



An accurate fuzzy rule-based classification systems for heart disease diagnosis



Khalid Bahani^{a,*}, Mohammed Moujabbir^b, Mohammed Ramdani^a

^aLaboratory of Computer Science, Faculty of Sciences and Techniques Mohammedia, Hassan II University, Morocco

^bLaboratory of Computer Science, Polydisciplinary Faculty of Khouribga, Sutan Moulay Slimane University, Morocco

ARTICLE INFO

Article history:

Received 9 January 2021

Revised 9 September 2021

Accepted 12 October 2021

Editor: DR B Gyampoh

Keywords:

Heart disease prediction

Fuzzy rule-based classification systems

Classification rules learning

Machine learning

Explainable artificial intelligence

ABSTRACT

Physicians and healthcare providers need to better understand the thought processes and methods used in clinical decision-making. This allows physicians to diagnose and detect diseases early, especially heart disease that causes death. The diversity and availability of healthcare data encourage clinicians to use healthcare applications in the diagnosis process. Most of these applications use machine learning techniques to make accurate and fast decisions. On the other hand, Explainability in healthcare applications increase the level of clinician confidence and reduces the risk of making wrong decisions, thus expands the scope and efficiency of healthcare applications. In this paper, we propose a novel data-driven method based on fuzzy clustering and linguistic modifiers to design a fuzzy rule-based classification system for heart disease diagnosis. The proposed system provides an interpretable knowledge base to explain the decision-making process. Regarding the experiment, we have used Cleveland, Hungarian and Va long beach heart disease datasets to compare the proposed method with five known machine learning methods for predicting heart disease: Artificial neural network, Support Vector Machine, K-Nearest Neighbor, Naïve Bayes, and Random Forest. The findings show that the proposed model is superior in terms of balancing interpretability and precision.

© 2021 The Author(s). Published by Elsevier B.V. on behalf of African Institute of Mathematical Sciences / Next Einstein Initiative.

This is an open access article under the CC BY-NC-ND license (<http://creativecommons.org/licenses/by-nc-nd/4.0/>)

Introduction

New technologies have led to the development of many fields such as medicine, industry, agriculture and commerce. Especially in the medical field, these technologies are used in the healthcare sector to provide better services at reasonable cost, and to develop hospital information systems by integrating tools ensuring comprehensive monitoring of patient health and clinical decision support [1]. The main purpose of clinical decision support tools is to aid healthcare by enabling analysis of patient data and using that information to help in formulating a diagnosis. Some of the benefits of clinical decision support tools are to help in clinical decision making, reduce misdiagnosis, reduce the risk of medication errors, and reduce the mortality rates [2]. The main objective of physicians around the world is to reduce mortality, but to achieve this goal, researchers and scientists must face many challenges. One of the biggest challenges is to treat diseases that in-

* Corresponding author.

E-mail address: khalid.bahani-etu@etu.univh2c.ma (K. Bahani).

crease mortality rates. In the literature, cardiovascular diseases are considered one of the deadliest diseases in the world, such as heart disease, cerebrovascular diseases and vascular diseases [3]. In 2016, the World Health Organization confirmed that 17.9 million people worldwide died from cardiovascular disease, accounting for 31% of all global deaths. 85% of these deaths are due to a heart attack and stroke [4]. The increase in mortality is due to risk factors that increase the probability of developing cardiovascular disease, such as age, family history of cardiovascular disease, gender, high cholesterol, high blood pressure, diabetes, smoking and obesity [5]. Avoiding risk factors decreases the probability of premature heart attacks and strokes, through healthier diets, regular physical activities, and not using tobacco products. It is also very important to check and control risk factors for heart disease and stroke such as high blood pressure, high cholesterol and high blood sugar and diabetes. Clinically, the early diagnosis of heart disease is essential to improve prevention and take more preventive measures to reduce the mortality of high-risk groups [6]. An accurate diagnosis analyzes the symptoms that differ from person to person based on age, weight, gender, and many other variables. The diversity and multiplicity of these variables require clinicians to spend a lot of time and energy studying them in order to make effective decisions. These reasons have prompted researchers to develop clinical decision support systems based on previous treatments, clinical records, statistics and information in the database [7]. Clinical decision support systems provide physicians and other health professionals with clinical decision support, that is, assistance with clinical decision-making tasks when it comes to their patients. In this context, machine learning and data mining technology effectively help heart disease data analysis and knowledge extraction [8]. Machine learning techniques (such as clustering, regression, and classification) have been widely used to predict heart disease; it is very important to make reliable predictions of heart disease to carry out appropriate treatments. Therefore, the ability to explain clinical decision-making has become the priority of a physician over accuracy, speed, and effectiveness [9]. Indeed, explaining why someone is classified as sick or for other reasons will increase the level of confidence of the physician and decrease the risk of making wrong decisions [10].

In order to achieve these goals, machine learning models often face the challenge of balancing interpretability (explainability) and accuracy. Currently, Explainable Artificial Intelligence (XAI) techniques are used to explain machine learning models, or in other words, to produce explainable models [11]. The objective of XAI is the end user who depends on the decisions or recommendations of AI systems, or the actions they take. Therefore, stakeholders must understand the logic of the systems they use. Fig. 1 illustrates the concept of XAI, which is to provide users with explanations allowing them to understand the general strengths and weaknesses of the system, to understand how it will behave in the future or in different situations, and to allow users to correct system errors [12]. One of the most transparent technologies in XAI is the rule-based model, in particular the fuzzy rule-based model, which allows the extracted knowledge to be reviewed for a dual purpose. On the one hand, to obtain a clear explanation of the inference process carried out by the system [13]. On the other hand, the ability to have confidence in the description of the rules and their relation to the problem to be solved. Fuzzy rule-based systems (FRBS) and fuzzy rule-based classification systems (FRBC) combine the ability to represent knowledge in a natural way for human understanding (with fuzzy rules), the power of fuzzy reasoning and the ability to probe complex problems. In this context, we raise the challenge to obtain a fuzzy rule-based classification system that can achieve a good compromise between interpretability and accuracy in predicting heart disease.

In this paper, we propose a fuzzy rule-based classification system for predicting heart disease through expanding the fuzzy linguistic rules learning method (based on subtractive clustering and linguistic modifiers) proposed in [14]. The novel method called "**Fuzzy classification Rules Learning through Clustering**" (FCRLC) learns fuzzy classification linguistic rules from data. The next section presents the related work. Section 3 explains the proposed methodology by presenting the dataset used in this study and the techniques for performance evaluation. Section 4 contains a preliminary of fuzzy rule-based classification system and the proposed method to learn linguistic fuzzy classification rules. In section 5, the results and the discussion are given.

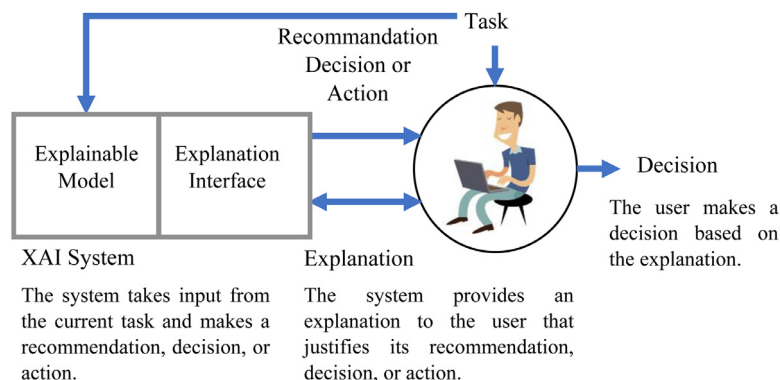


Fig. 1. The XAI concept.

Related work

Several studies have been used machine learning models, such as K-Nearest Neighbor (K-NN) [15], Naïve Bayes (NB) [16], Decision Trees (DT), Support Vector Machines (SVM) [17], and Artificial Neural Networks (ANN) [18], to identify heart disease. The available UCI Cleveland heart disease dataset [19], containing 76 attributes and 4 datasets, has been used in most studies. Studies using all features have used feature selection to improve the relevance of machine learning models, such as manual feature selection in [20]; Fast Correlation based Feature selection (FCBF) in [21], Least Absolute Shrinkage and Selection Operator (LASSO) in [22] and particle swarm optimization (PSO) in [23]. Most studies preprocess the data to remove missing values and use the features most relevant to heart disease (14 features). These studies aim to determine the most effective machine learning model for predicting heart disease, through improving of one or more machine learning algorithms, a hybridization of several algorithms, a comparative analysis of classification algorithms or using XAI techniques to explain machine learning models:

- 1 **Enhancing the machine learning model.** In [24], the author proposed a heart disease prediction system using the Multilayer Perceptron Neural Network (MLP) with backward propagation as a training algorithm, the model has a high accuracy of 93.39% for five neurons in the hidden layer. In [25], the author used logistic regression (LR) to predict heart disease. The LR algorithm is compared to four machine learning algorithms: NB, SVM, DT and K-NN. The LR algorithm achieves the best performance with an accuracy of 86.89%. In [20], the researcher proposed a prediction model based on Random Forest algorithm (RF) [26], the authors used manual feature selection (9 features) and 25% of database for testing. The RF algorithm achieves an accuracy of 97.56%.
- 2 **Hybrid algorithms.** In [27], the author combined several machine learning algorithms to propose a hybrid model for heart disease prediction. The algorithms involved are NB, SVM, K-NN, ANN, J48, and genetic algorithms (AG). The results showed that 89.2% accuracy was achieved by NB and SVM algorithm. In [28], the author proposed a hybrid model that combines naive Bayes algorithm and genetic algorithm, and compares the obtained model with weighted fuzzy rules and logistic regression models; the model achieves the best performance with an accuracy of 97.14%.
- 3 **Comparative analysis of classification algorithms.** In [29], A comparative study was carried out of the NB, DT, RF, SVM and LR algorithms in which the authors used a 10-fold cross-validation. The results demonstrated that DT and SVM are the two perfect algorithms with an accuracy of 93.19% and 92.30%, respectively. In [30], four machine learning algorithms are studied: RIPPER, DT, NN and SVM. To compare the performance of selected algorithms, the author identified three algorithms (K-NN, NB and MLP). The SVM algorithm achieves an accuracy of 90.00%. In [31], in order to propose a cloud-based system of heart disease prediction, the author combined two data sets: Cleveland and Va long beach. SVM, MLP, LR, NB and RF are the five algorithms involved in the comparative study. The best classifier is SVM, with an accuracy of 97.53%.
- 4 **XAI techniques to explain machine learning models.** Many studies in recent years have focused on explainable models in healthcare [9], particularly for heart disease prediction. In [32], The author used the Cleveland heart disease dataset to test feature-based and example-based XAI techniques. The author is interested in feature-based techniques such as Local Interpretable Model-agnostic Explanations (LIME) and Shapley Additive Explanations (SHAP), as well as example-based techniques like Anchors and Counterfactuals. The results show that Anchors, which are based on If-then rules, are more generalizable than LIME and SHAP. In [33], the author proposed an interpretable model based on Genetic Algorithms (GA) and Adaptive Neural Fuzzy Inference System (ANFIS) for predicting heart disease. 9-fold cross-validation was used in evaluation process. In addition, the author proposed an Importance Evaluation Function (IEF) to examine the importance of various features in predicting heart disease. The proposed model achieves an accuracy of 82.5%.

Few studies have also used a single dataset with limited features of heart disease using other data sources, such as [34–36].

We note that the fluctuating efficiency of machine learning algorithms in predicting heart disease, due to the different classification accuracies, cannot be generalized which reduces the confidence of physicians. To deal with these problems, explainable machine learning models improve transparency by indicating the reasoning behind a specific decision on the one hand, and knowledge of the relevant factors that influence the prediction of results on the other. Therefore, we propose an explainable model based on linguistic IF-THEN rules which provides textual explanations thus allows for informed decision-making.

Methodology

As shown in Fig. 2, we propose an explainable prediction system in three steps in order to prevent the presence of heart disease: The first step is preprocessing of dataset. The second step is the design of a fuzzy rule-based classification system using fuzzy clustering and linguistic modifiers. The final step is the design explanation interface.

Description and preprocessing of dataset

This paper uses three heart disease dataset extracted from the UCI machine learning repository [19] Cleveland, Hungarian and Va long beach. Cleveland dataset contains information about individuals who come to the hospital and have been

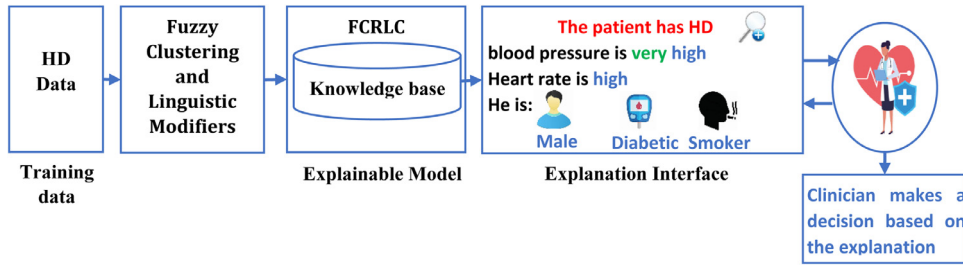


Fig. 2. Explainable prediction model of heart disease.

screened for heart disease. The dataset consists of 303 records and 14 attributes, six of which are numerical attributes, four of which are binary attributes and four of which are nominal attributes. The "One-Hot" method is used to encode nominal variables [38], since subtractive clustering algorithm requires that all input and output variables must be numeric. Due to the missing values, the size of the dataset is reduced to 297, where 137 have heart disease. A complete description of the used dataset is shown in the appendix (A) of this paper.

Due to the large number of missing values in the last three variables "Slope", "CA", and "THAL" in the Hungarian and Va long beach datasets, we decided to drop them from the prediction model. In order to have more instances we have combined Hungarian and Va long beach datasets into a single *CombinedHunVa* dataset with 358 records and 11 attributes.

Performance evaluation methods

In order to assess the validity of the predictive model, various measurements can be calculated such as sensitivity, specificity, accuracy and Receiver Operating Characteristics curve:

Specificity measures the proportion of negatives data that are correctly identified using Eq. (1).

$$Specificity = \frac{TN}{TN + FP} \tag{1}$$

Where **True Negative** (TN) means the number of negative data correctly labeled by the classifier and **False Positive** (FP) means the number of negative data that has been incorrectly labeled as positive.

Sensitivity measures the percentage of actual positives data that are correctly identified in Eq. (2) [39]:

$$Sensitivity = \frac{TP}{TP + FN} \tag{2}$$

Where **True Positive** (TP) and **False Negative** (FN) mean the number of positive data correctly labeled by the classifier, and the number of positive data that have been mislabeled as negative.

Accuracy measures the percentage of data points that are correctly identified in Eq. (3):

$$Accuracy = \frac{TN + TP}{TP + FP + TN + FN} \tag{3}$$

Receiver Operating Characteristics (ROC) curve is used to assess the predictive ability of different models. They are made by plotting the true positive rate against the false positive rate at different thresholds. Area Under the Curve (AUC) characterizes the ROC curve, the higher AUC value, the more efficient the classification performance.

Explainable model

Fuzzy rule-based classification system

This section contains a formal representation of linguistic classification rules learning. There are two main components of the FRBC knowledge base: The Database(DB) and the Rulebase (RB). The RB is a set of fuzzy IF-THEN rules. The DB defines the number of linguistic labels and their membership functions (MF) parameters for each linguistic variable.

Database of FRBC

Formally, linguistic fuzzy rule-based classifier has M attributes $a_j (a_j \in D), j = 1, \dots, M$ and one output $y (y \in Y)$; D_j and Y are the data domains of a_j and y respectively. Let $A_j = \{A_j^k / k = 1, \dots, S_j\}$ a set of fuzzy sets defined in D_j . Each fuzzy set A_j^k is associated with a linguistic label and is represented with its membership function $\mu_{A_j^k}(a_j) : D_j \rightarrow [0, 1]$. The output is associated with a collection of fuzzy singletons $A_0 = \{A_0^k / k = 1, \dots, L_0\}$ defined on the domain $Y = \{C_k / k = 1, \dots, L_0\}$ where C_k is a class label associated with A_0^k . The membership function of A_0^k is of the form:

$$\mu_{A_0^k} = \begin{cases} 1, & y = C_k \\ 0, & y \neq C_k \end{cases}$$

Rulebase of FRBC

Assume $S_{RB} = \{RF_i / i = 1, \dots, L\}$ is the set of fuzzy classification rules that make up the rule base, and each fuzzy rule is written as follows:

Rule RF_i : **IF** $(x^p_1 \text{ is } A_i^1) \text{ and } \dots \text{ and } (x^p_M \text{ is } A_i^M)$ **THEN** Class C_i **With** RW_i

where RF_i is the label of the i^{th} fuzzy rule, $X_p = (x^p_1, x^p_2, \dots, x^p_M)$ is an M -dimensional input data vector, $A_j^k \in A_j$ for $k = 1, \dots, M$; $C_i \in Y$ and RW_i is the rule weight ($RW_i \in [0, 1]$). The weight of each fuzzy base RF_i has a significant effect on the performance of fuzzy rule-based classification system [40]. Various specifications of the rule weight have been proposed and discussed in the literature [41] (such as heuristic methods), where the most common definition is the **fuzzy confidence value** or the **certainty factor** (CF) [42]:

$$RW_i = CF_i = \frac{\sum_{X_p \in \text{Class } C_i} \mu_{R_i}(X_p)}{\sum_p \mu_{R_i}(X_p)} \tag{4}$$

where $\mu_{R_i}(X_p)$ is the correspondence degree of the vector X_p calculated using the antecedent part of the fuzzy rule RF_i in Eq. (6).

Fuzzy reasoning method

The FRBC uses the RF_i rules from the knowledge base to determine the class of X_p . The vector X_p is attributed to the consequent class C_w of the rule R_w according to the formula (5). R_w is determined according to the correspondence degree $\mu_{R_w}(X_p)$ and its weight RW_w . The correspondence degree $\mu_{R_i}(X_p)$, defined by formula (6), of each rule RF_i is calculated using the conjunction operator (Product T-norms).

$$\mu_{R_w}(X_p) \cdot RW_w = \max\{\mu_{R_i}(X_p) \cdot RW_i / RF_i \in S_{RB}\} \tag{5}$$

$$\mu_{R_i}(X_p) = \mu_{A_i^1}(x^p_1) \cdot \dots \cdot \mu_{A_i^M}(x^p_M) \tag{6}$$

Fuzzy classification rules learning through clustering (FCRLC)

The purpose of this contribution is to build a FRBC for predicting heart disease with good trade-off between accuracy and interpretability. FCRLC method is an automated generation of a linguistic FRBC system based on data that integrates an embedded database learning wrapping Rulebase learning. Fig. 3 shows the architecture of FCRLC model, that contains three components: Database learning, Rulebase learning and Evaluation module. The Database learning is based on Multi-granularity fuzzy discretization algorithm to obtain uniform fuzzy partitions with Gaussian membership functions. In order to respect the complexity and the semantic constraints of interpretability in [43,44], the number of membership functions in each fuzzy partition must be between 2 and 9; and the maximum number of rules to be processed must be less than a threshold (NB_Max_Rules) determined by experimentation. Iteratively, the algorithm looks for the optimal Database. In which each iteration provides an intermediate Database. Each intermediate Database triggers Rulebase learning, that contains three components: Radius module, Subtractive clustering, and Rule module. Radius module calculates the radius r_a (a vector of scalars) using the parameters of Gaussian membership function (Mean and standard deviation). In order to extract fuzzy clusters, the Rulebase learning algorithm separates training data into groups according to their respective classes ($Data_C_i$ ($i = 1, \dots, L$)) and applies the subtractive clustering algorithm (using r_a) on each $Data_C_i$. The Rules module is based on these clusters ($Clusters_C_i$ is a set of clusters obtained from $Data_C_i$) to learn linguistic fuzzy classification rules in two steps:

- i Linguistic approximation of the fuzzy classification rules
- ii Improvement of accuracy in linguistic fuzzy classification rules with linguistic modifiers (applied to numerical attributes).

The third component is the Evaluation module in which the knowledge base is evaluated in a fuzzy rule-based classification system. The classification precision (accuracy) and the number of rules are calculated by Evaluation module; if the accuracy is increased and the number of rules is less than the threshold NB_Max_Rules , then the knowledge base is accepted; otherwise, it is rejected. The Database learning process stops when the optimal knowledge base is obtained.

The following sections detail the tasks of the Database learning unit and Rulebase learning unit.

Database learning

DB learning is based on Multi-granularity fuzzy discretization algorithm, in which the authors suppose that the fuzzy partitions are uniform and the Gaussian membership functions (MFs) define the meanings of each linguistic label. In order to select the optimal database, two issues to take into account: the error produced when applying the model to the training data and its complexity. In our case, classification precision (accuracy) and the complexity ($NBRules$ Numbers of fuzzy classification rules). The aim of multi-granularity fuzzy discretization algorithm is to precise the number of linguistic labels for each linguistic variable. Formally, consider a collection of N data points $\{x_1, x_2, \dots, x_N\}$ in an M -dimensional space. Let $V = \{v_1, v_2, \dots, v_M\}$ a set of linguistic variables, $\min(v_i)$ and $\max(v_i)$ are, respectively, the minimum and maximum values of universe of discourse of v_i , $NbMax$ (equal to 9) and $NbMin$ (equal to 2) are, respectively, the maximum and the minimum

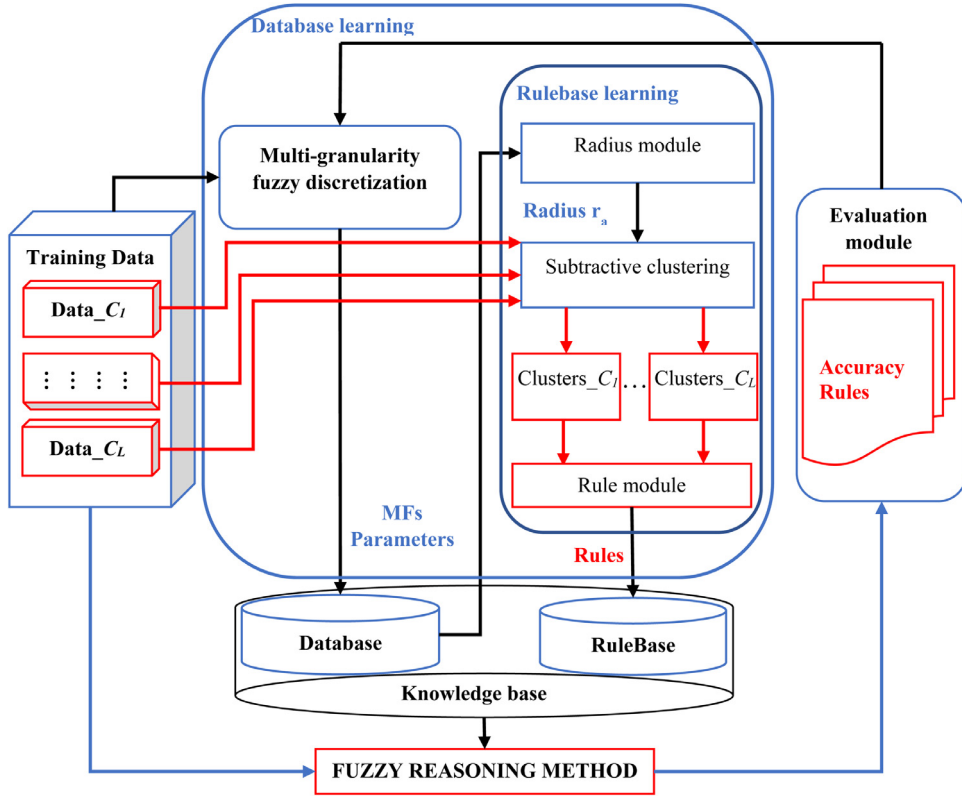


Fig. 3. FCRLC architecture.

numbers of linguistic labels per linguistic variable, and $Lb = \{(l_1, l_2, \dots, l_M) / l_i \in \{2, \dots, NbMax\} \text{ and } i = 1, \dots, M\}$ the set of M-tuples where l_i is the number of linguistic labels of v_i ($l_i \leq NbMax$). Lb define the search space and has a cardinality of $(NbMax-1)^M$. To deals with the complexity of Lb , the researchers determine an initial DB by searching in $\{(n_1, \dots, n_M) / n_i = NbMin \dots NbMax\}$ the optimal M-tuples ($OIDB = (n_{opt}^1, \dots, n_{opt}^M)$). Afterwards, the algorithm searches iteratively the final DB using $OIDB$: For example, in j^{th} dimension, the algorithm searches iteratively the optimum number (OPT_j) of linguistic labels (from $NbMin$ to $NbMax$) by fixing the other dimensions and replaces n_{opt}^j with OPT_j . The algorithm deals with the other dimensions in the same manner. The obtained DB is an intermediate DB (IDB). This process is repeated for each IDB until the final DB has been obtained. The Database learning algorithm is described in [14].

Rulebase learning

The Rulebase learning is based on subtractive clustering and linguistic modifiers. To extract fuzzy clusters from training data, Database learning algorithm separates training data into groups according to their respective classes C_f and applies the subtractive clustering algorithm on each $Data_{C_f}$ [45]. The subtractive clustering algorithm calculates the potential of data point x_i ($x_i \in Data_{C_f}$) with Eq. (7).

$$P_i = \frac{|Data_{C_f}|}{\sum_{j=1}^{|Data_{C_f}|} e^{-\alpha \|x_i - x_j\|^2}} \tag{7}$$

where $|Data_{C_f}|$ denotes the number of data points of $Data_{C_f}$, $\alpha = 4/r_a^2$ and r_a is the cluster radius, it is an M-dimensional vector of positive scalars which specifies the value of the radius in each dimension. The subtractive clustering algorithm uses a set of initial parameters: The cluster radius r_a , the accept ratio ($\epsilon = 0.5$), the reject ratio ($\epsilon = 0.15$) and the neighborhood of cluster ($r_b = 1.25 * r_a$). As showing in Fig. 3, the radius module uses the MFs parameters to calculate the radius r_a . In order to illustrate task radius module in j^{th} dimension, let $\{MFun_j^k / k = 1 \dots l_j\}$ the set of Gaussian membership functions obtained by uniform fuzzy discretization of v_j , the $MFun_j^k$ parameters are: C_j^k the mean and σ_j^k the standard deviation. The module calculates the j^{th} value r_a^j of r_a with Eq. (8) [46].

$$r_a^j = \frac{\sigma_{j^k} \sqrt{8}}{(\max(v_j) - \min(v_j))} \tag{8}$$

Experimentally, the default values of r_b , ϵ and ε affect the number of extracted clusters. Indeed, the constant values of initial parameters can produce excessive or insufficient number of clusters. It is necessary to adapt the values of initial parameters to the density of data points. Therefore, Database learning algorithm searches the adaptive value of r_b in $Sr_b = \{r_a * (1 + f/10) / f = 1...5\}$ (Sr_b is used to determine the good neighborhood of extracted clusters). Concerning ε value is calculated with the help of maximal and minimal potential (P_{max} and P_{min}): $\varepsilon = P_{min} / P_{max}$. Experimentally, $\epsilon = 0.5$ is a reasonable ratio to accept clusters [14].

In order to interpret the clusters centers X^c of each class C_f in fuzzy classification rules, Rule module projects X^c in all dimensions :

$$RuleRF_{X^c} : \mathbf{IF} (x_1 \text{ is } A_f^1) \text{ and } \dots \text{ and } (x_M \text{ is } A_f^M) \text{ THEN Class } C_f$$

where A_f^j is the fuzzy set defined by X^c on v_j (the j^{th} linguistic variables). The membership function of A_f^j is given by Eq. (9).

$$\mu_{A_f^j}(x) = e^{-\frac{(x-x_j^c)^2}{2\sigma_j^2}} \tag{9}$$

where σ_j and x_j^c are respectively the influence range and the j^{th} value of X^c .

Afterwards, the Rule module uses Euclidean distance to linguistically approximate the fuzzy rules and increases precision with linguistic modifiers (particularly, powered modifiers: $PM = \{\text{Very } (P = 2), \text{ Plus } (P = 1.25), \text{ Minus } (P = 0.75), \text{ More or less } (P = 0.5), \text{ slightly } (P = 1.7), \text{ and A little } (P = 1.3)\}$) using Hamming distance [47]. Eq. (10) illustrates the linguistic approximation of the cluster X^c :

$$T_j^C \leftarrow \underset{k = 1, \dots, l_j}{\operatorname{argmin}} (|x_j^C - C_j^k|) \tag{10}$$

With x_j^C is the j^{th} value of X^c and C_j^k the mean of $MFun_j^k$ and T_j^C is the linguistic label returned by Eq. (10). The generated fuzzy rules require an improvement of accuracy with linguistic modifiers due to the uniform fuzzy partition.

Eq. (11) calculates the Hamming distance between $\mu_{A_f^j}$, the MF of cluster X^c in j^{th} dimension, and $(MFun_j^C)^P$:

$$HD_P = \int_{\min(v_j)}^{\max(v_j)} |\mu_{A_f^j}(x) - (MFun_j^C)^P(x)| dx \tag{11}$$

where $MFun_j^C$ is the MF associated to the linguistic label T_j^C and P is the power value of the linguistic modifier.

The Rule module replaces the linguistic label T_j^C by the expression " $modifier_j^C T_j^C$ ". $modifier_j^C$ is obtained using Eq. (12)

$$P_{min} \leftarrow \underset{p \in PM}{\operatorname{argmin}} (HD_P) \tag{12}$$

where PM denotes the set of linguistic modifiers parameters and P_{min} is the value power of $modifier_j^C$.

Each linguistic fuzzy rule, in obtained Rulebase, includes M conditions. To simplify the Rulebase and take into account the improvement of accuracy simultaneously, we have reduced the number of conditions with *don't care* condition. The Rulebase learning algorithm is described in [14].

Results and discussion

This section involved discussion on FCRLC classification model compared to the most used machine learning models in the prediction of heart disease: Naive Bayes, K-Nearest Neighbor, Support Vector Machine, Random Forest and Artificial neural network. First, we describe the tools used to perform the experiments. In the second, we tested the performance of various machine learning algorithms on the Cleveland and *CombinedHunVa* heart disease datasets using various parameter values for each model. Table 1 lists the tuned parameters, with their meanings. In the third, we used the FCRLC knowledge base to create a clinician explanation interface.

Tools

The tools used to perform the experiments were Python libraries. We used Scikit-fuzzy, a fuzzy logic toolbox for SciPy, to build the FCRLC model [37]. Learning and optimization algorithms are implemented with Python. For SVM, ANN, NB, RF, and K-NN models are performed with Scikit-learn library. The hyperparameters of each model (SVM, ANN, K-NN and RF) are optimized with *GridSearchCV* method of Scikit-learn.

Results of 10-Fold cross-validation for classifiers performance

The experiment applies 10-fold cross validation methods to check the performance of six machine learning algorithms. In 10-fold cross validation approach, each dataset is randomly divided into two subsets: 90% of data for the training and 10% of data for testing. Moreover, different parameters values were tested for each classifier.

Table 1
Comparison algorithms and their tuned parameters.

Algorithms	Parameters
SVM	$C \in \{0.5, 1, 10, 100\}$ $\text{Gamma} \in \{1, 0.1, 0.01, 0.001, 0.0001\}$ $\text{Kernel} \in \{\text{'rbf'}, \text{'poly'}\}$
RF	$n_estimators \in \{10, 20, 30, 40, 50, 60, 70, 80, 90, 100\}$
K-NN	$K \in \{1, 2, \dots, 30\}$ $\text{Weights} \in \{\text{'uniform'}, \text{'distance'}\}$
ANN	$\text{hidden_layer_sizes} \in \{8, 12, 16\}$ $\text{activation} \in \{\text{'logistic'}, \text{'tanh'}, \text{'relu'}\}$ $\text{solver} \in \{\text{'lbfgs'}, \text{'sgd'}, \text{'adam'}\}$ $\text{alpha} \in \{0, 0.0005, 0.0001, 0.0005, 0.001\}$

'rbf': Radial Basis Function; 'poly': Polynomial; 'linear': Linear; 'uniform': Uniform weights; 'distance': Inverse Distance Weighting; 'logistic': Logistic sigmoid function; 'tanh': Hyperbolic Tan function; 'relu': Rectified Linear Unit function; 'lbfgs': Broyden Fletcher Goldfarb Shanno method; 'sgd': Stochastic Gradient Descent method; 'adam': Stochastic Gradient-based Optimization method.

Table 2
Comparison of developed models for Cleveland dataset.

Algorithms	Accuracy	Specificity	Sensitivity	AUC
NB	84.51%	84.84%	80.94%	90.14%
SVM (kernel = 'linear', C = 0.5, Gamma = 1)	84.19%	84.12%	78.80%	91.11%
ANN (activation = 'tanh', alpha = 0.0005, hidden_layer_sizes = 12, solver= 'adam')	83.87%	84.32%	78.85%	89.78%
FCRLC (NB_Max_Rules=30)	83.17%	83.17%	83.96%	85.48%
K-NN (k = 29)	82.15%	82.55%	77.31%	89.22%
RF (100 iterations)	82.17%	83.41%	79.51%	88.76%

Table 3
Comparison of developed models for CombinedHunVa dataset.

Algorithms	Accuracy	Specificity	Sensitivity	AUC
SVM (kernel = 'rbf', C = 100, Gamma = 0.1)	82.67%	84.32%	79.74%	88.69%
NB	82.33%	86.72%	83.70%	90.74%
K-NN (k = 30)	82.10%	83.46%	79.16%	88.55%
FCRLC (NB_Max_Rules=30)	80.46%	82.42%	79.82%	88.50%
ANN (activation = 'relu', alpha = 0.0005, hidden_layer_sizes = 8, solver= 'adam')	80.40%	83.29%	78.53%	87.64%
RF (90 iterations)	75.34%	79.31%	76.84%	86.37%

Results for Cleveland dataset

In Table 2, NB classifier shows good performance that has 84.51% classification accuracy, 84.84% specificity and 80.94 sensitivity. SVM classifier with kernel= 'linear', C = 0.5 and Gamma = 1 shows excellent performance which has accuracy 84.19%, specificity 84.12% and sensitivity 78.80%. Artificial neural network was trained with one hidden layer and different numbers of hidden neurons {8,12,16}. With 13 inputs and 12 neurons in hidden layer, ANN classifier achieved 83.87% accuracy, 84.32% specificity, and 78.85% sensitivity. For FCRL classifier, we performed experiments with different values of NB_Max_Rules = 20, 30, 40, 50 and 100. However, at NB_Max_Rules = 30, the performance of FCRLC was excellent. FCRLC has accuracy 83.17%, specificity 83.17% and sensitivity 83.96%. For K-NN classifier, we performed experiments with different values of k ∈ {1,2, ...,30}. However, at k = 29, the performance of K-NN was excellent. K-NN has accuracy 82.15%, specificity 82.55% and sensitivity 77.31%. Random forest classifier trained with 100 iterations has classification accuracy 82.17%, specificity 83.41%, and sensitivity 79.51%.

As shown in Fig. 4, NB classifier outperformed the other five classifiers in terms of accuracy, with classification accuracy more than 84.5%. The second important classifier is SVM with classification accuracy 84.19%. Other classifiers' accuracy was greater than 82%, and their ROC AUC values were greater than 85% and less than 92%, indicating that classifiers are more effective. The comparison emphasizes FCRLC's competitiveness based on accuracy and ROC AUC results.

Results for combinedhunva dataset

In Table 3, SVM with kernel 'rbf', C = 100 and Gamma = 0.1 showing good performance that has 82.67% classification accuracy, 84.32% specificity and 79.74% sensitivity. NB classifier has 82.33% accuracy, 86.72% specificity, and 83.70% sensitivity. The performance of K-NN was excellent at k = 30. K-NN has accuracy 82.10%, specificity 83.46% and sensitivity 79.16%. The performance of FCRLC was excellent with NB_Max_Rules = 30, indeed FCRLC has accuracy 80.46%, specificity 82.42% and

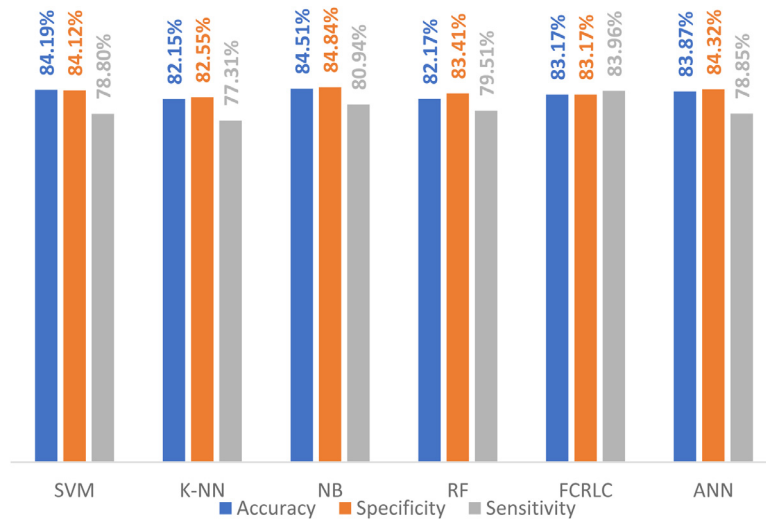


Fig. 4. Performance of different classifiers for Cleveland dataset.

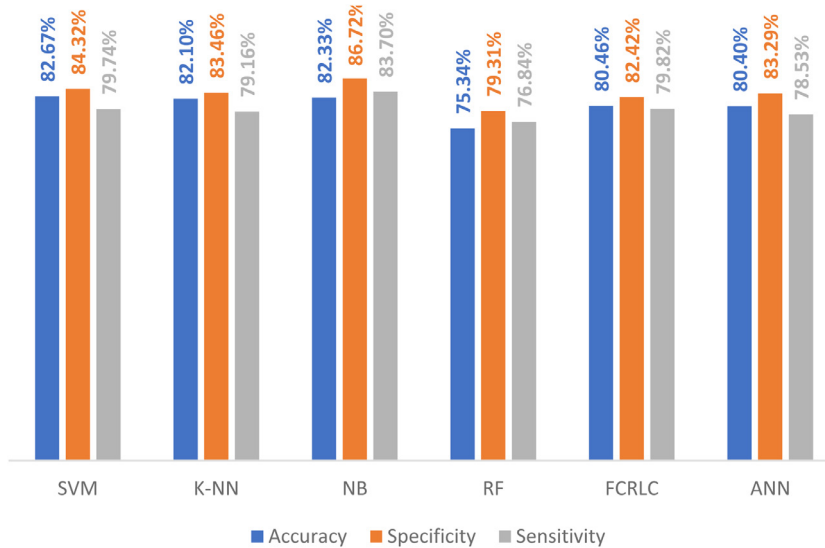


Fig. 5. Performance of different classifiers for CombinedHunVa dataset.

sensitivity 79.16%. ANN classifier (with 8 neurons in hidden layer) achieved 80.40% accuracy, 83.29% specificity and 78.53% sensitivity. Random forest classifier trained with 90 iterations has classification accuracy 75.34%, specificity 79.31%, and sensitivity 76.84%.

As shown in Fig. 5, in terms of accuracy, SVM outperformed the other five classifiers, with only minor differences between the NB and K-NN classifiers. The accuracy of the FCRLC and ANN classifiers exceeded 80%. The RF classifier had a poor performance in terms of classification accuracy, which was less than 76%. The ROC AUC values of classifiers are greater than 86% and less than 91%, indicating that classifiers are more effective.

Explainability of the FCRLC model

From the explainability point of view, NB, K-NN and FCRLC models are part of the transparent models, while the SVM, RF and ANN models belong to the set of opaque models. It is necessary to apply XAI techniques to machine learning models in order to discuss explainability. At this point, FCRLC outperforms the other five machine learning algorithms in terms of explainability by providing a simple and transparent linguistic knowledge base. Indeed, FCRLC offers a very simple database in which all input variables are discretized with uniform fuzzy partitions.

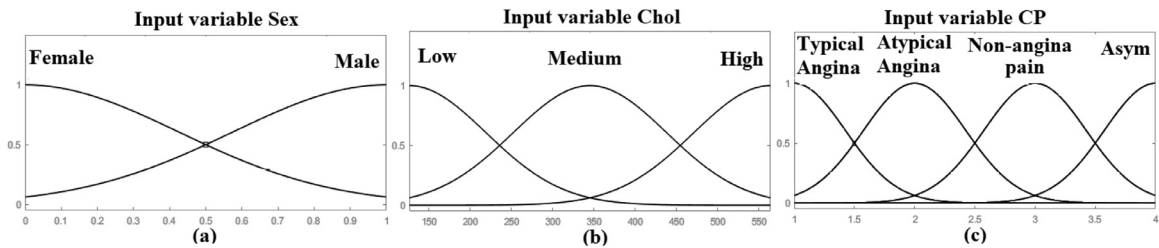


Fig. 6. The fuzzy partitions of linguistic variables Sex (a), Chol (b) et CP (c).

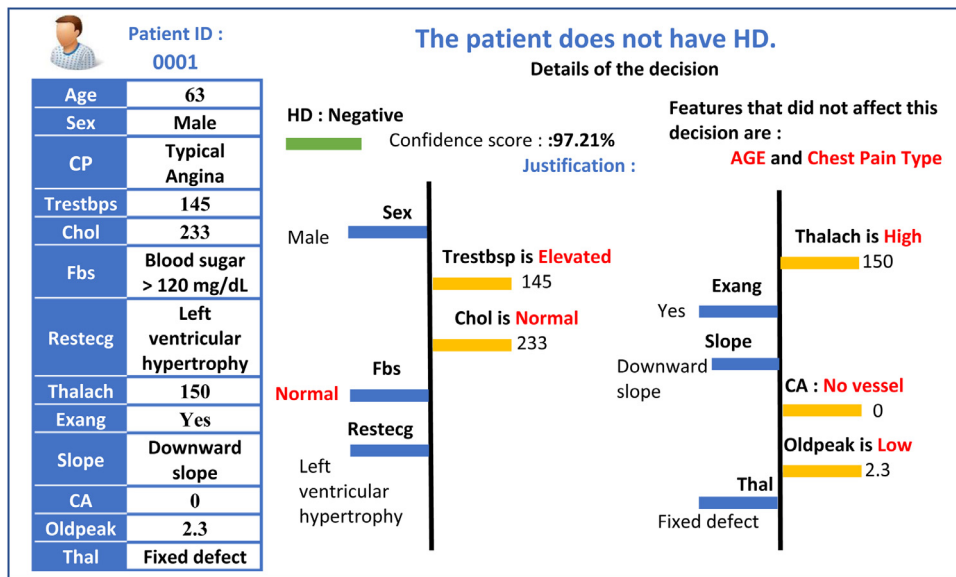


Fig. 7. Explanation interface of FCRLC.

Table 4 Database information.

Attributs	Number of linguistic labels	Lists of linguistic labels
Sex	2	{Female, Male}
Fbs	2	{Normal, High}
Exang	2	{Angina, No angina}
Age	3	{Adult, Mature, Old}
Trestbps	3	{Normal, Elevated,-High}
Chol	3	{Normal, Elevated, High}
Thalach	3	{Low, Medium, High}
Oldpeak	3	{Low, Medium, High}
RestEcg	3	{Normal, ST-T wave abnormality, left ventricular hypertrophy}
Slope	3	{Upward slope, Flat, downward slope}
Thal	3	{Normal, Fixed defect, Reversible defect}
CP	4	{Typical Angina, Atypical Angina, Non-Anginal Pain, Asymptomatic Pain}
CA	4	{No vessel, One vessel, two vessels, Three vessels}

In the case of Cleveland dataset (for example), Table 4 shows the number of membership functions and the list of linguistic labels for each linguistic variable. the variables Sex, Fbs and Exang have two linguistic labels, the variables Age, trestbps, Chol, thalach, Oldpeak, RestEcg and Slope have three linguistic labels, and the variables CP and CA have four linguistic labels.

As shown in Fig. 6, the variables Sex, Chol (Cholesterol level), and CP (Type of chest pain) were chosen to indicate the fuzzy partitions with 2, 3, and 4 linguistic labels, respectively.

Table 5 presents the FCRLC Rulebase, in which the linguistic modifiers are marked in bold and omitted conditions of a rule was described by "-" dashes. FCRLC Rulebase is composed of seven rules, three of which affirm the heart disease of a

Table 5
The FCRLC rule base.

Variables	Linguistic rules							
	R1	R2	R3	R4	R5	R6	R7	
Antecedents	Age	<i>More or less</i> Old	—	Mature	Mature	Mature	Mature	Mature
	Sexe	Female	Male	Female	Male	Male	Male	Male
	CP	Asymptomatic	—	Asymptomatic	Non-Anginal Pain	Asymptomatic	Asymptomatic	—
	Trestbps	<i>More or less</i> Normal	Elevated	Elevated	—	Elevated	Elevated	Normal
	Chol	<i>More or less</i> Normal	Normal	Normal	Normal	Elevated	Elevated	Elevated
	Fbs	Normal	Normal	Normal	Normal	Normal	Normal	Normal
	Restecg	Normal	left ventricular hypertrophy	Normal	Normal	left ventricular hypertrophy	Normal	Normal
	Thalach	<i>More or less</i> Medium	High	Medium	Medium	Medium	Medium	Medium
	Exang	No angina	No angina	No angina	No angina	Angina	Angina	Angina
	Slope	Flat	Flat	Upward slope	Upward slope	Flat	Flat	Flat
	CA	No vessel	No vessel	No vessel	No vessel	One vessel	No vessel	No vessel
	Oldpeak	<i>More or less</i> Medium	Low	Low	Low	Low	Low	Medium
	Thal	Normal	Normal	Normal	—	Fixed defect	Normal	Reversible defect
	Consequent	HD	Negative	Negative	Negative	Negative	Positive	Positive

Table 6
Example of a request for individual prediction.

Attribute	Value	Attribute	Value
Age	63	Thalach	150
Sex	Male	Exang	Yes
CP	Typical Angina	Slope	Downward slope
Trestbps	145	CA	0
Chol	233	Oldpeak	2.3
Fbs	Blood sugar > 120 mg/dL	Thal	Fixed defect
Restecg	Left ventricular hypertrophy		

patient. The linguistic form of rules allows experts and clinicians to analyze, criticize, accept, or reject the results provided by FCRLC when there is a risk of making wrong decisions.

Besides the readability of the linguistic rules, one or more explanatory interfaces can explain the decision taken by FCRLC and can contain automatic explanations through an automated reading of knowledge base in which developers can use Natural Language Generation (NLG) modules. Indeed, Fig. 7 shows an example of an FCRLC explanation interface for a patient whose information is detailed in

Table 6. The explanation interface shows that the decision was made with a certainty of 97.21% and provides decision details in the form of linguistic description involved the features which influenced the decision such as "Chol is normal" or "Oldpeak is Low" and the features which did not influence the decision in this case Age and CP.

Conclusion

Many health sectors are opting for machine learning methods to prevent heart disease in the early stages, that helps clinicians to make informed decisions. In this paper, using data-driven method based on fuzzy clustering and linguistic modifiers called: "Fuzzy Classification Rules Learning through Clustering" (FCRLC), a fuzzy rule-based classification system for heart disease diagnosis is proposed. The system has been evaluated on three datasets of heart disease and compared to five known classifiers, such as, K-NN, ANN, SVM, NB and RF. FCRLC has showing a good compromise between interpretability and accuracy and providing an interpretable knowledge base. FCRLC has three main advantages for providing explanations to stakeholders (end users, Experts and data scientists):

- i. Use the knowledge base in the explanation interface design phase to be exploited by users in the decision-making phase.
- ii. **Debugging models:** knowledge base can help detect issues in the data that standard model evaluation techniques would usually miss.
- iii. **Optimizing models:** In order to improve the FCRLC model, the expert or developer can easily manage the knowledge base by modifying the database and the Rulebase settings.

Future work is looks forward to optimizing the accuracy of FCRLC using new subsets to improve prediction results and providing interactive explanations in natural language with visualization as a complementary modality.

Appendix. A. Cleveland and Combinedhunva heart disease datasets

Table 7 describes the used attributes, in which five values are present in the expected attribute (HD) of the original dataset. A value of 0 means that there is no HD for the person, and a value of 1 to 4 means different HD levels. The goal of this study is to detect the presence or absence of HD. Therefore, the attribute (HD) is reclassified as a binary value, where 1 confirms the presence of HD for the person.

Table 7
Dataset information.

	Attributes	Descriptions	Type	Values
1	Age	The patient's age	Numeric	[29,77]
2	Sex	The gender of the patient	Binary	"Female"=0 "Male"=1
3	Type of Chest Pain (CP)	type of chest pain experienced by the individual	Nominal	"Typical Angina"=1 "Atypical Angina"=2 "Non-angina pain"=3 "Asymptomatic"=4
4	Resting blood pressure (Trestbps)	resting blood pressure value in mmHg	Numeric	[94, 200]
5	Serum cholesterol (Chol)	cholesterol levels in mg/dL	Numeric	[126, 564]
6	Fasting blood sugar (Fbs)	blood sugar levels	Binary	"otherwise" =0 "Blood sugar > 120 mg/dL" =1 "Normal"=0
7	Resting Ecg (Restecg)	Resting electrocardiographic results	Nominal	"having ST-T wave anomaly" =1 "left ventricular hypertrophy" =2
8	Maximum heart rate (Thalach)	maximum heart rate reached by the individual in number of samples	Numeric	[71, 202]
9	Exercise-induced angina (Exang)	provides information if exercise induces angina	Binary	"Yes"=0 "No"=1
10	Exercise peak ST segment (Slope)	displayed slope value on the ecg machine	Nominal	"upward slope" =1 "Flat"=2 "downward slope" =3
11	Number of large colorful ships (CA)	displays the number of vessels coloured by fluoroscopy	Numeric	[0,3]
12	St induced depression (Oldpeak)	shows the value of exercise-induced ST depression	Numeric	[0, 6.2]
13	Thallium Stress Test (Thal)	Stress test results	Nominal	"Normal"=3 "Fixed defect" =6 "Reversible defect" =7
14	Diagnosis of heart disease (HD)	provides information on whether the person has heart disease	Binary	"No"=0 "Yes"=1

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

- [1] K. Song, X. Zeng, Y. Zhang, J.D. Jonckheere, X. Yuan, L. Koehl, An interpretable knowledge-based decision support system and its applications in pregnancy diagnosis, *Knowl. Based Syst.* 221 (2021) 106835.
- [2] R.T. Sutton, D. Pincock, D.C. Baumgart, D.C. Sadowski, R.N. Fedorak, K.I. Kroeker, An overview of clinical decision support systems: benefits, risks, and strategies for success, *NPJ Digit. Med.* 3 (1) (2020) 1–10.
- [3] C. Liu, L. Du, S. Wang, L. Kong, S. Zhang, S. Li, W. Zhang, G. Du, Differences in the prevention and control of cardiovascular and cerebrovascular diseases, *Pharmacol. Res.* 170 (2021) 105737.
- [4] T. Keerthana, Heart disease prediction system using data mining method, *Int. J. Eng. Trends Technol.* 47 (6) (2017) 361–363.
- [5] H.E. Bays, P.R. Taub, E. Epstein, E.D. Michos, R.A. Ferraro, A.L. Bailey, H.M. Kelli, K.C. Ferdinand, M.R. Echols, H. Weintraub, J. Bostrom, H.M. Johnson, K.K. Hoppe, M.D. Shapiro, C.A. German, S.S. Virani, A. Hussain, C.M. Ballantyne, A.M. Agha, P.P. Toth, Ten things to know about ten cardiovascular disease risk factors, *Am. J. Prevent. Cardiol.* 5 (2021) 100149.
- [6] N.D. Wong, Cardiovascular risk assessment: the foundation of preventive cardiology, *Am. J. Prevent. Cardiol.* 1 (2020) 100008.
- [7] N. Mishra, S. Silakari, Predictive analytics: a survey, trends, applications, opportunities & challenges, *Int. J. Comput. Sci. Inf. Technol.* 3 (3) (2012) 4434–4438.
- [8] A. Solanki, M. Barot, Study of heart disease diagnosis by comparing various classification algorithms, *Int. J. Eng. Adv. Technol.* 8 (252) (2019) 40–42.
- [9] U. Pawar, D. O'Shea, S. Rea, R. O'Reilly, Explainable ai in healthcare, in: 2020 International Conference on Cyber Situational Awareness, Data Analytics and Assessment (CyberSA), 2020, pp. 1–2.
- [10] D. Dave, H. Naik, S. Singhal, P. Patel, Explainable AI meets healthcare: a study on heart disease dataset, *arXiv e-prints* (2020) arXiv-a2011.
- [11] A.B. Arrieta, N. Diaz-Rodriguez, J. Del Ser, A. Bennetot, S. Tabik, A. Barbadó, S. García, S. Gil-López, D. Molina, R. Benjamins, F. Herrera, Explainable Artificial Intelligence (XAI): concepts, taxonomies, opportunities and challenges toward responsible AI, *Inf. Fusion* 58 (2020) 82–115.
- [12] V. Belle and I. Papantonis, "Principles and practice of explainable machine learning," *arXiv preprint arXiv:2009.11698*, 2020.
- [13] B. Üstün, W. Melsens, L. Buydens, Visualisation and interpretation of support vector regression models, *Anal. Chim. Acta* 595 (1–2) (2007) 299–309.
- [14] K. Bahani, M. Moujabbar, M. Ramdani, Linguistic fuzzy rule learning through clustering for regression problems, *Int. J. Intell. Eng. Syst.* 13 (3) (2020) 80–89 Intelligent Networks and Systems Society.
- [15] N.S. Altman, An introduction to kernel and nearest-neighbor nonparametric regression, *Am Stat* 46 (3) (1992) 175–185.
- [16] I. Kononenko, Inductive and Bayesian learning in medical diagnosis, *Appl. Artif. Intell. Int. J.* 7 (4) (1993) 317–337.
- [17] V. Vapnik, *The Nature of Statistical Learning Theory*. Springer science & business media, 2013.
- [18] O.I. Abiodun, A. Jantan, A.E. Omolara, K.V. Dada, N.A. Mohamed, H. Arshad, State-of-the-art in artificial neural network applications: a survey, *Heliyon* 4 (11) (2018) e00938.
- [19] D. Robert, Heart Disease Data Set, 1988.

- [20] D. Annepu, G. Gowtham, Cardiovascular disease prediction using machine learning techniques, *Int. Res. J. Eng. Technol.* 6 (4) (2019) 3963–3971.
- [21] Y. Khourdifi, M. Bahaj, Heart disease prediction and classification using machine learning algorithms optimized by particle swarm optimization and ant colony optimization, *Int. J. Intell. Eng. Syst.* 12 (1) (2019) 242–252.
- [22] A.U. Haq, J.P. Li, M.H. Memon, S. Nazir, R. Sun, A hybrid intelligent system framework for the prediction of heart disease using machine learning algorithms, *Mob. Inf. Syst.* 2018 (2018).
- [23] U.N. Dulhare, "Prediction system for heart disease using Naive Bayes and particle swarm optimization," 2018.
- [24] K. Subhadra, B. Vikas, Neural network based intelligent system for predicting heart disease, *Int. J. Innov. Technol. Explor. Eng.(IJITEE)* 8 (5) (2019) 484–487.
- [25] R. Prasad, P. Anjali, S. Adil, N. Deepa, Heart disease prediction using logistic regression algorithm using machine learning, *Int. J. Eng. Adv. Technol.* 8 (2019) 659–662.
- [26] T.K. Ho, Random decision forests, in: *Proceedings of 3rd International Conference on Document Analysis and Recognition*, 1, 1995, pp. 278–282.
- [27] M. Tarawneh, O. Embarak, Hybrid Approach for Heart Disease Prediction Using Data Mining Techniques, 2019, pp. 447–454.
- [28] N. Singh, P. Firozpur, S. Jindal, Heart disease prediction system using hybrid technique of data mining algorithms, *Int. J. Adv. Res. Ideas Innov. Technol.* 4 (2) (2018) 982–987.
- [29] F.S. Alotaibi, Implementation of machine learning model to predict heart failure disease, *Int. J. Adv. Comput. Sci. Appl.*, 10, 2019.
- [30] A. Ullah, N.M. Nawari, A. Shahzad, S.N. Khan, M. Aamir, An e-learning system in Malaysia based on green computing and energy level, *JOIV* 1 (4–2) (2017) 184–187.
- [31] S. Nashif, M.R. Raihan, M.R. Islam, M.H. Imam, Heart disease detection by using machine learning algorithms and a real-time cardiovascular health monitoring system, *World J. Eng. Technol.* 6 (4) (2018) 854–873.
- [32] D. Dave, H. Naik, S. Singhal, P. Patel, Explainable AI meets healthcare: a study on heart disease dataset, *Comput. Res. Reposit.* (2020) abs/2011.03195.
- [33] M. Aghamohammadi, M. Madan, J.K. Hong, I. Watson, Predicting heart attack through explainable artificial intelligence, in: *International Conference on Computational Science*, 2019, pp. 633–645.
- [34] H. Ayatollahi, L. Gholamhosseini, M. Salehi, Predicting coronary artery disease: a comparison between two data mining algorithms, *BMC Public Health* 19 (1) (2019) 1–9.
- [35] A. Lakshmanarao, Y. Swathi, P.S.S. Sundareswar, Machine learning techniques for heart disease prediction, *Int. J. Sci. Technol. Res.* 8 (11) (2019) 374–377.
- [36] M. Shamsollahi, A. Badiie, M. Ghazanfari, Using combined descriptive and predictive methods of data mining for coronary artery disease prediction: a case study approach, *J. AI Data Min.* 7 (1) (2019) 47–58.
- [37] J. Warner, J. Sexauer, scikit-fuzzy, twmeggs, alexsavio, A.Unnikrishnan, G. Castelão, F.A. Pontes, T. Uelwer, pd2f, laurazh, F. Batista, alexbuy, W.V.den Broeck, W. Song, T.G. Badger, R.A.M. Pérez, J.F. Power, H. Mishra, G.O. Trullols, A. Hörteborn, and 99991, "JDWarner/scikit-fuzzy: Scikit-Fuzzy version 0.4.2." Zenodo, Nov- 2019.
- [38] P. Cerda, G. Varoquaux, B. Kégl, Similarity encoding for learning with dirty categorical variables, *Mach. Learn.* 107 (8) (2018) 1477–1494.
- [39] S.V. Stehman, Selecting and interpreting measures of thematic classification accuracy, *Remote Sens. Environ.* 62 (1) (1997) 77–89.
- [40] H. Ishibuchi, T. Nakashima, Effect of rule weights in fuzzy rule-based classification systems, *IEEE Trans. Fuzzy Syst.* 9 (4) (2001) 506–515.
- [41] H. Ishibuchi, T. Yamamoto, Rule weight specification in fuzzy rule-based classification systems, *IEEE Trans. Fuzzy Syst.* 13 (4) (2005) 428–435.
- [42] O. Córdón, M.J. Del Jesus, F. Herrera, A proposal on reasoning methods in fuzzy rule-based classification systems, *Int. J. Approx. Reason.* 20 (1) (1999) 21–45.
- [43] J.V. de Oliveira, Semantic constraints for membership function optimization, *IEEE Trans. Syst., Man Cybern.–Part A* 29 (1) (1999) 128–138.
- [44] C. Mencar, A.M. Fanelli, Interpretability constraints for fuzzy information granulation, *Inf. Sci.* 178 (24) (2008) 4585–4618.
- [45] S. Chiu, Extracting fuzzy rules from data for function approximation and pattern classification, *Fuzzy Inf. Eng.* 9 (1997) 149–162 chapter.
- [46] K. Bahani, M. Moujabbar, M. Ramdani, Fuzzy Rule Learning with Linguistic Modifiers, in: *Proceedings of the 12th International Conference on Intelligent Systems: Theories and Applications*, 2018, pp. 1–6.
- [47] S. Pandit, S. Gupta, A comparative study on distance measuring approaches for clustering, *Int. J. Res. Comput. Sci.* 2 (1) (2011) 29–31.