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Evaluation of insurance companies considering uncertainty: A multi-objective network data envelopment analysis model with negative data and undesirable outputs

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ABSTRACT

Uncertainty is an important issue to consider when evaluating entities in both public and private sectors. On the other hand, many operations have more than one stage process when some inputs are fed to the system to produce a number of intermediate measures. The intermediate measures are then transformed into final products in the subsequent stages. The composition method in network data envelopment analysis (NDEA) is a popular method for measuring the efficiency of a two-stage process. The composition method is fractional bi-objective programming that is solved by non-linear programming techniques such as bisection search. In this paper, the two-stage NDEA is extended with negative data and undesirable outputs. First, we propose an alternative linear model based on the goal programming technique to avoid complex non-linear calculations. Then, we use a method to transform negative data into positive and undesirable outputs into desirable ones. Finally, we develop the proposed model using the fuzzy α-cut approach in order to incorporate data uncertainty in the linear goal programming (GP) model. To validate the accuracy of the proposed model, a numerical example is solved. To show the applicability of the proposed model, a real case of 22 insurance companies is examined. We also perform a comparative analysis to specify the benchmark and inefficient companies. Comparative analysis can help managers to recognize where improvement should be investigated with priority.

1. Introduction

Most organizations today operate in a competitive and dynamic environment, an environment whose variables are constantly changing, and it is challenging to predict these changes. On the other hand, organizations spend a lot of time and money to achieve their goals. Therefore, evaluating the performance of organizations in achieving their goals and understanding the organization's position in a complex and dynamic environment is very important for managers.

As one of the financial institutions, the insurance industry has a special place in economic growth and development, so that performance of this sector will stimulate other economic sectors. The small amount of money paid as a premium to insurance companies constitutes a sizeable financial capital that contributes to the development of the economy and also provides financial security for insured customers against various events leading to damages. Therefore, one of the factors of the economic growth of any country depends on the development of the insurance industry of that country. One cannot expect a country to achieve economic excellence until the insurance industry can provide the necessary conditions for the safe presence of domestic and foreign investors in various economic sectors.

During the last decade, Iran's insurance industry has undergone significant changes such as the entry of the private sector into the market, leading to the abolition of government monopoly, implementing the policy of privatization, liberalization, and deregulation. According to the developments and government policies based on the promotion and improvement of the country's insurance industry and creating a competitive environment, insurance companies require to design and implement a comprehensive system to evaluate their performance to improve the quality of their services and to determine their current position in comparison with competitors, which helps them to survive and progress in today's competitive world.

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The main question of this study is: which insurance company in Iran is overall efficient? In addition, there are two related questions to the main question: which insurance company in Iran is efficient profitability? And which insurance company in Iran is efficient operationally? Several methods have been introduced to evaluate the performance of organizations, including parametric and non-parametric models. Due to the limitations of parametric methods in evaluating financial institutions' performance [1,2], researchers have developed non-parametric frontier-based models capable of estimating the efficiency of Decision-Making Units (DMUs) without pre-defined production functions. Frontier-based models measure the relative efficiency of a group of homogeneous DMUs. In other words, frontier-based models assess how well DMUs operate compared to each other in the homogeneous condition through determining the efficient frontier associated with the best DMU. Data Envelopment Analysis (DEA) introduced by Charnes et al. [3]; as a non-parametric frontier-based model, is one of the most well-known and popular methods with a wide spectrum of applications to calculate the efficiency of homogeneous DMUs.

DEA method has been widely used for calculating the organizational performance of different sets of DMUs such as schools, hospitals, banks, and insurance companies in the literature [4]. Emrouznejad and Yang [4] presented an extensive listing of DEA-related studies from 1978 to the end of 2016. DEA contributes a fair measure of the performance of each DMU compared to similar DMUs [5]. This method's main advantage over other methods is its ability to examine the relative efficiency of DMUs with multiple inputs and multiple outputs simultaneously, without using a predefined production function. Basic DEA models treat DMU as a black box without considering its internal relations. However, in some real-world cases such as insurance companies, the workflow consists of multiple stages. In other words, each DMU consumes some inputs to produce intermediate products in one stage, then the intermediate products are used to generate the final outputs. Network DEA (NDEA) models are developed to evaluate such DMUs with multiple stages [6].

This study evaluates the efficiency of Iranian insurance companies using a two-stage NDEA model. The considered model uses multiple inputs in the first stage to produce multiple intermediate measures. Intermediate measures are then fed to the second stage to generate the final outputs. Furthermore, an independent input is also fed to the second stage, which has no relation with the first stage. The first stage calculates the operational efficiency and the second stage calculates the profitability efficiency. In a recent study by Ref. [7]; they developed a multi-objective non-linear programming model to estimate the stage efficiencies and then calculate the overall efficiencies by multiplying the stage efficiencies. This approach has two main drawbacks: 1-The proposed multi-objective model is a non-linear program that needs complex multi-step calculations 2- The developed model cannot handle data uncertainty. Also, the proposed model needs to rewrite to include the negative data. Thus, we propose an alternative method based on the goal programming approach to develop a single objective linear model equivalent to the multi-objective non-linear model proposed by Ref. [7]. Necessary data transformation proposed by Koopmans [8] and Tone et al. [9] is applied to convert negative data into positive and undesirable outputs into desirable. We also employed a fuzzy technique based on the $\alpha - cut$ approach to include data uncertainty into the proposed model.

The remainder of this study is as follows: The related literature is surveyed in section 2. In section 3, the proposed single objective fuzzy goal programming DEA is described. A numerical example is also presented in this section to investigate the validity of the proposed model. The case study and the achieved results are explained in section 4. In section 5, the conclusion of this paper is presented.

2. Literature review

The insurance industry can be easily fluctuated by diverse challenges despite its profitability market. Eling and Luhnen [10] declared that insurance firms compete and operate in a swiftly developing environment that requires an objective and reliable estimation of individual performance and efficiency. Among various methods that have been developed for performance evaluation and comparison, DEA has been extensively used in the literature to calculate the relative efficiency of insurance firms. This section provides a brief survey of the studies that have been carried out about the insurance industry in various countries in previous years, irrespective of the approach adopted.

Tone and Sahoo [11] developed a new variant of the DEA model to study the efficiency of life insurance corporations in India. Hwang and Tong-Liang [12] measured the managerial efficiency of non-life insurance companies using a two-stage DEA introduced by Seiford and Zhu [13]. They also applied a Tobit regression model to investigate factors that notably affect managerial efficiency. Gharakhani et al. [14] used the common weight method in dynamic network DEA to achieve appropriate weights for each input/output variable, and then they used the goal programming approach to estimate the efficiency of Iranian insurance companies [15]. used DEA to measure the efficiency score of 53 Chinese property insurance companies. They also used Tobit regression models to check which factors had the most effect on profitability. Kao and Hwang [16] stated that the process of insurance companies could be divided into two sub-processes. They proposed a novel relational DEA approach by defining a series relationship between two sub-stages. Under this framework, the efficiency of the whole process can be decomposed into the product of the efficiencies of the two sub-processes instead of calculating each stage's efficiency independently. Kao and Liu [17] used a fuzzy two-stage network DEA to calculate the efficiency of non-life insurance companies in Taiwan under uncertain conditions.

With respect to the efficiency fluctuations in different years, it can be concluded that a company's inefficiency in one year does not mean the general inefficiency of that company and vice versa. Hence, Sinha (2015) proposed a dynamic slack-based DEA model to evaluate Indian life insurance companies during a seven-year period using a link variable to make a common benchmark between different years. In their study, Wanke and Barros [18] indicated that heterogeneity in the insurance sector can impact performance. They developed an approach that integrates the two-stage DEA with data mining to assess the efficiency of insurance companies and determine the major efficiency drivers. Ertuğrul et al. [19] applied both DEA-CCR and DEA-BCC models to evaluate 12 insurance companies in Turkey. Nasseri et al. [20] proposed fuzzy stochastic DEA model to deal with randomness and fuzziness of parameters. They employed insurance companies as an example to show validation of the proposed model.

Changes in the business environment such as deregulation and widespread economic changes can significantly affect the efficiency of insurance companies. Eling and Schaper [21] addressed this issue in their study by evaluating 970 insurance companies from 14 European countries. The findings showed that the three main drivers of efficiency in the insurance industry were general economic, capital market, and insurance market conditions. Nourani et al. [22] decomposed the insurance companies into two functional divisions, premium accumulation and investment capability. Then, they applied the dynamic network DEA model to assess Malaysian insurance companies as a case study. Almulhim [23] examined the efficiency of Saudi Arabia's insurance by using a two-stage DEA model. Unlike previous studies which considered insurance companies in operation and profitability stages, he designed a structure of two production stages for insurance companies and accordingly, defined the leader stage.

Another crucial factor affecting insurance companies' efficiency is risk management. Kuo et al. [24] employed truncated regression to examine the relationship between risk management committee structure and general insurers' operating efficiency. They applied a two-stage dynamic NDEA based on decomposition paradigm to calculate marketing and profitability efficiency at stages one and two, respectively. In

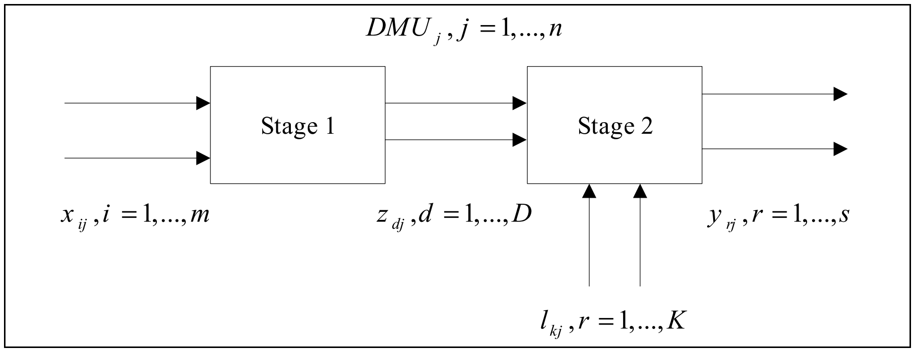

Fig. 1. The overall structure of two-stage DEA with independent inputs at the second stage.

another study [25], utilized a basic DEA to evaluate 17 insurance companies in the Czech. However, it is suggested in most studies that the efficiency of insurance companies should be evaluated in a two-stage process [16]. Mavi et al. [26] introduced a novel method to find the common weights in a two-stage NDEA based on goal programming approach considering undesirable inputs, intermediate products, and the outputs in the presence of big data. In their paper, Anandarao et al. [27] applied a two-stage relational DEA to evaluate the efficiency of Indian life insurance companies. They indicated that the main advantage of this method is that it identifies the inefficient stage of the process. Kaffash et al. [28] reviewed 132 data envelopment analysis studies in the insurance industry. They indicated that newly-developed DEA models were less adopted in applications. Also, the two-stage approach was dominant in exploring the efficiency of insurance firms.

lo Storto (2020) developed a two-stage network DEA to measure 103 major Italian municipalities. The efficiency of the first stage shows how productive the municipalities have been in using the available financial resources to create welfare facilities. The efficiency of the second stage determines the extent to which municipalities have been able to make the welfare facilities of the first stage available to the public. Shi et al. [29] incorporated the undesirable outputs into a DEA model by introducing a new slacks-based measure network DEA approach to evaluate the performance of Chinese commercial banks.

With respect to the scope of applied methodologies and the assumptions found in the insurance literature, this paper attempts to fill the gaps from previous studies. First, our study continues to the very few papers that have evaluated insurance companies in Iran. Besides, this paper differs from previous studies in terms of the type of data used. Most studies on insurance performance evaluation do not include negative data and undesirable outputs in the model. Moreover, our study includes data uncertainty in the evaluation process. It is notable that the undesirable output used in this paper is itself negative. In addition, this paper uses a new bi-objective composition linear NDEA model to evaluate Iranian insurance companies. Most of the previous studies applied the multiplicative decomposition NDEA model, which is proposed by Kao and Hwang [16]. According to Ref. [30]; the efficiency estimates obtained by the multiplicative method are not unique. Briefly, this study applied a new single objective composition NDEA model to evaluate Iranian insurance companies with negative data and undesirable output.

3. Methodology

Data Envelopment Analysis, introduced by Charnes et al. [3]; measures the relative efficiency among a set of homogenous decision-making units (DMUs) using a linear programming method. DEA can measure DMUs' efficiency with multiple inputs and multiple outputs without using predefined functions, making it an appropriate tool for evaluating insurance companies' efficiency. The DEA approach is based on the hypothesis that in each DMU, a number of inputs are converted to outputs without considering any internal relations between inputs and outputs. In fact, the DEA approach treats DMUs as black boxes and ignores how inputs are converted to outputs. However, in some cases, such as insurance companies, DEA models consist of two stages. There are intermediate measures that are considered inputs in one stage and outputs in another stage. Some researchers proposed different two-stage DEA models to consider DMUs as a network structures. Cook et al. [31] classified two-stage DEA models into four groups: game-theoretic approach, standard DEA approach, efficiency decomposition approach, and network-DEA approach.

Kao and Hwang [16] introduced the multiplicative efficiency-decomposition approach to model a two-stage NDEA. Their proposed method utilizes a simple geometric mean approach, employing the product of the efficiencies from the two stages. Readers can refer to Kao and Hwang [16,30] for more details. According to Ref. [30]; the main weakness of Kao and Hwang [16] approach is that the decomposition of the overall efficiency to the stage efficiencies is not unique. Another approach known as additive efficiency-decomposition is proposed by Chen et al. [32]. They defined the overall efficiency score for a DMU as a weighted sum of the efficiencies for each stage, rather than using a simple product of those efficiencies [30]. indicated that the main weakness of this approach is that it biases the efficiency calculation in favor of the second stage [7]. introduced a composition two-stage NDEA approach in which the stage efficiencies are measured without a prior definition of overall efficiency. In this approach, after measuring the stage efficiencies, the overall efficiency is calculated by aggregating the stage efficiencies additively or multiplicatively. In the following, we briefly discuss the composition approach.

Consider Fig. 1 as the structure of the two-stage NDEA model. Assume we have n DMUs, and each DMU consumes m inputs $(x_{ij},\ i = 1,\dots m)$ to produce D outputs $(z_{dj},\ d = 1,\dots D)$ at the stage one. The outputs of the stage one referred as intermediate measures are then used as the inputs for the second stage to produce s outputs $(y_{rj},\ r = 1, \dots s)$. In this system, each DMU converts m inputs to s outputs using D intermediate measures. Also, K extra independent inputs are fed to the second stage beyond the intermediate measures. In this case, the stage efficiencies and the overall efficiency of DMU_j are calculated as follows:

$$e_j^1 = \frac{\sum_{d=1}^{D} w_d z_{dj}}{\sum_{i=1}^{m} v_i x_{ij}} \tag{1}$$

$$e_j^2 = \frac{\sum_{r=1}^{s} u_r y_{rj}}{\sum_{d=1}^{D} w_d z_{dj} + \sum_{k=1}^{K} \gamma_k l_{kj}} \tag{2}$$

$$e_j^{total} = e_j^1 \times e_j^2 \tag{3}$$

The basic input-oriented CRS-DEA model is applied to separately measure the efficiencies of the stage 1 and stage 2 of DMU_o. Thus, we have the following two models that should be solved for each DMU:

$$
\begin{aligned}
& E_o^1 = \max \frac{\sum_{d=1}^{D} w_d z_{do}}{\sum_{i=1}^{m} v_i x_{io}} \\
& s.t. \\
& \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1, \dots, n \\
& w_d \ge \varepsilon, \ v_i \ge \varepsilon
\end{aligned}
\tag{4}
$$

$$
\begin{aligned}
& E_o^2 = \max \frac{\sum_{r=1}^{s} u_r y_{ro}}{\sum_{d=1}^{D} w_d z_{do} + \sum_{k=1}^{K} \gamma_k l_{ko}} \\
& s.t. \\
& \sum_{r=1}^{s} u_r y_{rj} - \left(\sum_{d=1}^{D} w_d z_{dj} + \sum_{k=1}^{K} \gamma_k l_{kj} \right) \le 0, \quad j = 1, \dots, n \\
& u_r \ge \varepsilon, \ w_d \ge \varepsilon, \ \gamma_k \ge \varepsilon
\end{aligned}
\tag{5}
$$

To calculate the efficiency of the stages [7], introduced a nonlinear NDEA model as follows:

$$
\begin{aligned}
& \min \delta \\
& s.t. \\
& E_o^1 - \sum_{d=1}^{D} w_d z_{do} \le \delta \\
& \left(E_o^2 - \delta\right) \left(\sum_{d=1}^{D} w_d z_{do} + \sum_{k=1}^{K} \gamma_k l_{ko} \right) - \sum_{r=1}^{s} u_r y_{ro} \le 0 \\
& \sum_{i=1}^{m} v_i x_{io} = 1 \\
& \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \quad j = 1, \dots, n \\
& \sum_{r=1}^{s} u_r y_{rj} - \left(\sum_{d=1}^{D} w_d z_{dj} + \sum_{k=1}^{K} \gamma_k l_{kj} \right) \le 0, \quad j = 1, \dots, n \\
& w_d \ge \varepsilon, \ v_i \ge \varepsilon, \ u_r \ge \varepsilon, \ \gamma_k \ge \varepsilon, \ \delta \ge 0
\end{aligned}
\tag{6}
$$

Model (6) is a non-linear multi-objective programming and has four main drawbacks. 1-the proposed approach is intensive computationally 2-the developed model is a non-linear program 3-the proposed model considers neither negative data nor undesirable parameters 4-the data uncertainty is not included in the model. In the following section, first, we discuss a method to convert negative data into positive data. We then apply a goal programming approach to avoid solving non-linear programs and avoid intensive computations. Moreover, we use the $\alpha-$ *cut* method to extend the developed goal programing in order to include the data uncertainty in the calculations.

3.1. The proposed composition two-stage NDEA model with negative data and undesirable output

Here, a new single objective function model is proposed to measure stage efficiencies of DMUs with less computational intensity. In real case studies, data can be either positive or negative. Also, the considered outputs in the model can be undesirable. In our proposed approach, the model can contain both negative data and undesirable outputs. However, network-DEA lacks the power to handle negative data and undesirable outputs. To overcome this issue, first, we multiply the values of undesirable outputs by -1, in which case the undesirable outputs become desirable [8], and then we use a method proposed by Tone et al. [9] to deal with data negativity. Assume that X is a matrix of input parameters containing both positive and negative values.

$$X = \left(x_{ij}\right) \in R^{m \times n} \tag{7}$$

The minimum of each input can be defined as follows:

$$\theta_i = \min\{x_{i1}, \ x_{i2}, \dots, x_{in}\} \ (i = 1, \dots, m) \tag{8}$$

When $\theta_i > 0$, no data transformation is needed, because all the values are positive and non-zero. If one or several data of x_i is equal to zero, then all the values of x_i should be perturbed by a small amount $\sigma_i > 0$ to avoid dividing by zero when is calculating efficiency scores. Such data transformation is only needed when x_i is an input parameter. If $\theta_i < 0$, it means that one or several data of x_i is negative; Hence, all the data within x_i should be translated by an amount big enough to make all values strictly positive. The transformation process can be summarized as follows:

$$if \ \theta_i > 0, \ then \ x_i^{\min} = 0 \tag{9}$$

$$if \ \theta_i = 0, \ then \ x_i^{\min} = -\sigma_i \tag{10}$$

$$if \ \theta_i < 0, \ then \ x_i^{\min} = \theta_i(1 + \tau_i) \tag{11}$$

which τ_i is a small value, e.g. $\tau_i = 0.01$. Then new inputs are calculated as $\overline{x}_{ij} = x_{ij} - x_i^{\min} \ (\forall i, j)$ that can be considered as new positive inputs in the DEA model. For negative outputs, a similar way in applied. After handling the negative data and undesirable outputs, in this section the proposed model is introduced. The new model identifies the DMU as efficient if and only if both stages are efficient. Consider that both objective functions $\sum_{d=1}^{D} w_d z_{do}$ and $\sum_{r=1}^{s} u_r y_{ro} / (\sum_{d=1}^{D} w_d z_{do} + \sum_{k=1}^{K} \gamma_k l_{ko})$, presented in models (4) and (5), should be maximized. According to Despotis et al. (2016), it is assumed that the denominator of the objective function of model (4) is equal to 1. Hence, it needs to maximize $\sum_{d=1}^{D} w_d z_{do}$ as the objective function of the model (4). A linear goal programming (GP) method introduced by Charnes et al. [33] is applied to consider these two objectives simultaneously. GP is only applicable when the optimal points are known in advance. Since the objectives are the stage efficiencies score in our case, it is clear that the optimal value of each objective is equal to 1. Thus, a deviation variable from the ideal point is defined for each objective function, then a linear GP is employed to seek a solution that minimizes the sum of the deviations from full stage efficiencies scores.

Assume that $\sum_{d=1}^{D} w_d z_{do}$ and $\sum_{r=1}^{s} u_r y_{ro} / (\sum_{d=1}^{D} w_d z_{do} + \sum_{k=1}^{K} \gamma_k l_{ko})$ are the two objective functions to be optimized. The ideal value for each objective function is equal to 1. Consider that variables (dev_1^{o+}, dev_1^{o-}) and (dev_2^{o+}, dev_2^{o-}) are the deviations of DMU_o from stage one and stage two efficiencies, respectively. The goal is to determine the optimal weights of inputs, intermediate measures, and outputs while minimizing the summation of (dev_1^{o+}, dev_1^{o-}) and (dev_2^{o+}, dev_2^{o-}).

As discussed before, the ideal value for the first objective function ($\sum_{d=1}^{D} w_d z_{do}$) is equal to 1; namely $\sum_{d=1}^{D} w_d z_{do} = 1$. It is clear that the deviation of the first objective function from its ideal level should be minimized. According to the goal programming framework, the negative and positive deviations from this objective function can be shown as $\sum_{d=1}^{D} w_d z_{do} + dev_1^{o-} - dev_1^{o+} = 1$. In optimal solution, if dev_1^{o-} be greater than zero ($dev_1^{o-} > 0$), the positive deviation will be zero and the efficiency will be less than 1. On the other hand, if $dev_1^{o+} > 0$, the objective function of the model (4) which is the efficiency of DMU under consideration, will be greater than 1. Hence, it is necessary to minimize both negative and positive deviation, simultaneously. Similarly, the optimal value for the second objective function is equal to one. Therefor we have $\sum_{r=1}^{s} u_r y_{ro} / (\sum_{d=1}^{D} w_d z_{do} + \sum_{k=1}^{K} \gamma_k l_{ko}) = 1$. Multiplying both sides by $(\sum_{d=1}^{D} w_d z_{do} + \sum_{k=1}^{K} \gamma_k l_{ko})$, we have $\sum_{r=1}^{s} u_r y_{ro} - (\sum_{d=1}^{D} w_d z_{do} + \sum_{k=1}^{K} \gamma_k l_{ko}) = 0$. By adding the negative and positive deviations to the second objective function, it can be written as $\sum_{r=1}^{s} u_r y_{ro} - (\sum_{d=1}^{D} w_d z_{do} + \sum_{k=1}^{K} \gamma_k l_{ko}) + dev_2^{o-} - dev_2^{o+} = 0$. Similar to the first objective function, both negative and positive deviations should be minimized. Finally, the GP proposed model can be formulated as the model (12). The model (12) is a single objective linear model and should be run for each DMU separately.

$$
\begin{aligned}
& Min \quad z = dev_1^{o+} + dev_1^{o-} + dev_2^{o+} + dev_2^{o-} \\
& s.t. \\
& \sum_{d=1}^{D} w_d z_{do} - dev_1^{o+} + dev_1^{o-} = 1, \\
& \sum_{r=1}^{s} u_r y_{ro} - \left(\sum_{d=1}^{D} w_d z_{do} + \sum_{k=1}^{K} \gamma_k l_{ko} \right) - dev_2^{o+} + dev_2^{o-} = 0, \\
& \sum_{i=1}^{m} v_i x_{io} = 1, \\
& \sum_{d=1}^{D} w_d z_{dj} - \sum_{i=1}^{m} v_i x_{ij} \le 0, \qquad j = 1, \dots, n \\
& \sum_{r=1}^{s} u_r y_{rj} - \left(\sum_{d=1}^{D} w_d z_{dj} + \sum_{k=1}^{K} \gamma_k l_{kj} \right) \le 0, \qquad j = 1, \dots, n \\
& v_i \ge \varepsilon, w_d \ge \varepsilon, u_r \ge \varepsilon, \gamma_k \ge \varepsilon \\
& dev_1^{o+}, dev_1^{o-}, dev_2^{o+}, dev_2^{o-} \ge 0
\end{aligned}
\tag{12}
$$

Model (12) is a GP version of models (3) and (4). Since models (3) and (4) are feasible DEA models, hence, the proposed model (12) is also feasible. In fact, the constraint (3) to (5) of the models (12) are DEA system constraints and they construct solution space. The constraints (1) and (2) are goal programming constraints and by adjusting deviations, they are moved to the solution space. Once the model (12) is solved for each DMU, the non-dominated weights of inputs, intermediate measures, and outputs are achieved. Then, the stage efficiencies and the overall efficiency can be calculated as follows:

$$
e_o^1 = \frac{\sum_{d=1}^{D} w_d^* z_{do}}{\sum_{i=1}^{m} v_i^* x_{io}} \tag{13}
$$

$$
e_o^2 = \frac{\sum_{r=1}^{s} u_r^* y_{ro}}{\sum_{d=1}^{D} w_d^* z_{do} + \sum_{k=1}^{K} \gamma_k^* l_{ko}} \tag{14}
$$

$$
e_o^{total} = e_o^1 . \, e_o^2 \tag{15}
$$

3.2. The proposed fuzzy composition two-stage NDEA model

The developed model by Ref. [7] lacks the power to manage data uncertainty. However, in most real-world cases, the data may be imprecise. Here, we apply a fuzzy technique developed by Saati et al. [34] to turn Model (24) into a fuzzy DEA goal programming. First, a brief review of fuzzy sets definitions and terms have been explained in the following. For more details, the readers can refer to Zimmermann [35] and Omrani et al. (2022).

Denote the source set of U and its fuzzy subset of $\widetilde{A}$. The elements from U that have the minimum membership degree of α $(0 \le \alpha \le 1)$ in the fuzzy subset of $\widetilde{A}$ are named *α-cut* of A and shown with A_α.

$$
A_\alpha = \{x \in X | \mu_{\widetilde{A}}(x) \ge \alpha\} \tag{16}
$$

α-cuts describe fuzzy sets using definite sets. *α-cut* set of A_α is the crisp interval numbers of the which can be shown as follows:

$$
A_\alpha = \left[\min_A \{x \in X | \mu_{\widetilde{A}}(x) \ge \alpha\} \, , \, \max_A \{x \in X | \mu_{\widetilde{A}}(x) \ge \alpha\} \right] \tag{17}
$$

Each *α-cut* of a triangle fuzzy number can be written as follows:

$$
\begin{aligned}
A_\alpha &= \left[\min_A \{x \in X | \mu_{\widetilde{A}}(x) \ge \alpha\} \, , \, \max_A \{x \in X | \mu_{\widetilde{A}}(x) \ge \alpha\} \right] = \left[a_1^{(\alpha)}, a_3^{(\alpha)} \right] \\
A_\alpha &= [(a_2 - a_1)\alpha + a_1 \, , \, a_3 - (a_3 - a_2)\alpha]
\end{aligned}
\tag{18}
$$

It is assumed that all the data corresponding to input and output parameters poses non-negative crisp values. The conventional input-oriented DEA model with fuzzy data to calculate the efficiency of DMU_o is as follows, where ~ indicates the fuzziness:

$$
\begin{aligned}
& Min \quad z = dev_1^{o+} + dev_1^{o-} + dev_2^{o+} + dev_2^{o-} \\
& s.t. \\
& \sum_{d=1}^{D} w_d \widetilde{z}_{do} - dev_1^{o+} + dev_1^{o-} = 1, \\
& \sum_{r=1}^{s} u_r \widetilde{y}_{ro} - \left(\sum_{d=1}^{D} w_d \widetilde{z}_{do} + \sum_{k=1}^{K} \gamma_k \widetilde{l}_{ko} \right) - dev_2^{o+} + dev_2^{o-} = 0, \\
& \sum_{i=1}^{m} v_i \widetilde{x}_{io} = \widetilde{1}, \\
& \sum_{d=1}^{D} w_d \widetilde{z}_{dj} - \sum_{i=1}^{m} v_i \widetilde{x}_{ij} \le 0, \qquad j = 1, \dots, n \\
& \sum_{r=1}^{s} u_r \widetilde{y}_{rj} - \left(\sum_{d=1}^{D} w_d \widetilde{z}_{dj} + \sum_{k=1}^{K} \gamma_k \widetilde{l}_{kj} \right) \le 0, \qquad j = 1, \dots, n \\
& v_i \ge \varepsilon, w_d \ge \varepsilon, u_r \ge \varepsilon, \gamma_k \ge \varepsilon \\
& dev_1^{o+}, dev_1^{o-}, dev_2^{o+}, dev_2^{o-} \ge 0
\end{aligned}
\tag{19}
$$

In this approach, triangular fuzzy numbers are used to indicate the fuzziness of the data. Let $\widetilde{x}_{ij} = (x_{ij}^m, x_{ij}^l, x_{ij}^u)$, $\widetilde{z}_{dj} = (z_{dj}^m, z_{dj}^l, z_{dj}^u)$, $\widetilde{l}_{kj} = (l_{kj}^m, l_{kj}^l, l_{kj}^u)$,and $\widetilde{y}_{rj} = (y_{rj}^m, y_{rj}^l, y_{rj}^u)$ are triangular fuzzy numbers. Therefore, model (19) can be formulated as follows:

$$\begin{aligned}
&Min \quad z = dev_1^{o+} + dev_1^{o-} + dev_2^{o+} + dev_2^{o-}\\
&s.t.\\
&\sum_{d=1}^{D} w_d\left(z_{do}^m, z_{do}^l, z_{do}^u\right) - dev_1^{o+} + dev_1^{o-} = 1,\\
&\sum_{r=1}^{s} u_r\left(y_{ro}^m, y_{ro}^l, y_{ro}^u\right) - \left(\sum_{d=1}^{D} w_d\left(z_{do}^m, z_{do}^l, z_{do}^u\right) + \sum_{k=1}^{K} \gamma_k\left(l_{ko}^m, l_{ko}^l, l_{ko}^u\right)\right) - dev_2^{o+} + dev_2^{o-} = 0,\\
&\sum_{i=1}^{m} v_i\left(x_{io}^m, x_{io}^l, x_{io}^u\right) = 1, 1^l, 1^u\Big),\\
&\sum_{d=1}^{D} w_d\left(z_{dj}^m, z_{dj}^l, z_{dj}^u\right) - \sum_{i=1}^{m} v_i\left(x_{ij}^m, x_{ij}^l, x_{ij}^u\right) \le 0, \qquad j = 1, \dots, n\\
&\sum_{r=1}^{s} u_r\left(y_{rj}^m, y_{rj}^l, y_{rj}^u\right) - \left(\sum_{d=1}^{D} w_d\left(z_{dj}^m, z_{dj}^l, z_{dj}^u\right) + \sum_{k=1}^{K} \gamma_k\left(l_{kj}^m, l_{kj}^l, l_{kj}^u\right)\right) \le 0, \qquad j = 1, \dots, n\\
&v_i \ge \varepsilon, w_d \ge \varepsilon, u_r \ge \varepsilon, \gamma_k \ge \varepsilon\\
&dev_1^{o+}, dev_1^{o-}, dev_2^{o+}, dev_2^{o-} \ge 0
\end{aligned} \tag{20}$$

Using the $\alpha - cut$ method, the model (20) is changed to model (21) as follows:

$$\begin{aligned}
&Min \quad z = dev_1^{o+} + dev_1^{o-} + dev_2^{o+} + dev_2^{o-}\\
&s.t.\\
&\sum_{d=1}^{D} w_d\left[\alpha z_{do}^m + (1-\alpha) z_{do}^l, \alpha z_{do}^m + (1-\alpha) z_{do}^u\right] - dev_1^{o+} + dev_1^{o-} = 1,\\
&\sum_{r=1}^{s} u_r\left[\alpha y_{ro}^m + (1-\alpha) y_{ro}^l, \alpha y_{ro}^m + (1-\alpha) y_{ro}^u\right]\\
&-\Bigg(\sum_{d=1}^{D} w_d\left[\alpha z_{do}^m + (1-\alpha) z_{do}^l, \alpha z_{do}^m + (1-\alpha) z_{do}^u\right]\\
&+\sum_{k=1}^{K} \gamma_k\left[\alpha l_{ko}^m + (1-\alpha) l_{ko}^l, \alpha l_{ko}^m + (1-\alpha) l_{ko}^u\right]\Bigg) - dev_2^{o+} + dev_2^{o-} = 0,\\
&\sum_{i=1}^{m} v_i\left[\alpha x_{io}^m + (1-\alpha) x_{io}^l, \alpha x_{io}^m + (1-\alpha) x_{io}^u\right] = \left[\alpha + (1-\alpha)1^l, \alpha + (1-\alpha)1^u\right],\\
&\sum_{d=1}^{D} w_d\left[\alpha z_{dj}^m + (1-\alpha) z_{dj}^l, \alpha z_{dj}^m + (1-\alpha) z_{dj}^u\right]\\
&-\sum_{i=1}^{m} v_i\left[\alpha x_{ij}^m + (1-\alpha) x_{ij}^l, \alpha x_{ij}^m + (1-\alpha) x_{ij}^u\right] \le 0, \qquad j\\
&= 1, \dots, n \sum_{r=1}^{s} u_r\left[\alpha y_{rj}^m + \left(1-\alpha\right) y_{rj}^l, \alpha y_{rj}^m + \left(1-\alpha\right) y_{rj}^u\right] - \Bigg(\sum_{d=1}^{D} w_d\Big[\alpha z_{dj}^m\\
&\quad + \left(1-\alpha\right) z_{dj}^l, \alpha z_{dj}^m + \left(1-\alpha\right) z_{dj}^u\Big] + \sum_{k=1}^{K} \gamma_k\Big[\alpha l_{kj}^m + \left(1-\alpha\right) l_{kj}^l, \alpha l_{kj}^m + \Big(1\\
&\quad - \alpha\Big) l_{kj}^u\Big]\Bigg)\\
&\le 0, \qquad j = 1, \dots, n v_i \ge \varepsilon, w_d \ge \varepsilon, u_r \ge \varepsilon, \gamma_k \ge \varepsilon dev_1^{o+}, dev_1^{o-}, dev_2^{o+}, dev_2^{o-}\\
&\ge 0
\end{aligned} \tag{21}$$

The next step is to define variables in the intervals to satisfy the set of constraints while maximizing the objective function. Assume that $\overline{x}_{ij} = v_i \widehat{x}_{ij}$, $\overline{z}_{dj} = w_d \widehat{z}_{dj}$, $\overline{l}_{kj} = \gamma_k \widehat{l}_{kj}$, and $\overline{y}_{rj} = u_r \widehat{y}_{rj}$,where $\widehat{x}_{ij} \in [\alpha x_{ij}^m + (1-\alpha) x_{ij}^l, \alpha x_{ij}^m + (1-\alpha) x_{ij}^u]$, $\widehat{z}_{dj} \in [\alpha z_{dj}^m + (1-\alpha) z_{dj}^l, \alpha z_{dj}^m + (1-\alpha) z_{dj}^u] \widehat{l}_{kj} \in [\alpha l_{kj}^m + (1-\alpha) l_{kj}^l, \alpha l_{kj}^m + (1-\alpha) l_{kj}^u]$ and $\widehat{y}_{rj} \in [\alpha y_{rj}^m + (1-\alpha) y_{rj}^l, \alpha y_{rj}^m + (1-\alpha) y_{rj}^u]$. Using these variables, the model (21) will become linear problem as follows:

$$\begin{aligned}
&Min \quad z = dev_1^{o+} + dev_1^{o-} + dev_2^{o+} + dev_2^{o-}\\
&s.t.\\
&\sum_{d=1}^{D} \overline{z}_{do} - dev_1^{o+} + dev_1^{o-} = 1,\\
&\sum_{r=1}^{s} \overline{y}_{ro} - \left(\sum_{d=1}^{D} \overline{z}_{do} + \sum_{k=1}^{K} \overline{l}_{ko}\right) - dev_2^{o+} + dev_2^{o-} = 0,\\
&\sum_{i=1}^{m} \overline{x}_{io} = L,\\
&\sum_{d=1}^{D} \overline{z}_{kj} - \sum_{i=1}^{m} \overline{x}_{ij} \le 0, \qquad j = 1, \dots, n\\
&\sum_{r=1}^{s} \overline{y}_{rj} - \left(\sum_{d=1}^{D} \overline{z}_{kj} + \sum_{k=1}^{K} \overline{l}_{kj}\right) \le 0, \qquad j = 1, \dots, n\\
&v_i\left(\alpha x_{ij}^m + (1-\alpha) x_{ij}^l\right) \le \overline{x}_{ij} \le v_i\left(\alpha x_{ij}^m + (1-\alpha) x_{ij}^u\right) \quad \forall i, j\\
&w_d\left(\alpha z_{dj}^m + (1-\alpha) z_{dj}^l\right) \le \overline{z}_{dj} \le w_d\left(\alpha z_{dj}^m + (1-\alpha) z_{dj}^u\right) \quad \forall d, j\\
&u_r\left(\alpha y_{rj}^m + (1-\alpha) y_{rj}^l\right) \le \overline{y}_{rj} \le u_r\left(\alpha y_{rj}^m + (1-\alpha) y_{rj}^u\right) \quad \forall r, j\\
&\gamma_k\left(\alpha l_{kj}^m + (1-\alpha) l_{kj}^l\right) \le \overline{l}_{kj} \le \gamma_k\left(\alpha l_{kj}^m + (1-\alpha) l_{kj}^u\right) \quad \forall k, j\\
&\alpha + (1-\alpha)1^l \le L \le \alpha + (1-\alpha)1^u\\
&v_i \ge \varepsilon, w_d \ge \varepsilon, u_r \ge \varepsilon, \gamma_k \ge \varepsilon\\
&dev_1^{o+}, dev_1^{o-}, dev_2^{o+}, dev_2^{o-} \ge 0
\end{aligned} \tag{22}$$

If $1^u > 1$ then, there is a possibility that some of the DMUs achieve an efficiency score greater than 1. Thus, it must be equal to 1. As a result, the last constraint of the model (22) can be written as $\alpha + (1 - \alpha)1^l \le L \le 1$. Considering the objective function, it can be concluded that $L = 1$. Therefore the model (22) becomes:

Table 1
Results of [7] model and proposed model (12).

	[7]			Model (12)		
DMU	e^1	e^2	$e^{total} = e^1 . e^2$	e^1	e^2	$e^{total} = e^1 . e^2$
1	1	0.160	0.160	1	0.160	0.160
2	1	0.249	0.249	1	0.209	0.209
3	0.931	0.504	0.470	0.859	0.537	0.461
4	0.702	0.530	0.372	0.713	1	0.713
5	0.670	0.390	0.261	0.638	0.344	0.219
6	0.567	1	0.567	0.553	1	0.553
7	0.918	0.230	0.211	1	0.212	0.212
8	**1**	**1**	**1**	**1**	**1**	**1**
9	0.940	1	0.940	0.889	1	0.889
10	**1**	**1**	**1**	**1**	**1**	**1**
11	0.889	0.835	0.742	0.885	0.512	0.453
12	0.928	0.265	0.246	0.864	0.238	0.205
13	0.850	0.736	0.626	0.815	0.568	0.463
14	0.855	0.285	0.243	0.900	0.277	0.249
15	0.992	0.370	0.367	0.940	0.237	0.223
16	0.923	1	0.923	0.925	1	0.925
17	0.557	0.993	0.553	0.564	0.893	0.504
18	0.699	0.501	0.350	0.462	0.804	0.372
19	0.681	0.358	0.244	0.606	0.370	0.224
20	0.457	1	0.457	0.401	0.996	0.399
21	0.685	0.825	0.566	0.710	0.766	0.544
22	0.550	0.534	0.293	0.408	0.394	0.160
23	0.944	0.195	0.184	1	0.275	0.275
24	0.976	0.457	0.446	1	0.447	0.447
25	0.976	0.592	0.577	0.932	1	0.932
26	0.787	0.830	0.653	0.680	0.934	0.634
27	**1**	**1**	**1**	**1**	**1**	**1**
28	0.360	1	0.360	0.360	1	0.360
29	0.430	1	0.430	0.559	1	0.559
30	**1**	**1**	**1**	**1**	**1**	**1**

$$
\begin{aligned}
& Min \quad z = dev_1^{o+} + dev_1^{o-} + dev_2^{o+} + dev_2^{o-} \\
& s.t. \\
& \sum_{d=1}^{D} \bar{z}_{do} - dev_1^{o+} + dev_1^{o-} = 1, \\
& \sum_{r=1}^{s} \bar{y}_{ro} - \left(\sum_{d=1}^{D} \bar{z}_{do} + \sum_{k=1}^{K} \bar{l}_{ko} \right) - dev_2^{o+} + dev_2^{o-} = 0, \\
& \sum_{i=1}^{m} \bar{x}_{io} = 1, \\
& \sum_{d=1}^{D} \bar{z}_{kj} - \sum_{i=1}^{m} \bar{x}_{ij} \leq 0, \qquad \mathrm{j} = 1, \dots, \mathrm{n} \\
& \sum_{r=1}^{s} \bar{y}_{rj} - \left(\sum_{d=1}^{D} \bar{z}_{kj} + \sum_{k=1}^{K} \bar{l}_{kj} \right) \leq 0, \qquad \mathrm{j} = 1, \dots, \mathrm{n} \\
& v_i \left(\alpha x_{ij}^m + (1-\alpha) x_{ij}^l \right) \leq \bar{x}_{ij} \leq v_i \left(\alpha x_{ij}^m + (1-\alpha) x_{ij}^u \right) \quad \forall i, j \\
& w_d \left(\alpha z_{dj}^m + (1-\alpha) z_{dj}^l \right) \leq \bar{z}_{dj} \leq w_d \left(\alpha z_{dj}^m + (1-\alpha) z_{dj}^u \right) \quad \forall d, j \\
& u_r \left(\alpha y_{rj}^m + (1-\alpha) y_{rj}^l \right) \leq \bar{y}_{rj} \leq u_r \left(\alpha y_{rj}^m + (1-\alpha) y_{rj}^u \right) \quad \forall r, j \\
& \gamma_k \left(\alpha l_{kj}^m + (1-\alpha) l_{kj}^l \right) \leq \bar{l}_{kj} \leq \gamma_k \left(\alpha l_{kj}^m + (1-\alpha) l_{kj}^u \right) \quad \forall k, j \\
& \mathrm{v}_i \geq \varepsilon, \mathrm{w}_d \geq \varepsilon, \mathrm{u}_r \geq \varepsilon, \gamma_k \geq \varepsilon \\
& dev_1^{o+}, dev_1^{o-}, dev_2^{o+}, dev_2^{o-} \geq 0
\end{aligned}
\tag{23}
$$

The model (23) is a linear programming and can be easily used for estimating efficiency scores in a fuzzy environment. Once the model (23) is solved for each DMU, the weights of inputs, intermediate measures, and outputs are achieved. Then, the stage efficiencies and the overall efficiency can be calculated using equations (13)–(15). Like model (12), model (23) is also feasible. If $\alpha = 1$, then the model (23) is converted to the model (12) which is feasible. For instance, the constraint $v_i(\alpha x_{ij}^m + (1-\alpha) x_{ij}^l) \leq \bar{x}_{ij} \leq v_i(\alpha x_{ij}^m + (1-\alpha) x_{ij}^u)$ is changed to $v_i x_{ij}^m \leq \bar{x}_{ij} \leq v_i x_{ij}^m \; \bar{x}_{ij} = v_i x_{ij}^m$. On the other hand, If $\alpha = 0$, the solution space of the model (23) will be greater than the solution space of the model (12), which means the model (23) is feasible, too.

3.3. Numerical example

To examine the validity of the proposed method, we use the original data presented in a work of Li et al. [36] to calculate the stage efficiencies along with the overall efficiency using the proposed model (12). We then compare our obtained results with the ones which [7] achieved using their own developed model. To compare the results, we apply Spearman correlation coefficient to identify the correlation between the efficiency scores. This case study is about evaluating the regional R&D process of 30 Provincial Level Regions in China. Stage-1 describes technology development, and stage-2 represents the economic application. The stage-1 inputs are R&D personnel (X1), R&D expenditure (X2), and the proportion of regional science and technology funds in regional total financial expenditure (X3). The outputs (intermediate measures) of stage-1, which are inputs to stage-2, are number of patents (Z1) and number of papers (Z2). The extra input to stage-2 is contract value in the technology market (L). The final outputs are GDP (Y1), total exports (Y2), urban per capita annual income (Y3), and gross output of high-tech industry (Y4). The reader is referred to Li et al. [36] for the complete data set. The results of efficiency scores obtained from both models are presented in Table 1. The Spearman correlations between ranks generated by Despotis et al. (2012) model and model (12) for the stage 1, stage 2 and overall are 0.932, 0.898, and 0.887, respectively. All the Spearman correlations coefficients are significant at 0.01 level which proves the validity of our proposed model.

4. Application in insurance companies

We here apply the proposed model to the 22 Iranian insurance companies. The activity of insurance companies in Iran goes back to early 1910. Until 1935, 29 foreign insurance companies started their business in Iran. In 1936, the first Iranian insurance company was established in the country, which overshadowed the activities of foreign insurance companies and, by supporting insurance laws, was able to change the atmosphere of the country towards Iranian insurance companies and close foreign insurance companies.1935, 29 foreign insurance companies started their business in Iran. In 1937, the insurance law was passed in Iran, and from 1950, private insurance companies were established in Iran, so that eight private companies were operating until 1964. In 1952, laws were established that restricted insurance activities for foreign companies, which resulted in the closure of these companies and the growth of domestic insurance companies. Between 2000 and 2010, the establishment of insurance companies grew significantly, and so far, about 30 private and public insurance companies are operating in Iran. Among these 30 insurance companies, we only select 22 of them due to data availability.

We divide the process of insurance companies into two stages (see Fig. 2). In the first stage, insurance companies seek to attract more customers by using their facilities and labor force. In the second stage, insurance companies seek to generate maximum profits by investing in various economic sectors. Three inputs are used in the first stage, which are characterized by operational cost, number of staff, and number of branches of each insurance company. There are three intermediate measures characterized by direct written premiums, number of issued insurances, and the growth rate of issued insurances. There is only one independent input to the second stage characterized by investment. The whole process generates two outputs which are net profit and the growth rate of insurance claims. Operational cost includes the salaries of the employees and all other different variable costs incurred during the whole year. Direct written premiums are the premiums received from insured clients. The growth rate of issued insurance is determined by the difference between the number of insurances issued in the previous year and the current year. Each insurance company is allowed to use part of

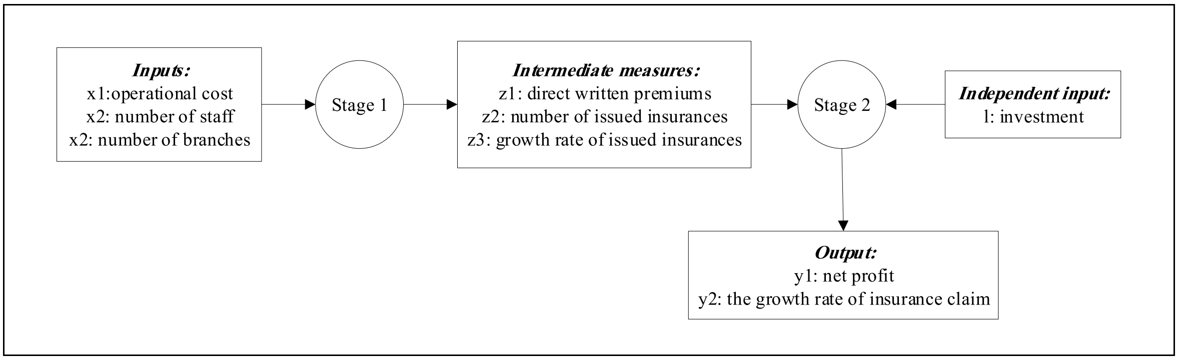

Fig. 2. Process of insurance companies.

Table 2
Raw data for Iranian insurance companies.

	Inputs			Intermediate measures			independent input	Outputs	
DMU	x_1: Operational cost Million Rials	x_2: Number of staff	x_3: Number of branches	z_1: Direct written premiums	z_2: Number of issued insurances	z_3: growth rate of issued insurance (%)	l_1: Investment Million Rials	y_1: Net profit Million Rials	y_2: growth rate of insurance claims (%)
Asia	3029460	2645	103	58855.30	6265739	−0.1	32546257	5408588	3.4
Alborz	1763981	1354	56	31316.3	2704396	−7.4	17540628	707112	−23.2
Dana	2856059	2085	80	47527.00	5093097	36.3	6721245	342174	12.2
Moalem	1195805	791	62	23714.6	2898501	−3	6260610	411912	282.1
Parsian	1256380	870	70	28536.7	2766107	5.8	10873080	911950	−18.1
Razi	761955	652	42	10148.8	1145922	−27.7	3090854	295235	−11.8
Kar-afarin	649568	734	57	12241.2	591098	−18.9	21866546	259079	−20
Sina	756256	430	61	13391.5	672469	3.2	4464151	411836	46.1
Mellat	591599	550	10	14267.6	1725694	−3.6	8383923	651719	89
Hafez	114166	100	10	534.2	138088	−42.7	99132	−10110	−53.7
Dey	674175	533	49	41097.9	432112	2.1	2118501	1791775	−15.7
Saman	1051675	804	33	13534	1900781	23.1	12083293	413707	−26.9
Novin	604222	694	62	9910.1	644257	12.1	6780302	302867	−1.2
Pasargad	1624298	1077	85	32985.6	2818497	−0.2	44967451	1991520	93.5
Mihan	76541	341	40	3033.9	421707	16.7	589198	11302	54.2
Kosar	1104741	1138	36	24510.1	2077924	4.4	10928576	1307220	31.7
Ma	574937	462	46	10194.1	952507	9.2	13675357	631188	71.7
Arman	436715	518	37	4078.3	310070	92.4	1131450	61671	−45.6
Taavon	263001	300	33	3798.9	197372	−43.4	1620630	75629	−23.1
Sarmad	676778	434	41	9660.8	307892	13.5	4051851	416408	119.7
Tejarat-no	399543	317	32	5845.7	438847	115.2	3600191	291420	136.4
Hekmat-saba	207148	278	25	1873.9	184346	156.1	851253	16969	359.6

the premium income received to invest in the capital market, referred to as the investment in this process. Net profit equals income from premiums received plus return on investment. The growth rate of insurance claims is calculated by comparing the total insurance claims that each company paid during the previous year and the current year. It should be noted that the growth rate of issued insurances, net profit, and the growth rate of the insurance claims contain negative data. Also, the growth rate of the insurance is an undesirable output. According to the selected variables, the first stage is named as operational stage and the efficiency related to this stage is operational efficiency. In addition, the second stage is called the profitability stage and the efficiency related to it is profitability efficiency. Referring to equations (13)–(15), overall efficiency is calculated by multiplying these two efficiencies. The raw data are collected for the year 2009 and shown in Table 2.

The proposed fuzzy model (23) is solved with different values of α for each *DMU* separately. The values of efficiencies at the different levels of $\alpha \in (0,1]$ are evaluated and the results are reported in Table 3. It should be noted $\alpha = 1$ means evaluating efficiencies using crisp data. The graphical representation of efficiency results using fuzzy data at the different levels of $\alpha \in (0,1]$ is shown in Fig. 3. These figure imply that with variation in the satisfaction level of α, the efficiency of almost every DMU varies. Therefore, the achieved efficiency scores using fuzzy data are more realistic compared to the results obtained using definite data. It can be observed that *Moalem* and *Arman* obtained the highest and the lowest operational efficiency, respectively. *Dey* and *Dana* achieved the highest and the lowest profitability efficiency, respectively. *Dey* insurance company is the overall efficiency DMU.

To summarize the results, we only discuss the achieved efficiency scores for $\alpha = 0.6$. Table 4 reports the efficiencies and ranks of the insurance companies based on our developed fuzzy goal programming

Table 3
The results of the fuzzy NDEA model (23) for different α

DMU	$\alpha = 0.2$			$\alpha = 0.4$			$\alpha = 0.6$			$\alpha = 0.8$			$\alpha = 1$		
	e^1	e^2	e^{total}	e^1	e^2	e^{total}	e^1	e^2	e^{total}	e^1	e^2	e^{total}	e^1	e^2	e^{total}
Asia	0.787	0.243	0.191	0.773	0.234	0.181	0.760	0.225	0.171	0.745	0.216	0.161	0.730	0.208	0.152
Alborz	0.640	0.084	0.054	0.628	0.078	0.049	0.617	0.073	0.045	0.608	0.069	0.042	0.599	0.066	0.039
Dana	0.751	0.069	0.052	0.742	0.068	0.050	0.733	0.066	0.049	0.725	0.065	0.047	0.716	0.063	0.045
Moalem	0.912	0.180	0.164	0.938	0.170	0.160	0.965	0.162	0.156	0.992	0.153	0.152	1.000	0.135	0.135
Parsian	0.953	0.356	0.339	0.971	0.339	0.329	0.987	0.323	0.319	0.978	0.287	0.280	0.927	0.218	0.202
Razi	0.570	0.389	0.222	0.554	0.377	0.209	0.539	0.366	0.197	0.467	0.394	0.184	0.458	0.380	0.174
Kar-afarin	0.435	0.371	0.161	0.436	0.350	0.153	0.437	0.332	0.145	0.438	0.314	0.138	0.439	0.296	0.130
Sina	0.658	0.506	0.333	0.648	0.505	0.327	0.639	0.504	0.322	0.630	0.502	0.316	0.620	0.502	0.311
Mellat	1.000	0.622	0.622	1.000	0.585	0.585	1.000	0.550	0.550	1.000	0.516	0.516	1.000	0.482	0.482
Hafez	0.468	1.000	0.468	0.459	1.000	0.459	0.451	1.000	0.451	0.442	1.000	0.442	0.433	1.000	0.433
Dey	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000	1.000
Saman	0.591	0.358	0.212	0.603	0.341	0.206	0.621	0.318	0.198	0.639	0.296	0.189	0.657	0.276	0.182
Novin	0.445	0.546	0.243	0.441	0.529	0.233	0.438	0.511	0.224	0.435	0.494	0.215	0.432	0.476	0.206
Pasargad	0.757	0.308	0.233	0.730	0.183	0.134	0.726	0.117	0.085	0.726	0.092	0.067	0.725	0.078	0.057
Mihan	1.000	0.329	0.329	1.000	0.312	0.312	1.000	0.296	0.296	1.000	0.282	0.282	1.000	0.268	0.268
Kosar	0.818	0.675	0.552	0.788	0.630	0.496	0.740	0.505	0.374	0.691	0.375	0.259	0.643	0.243	0.156
Ma	0.648	0.708	0.459	0.646	0.676	0.437	0.644	0.644	0.415	0.642	0.615	0.395	0.640	0.586	0.375
Arman	0.285	0.566	0.161	0.285	0.537	0.153	0.285	0.510	0.145	0.286	0.481	0.138	0.286	0.458	0.131
Taavon	0.358	0.813	0.291	0.357	0.764	0.273	0.355	0.722	0.256	0.353	0.680	0.240	0.352	0.641	0.226
Sarmad	0.382	0.823	0.314	0.381	0.801	0.305	0.380	0.779	0.296	0.379	0.755	0.286	0.378	0.733	0.277
Tejarat-no	0.490	0.709	0.348	0.488	0.688	0.336	0.485	0.669	0.325	0.506	0.623	0.315	0.502	0.612	0.307
Hekmat-saba	0.318	0.177	0.056	0.311	0.175	0.054	0.305	0.169	0.051	0.299	0.166	0.050	0.294	0.158	0.047

presented in the model (23). *Mellat* and *Mihan* obtained the full operational efficiency score of 1 at the first stage. Hafez is determined as a profitability efficient DMU at stage two. Dey achieved an efficiency score of 1 at both stages, which makes it an overall efficient DMU. From Table 3, it can be concluded that the average efficiency of the first and second stages are 0.641 and 0.447, respectively, which indicates that Iranian insurance companies lack desirable efficiencies and require significant improvements. Comparing stage one and stage two, it is observed that insurance companies have better performance in stage one, which shows that these companies are successful in attracting customers using their available facilities and labors. However, they have failed to manage the gained capital to generate profit properly.

The pairwise comparison between operational (stage 1) and profitability (stage 2) efficiencies is plotted in Fig. 4. It can be observed that only five DMUs are identified as star DMUs. In Figs. 4 and 9 out of 22 DMUs are located at quadrant 4. These are the insurance companies that only need to improve their profitability efficiency through better investment management. Similarly, the DMUs plotted in quadrant 2 should only focus on improving their operational efficiency. *Sina*, *Novin*, *Kosar*, and *Arman* are at the edge of being a weak DMU in terms of

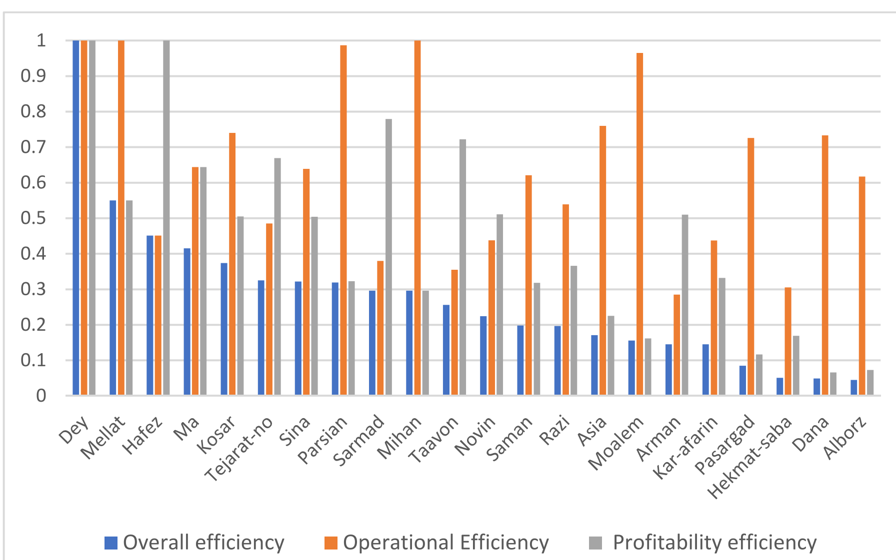

Fig. 3. Efficiency scores for different α

Table 4
The efficiency and ranks of DMUs generated by the model (23) for. $\alpha = 0.6$

DMU	e^1	Rank	e^2	Rank	e^{total}	Rank
Asia	0.760	4	0.225	16	0.171	15
Alborz	0.617	11	0.073	20	0.045	22
Dana	0.733	6	0.066	21	0.049	21
Moalem	0.965	3	0.162	18	0.156	16
Parsian	0.987	2	0.323	13	0.319	8
Razi	0.539	12	0.366	11	0.197	14
Kar-afarin	0.437	16	0.332	12	0.145	18
Sina	0.639	9	0.504	10	0.322	7
Mellat	1	1	0.550	6	0.550	2
Hafez	0.451	14	1	1	0.451	3
Dey	**1**	**1**	**1**	**1**	**1**	**1**
Saman	0.621	10	0.318	14	0.198	13
Novin	0.438	15	0.511	7	0.224	12
Pasargad	0.726	7	0.117	19	0.085	19
Mihan	1	1	0.296	15	0.296	9
Kosar	0.740	5	0.505	9	0.374	5
Ma	0.644	8	0.644	5	0.415	4
Arman	0.285	20	0.510	8	0.145	17
Taavon	0.355	18	0.722	3	0.256	11
Sarmad	0.380	17	0.779	2	0.296	10
Tejarat-no	0.485	13	0.669	4	0.325	6
Hekmat-saba	0.305	19	0.169	17	0.051	20

profitability efficiency. Therefore, the managers of these 4 insurance companies should be more sensitive to investment management. The pairwise comparison between operational efficiency and the overall efficiency can be plotted. Based on the comparison between operational and overall efficiencies, although more than half of the companies obtained a satisfactory operational efficiency, only two DMUs are determined as the star DMUs. This proves that Iranian insurance companies' main weakness is their inability to manage gained investments to generate profit.

Providing such graph analysis can help managers to recognize where improvement should be investigated with priority. For instance, *Moalem* obtained a satisfactory efficiency score for operational efficiency while its profitability efficiency is very low; Thus, increasing the net profit is the first priority for this DMU. In another example, it can be observed that with minor modification at the stage one process, *Tejarat-no* can reach high profits. In order to better understand the managerial application of this study, we examined three random bank branches and analyzed their efficiency. Fig. 5 illustrates the performance of three insurance companies in terms of stage efficiencies and overall efficiency. It can be seen that none of the branches received all three desirable performances at the same time. *Mihan* has a satisfactory operational efficiency, while *Taavon* obtained a desired profitability efficiency. *Hekmat-saba* is inefficient with respect to all three discussed efficiencies. Fig. 5 tells the managers that *Mihan* needs to identify the reasons for the poor performance of stage two and take the necessary actions to improve its overall efficiency. Similarly, *Taavon* only requires attention in the first stage to reach a desirable overall efficiency. *Hekmat-saba* is in a critical situation and if the necessary measures are not taken, it may face irreparable consequences.

5. Conclusions and direction for future studies

In an earlier study by Ref. [7]; a multi-objective non-linear model is proposed for integrating the stages in a two-stage DEA process to calculate the overall efficiency. Their methodology adopts a multi-step composition approach, which leads to complex non-linear calculations to evaluate the stage efficiencies and overall efficiency without considering either negative data or undesirable output. To overcome these shortcomes, the current study suggested an alternative method to modify the work of [7] for efficiency assessment. We transformed negative data into positive data using an approach developed by Tone et al. [9]. Then a single objective linear model was developed equivalent to the multi-objective non-linear model proposed by Ref. [7]; using a goal programming approach. The validation of the proposed model was confirmed through a numerical example. Finally, we incorporate the data uncertainty in the developed goal programming model using a fuzzy approach based on the $\alpha - cut$ technique. After presenting the obtained results for different α values, we performed a comparative

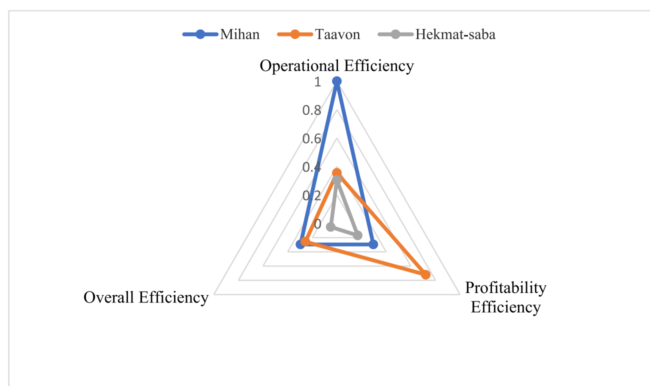

Fig. 5. Performance profile of three insurance companies.

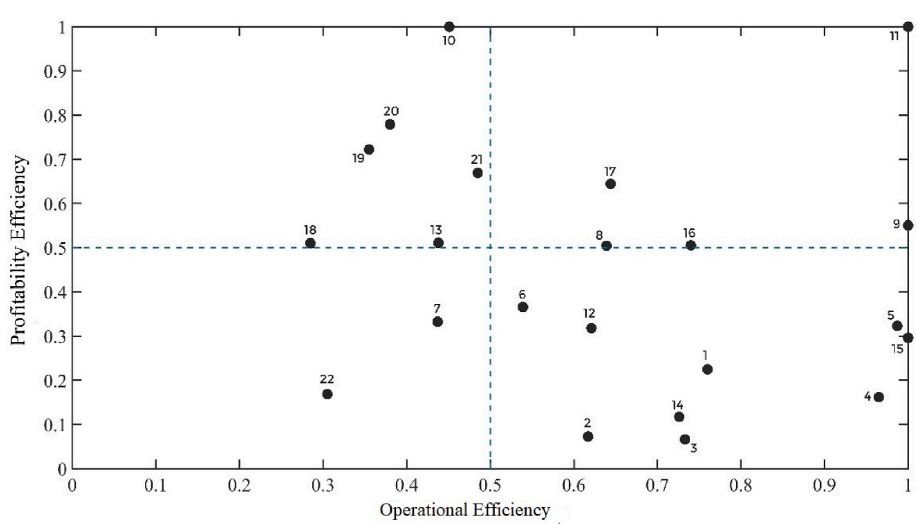

Fig. 4. Operational efficiency versus profitability efficiency.

analysis which provides useful managerial information for insurance companies managers since they can easily identify where improvements should be carried out with priority.

Credit author Statement

Hashem Omrani: Conceptualization, Methodology, Supervision, Validation, Writing - review & editing. **Ali Emrouznejad**: Formal analysis, Investigation, Methodology, Supervision, Writing - review & editing. **Meisam Shamsi:** Data curation, Methodology, Software, Writing - original draft.**Pegah Fahimi**: Data curation, Formal analysis, Writing - original draft.

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