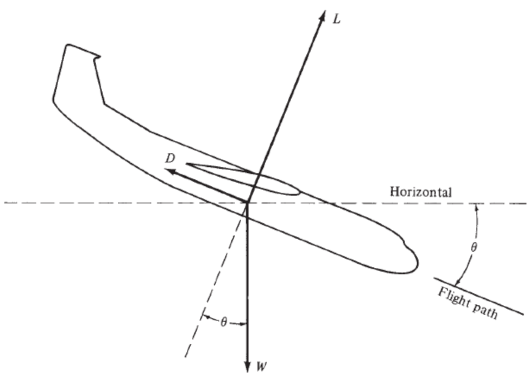
GLIDING FLIGHT

$$D = W \sin\theta$$

$$L = W \cos\theta$$



$$\frac{\sin\theta}{\cos\theta} = \frac{D}{L}$$

$$\boxed{\tan\theta = \frac{1}{L/D}}$$

EXAMPLE 6.12

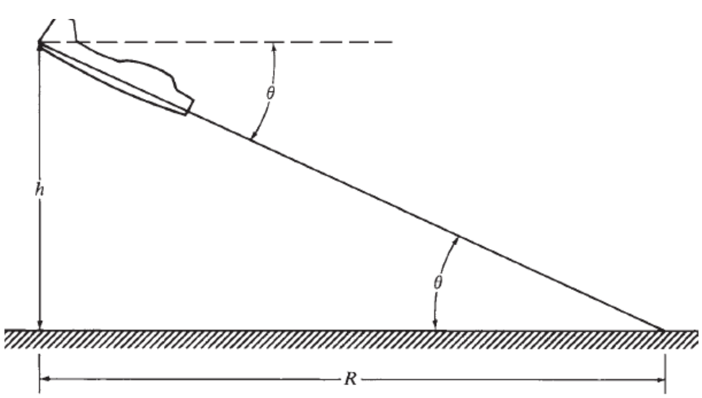
The maximum lift-to-drag ratio for the CP-1 is 13.6. Calculate the minimum glide angle and the maximum range measured along the ground covered by the CP-1 in a power-off glide that starts at an altitude of 10,000 ft.

■ **Solution**

The minimum glide angle is obtained from Eq. (6.56) as

$$\tan\theta_{\min} = \frac{1}{(L/D)_{\max}} = \frac{1}{13.6}$$

$$\boxed{\theta_{\min} = 4.2^\circ}$$



$$R = \frac{h}{\tan\theta} = h\frac{L}{D}$$

$$R_{\max} = h\left(\frac{L}{D}\right)_{\max} = 10{,}000(13.6)$$

$$\boxed{R_{\max} = 136{,}000 \text{ ft} = 25.6 \text{ mi}}$$

EXAMPLE

Repeat Example 6.12 for the CJ-1, for which the value of $(L/D)_{max}$ is 16.9.

■ **Solution**

$$\tan\theta_{min} = \frac{1}{(L/D)_{max}} = \frac{1}{16.9}$$

$$\boxed{\theta_{min} = 3.39°}$$

$$R_{max} = h\left(\frac{L}{D}\right)_{max} = 10{,}000(16.9)$$

$$\boxed{R_{max} = 169{,}000 \text{ ft} = 32 \textbf{ mi}}$$

Note the obvious fact that the CJ-1, with its higher value of $(L/D)_{max}$, is capable of a larger glide range than the CP-1.

ABSOLUTE AND SERVICE CEILINGS

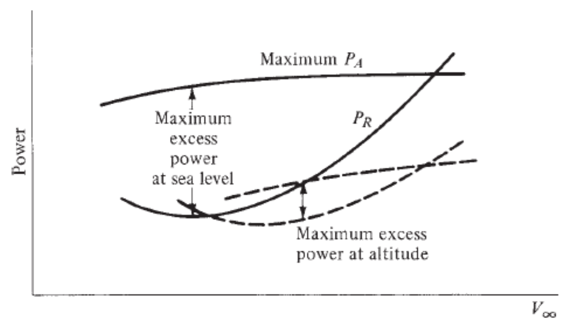


Figure 6.36 Variation of excess power with altitude.

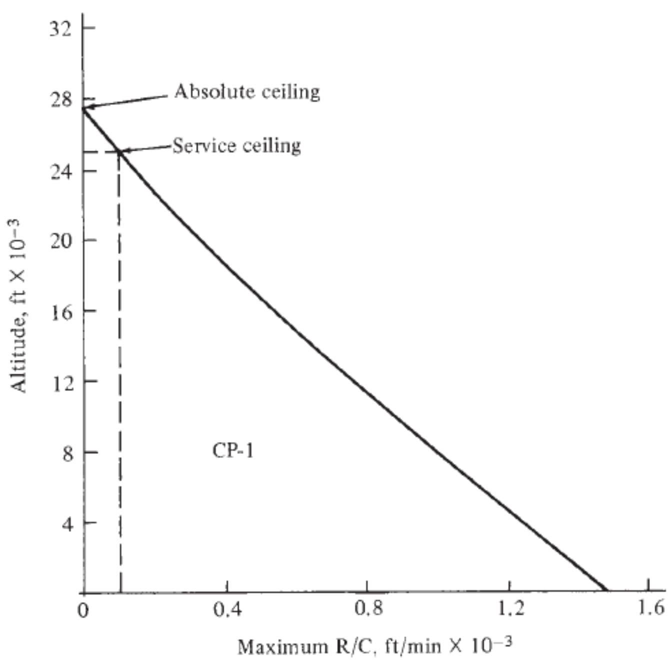


Figure 6.37 Determination of absolute and service ceilings for the CP-1.

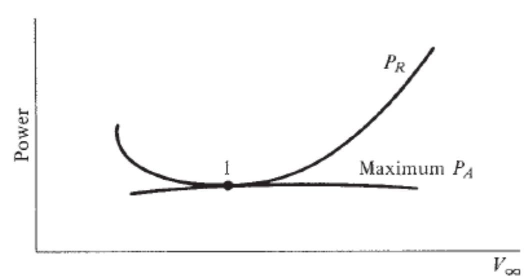


Figure 6.38 Power-required and power-available curves at the absolute ceiling.

1. Using the technique of Sec. 6.8, calculate values of maximum R/C for a number of different altitudes.
2. Plot maximum rate of climb versus altitude, as shown in Fig. 6.37.
3. Extrapolate the curve to 100 ft/min and 0 ft/min to find the service and absolute ceilings, respectively, as also shown in Fig. 6.37.

EXAMPLE 6.16

Derive a closed-form equation for the absolute ceiling of a given airplane as a function of the wing loading, W/S, and the power loading, $W/(P_{A,0})_{max}$, where $(P_{A,0})_{max}$ is the maximum power available at sea level.

■ Solution

Examining Fig. 6.38, we see that when an airplane is flying at its absolute ceiling, the minimum power required, $(P_{R,alt})_{min}$, is equal to the maximum power available, $(P_{A,alt})_{max}$; this condition is shown as point 1 in Figure 6.38, and is given by

$$(P_{R,alt})_{min} = (P_{A,alt})_{max} \tag{E 6.15.1}$$

The altitude effect on power available is discussed in Sec. 6.7, where we assume that P_A is proportional to the ambient density. This is a reasonable approximation for a turbojet engine, and for an unsupercharged reciprocating engine. Thus

$$(P_{A,alt})_{max} = \frac{\rho}{\rho_0}(P_{A,0})_{max} \tag{E 6.15.2}$$

From Example 6.6, we have the relation for the minimum power required at altitude, $(P_{R,alt})_{min}$, in terms of the minimum power required at sea level, $(P_{R,0})_{min}$, namely Eq. (E 6.6.1).

$$(P_{R,alt})_{min} = \left(\frac{\rho_0}{\rho}\right)^{1/2} (P_{R,0})_{min} \tag{E 6.6.1}$$

Substituting Eqs. (E 6.15.2) and (E 6.6.1) into (E 6.15.1), we have

$$\left(\frac{\rho}{\rho_0}\right)^{1/2} (P_{R,0})_{min} = \frac{\rho}{\rho_0}(P_{A,0})_{max}$$

or

$$(P_{R,0})_{min} = \left(\frac{\rho}{\rho_0}\right)^{1.5} (P_{A,0})_{max} \tag{E6.15.3}$$

where the density ρ is the density at the absolute ceiling.

An approximation for ρ/ρ_0 as a function of h is given in Example 3.2 as

$$\frac{\rho}{\rho_0} = e^{-\frac{h}{29{,}800}} \tag{3.16}$$

where h is in feet. Substituting Eq. (3.16) into (E 6.15.3), where now h pertains to the absolute ceiling, denoted by H, we have

$$(P_{R,0})_{\min} = e^{\frac{-1.5H}{29,800}}(P_{A,0})_{\max}$$

$$(P_{R,0})_{\min} = e^{\frac{-H}{19,867}}(P_{A,0})_{\max}$$

$$e^{\frac{-H}{19,867}} = \frac{(P_{R,0})_{\min}}{(P_{A,0})_{\max}}$$

$$-\frac{H}{19,867} = \ell n\left[\frac{(P_{R,0})_{\min}}{(P_{A,0})_{\max}}\right]$$

$$H = -19,867\ \ell n\left[\frac{(P_{R,0})_{\min}}{(P_{A,0})_{\max}}\right] \qquad \text{(E 6.15.4)}$$

Returning to Eq. (6.27) for P_R, at sea level,

$$P_{R,0} = \sqrt{\frac{2W^3}{\rho_0 S}}\left(\frac{1}{C_L^{3/2}/C_D}\right) \qquad \text{(E 6.15.5)}$$

Minimum power required occurs when the airplane is flying at $\left(\frac{C_L^{3/2}}{C_D}\right)_{\max}$. Hence, from Eq. (E 6.15.5),

$$(P_{R,0})_{\min} = \sqrt{\frac{2W^3}{\rho_0 S}}\frac{1}{(C_L^{3/2}/C_D)_{\max}} \qquad \text{(E 6.15.6)}$$

Reaching ahead for a result from Sec. 6.14 where we prove that the value of $\left(\frac{C_L^{3/2}}{C_D}\right)_{\max}$ for a given airplane is simply an aerodynamic property of the airplane, namely from Eq. (6.87),

$$\left(\frac{C_L^{3/2}}{C_D}\right)_{\max} = \frac{(3\ C_{D,0}\ \ \pi e\,\text{AR})^{3/4}}{4C_{D,0}}$$

Substituting this result into Eq. (E 6.15.6), we get

$$(P_{R,0})_{\min} = \sqrt{\frac{2W^3}{\rho_0 S}}\left[\frac{4C_{D,0}}{(3\ C_{D,0}\ \pi e\,\text{AR})^{3/4}}\right]$$

or

$$(P_{R,0})_{\min} = W\sqrt{\frac{2}{\rho_0}\left(\frac{W}{S}\right)}\left[\frac{0.7436\ C_{D,0}{}^{1/4}}{(e\,\text{AR})^{3/4}}\right] \qquad \text{(E 6.15.7)}$$

Substituting Eq. (E 6.15.7) into (E 6.15.4), we have

$$\boxed{H = -19{,}867\, \ell n \left\{ \frac{W}{(P_{A,0})_{\max}} \right\} \sqrt{\frac{2}{\rho_0}\left(\frac{W}{S}\right)} \left[\frac{0.7436\, C_{D,0}^{\;1/4}}{(e\,\mathrm{AR})^{3/4}} \right]} \qquad \text{(E 6.15.8)}$$

Eq. (E 6.15.8) is a closed-form analytical equation for the absolute ceiling H, where H is in feet.

TIME TO CLIMB

$$\mathrm{R/C} = dh/dt$$

$$dt = \frac{dh}{\mathrm{R/C}}$$

$$t = \int_{h_1}^{h_2} \frac{dh}{\mathrm{R/C}}$$

اگر $h_1 = 0$

$$t = \int_{0}^{h_2} \frac{dh}{\mathrm{R/C}}$$

EXAMPLE

Calculate and compare the time required for (*a*) the CP-1 and (*b*) the CJ-1 to climb to 20,000 ft.

■ Solution

a. For the CP-1, from Eq. (6.58), the time to climb is equal to the shaded area under the curve shown in Fig. 6.40. The resulting area gives time to climb as $\boxed{27.0\text{ min}}$.

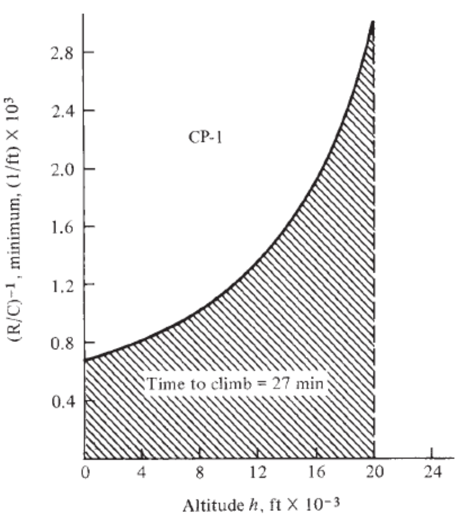


Figure 6.40 Determination of time to climb for the CP-1.

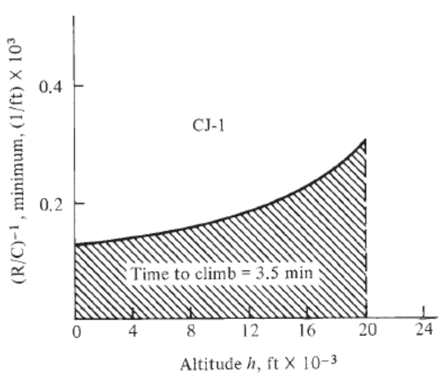


Figure 6.41 Determination of time to climb for the CJ-1.

RANGE AND ENDURANCE: PROPELLER-DRIVEN AIRPLANE

Physical Considerations

SFC : میزان سوخت مصرفی به ازای تولید واحد توان در واحد زمان

$$\text{SFC} = \frac{\text{lb of fuel}}{(\text{bhp})(\text{h})}$$

In this development, the specific fuel consumption is couched in units that are consistent:

$$\frac{\text{lb of fuel}}{(\text{ft}\cdot\text{lb/s})(\text{s})} \quad \text{or} \quad \frac{\text{N of fuel}}{(\text{J/s})(\text{s})}$$

For convenience and clarification, c will designate the specific fuel consumption with consistent units.

Consider the product $cP\,dt$, where P is engine power and dt is a small increment of time. The units of this product are (in the English engineering system)

$$cP\,dt = \frac{\text{lb of fuel}}{(\text{ft}\cdot\text{lb/s})(\text{s})}\frac{\text{ft}\cdot\text{lb}}{\text{s}}(\text{s}) = \text{lb of fuel}$$

$$W_1 = W_0 - W_f$$

$$dW_f = dW = -cP\,dt$$

$$dt = -\frac{dW}{cP}$$

$$\int_0^E dt = -\int_{W_0}^{W_1} \frac{dW}{cP}$$

$$\boxed{E = \int_{W_1}^{W_0} \frac{dW}{cP}}$$

$$V_\infty\,dt = -\frac{V_\infty\,dW}{cP}$$

$$ds = -\frac{V_\infty\,dW}{cP}$$

$$\int_0^R ds = -\int_{W_0}^{W_1} \frac{V_\infty \, dW}{cP}$$

$$\boxed{R = \int_{W_1}^{W_0} \frac{V_\infty \, dW}{cP}}$$

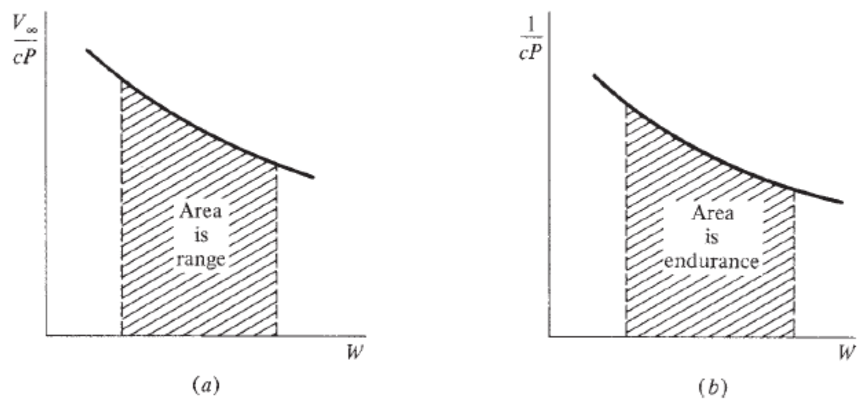
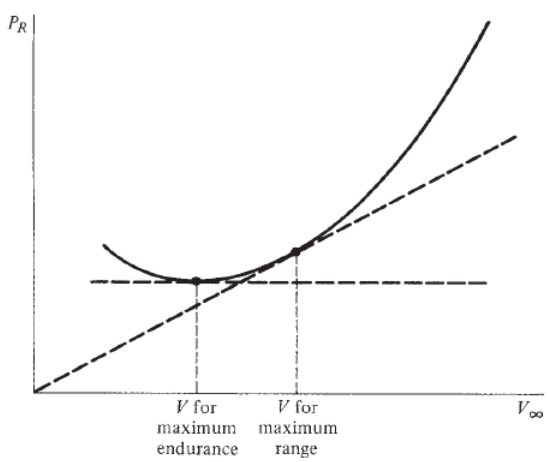


Figure 6.43 Determination of range and endurance.

Maximum endurance for a propeller-driven airplane occurs when the airplane is flying at minimum power required.

Maximum endurance for a propeller-driven airplane occurs when the airplane is flying at a velocity such that $C_L^{3/2}/C_D$ is at its maximum.

Maximum range for a propeller-driven airplane occurs when the airplane is flying at a velocity such that C_L/C_D is at its maximum.



For level, unaccelerated flight, we demonstrated in Sec. 6.5 that $P_R = DV_\infty$. To maintain steady conditions, the pilot has adjusted the throttle so that power available from the engine–propeller combination is just equal to the power required: $P_A = P_R = DV_\infty$. In Eq. (6.59), P is the brake power output of the engine itself. Recall from Eq. (6.31) that $P_A = \eta P$, where η is the propeller efficiency. Thus

$$P = \frac{P_A}{\eta} = \frac{DV_\infty}{\eta} \tag{6.64}$$

Substitute Eq. (6.64) into (6.63):

$$R = \int_{W_1}^{W_0} \frac{V_\infty dW}{cP} = \int_{W_1}^{W_0} \frac{V_\infty \eta\, dW}{cDV_\infty} = \int_{W_1}^{W_0} \frac{\eta\, dW}{cD} \tag{6.65}$$

$$R = \int_{W_1}^{W_0} \frac{\eta}{cD} \frac{W}{W} dW = \int_{W_1}^{W_0} \frac{\eta}{c} \frac{L}{D} \frac{dW}{W}$$

$$R = \frac{\eta}{c} \frac{C_L}{C_D} \int_{W_1}^{W_0} \frac{dW}{W}$$

$$\boxed{R = \frac{\eta}{c} \frac{C_L}{C_D} \ln \frac{W_0}{W_1}}$$ *Breguet range formula*

1. The largest possible propeller efficiency η.
2. The lowest possible specific fuel consumption c.
3. The highest ratio of W_0/W_1, which is obtained with the largest fuel weight W_F.
4. Most importantly, flight at maximum L/D. This confirms our argument in Sec. 6.12.1 that for *maximum range,* we must fly at maximum L/D. Indeed, the Breguet range formula shows that range is directly proportional to L/D. This clearly explains why high values of L/D (high aerodynamic efficiency) have always been important in the design of airplanes. This importance was underscored in the 1970s by the increasing awareness of the need to conserve fuel.

A similar formula can be obtained for endurance. If we recall that $P = DV_\infty/\eta$ and that $W = L$, Eq. (6.60) becomes

$$E = \int_{W_1}^{W_0} \frac{dW}{cP} = \int_{W_1}^{W_0} \frac{\eta}{c} \frac{dW}{DV_\infty} = \int_{W_1}^{W_0} \frac{\eta}{c} \frac{L}{DV_\infty} \frac{dW}{W}$$

Because $L = W = \frac{1}{2}\rho_\infty V_\infty^2 S C_L, V_\infty = \sqrt{2W/(\rho_\infty S C_L)}$. Thus

$$E = \int_{W_1}^{W_0} \frac{\eta}{c} \frac{C_L}{C_D} \sqrt{\frac{\rho_\infty S C_L}{2}} \frac{dW}{W^{3/2}}$$

$$E = -2\frac{\eta}{c}\frac{C_L^{3/2}}{C_D}\left(\frac{\rho_\infty S}{2}\right)^{1/2}[W^{-1/2}]_{W_1}^{W_0}$$

$$\boxed{E = \frac{\eta}{c}\frac{C_L^{3/2}}{C_D}(2\rho_\infty S)^{1/2}(W_1^{-1/2} - W_0^{-1/2})}$$ *Breguet endurance formula*

1. The highest propeller efficiency η.
2. The lowest specific fuel consumption c.
3. The highest fuel weight W_f, where $W_0 = W_1 + W_f$.
4. Flight at maximum $C_L^{3/2}/C_D$. This confirms our argument in Sec. 6.12.1 that for *maximum endurance,* we must fly at maximum $C_L^{3/2}/C_D$.
5. Flight at sea level, because $E \propto \rho_\infty^{1/2}$, and ρ_∞ is largest at sea level.

EXAMPLE 6.19

Estimate the maximum range and maximum endurance for the CP-1.

■ Solution

The Breguet range formula is given by Eq. (6.67) for a propeller-driven airplane. This equation is

$$R = \frac{\eta}{c}\frac{C_L}{C_D}\ln\frac{W_0}{W_1}$$

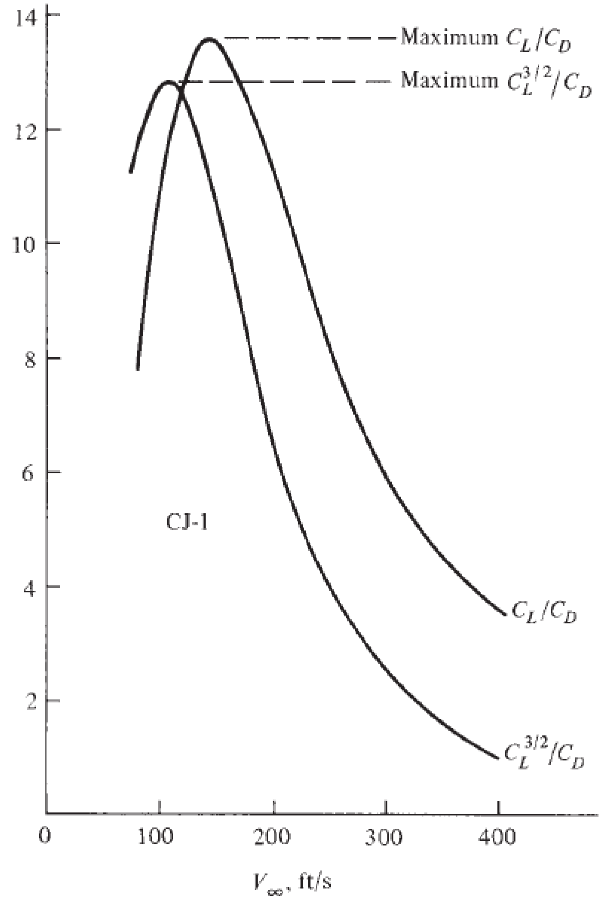
with the specific fuel consumption c in consistent units, say (lb fuel)/(ft · lb/s)(s) or simply per foot. However, in Example 6.1 the SFC is given as 0.45 lb of fuel/(hp)(h). This can be changed to consistent units:

$$c = 0.45\frac{\text{lb}}{(\text{hp})(\text{h})}\frac{1\text{ hp}}{550\text{ ft}\cdot\text{lb/s}}\frac{1\text{ h}}{3600\text{ s}} = 2.27\times10^{-7}\text{ ft}^{-1}$$

In Example 6.1 the variation of $C_L/C_D = L/D$ was calculated versus velocity. The variation of $C_L^{3/2}/C_D$ can be obtained in the same fashion. The results are plotted in Fig. 6.44.

From these curves,

$$\max\left(\frac{C_L}{C_D}\right) = 13.62 \qquad \max\left(\frac{C_L^{3/2}}{C_D}\right) = 12.81$$



The gross weight of the CP-1 is $W_0 = 2950$ lb. The fuel capacity given in Example 6.1 is 65 gal of aviation gasoline, which weighs 5.64 lb/gal. Hence, the weight of the fuel $W_p = 65(5.64) = 367$ lb. Thus, the empty weight $W_1 = 2950 - 367 = 2583$ lb.

Returning to Eq. (6.67), we have

$$R = \frac{\eta}{c}\frac{C_L}{C_D}\ln\frac{W_0}{W_1} = \frac{0.8}{2.27\times10^{-7}}(13.62)\left(\ln\frac{2950}{2583}\right)$$

$$\boxed{R = 6.38\times10^6 \text{ ft}}$$

Because 1 mi = 5280 ft,

$$R = \frac{6.38\times10^6}{5280} = \boxed{1207 \text{ mi}}$$

The endurance is given by Eq. (6.68):

$$E = \frac{\eta}{c}\frac{C_L^{3/2}}{C_D}(2\rho_\infty S)^{1/2}(W_1^{-1/2} - W_0^{-1/2})$$

Because of the explicit appearance of ρ_∞ in the endurance equation, maximum endurance will occur at sea level, $\rho_\infty = 0.002377$ slug/ft³. Hence

$$E = \frac{0.8}{2.27\times10^{-7}}(12.81)[2(0.002377)(174)]^{1/2}\left(\frac{1}{2583^{1/2}} - \frac{1}{2950^{1/2}}\right)$$

$$\boxed{E = 5.19 \times 10^4 \text{ s}}$$

Because 3600 s = 1 h,

$$E = \frac{5.19\times10^4}{3600} = \boxed{14.4\,\text{h}}$$

6.13 RANGE AND ENDURANCE: JET AIRPLANE

For a jet airplane, the specific fuel consumption is defined as the *weight of fuel consumed per unit thrust per unit time.* Note that *thrust* is used here, in contradistinction to *power,* as in the previous case for a reciprocating engine–propeller combination. The fuel consumption of a jet engine physically depends on the thrust produced by the engine, whereas the fuel consumption of a reciprocating engine physically depends on the brake power produced. It is this simple difference that leads to different range and endurance formulas for a jet airplane. In the literature, *thrust-specific fuel consumption (TSFC)* for jet engines is commonly given as

$$\text{TSFC} = \frac{\text{lb of fuel}}{(\text{lb of thrust})(\text{h})}$$

Note the nonconsistent unit of time.

$$\frac{T_R}{V_\infty} = \frac{D}{V_\infty} = \frac{\frac{1}{2}\rho_\infty V_\infty^2 S C_D}{V_\infty} = \frac{1}{2}\rho_\infty V_\infty S C_D$$

Because $V_\infty = \sqrt{2W/(\rho_\infty S C_L)}$, we have

$$\frac{T_R}{V_\infty} = \frac{1}{2}\rho_\infty S\sqrt{\frac{2W}{\rho_\infty S C_L}}\,C_D \propto \frac{1}{C_L^{1/2}/C_D}$$

6.13.2 Quantitative Formulation

Let c_t be the thrust-specific fuel consumption in consistent units:

$$\frac{\text{lb of fuel}}{(\text{lb of thrust})(\text{s})} \quad \text{or} \quad \frac{\text{N of fuel}}{(\text{N of thrust})(\text{s})}$$

Let dW be the elemental change in weight of the airplane due to fuel consumption over a time increment dt. Then

$$dW = -c_t T_A \, dt$$

or

$$dt = \frac{-dW}{c_t T_A} \tag{6.69}$$

$$E = -\int_{W_0}^{W_1} \frac{dW}{c_t T_A}$$

Recalling that $T_A = T_R = D$ and $W = L$, we have

$$E = \int_{W_1}^{W_0} \frac{1}{c_t} \frac{L}{D} \frac{dW}{W}$$

With the assumption of constant c_t and $C_L/C_D = L/D$, Eq. (6.71) becomes

$$\boxed{E = \frac{1}{c_t} \frac{C_L}{C_D} \ln \frac{W_0}{W_1}}$$

1. Minimum thrust-specific fuel consumption c_t.
2. Maximum fuel weight W_f.
3. Flight at maximum L/D. This confirms our argument in Sec. 6.13.1 that for maximum endurance for a jet, we must fly so that L/D is at its maximum.

$$ds = V_\infty dt = -\frac{V_\infty dW}{c_t T_A}$$

$$R = \int_0^R ds = -\int_{W_0}^{W_1} \frac{V_\infty dW}{c_t T_A}$$

$$R = \int_{W_1}^{W_0} \frac{V_\infty}{c_t} \frac{C_L}{C_D} \frac{dW}{W}$$

Because $V_\infty = \sqrt{2W/(\rho_\infty S C_L)}$, Eq. (6.75) becomes

$$R = \int_{W_1}^{W_0} \sqrt{\frac{2}{\rho_\infty S}} \frac{C_L^{1/2}/C_D}{c_t} \frac{dW}{W^{1/2}}$$

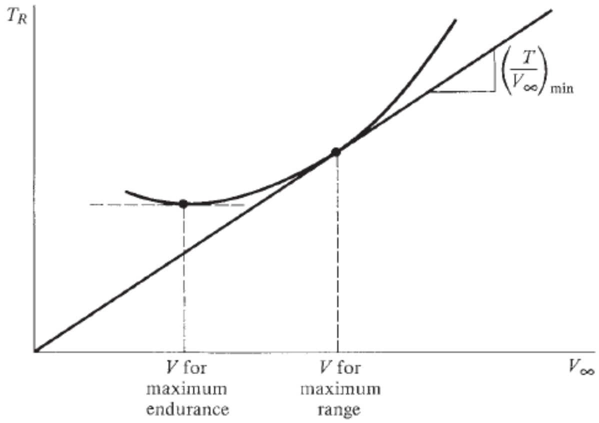
$$R = \sqrt{\frac{2}{\rho_\infty S}} \frac{C_L^{1/2}}{C_D} \frac{1}{c_t} \int_{W_1}^{W_0} \frac{dW}{W^{1/2}}$$

$$\boxed{R = 2\sqrt{\frac{2}{\rho_\infty S}} \frac{1}{c_t} \frac{C_L^{1/2}}{C_D} (W_0^{1/2} - W_1^{1/2})}$$

1. Minimum thrust-specific fuel consumption c_t.
2. Maximum fuel weight W_f.
3. Flight at maximum $C_L^{1/2}/C_D$. This confirms our argument in Sec. 6.13.1 that for maximum range, a jet must fly at a velocity such that $C_L^{1/2}/C_D$ is at its maximum.
4. Flight at high altitudes—that is, low ρ_∞. Of course Eq. (6.77) says that R becomes infinite as ρ_∞ decreases to zero (that is, as we approach outer space). This is physically ridiculous, however, because an airplane requires the atmosphere to generate lift and thrust. Long before outer space is reached, the assumptions behind Eq. (6.77) break down. Moreover, at extremely high altitudes ordinary turbojet performance deteriorates and c_t begins to increase. All we can conclude from Eq. (6.77) is that range for a jet is poorest at sea level and increases with altitude up to a point. Typical cruising altitudes for subsonic commercial jet transports are from 30,000 to 40,000 ft; for supersonic transports they are from 50,000 to 60,000 ft.

Maximum endurance for a jet airplane occurs when the airplane is flying at the minimum thrust required.

Maximum endurance for a jet airplane occurs when the airplane is flying at a velocity such that C_L/C_D is at its maximum.



EXAMPLE 6.20

Estimate the maximum range and endurance for the CJ-1.

■ **Solution**

From the calculations of Example 6.1, the variation of C_L/C_D and $C_L^{1/2}/C_D$ can be plotted versus velocity, as given in Fig. 6.46. From these curves, for the CJ-1,

$$\max\left(\frac{C_L^{1/2}}{C_D}\right) = 23.4$$

$$\max\left(\frac{C_L}{C_D}\right) = 16.9$$

In Example 6.1 the specific fuel consumption is given as TSFC = 0.6 (lb fuel)/(lb thrust)(h). In consistent units,

$$c_t = 0.6 \frac{\text{lb}}{(\text{lb})(\text{h})} \frac{1\,\text{h}}{3600\ \text{s}} = 1.667 \times 10^{-4}\ \text{s}^{-1}$$

Also, the gross weight is 19,815 lb. The fuel capacity is 1119 gal of kerosene, where 1 gal of kerosene weighs 6.67 lb. Thus $W_f = 1119(6.67) = 7463$ lb. Hence the empty weight is $W_1 = W_0 - W_f = 19{,}815 - 7463 = 12{,}352$ lb.

The range of a jet depends on altitude, as shown by Eq. (6.77). Assume that the cruising altitude is 22,000 ft, where $\rho_\infty = 0.001184$ slug/ft^3. From Eq. (6.77), using information from Example 6.1, we obtain

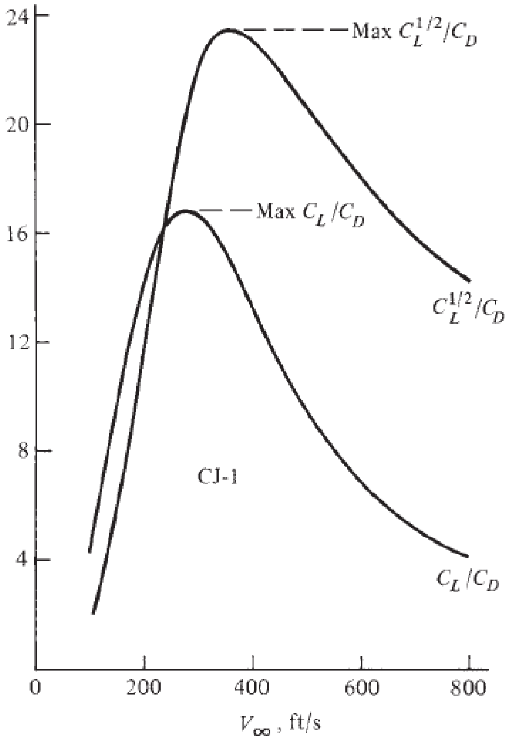


Figure 6.46 Aerodynamic ratios for the CJ-1 at sea level.

$$R = 2\sqrt{\frac{2}{\rho_\infty S}}\frac{1}{c_t}\frac{C_L^{1/2}}{C_D}\left(W_0^{1/2} - W_1^{1/2}\right)$$

$$= 2\sqrt{\frac{2}{0.001184(318)}}\left(\frac{1}{1.667\times10^{-4}}\right)(23.4)(19{,}815^{1/2} - 12{,}352^{1/2})$$

$$\boxed{R = 19.2\times10^{6}\,\text{ft}}$$

In miles,

$$R = \frac{19.2\times10^{6}}{5280} = \boxed{3630\ \text{mi}}$$

The endurance can be found from Eq. (6.72):

$$E = \frac{1}{c_t}\frac{C_L}{C_D}\ln\frac{W_0}{W_1} = \frac{1}{1.667\times10^{-4}}(16.9)\left(\ln\frac{19{,}815}{12{,}352}\right)$$

$$\boxed{E = 4.79\times10^{-4}\,\text{s}}$$

or in hours
$$E = \frac{4.79\times10^{4}}{3600} = \boxed{13.3\ \text{h}}$$

RELATIONS BETWEEN $C_{D,0}$ AND $C_{D,i}$

For example, consider maximum L/D. Recalling that $C_D = C_{D,0} + C_L^2/(\pi e \mathrm{AR})$, we can write

$$\frac{C_L}{C_D} = \frac{C_L}{C_{D,0} + C_L^2/(\pi e \mathrm{AR})} \tag{6.78}$$

For maximum C_L/C_D, differentiate Eq. (6.78) with respect to C_L and set the result equal to 0:

$$\frac{d(C_L/C_D)}{dC_L} = \frac{C_{D,0} + C_L^2/(\pi e \mathrm{AR}) - C_L[2C_L/(\pi e \mathrm{AR})]}{[C_{D,0} + C_L^2/(\pi e \mathrm{AR})]^2} = 0$$

Thus
$$C_{D,0} + \frac{C_L^2}{\pi e \mathrm{AR}} - \frac{2C_L^2}{\pi e \mathrm{AR}} = 0$$

or
$$C_{D,0} = \frac{C_L^2}{\pi e \mathrm{AR}}$$

$$\boxed{C_{D,0} = C_{D,i}} \quad \text{for} \quad \left(\frac{C_L}{C_D}\right)_{\max} \tag{6.79}$$

$$\boxed{C_{D,0} = C_{D,i}} \quad \text{for} \quad \left(\frac{C_L}{C_D}\right)_{\max}$$

$$\boxed{C_{D,0} = \frac{1}{3}C_{D,i}} \quad \text{for} \quad \left(\frac{C_L^{3/2}}{C_D}\right)_{\max}$$

$$\boxed{C_{D,0} = 3C_{D,i}} \quad \text{for} \quad \left(\frac{C_L^{1/2}}{C_D}\right)_{\max}$$

First consider again the case of maximum C_L/C_D. From Eq. (6.79),

$$C_{D,0} = C_{D,i} = \frac{C_L^2}{\pi e \mathrm{AR}}$$

Thus
$$C_L = \sqrt{\pi e \mathrm{AR} C_{D,0}}$$

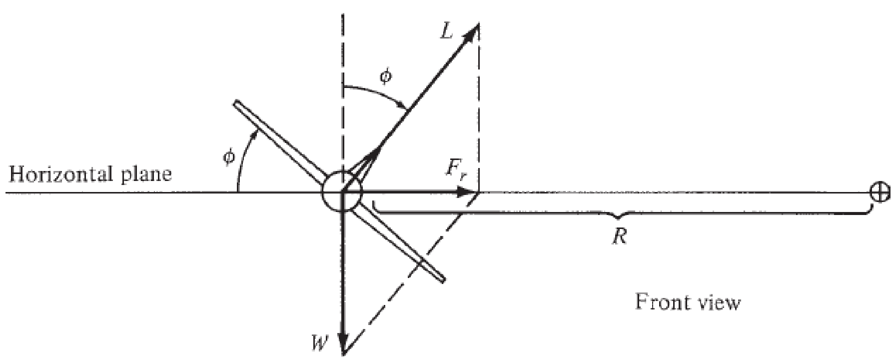
$$\frac{C_L}{C_D} = \frac{C_L}{2C_L^2/(\pi e \mathrm{AR})} = \frac{\pi e \mathrm{AR}}{2C_L} = \frac{\pi e \mathrm{AR}}{2\sqrt{\pi e \mathrm{AR} C_{D,0}}}$$

$$\boxed{\left(\frac{C_L}{C_D}\right)_{\max} = \frac{(C_{D,0}\pi e \mathrm{AR})^{1/2}}{2C_{D,0}}}$$

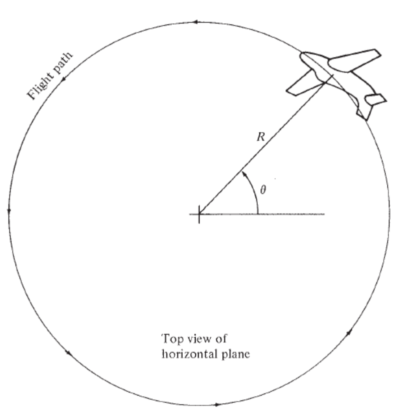
$$\boxed{\left(\frac{C_L^{1/2}}{C_D}\right)_{\max} = \frac{\left(\frac{1}{3}C_{D,0}\pi e \mathrm{AR}\right)^{1/4}}{\frac{4}{3}C_{D,0}}}$$

$$\boxed{\left(\frac{C_L^{3/2}}{C_D}\right)_{\max} = \frac{(3C_{D,0}\pi e \mathrm{AR})^{3/4}}{4C_{D,0}}}$$

TURNING FLIGHT AND THE V–n DIAGRAM



$$L\cos\phi = W$$



$$F_r = \sqrt{L^2 - W^2}$$

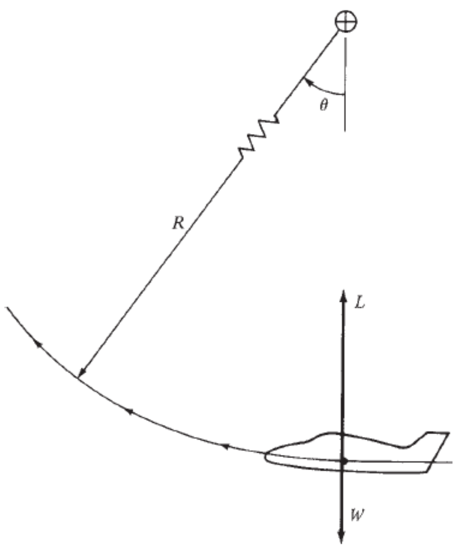
$$n \equiv \frac{L}{W} \qquad F_r = W\sqrt{n^2 - 1}$$

$$F_r = m\frac{V_\infty^2}{R} = \frac{W}{g}\frac{V_\infty^2}{R}$$

$$\boxed{R = \frac{V_\infty^2}{g\sqrt{n^2 - 1}}}$$

1. The highest possible load factor (that is, the highest possible L/W).
2. The lowest possible velocity.

Consider another case of turning flight



$$F_r = L - W = W(n-1)$$

$$F_r = m\frac{V_\infty^2}{R} = \frac{W}{g}\frac{V_\infty^2}{R}$$

$$\boxed{R = \frac{V_\infty^2}{g(n-1)}}$$

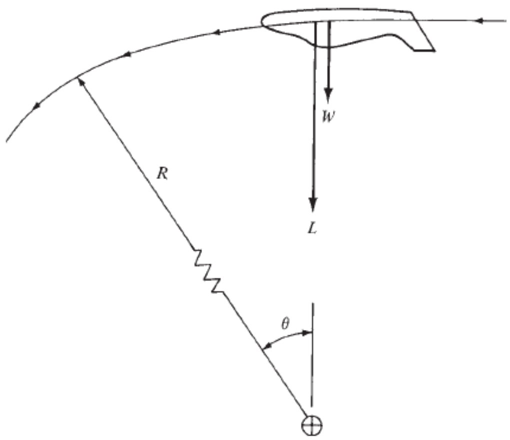


Figure 6.54 The pull-down maneuver.

$$R = \frac{V_\infty^2}{g(n+1)}$$

Considerations of turn radius and turn rate are particularly important to military fighter aircraft; everything else being equal, airplanes with the smallest R and largest ω will have definite advantages in air combat. High-performance fighter aircraft are designed to operate at high load factors—typically from 3 to 10. When n is large, then $n + 1 \approx n$ and $n - 1 \approx n$; for such cases Eqs. (6.118), (6.119), and (6.122) to (6.125) reduce to

$$R = \frac{V_\infty^2}{gn} \tag{6.126}$$

$$L = \tfrac{1}{2}\rho_\infty V_\infty^2 S C_L$$

then

$$V_\infty^2 = \frac{2L}{\rho_\infty S C_L} \tag{6.128}$$

Substituting Eqs. (6.128) and (6.115) into Eqs. (6.126) and (6.127), we obtain

$$R = \frac{2L}{\rho_\infty S C_L g(L/W)} = \frac{2}{\rho_\infty C_L g}\frac{W}{S} \tag{6.129}$$

$$\frac{W}{S} \equiv \text{wing loading}$$

Airplane	W/S, lb/ft²
Wright Flyer (1903)	1.2
Beechcraft Bonanza	18.8
McDonnell Douglas F-15	66
General Dynamics F-16	74

$$R_{\min} = \frac{2}{\rho_\infty g C_{L,\max}} \frac{W}{S}$$

$$n = \frac{L}{W} = \frac{\frac{1}{2}\rho_\infty V_\infty^2 S C_L}{W}$$

$$n_{\max} = \frac{1}{2}\rho_\infty V_\infty^2 \frac{C_{L,\max}}{W/S}$$

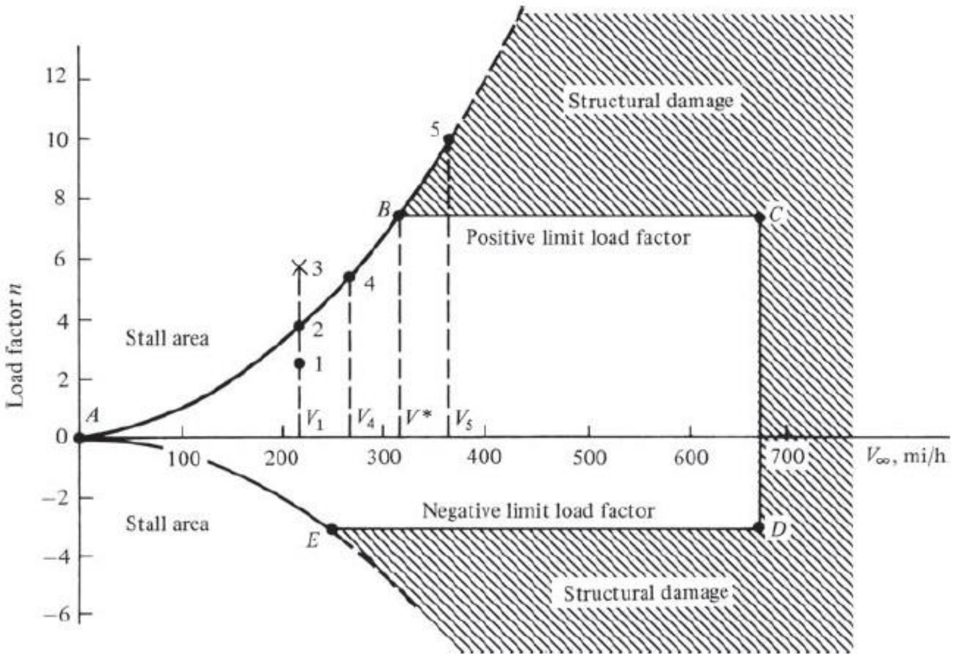
V–n diagram

However, n_{max} cannot be allowed to increase indefinitely. Beyond a certain value of load factor, defined as the *positive limit load factor* and shown as the horizontal line *BC* in Fig. 6.55, structural damage may occur to the aircraft. The velocity corresponding to point *B* is designated as V^*. At velocities higher than V^*, say V_5, the airplane must fly at values of C_L less than $C_{L,max}$ so that the positive limit load factor is not exceeded. If flight at $C_{L,max}$ is obtained at velocity V_5, corresponding to point 5 in Fig. 6.55, then structural damage will occur. The right side of the V–n diagram, line *CD*, is a high-speed limit. At velocities greater than this, the dynamic pressure becomes so large that again structural damage may occur to the airplane.

(This maximum velocity limit is, by design, much larger than the level-flight V_{max} calculated in Secs. 6.4 to 6.6. In fact, the structural design of most airplanes is such that the maximum velocity allowed by the V–n diagram is sufficiently greater than the maximum diving velocity for the airplane.) Finally, the bottom part of the V–n diagram, given by curves *AE* and *ED* in Fig. 6.55, corresponds to negative absolute angles of attack—that is, negative load factors. Curve *AE* defines the stall limit. (At absolute angles of attack less than zero, the lift is negative and acts in the downward direction. If the wing is pitched downward to a large enough negative angle of attack, the flow will separate from the bottom surface of the wing and the downward-acting lift will decrease in magnitude; that is, the wing *stalls*.) Line *ED* gives the *negative limit load factor*, beyond which structural damage will occur.

As a final note concerning the V–n diagram, consider point *B* in Fig. 6.55. This point is called the *maneuver point*. At this point both C_L and n are simultaneously at their highest possible values that can be obtained anywhere throughout the allowable flight envelope of the aircraft. Consequently, from Eqs. (6.131) and (6.132), this point corresponds simultaneously to the smallest possible turn radius and the largest possible turn rate for the airplane. The velocity corresponding to point *B* is called the *corner velocity* and is designated by V^* in Fig. 6.55. We can obtain the corner velocity by solving Eq. (6.133) for velocity, yielding

$$V^* = \sqrt{\frac{2n_{\max}}{\rho_\infty C_{L,\max}} \frac{W}{S}} \tag{6.134}$$



EXAMPLE

Consider the CJ-1 (Example 6.1) in a level turn at sea level. Calculate the minimum turn radius and the maximum turn rate. The maximum load factor and lift coefficient (with no flap deflection) are 5 and 1.4, respectively.

■ **Solution**

The minimum turn radius and maximum turn rate are obtained when the flight velocity is the corner velocity, V^*, given by Eq. (6.134):

$$V^* = \sqrt{\frac{2n_{\max}\left(\dfrac{W}{S}\right)}{\rho_\infty C_{L,\max}}} \tag{6.134}$$

The wing loading for the CJ-1 with a full fuel load is

$$\frac{W}{S} = \frac{19{,}815}{318} = 62.3 \text{ lb/ft}^2$$

Thus, from Eq. (6.134),

$$V^* = \sqrt{\frac{2n_{\max}\left(\dfrac{W}{S}\right)}{\rho_\infty C_{L,\max}}} = \sqrt{\frac{2(5)(62.3)}{(0.002377)(1.4)}} = 43.6 \text{ ft/s}$$

From Eq. (6.118), with $V_\infty = V^*$ and $n = n_{\max}$, we have

$$R_{\min} = \frac{(V^*)^2}{g\sqrt{n_{\max}^2 - 1}} = \frac{(43.6)^2}{32.2\sqrt{(5)^2 - 1}} = \boxed{1186 \text{ ft}}$$