Beam Theory

4BY372

Assignment No. 1 (2021)

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Tasks for Assignment No. 1

The first assignment comprises two parts in which the students are required to develop two Matlab functions.

Task 1a

A Matlab function named **`secgeom`** for calculation of geometrical cross-sectional properties for any thin-walled cross section made up of *n* arbitrarily oriented and connected thin-walled rectangular strips shall be developed.

The function shall provide:

1. The coordinates of the center of gravity with respect to coordinate system employed by the user. (The x-axis shall be directed in the longitudinal direction of the beam, i.e. perpendicular to the beam cross section.)
2. The second moments of inertia (also mixed moment of inertia) with respect to the coordinate system used,
3. The angle between the axes of the coordinate system used and the principal axes, and the second moments of inertia with respect to the principal axes.

The function shall also provide a figure showing:

1. The given thin-walled cross section itself, with its numbered nodes, drawn with lines with a line width of '4' or similar, (hint! `'linewidth',4`)
2. The calculated position of the center of gravity (CG) marked with (*) or similar, and the calculated principal axes represented by arrows.
3. The original coordinate system used, represented by arrows.

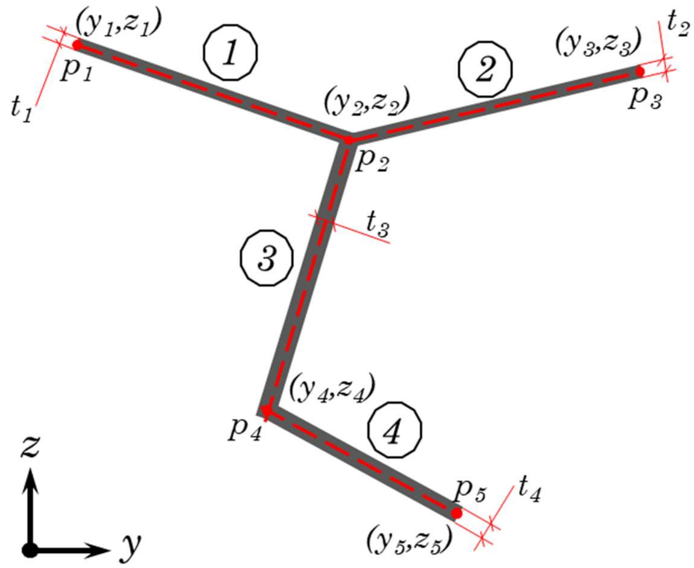
The task is defined in more detail by the *draft function manual* page provided below.

Draft function Manual

The *draft function manual* given below is a starting point for the full final manual page that you shall submit as part of the assignment report. The final function manual page/pages shall correspond to those corresponding to the CALFEM functions in the CALFEM manual.

Task 1a) `secgeom`

Purpose: Calculate the geometrical cross-sectional properties for any thin-walled cross section made up of *n* arbitrarily oriented and connected thin-walled rectangular strips as shown in the figure, and draw the cross-section along with original and calculated principal axes.



Syntax:

```
[CG,A,I,phi]= secgeom(scoord,sectopo,th,plotpar)
```

Description: `secgeom` computes the center of gravity, the directions of principal axes and second moments of inertia, with respect to both the original and the principal axes, for any given arbitrary thin-walled cross-section.

Inputs: scoord = [y_1 z_1;
y_2 z_2;
y_3 z_3;
.. ..;
y_k z_k],

: section nodal coordinates with respect to the employed coordinate system.

```
sectopo = [1 p1 p2;
           2 p2 p3;
           3 p2 p4;
           4 p4 p5;
           .  .  .;
           n pi  pj],
        : section topology matrix.

     th = [t1; t2; t3; ..; tn],
         : thicknesses for the n rectangular strips.
```

Outputs:

```
     CG = coordinates of center of gravity [Yc Zc].
      A = Total cross sectional area.
      I = [Iy Iz Iyz Ieta Izeta],
         : Moments of inertia with respect to original axes
           and principal axes.
    phi = Angle (counter clockwise direction) between the
          original Y-axis and the first principal axis
          (eta-axis).
```

Task 1b

A Matlab function named `secdat.m` for calculation of cross sectional properties of open thin-walled cross sections of specific types shall be developed.

The function shall provide:

1. Cross sectional area, second moments of inertia, torsion constant and sectorial moment of inertia for thin-walled cross sections of type 'I', 'T', 'L', 'Z', 'C' and 'R'.

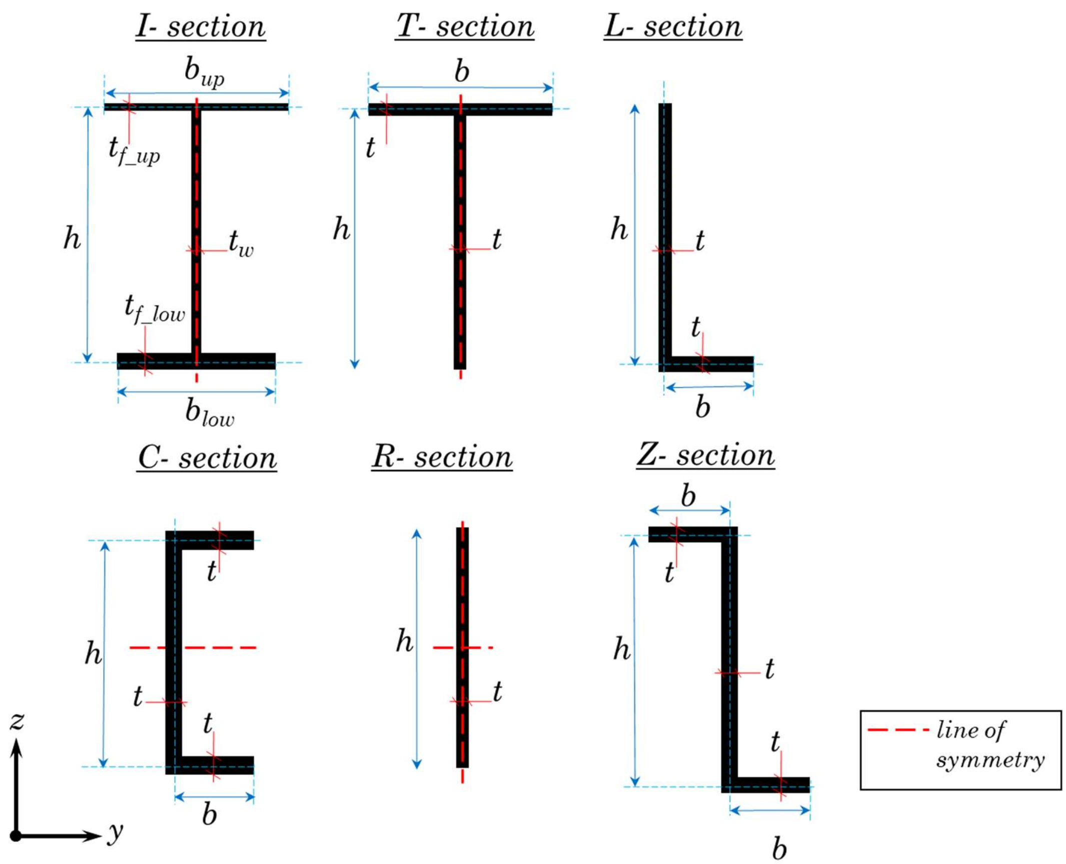
Below a *draft function manual*, defining the task in detail, is provided. Closed form expressions for cross sectional properties for different types of cross sections are provided in a formulary in Appendix.

Draft function Manual

The *draft function manual* given below is a starting point for the full final manual page that you shall submit as part of the assignment report. The final function manual page/pages shall correspond to those corresponding to the CALFEM functions in the CALFEM manual.

Task 1b) secdat

Purpose: Calculate the cross-sectional properties for thin-walled cross-sections of selected specific type, out of those shown in the figure.



Syntax:

```
[sp] = secdat(type,sdim)
```

Description: **secdat** computes cross-sectional area, second moments of inertia with respect to the coordinate system defined in the figure, St. Venant torsion constant and sectorial moment of inertia for a thin-walled cross-section of specified type.

Inputs:

```
type:  ['type'] provide the type of cross-section, out of
       those specified in Fig2 (i.e. type = I, T, L, Z, C
       or R) to be considered.
sdim = [b h t],
```

```
if type == 'I',
  : b = [b_up b_low]
  : h = [h]
  : t = [tf_up tf_low t_w]
 else,
  : b = [b]
  : h = [h]
  : t = [t]
```

Outputs: `sp = [Iy Iz Iyz Kv Iw A],`

```
: Kv = St. Venant torsion constant.
: Iw = Sectorial moment of inertia.
```

TABLE 10.2 Formulas for torsional properties and stresses in thin-walled open cross sections

NOTATION: Point 0 indicates the shear center. $e =$ distance from a reference to the shear center; $K =$ torsional stiffness constant (length to the fourth power); $C_w =$ warping constant (length to the sixth power); $\tau_1 =$ shear stress due to torsional rigidity of the cross section (force per unit area); $\tau_2 =$ shear stress due to warping rigidity of the cross section (force per unit area); $\sigma_x =$ bending stress due to warping rigidity of the cross section (force per unit area); $E =$ modulus of elasticity of the material (force per unit area); and $G =$ modulus of rigidity (shear modulus) of the material (force per unit area)

The appropriate values of θ', θ'', and θ''' are found in Table 10.3 for the loading and boundary restraints desired

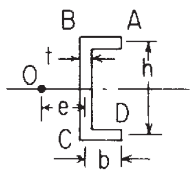
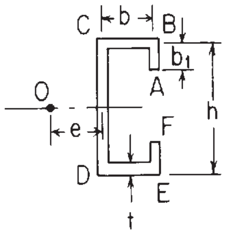
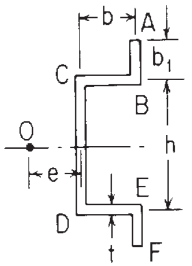
Cross section, reference no.	Constants	Selected maximum values
1. Channel 	$e = \dfrac{3b^2}{h+6b}$ $K = \dfrac{t^3}{3}(h+2b)$ $C_w = \dfrac{h^2b^3t}{12}\dfrac{2h+3b}{h+6b}$	$(\sigma_x)_{\max} = \dfrac{hb}{2}\dfrac{h+3b}{h+6b}E\theta''$ throughout the thickness at corners A and D $(\tau_2)_{\max} = \dfrac{hb^2}{4}\left(\dfrac{h+3b}{h+6b}\right)^2 E\theta'''$ throughout the thickness at a distance $b\dfrac{h+3b}{h+6b}$ from corners A and D $(\tau_1)_{\max} = tG\theta'$ at the surface everywhere
2. C-section 	$e = b\dfrac{3h^2b+6h^2b_1-8b_1^3}{h^3+6h^2b+6h^2b_1+8b_1^3-12hb_1^2}$ $K = \dfrac{t^3}{3}(h+2b+2b_1)$ $C_w = t\left[\dfrac{h^2b^2}{2}\left(b_1+\dfrac{b}{3}-e-\dfrac{2eb_1}{b}+\dfrac{2b_1^2}{h}\right) + \dfrac{h^2e^2}{2}\left(b+b_1+\dfrac{h}{6}-\dfrac{2b_1^2}{h}\right)+\dfrac{2b_1^3}{3}(b+e)^2\right]$	$(\sigma_x)_{\max} = \left[\dfrac{h}{2}(b-e)+b_1(b+e)\right]E\theta''$ throughout the thickness at corners A and F $(\tau_2)_{\max} = \left[\dfrac{h}{4}(b-e)(2b_1+b-e)+\dfrac{b_1^2}{2}(b+e)\right]E\theta'''$ throughout the thickness on the top and bottom flanges at a distance e from corners C and D $(\tau_1)_{\max} = tG\theta'$ at the surface everywhere
3. Hat section 	$e = b\dfrac{3h^2b+6h^2b_1-8b_1^3}{h^3+6h^2b+6h^2b_1+8b_1^3+12hb_1^2}$ $K = \dfrac{t^3}{3}(h+2b+2b_1)$ $C_w = t\left[\dfrac{h^2b^2}{2}\left(b_1+\dfrac{b}{3}-e-\dfrac{2eb_1}{b}-\dfrac{2b_1^2}{h}\right) + \dfrac{h^2e^2}{2}\left(b+b_1+\dfrac{h}{6}+\dfrac{2b_1^2}{h}\right)+\dfrac{2b_1^3}{3}(b+e)^2\right]$	$\sigma_x = \left[\dfrac{h}{2}(b-e)-b_1(b+e)\right]E\theta''$ throughout the thickness at corners A and F $\sigma_x = \dfrac{h}{2}(b-e)E\theta''$ throughout the thickness at corners B and E $\tau_2 = \left[\dfrac{h^2(b-e)^2}{8(b+e)}+\dfrac{b_1^2}{2}(b+e)-\dfrac{hb_1}{2}(b-e)\right]E\theta'''$ throughout the thickness at a distance $\dfrac{h(b-e)}{2(b+e)}$ from corner B toward corner A $\tau_2 = \left[\dfrac{b_1^2}{2}(b+e)-\dfrac{hb_1}{2}(b-e)-\dfrac{h}{4}(b-e)^2\right]E\theta'''$ throughout the thickness at a distance e from corner C toward corner B $\tau_1 = tG\theta'$ at the surface everywhere

TABLE 10.2 Formulas for torsional properties and stresses in thin-walled open cross sections (*Continued*)

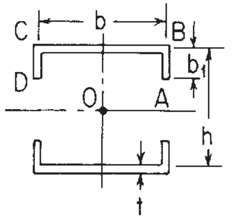
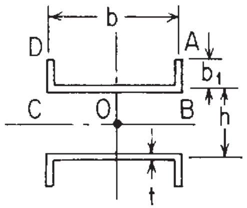
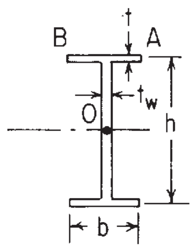
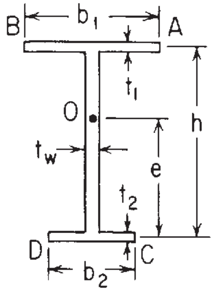
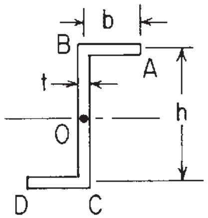
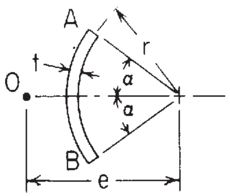
Cross section, reference no.	Constants	Selected maximum values
4. Twin channel with flanges inward 	$K = \frac{t^3}{3}(2b + 4b_1)$ $C_w = \frac{tb^2}{24}(8b_1^3 + 6h^2b_1 + h^2b + 12b_1^2h)$	$(\sigma_x)_{\max} = \frac{b}{2}\left(b_1 + \frac{h}{2}\right)E\theta''$ throughout the thickness at points A and D $(\tau_2)_{\max} = \frac{-b}{16}(4b_1^2 + 4b_1h + hb)E\theta'''$ throughout the thickness midway between corners B and C $(\tau_1)_{\max} = tG\theta'$ at the surface everywhere
5. Twin channel with flanges outward 	$K = \frac{t^3}{3}(2b + 4b_1)$ $C_w = \frac{tb^2}{24}(8b_1^3 + 6h^2b_1 + h^2b - 12b_1^2h)$	$(\sigma_x)_{\max} = \frac{hb}{4}E\theta''$ throughout the thickness at points B and C if $h > b_1$ $(\sigma_x)_{\max} = \left(\frac{hb}{4} - \frac{bb_1}{2}\right)E\theta''$ throughout the thickness at points A and D if $h < b_1$ $(\tau_2)_{\max} = \frac{b}{4}\left(\frac{h}{2} - b_1\right)^2 E\theta'''$ throughout the thickness at a distance $\frac{h}{2}$ from corner B toward point A if $b_1 > \frac{h}{2}\left(1 + \sqrt{\frac{1}{2} + \frac{b}{2h}}\right)$ $(\tau_2)_{\max} = \frac{b}{4}\left(b_1^2 - \frac{hb}{4} - hb_1\right)E\theta'''$ throughout the thickness at a point midway between corners B and C if $b_1 < \frac{h}{2}\left(1 + \sqrt{\frac{1}{2} + \frac{b}{2h}}\right)$ $(\tau_1)_{\max} = tG\theta'$ at the surface everywhere
6. Wide flanged beam with equal flanges 	$K = \frac{1}{3}(2t^3b + t_w^3h)$ $C_w = \frac{h^2tb^3}{24}$	$(\sigma_x)_{\max} = \frac{hb}{4}E\theta''$ throughout the thickness at points A and B $(\tau_2)_{\max} = -\frac{hb^2}{16}E\theta'''$ throughout the thickness at a point midway between A and B $(\tau_1)_{\max} = tG\theta'$ at the surface everywhere

TABLE 10.2 Formulas for torsional properties and stresses in thin-walled open cross sections (*Continued*)

7. Wide flanged beam with unequal flanges 	$e = \frac{t_1 b_1^3 h}{t_1 b_1^3 + t_2 b_2^3}$ $K = \frac{1}{3}(t_1^3 b_1 + t_2^3 b_2 + t_w^3 h)$ $C_w = \frac{h^2 t_1 t_2 b_1^3 b_2^3}{12(t_1 b_1^3 + t_2 b_2^3)}$	$(\sigma_x)_{max} = \frac{hb_1}{2} \frac{t_2 b_2^3}{t_1 b_1^3 + t_2 b_2^3} E\theta''$ throughout the thickness at points A and B if $t_2 b_2^2 > t_1 b_1^2$ $(\sigma_x)_{max} = \frac{hb_2}{2} \frac{t_1 b_1^3}{t_1 b_1^3 + t_2 b_2^3} E\theta''$ throughout the thickness at points C and D if $t_2 b_2^2 < t_1 b_1^2$ $(\tau_2)_{max} = \frac{-1}{8} \frac{h t_2 b_2^3 b_1^2}{t_1 b_1^3 + t_2 b_2^3} E\theta'''$ throughout the thickness at a point midway between A and B if $t_2 b_2 > t_1 b_1$ $(\tau_2)_{max} = \frac{-1}{8} \frac{h t_1 b_1^3 b_2^2}{t_1 b_1^3 + t_2 b_2^3} E\theta'''$ throughout the thickness at a point midway between C and D if $t_2 b_2 < t_1 b_1$ $(\tau_1)_{max} = t_{max} G\theta'$ at the surface on the thickest portion
8. *Z*-section 	$K = \frac{t^3}{3}(2b + h)$ $C_w = \frac{th^2 b^3}{12}\left(\frac{b + 2h}{2b + h}\right)$	$(\sigma_x)_{max} = \frac{hb}{2} \frac{b + h}{2b + h} E\theta''$ throughout the thickness at points A and D $(\tau_2)_{max} = \frac{-hb^2}{4}\left(\frac{b + h}{2b + h}\right)^2 E\theta'''$ throughout the thickness at a distance $\frac{b(b + h)}{2b + h}$ from point A $(\tau_1)_{max} = tG\theta'$ at the surface everywhere
9. Segment of a circular tube  (*Note:* If t/r is small, α can be larger than π to evaluate constants for the case when the walls overlap)	$e = 2r \frac{\sin\alpha - \alpha\cos\alpha}{\alpha - \sin\alpha\cos\alpha}$ $K = \frac{2}{3} t^3 r\alpha$ $C_w = \frac{2tr^5}{3}\left[\alpha^3 - 6\frac{(\sin\alpha - \alpha\cos\alpha)^2}{\alpha - \sin\alpha\cos\alpha}\right]$	$(\sigma_x)_{max} = (r^2\alpha - re\sin\alpha)E\theta''$ throughout the thickness at points A and B $(\tau_2)_{max} = r^2\left[e(1 - \cos\alpha) - \frac{r\alpha^2}{2}\right]E\theta'''$ throughout the thickness at midlength $(\tau_1)_{max} = tG\theta'$ at the surface everywhere