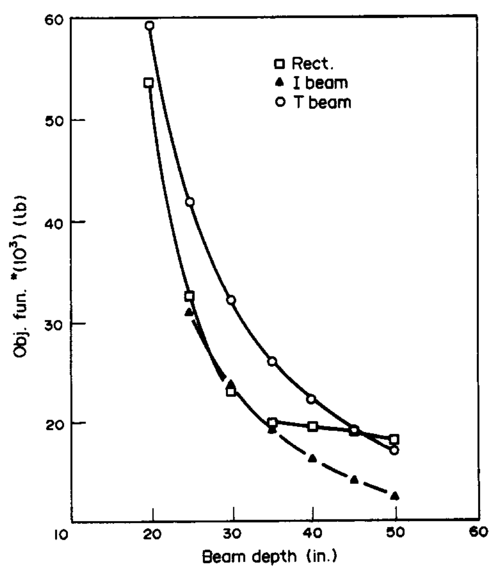

Fig. 11. Beam depth effect on the weight of the beams.

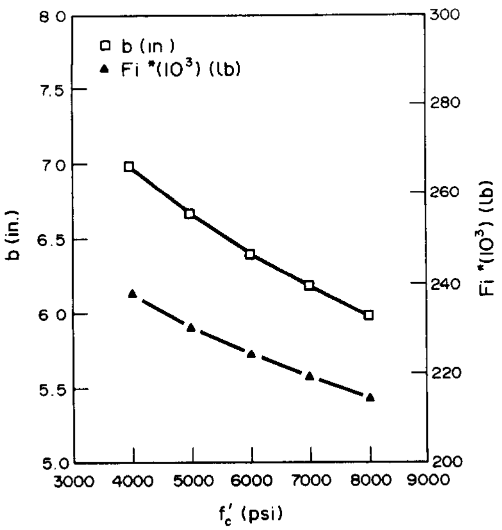

Fig. 12. Effect on f'_c on the rectangular section beam.

4. The example design problems, although limited in number, show that the minimum cost and minimum weight optimizations give the same results, for beams of rectangular sections, for $L < 85$ ft.

5. For I-beams where $C_s/C_c < 25$ and T-beams where $C_s/C_c < 35$ the results of minimum cost and minimum weight optimizations give the same results.

6. b and F_i values are sensitive to changes in span, L, as expected.

7. Again as expected, b and F values are decreased with a decrease in beam height, h. However, especially for rectangular section beams, this decrease is quite small for $h > L/24$.

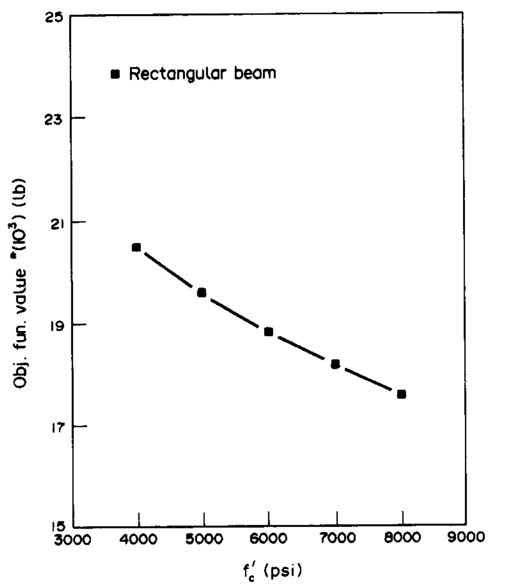

Fig. 13. Effect of f'_c on the weight of the rectangular section beam.

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