Fuzzy Disturbance Observer-Based Sliding Mode Control for Liquid-Filled Spacecraft With Flexible Structure Under Control Saturation

LIQIAN DOU[ID]**[1], MIAOMIAO DU[1], XIUYUN ZHANG**[ID]**[1], HUI DU[1], AND WENJING LIU[2]**

[1]School of Electrical and Information Engineering, Tianjin University, Tianjin 300072, China
[2]Science and Technology on Space Intelligent Control Laboratory, Beijing Institute of Control Engineering, Beijing 100101, China

Corresponding author: Xiuyun Zhang (zxy_11@tju.edu.cn)

ABSTRACT This paper investigates a fuzzy disturbance observer (FDO)-based terminal sliding mode control (TSMC) strategy for the liquid-filled spacecraft with flexible structure(LFS-FS) under control saturation. Firstly, a novel FDO is designed to estimate the lumped uncertainty, including the inertia uncertainty, external disturbance, the coupling of liquid slosh and flexible structure(LF), as well as the parts that exceed control saturation. The merits of the FDO lie in that estimation error can be arbitrarily small by adjusting the designed parameters and the prior information is not required. Then, based on the estimation of FDO, a finite-time TSMC is designed, which has more satisfactory control performance, such as chattering reduction and fast convergence speed. The stability of the closed-loop system is proved strictly by Lyapunov theory. Finally, numerical simulations are presented to demonstrate the effectiveness of the proposed method.

INDEX TERMS Spacecraft, flexible vibration, liquid fuel slosh, control saturation, fuzzy disturbance observer, terminal sliding mode control.

I. INTRODUCTION

Over the past few decades, the space technology development is recognized as an important part for national security, earth observation, planetary exploration and so on. Since it has been rapidly developed for the control strategy of rigid spacecraft, many methods have been introduced to enhance the stability of the control effect, such as sliding mode control(SMC) [1]–[4], adaptive control [5], [6], robust control [7]–[9]. However, The missions of the spacecraft are becoming increasingly complex, flexible appendages [10] and liquid storage chamber [11] are widely adopted on the spacecraft, which not only change the control mode but also make the control more difficult. For example, there are some features of highly flexibility and low damping for flexible appendages such as solar panels and long antennas [12]. Hence, the control performance and stability of LFS-FS would be deteriorated due to the coupling between the rigid body and the flexible appendages [13]–[15]. In addition, because the space task becomes more complex, longer

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residence time and much more liquid fuel are needed for the spacecraft [16], [17]. However, due to the sloshing of the liquid fuel, the maneuverability of spacecraft is inevitably influenced, which causes the bad control performance and even the failure task [18], [19]. Therefore, the attitude stability control for LFS-FS with model uncertainties and external disturbance have been one of the important research topics.

What's more, as is known to all, the control torque produced by the actuator is usually not infinite in practical applications, which will no doubt cause the control saturation problems and lead substantial degradation to the system [20]–[24]. In [25], American researcher RD Robinett discussed a generalized feedback control law design method under control saturation constraints as early as 1997. Then in [26], [27], the Boskovic in Yale taked the control saturation problem into account in the design of the control law for spacecraft and designed a control algorithm based on variable structure. In [28], Hu *et al.* investigated a finite-time controller under input saturation based on a second-order disturbance observer and the adaptive control.

Although Boskovicv *et al.* [26], [27], Hu *et al.* [21], Xiao *et al.* [22], Hu *et al.* [28], Xiao *et al.* [29], Hu *et al.* [30],

Zou *et al.* [31], Ruiter [32], and Zhu *et al.* [33], Xia *et al.* [34], Lu and Xia [35] have developed some controllers to effectively deal with the constrained actuator output, hardly any study discussed the control algorithm for LFS-FS under control saturation constrain and thus cannot meet the demands of aerospace task for the future. For the above reasons, this paper will propose a control method for the stability control problem of LFS-FS with actuator saturation.

The disturbance observer, is widely used for nonlinear control problems with external disturbances and uncertainties. Reference [36], [37] proposed extended state observer-based adaptive and robust control methods, which estimated the unmeasurable system states and the additive disturbances effectively. A novel adaptive sliding mode disturbance observer is proposed in [38], which could achieve the precise and fast trajectory tracking control for space manipulator after capturing an uncertain space target. And the FDO has universal approximation capability for the system with unknown disturbances and uncertainties [39]. Therefore, it is widely employed to counteract the system disturbances and improve controller robustness. A novel control method for the flight simulator was proposed in [40], in which a FDO was designed to compensate the external disturbances. The unknown uncertainty and disturbance of nonlinear system were estimated by the FDO adopted in [41], [42], and the simulations are carried out to verify the effectiveness and the applicability of the FDO for the nonlinear control system with uncertainty.

Furthermore, in nonlinear control designs, SMC is a well-known and powerful control scheme that has been widely used. In [43]–[45] the SMC was adopted to design an effective and stable controller for nonlinear systems. The hierarchical SMC was employed to design the controller for the space vehicle in [46]. In [6], a second-order SMC method for spacecraft was presented to guarantee control error can converge to zero in the finite time. SMC has some advantages such that it is not sensitive to parameters change. And, the TSMC adds terminal items based on SMC, which makes the control system have faster convergence speed and ensures that the system converges in a finite time.

In this work, the integration of FDO and TSMC is proposed to solve the attitude stability problem for the LFS-FS under control saturation, uncertainties and disturbances, which can ensure the control system of the LFS-FS is stable. The main contributions are summarized as follows:

1) From the theoretic aspect, an FDO-based TSMC strategy is proposed in this paper. The FDO, designed by the fuzzy logic system (FLS), can approximate the uncertainties and various types of disturbances, which provides freedom from derivative of disturbance bound assumption. What's more, the FDO-based TSMC strategy is proposed to guarantee the asymptotically stability, and reject the lumped uncertainty with lower chattering and higher accuracy, which has been rigorously proved by the Lyapunov theory.
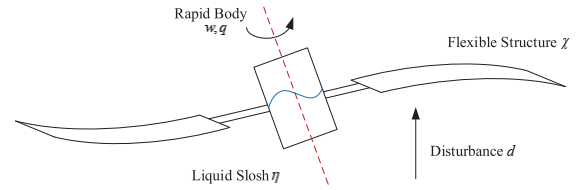

FIGURE 1. The diagram of LFS-FS.

2) From the engineering application aspect, the integrated design of FDO and TSMC can provide the fast and high precision attitude stabilization control of the complex spacecraft, which is coupled with liquid slosh and flexible vibration, even in the presence of external disturbances and control saturation. What's more, the simulations and comparison analysis are presented to verify the effectiveness and better performance of the proposed method, which can reduce the chattering and accelerate the convergence speed effectively.

The outline of this paper is as follows. At first, the mathematical model of LFS-FS under control input constraint is introduced in Section II. Then, FLS is introduced and the designed FDO is adopted to estimate the lumped uncertainty in Section III. In Section IV, the TSMC based on the estimations of FDO is proposed and the Lyapunov function is adopted to prove that the controller system is stable. Simulation results and analysis are shown in Section V. The paper ends with the conclusions in Section VI.

II. PROBLEM FORMULATION
A. MATHEMATICAL SYSTEM MODEL
The diagram of LFS-FS is shown in Figure 1. The dynamic equations can be described by [16], [47]

$$\begin{cases} \dot{q}_v = \dfrac{1}{2}(q_0 I_3 + q_v^\times)\omega \\ \dot{q}_0 = -\dfrac{1}{2}q_v^T \omega \end{cases} \tag{1}$$

$$\begin{cases} \ddot{\chi} + C_1\dot{\chi} + K_1\chi + N\dot{\omega} = 0 \\ M_0\ddot{\eta} + C_2\dot{\eta} + K_2\eta + M\dot{\omega} = 0 \end{cases} \tag{2}$$

$$J\dot{\omega} + N^T\ddot{\chi} + M^T\ddot{\eta} = -\omega^\times(J\omega + N^T\dot{\chi} + M^T\dot{\eta}) + u + d \tag{3}$$

where the unit quaternion $q = [q_0 \ q_v^T]^T = [q_0 \ q_1 \ q_2 \ q_3]^T$, q_0 is the scalar part of q and q_v is the vector part, and satisfies $q_v^T q_v + q_0^2 = 1$; $\omega \in R^3$ is the angular velocity of the body fixed frame with respect to the inertia reference frame, χ and η are the modal coordinate vectors of flexible structure and liquid slosh respectively; u, d represent the control torque and external disturbance, respectively; The inertia matrix J is described as $J = J_0 + \Delta J$, where the nominal part and the uncertain part are represented by J_0 and ΔJ; $K_i, C_i(i = 1, 2)$ denote the stiffness and damping matrices, respectively; $M, N \in R^{4\times3}$ are the coupling matrixs of the liquid slosh and flexible structure between rigid dynamics, respectively. And the expressions of M and the quality matrix M_0 of liquid slosh

are given as follows.

$$M = \begin{bmatrix} 0 & m_1 b_1 & 0 \\ -m_1 b_1 & 0 & 0 \\ 0 & m_2 b_2 & 0 \\ -m_2 b_2 & 0 & 0 \end{bmatrix}$$

$$M_0 = \begin{bmatrix} m_1 & 0 & 0 & 0 \\ 0 & m_1 & 0 & 0 \\ 0 & 0 & m_2 & 0 \\ 0 & 0 & 0 & m_2 \end{bmatrix} \quad (4)$$

where $m_i, b_i (i = 1, 2)$ represent the quality of the sloshing liquid mass and the distance between each sloshing mass and the center mass of the total spacecraft, respectively. The $q_v^\times \in R^{3 \times 3}$ is a cross product matrix, which can be defined as follows.

$$q_v^\times = \begin{bmatrix} 0 & -q_3 & q_2 \\ q_3 & 0 & -q_1 \\ -q_2 & q_1 & 0 \end{bmatrix} \quad (5)$$

B. CONTROL-ORIENTED MODEL

Consider the situation of actuator input saturation, the dynamic model (3) is expressed as

$$J\dot{\omega} + N^T\ddot{\chi} + M^T\ddot{\eta} = -\omega^\times (J\omega + N^T\dot{\chi} + M^T\dot{\eta}) + sat(u) + d \quad (6)$$

where $sat(\cdot)$ represents the nonlinear saturation characteristic of the actuators, defined as

$$sat(u_i) = \begin{cases} u_{\max}, & u_i > u_{\max} \\ u_i, & u_{\min} \le u_i \le u_{\max} \\ u_{\min}, & u_i < u_{\min} \end{cases} \quad (7)$$

where $u_{\max}$ and $u_{\min}$ are the upper bound and lower bound of actuator torque respectively.

Design a virtual variable $\Delta u = (\Delta u_1, \Delta u_2, \Delta u_3) \in R^3$, in which $\Delta u_i, i = 1, 2, 3$ is given as

$$\Delta u_i = \begin{cases} u_{\max} - u_i, & u_i > u_{\max} \\ 0, & u_{\min} \le u_i \le u_{\max} \\ u_{\min} - u_i, & u_i < u_{\min} \end{cases} \quad (8)$$

Thus (6) is rewritten as

$$J\dot{\omega} + N^T\ddot{\chi} + M^T\ddot{\eta} = -\omega^\times (J\omega + N^T\dot{\chi} + M^T\dot{\eta}) + u + \Delta u + d \quad (9)$$

Define $C = \omega^\times J_0, D = I_3$ and the lumped uncertainty

$$\Xi = -\omega^\times \Delta J \omega - \Delta J \dot{\omega} - \omega^\times N^T \dot{\chi} - \omega^\times M^T \dot{\eta} \\ - N^T \ddot{\chi} - M^T \ddot{\eta} + d + \Delta u$$

thus we have

$$J_0 \dot{\omega} = -C\omega + Du + \Xi \quad (10)$$

In this work, our goal is to achieve the attitude stabilization in the presence of inertia uncertainty, external disturbances and LF with bounded control input. The main objective is to

design a FDO-based TSMC scheme to guarantee the attitude stabilization of complex spacecraft system (1) (2) and (9), which means the q and ω will converge to an arbitrary small set containing the origin.

III. FUZZY DISTURBANCE OBSERVER

To give a better understanding of FDO, the FLS is briefly introduced firstly in this section. After that, we designed a new fuzzy disturbance observer to estimate the whole uncertainties Ξ, which could realize an arbitrarily small estimation error by adjusting the designed parameters.

A. DESCRIPTION OF THE FUZZY LOGIC SYSTEM

The FLS contains five parts: fuzzifier, rule bases, inference engine, type-reducer, and defuzzifier. The fuzzy inference engine uses IF-THEN rules to map from an input compact set $X = [x_1, x_2, \cdots, x_n] \in R^n$ to an output compact set $Y \in R$. The fuzzy rules can be described as

$$R^i : IF \ x_1 \ is \ A_1^i, and \dots and, \ x_n \ is \ A_n^i, \ then \ y \ is \ Y \quad (11)$$

where $A_1^i, A_2^i, \dots, A_n^i$ are fuzzy variables and Y is a fuzzy singleton.

Using center-average defuzzifier and singleton fuzzifier, the output of FLS is represented as

$$y(x) = \frac{\sum_{j=1}^p h_j [\prod_{i=1}^n \mu_{A_i^j}(x_i)]}{\sum_{j=1}^p [\prod_{i=1}^n \mu_{A_i^j}(x_i)]}$$

$$= \sum_{j=1}^p \theta_j \xi_j(x) = \theta^T \xi(x) \quad (12)$$

where p represents the number of fuzzy logic rules, $\theta^T = (h_1, h_2, \cdots, h_p)^T$ is the vector of adjustable parameters, $\xi_j(x) = \frac{\prod_{i=1}^n \mu_{A_i^j}(x_i)}{\sum_{j=1}^p (\prod_{i=1}^n \mu_{A_i^j}(x_i))}$ denotes the vector of fuzzy basis function, and the $\mu_{A_i^j}(x_i)$ denotes the membership function.

Assumption 1 [48], [49]: Let $f(x)$ be a continuous function defined on a compact set M_x. Then, for any constant $\varepsilon > 0$, there exists a fuzzy logic system (12) such that $\sup_{x \in M_x} |f(x) - \theta^T \xi| < \varepsilon$.

B. FDO DESIGN FOR SPACECRAFT ATTITUDE CONTROL SYSTEM

In this work, we design the fuzzy disturbance observer for spacecraft attitude control system. Based on the (10) and (12), the FDO is constructed as

$$\dot{z} = -\sigma z + \sigma J_0 \omega - C\omega + Du + \hat{\Xi} \quad (13)$$

The estimation of the lumped uncertainty Ξ is given as

$$\hat{\Xi} = \hat{\theta}^T \xi \quad (14)$$

thus the FDO can be written as:

$$\dot{z} = -\sigma z + \sigma J_0 \omega - C\omega + Du + \hat{\theta}^T \xi \quad (15)$$

where z denotes the observer state, the designed constant $\sigma > 0$, $\hat{\Xi}$ is the estimate value of the lumped uncertainty Ξ, $\hat{\theta}$ is achieved by an adaptation law which is expressed as

$$\dot{\hat{\theta}} = \kappa_0 \xi (e + \gamma_0 \varepsilon^*) \tag{16}$$

where the constants $\kappa_0 > 0, \gamma_0 > 0$ are the designed values.

Define the estimated error $e = J_0 \omega - z$, the dynamics of the observation estimation errors is described as

$$\dot{e} = J_0 \dot{\omega} - \dot{z} = -\sigma e + \Xi - \hat{\Xi} \tag{17}$$

According to the Assumption 1, Ξ satisfies the following form

$$\Xi = \theta^{*T} \xi + \varepsilon \tag{18}$$

where θ^* is the optimal parameter vector. The ε denotes approximation error of FLS, which is bounded by $\bar{\varepsilon}$.

Substituting the (14) and (18) into (17), $\dot{e}$ can be rewritten as

$$\dot{e} = -\sigma e - \tilde{\theta}^T \xi + \varepsilon \tag{19}$$

where $\tilde{\theta} = \hat{\theta} - \theta^*$ denotes the parameter estimation error. The e is exponentially convergent, when $\tilde{\theta}$ approaches 0.

Define the disturbance reconstruction error as $\varepsilon^* = -\tilde{\theta}^T \xi + \varepsilon$, let $m = -\tilde{\theta}^T \xi$, then the (19) is transformed into $\dot{e} = -\sigma e + \varepsilon^*$, and thus we have the reconstructed error formulated as:

$$\varepsilon^* = \sigma e + \dot{e} \tag{20}$$

Theorem 1: Consider the (9) and the FDO (13). If the adaptive law of the parameter vector $\hat{\theta}$ is chosen as (16), and the parameters $\kappa_0 > 0$, $\gamma_0 > 0$, then e is uniformly stable converging to a small region.

Proof: The following Lyapunov function candidate is chosen:

$$V_F = \frac{1}{2} e^2 + \frac{1}{2\kappa_0} \tilde{\theta}^T \tilde{\theta} \tag{21}$$

Taking the time derivative of (21), and combining (19) and (16), $\dot{V}_F$ satisfies:

$$\begin{aligned}
\dot{V}_F &= e\dot{e} + \frac{1}{\kappa_0} \tilde{\theta}^T \dot{\hat{\theta}} \\
&= e(-\sigma e - \tilde{\theta}^T \xi + \varepsilon) + \tilde{\theta}^T \xi(e + \gamma_0 \varepsilon^*) \\
&= -\sigma e^2 + e\varepsilon + \gamma_0 \tilde{\theta}^T \xi \varepsilon^* \\
&= -\sigma e^2 + e\varepsilon - m\gamma_0(m + \varepsilon) \\
&= -\sigma e^2 + e\varepsilon - \gamma_0 m^2 - \gamma_0 m\varepsilon
\end{aligned} \tag{22}$$

It is noted that the following inequalities satisfy

$$\begin{aligned}
e\varepsilon &\le \frac{1}{2}\sigma e^2 + \frac{1}{2\sigma}\bar{\varepsilon}^2 \\
m\varepsilon &\le \frac{1}{2}m^2 + \frac{1}{2}\bar{\varepsilon}^2
\end{aligned} \tag{23}$$

Then, $\dot{V}_F$ in (22) can be transformed into following form

$$\begin{aligned}
\dot{V}_F &\le -\sigma e^2 + \frac{1}{2}\sigma e^2 + \frac{1}{2\sigma}\bar{\varepsilon}^2 - \gamma_0 m^2 + \frac{\gamma_0}{2}m^2 + \frac{\gamma_0}{2}\bar{\varepsilon}^2 \\
&= -\frac{1}{2}\sigma e^2 + \frac{1}{2\sigma}\bar{\varepsilon}^2 - \frac{\gamma_0}{2}m^2 + \frac{\gamma_0}{2}\bar{\varepsilon}^2
\end{aligned} \tag{24}$$

Integrating both sides of (24) from 0 to T yields

$$\begin{aligned}
\frac{1}{2}\int_0^T &\sigma e^2 dt + \frac{1}{2}\int_0^T \gamma_0 m^2 dt \\
&\le V(0) - V(T)t + \int_0^T \frac{1}{2\sigma}\bar{\varepsilon}^2 d + \int_0^T \frac{\gamma_0}{2}\bar{\varepsilon}^2 dt \\
&\le V(0) + \int_0^T (\frac{1}{2\sigma} + \frac{\gamma_0}{2})\bar{\varepsilon}^2 dt
\end{aligned} \tag{25}$$

which is equivalent to

$$\begin{aligned}
\int_0^T &\sigma e^2 dt + \int_0^T \gamma_0 m^2 dt \\
&\le e^2(0) + \frac{1}{\kappa_0}\tilde{\theta}(0)^T\tilde{\theta}(0) + \int_0^T (\gamma_0 + \frac{1}{\sigma})\bar{\varepsilon}^2 dt
\end{aligned} \tag{26}$$

The robust performance of (26) can be explained by Barbalat's lemma [50]. If $\varepsilon \in L_2$, i.e. $\int_0^\infty \varepsilon^2 dt < \infty$, then $e \in L_2$ and $m \in L_2$. This means $\lim_{t \to \infty} \|e\| = 0$ and $\lim_{t \to \infty} \|m\| = 0$. Even though $\varepsilon \notin L_2$, e^2 is bounded by ε^2. Therefore, the observation error can be reduced to any small value with adjusting the σ and γ_0. Thus, $\hat{\Xi}$ can estimate Ξ with arbitrarily small error.

IV. COTROLLER DESIGN FOR SPACECRAFT SYSTEM
A. FDO-BASED TSMC DESIGN
In the subsection, a FDO-based terminal sliding mode controller is designed for the LFS-FS with inertia uncertainty and external disturbance under actuator input saturation, which can guarantee the stabilization of the spacecraft system with high speed and precision.

The following sliding mode surface is designed:

$$s = \omega + k \cdot \beta(q_v) \tag{27}$$

where $s = [s_1\ s_2\ s_3]^T \in R^3$, $k > 0$ is user-designed constant, and $\beta(q_v) = [\beta(q_1)\ \beta(q_2)\ \beta(q_3)]^T$ satisfies the following form:

$$\beta(q_i) = \begin{cases} sig^r(q_i) & if\ \bar{s}_i = 0\ or\ \bar{s}_i \ne 0,\ |q_i| > v \\ a_{11}q_i + a_{12}sig^2(q_i) & if\ \bar{s}_i \ne 0,\ |q_i| \le v \end{cases} \tag{28}$$

where $\bar{s}_i = \omega_i + k \cdot sig^r(q_i)$, $sig^r(q_i) = |q_i|^r sign(q_i)$, $i = 1, 2, 3$, $0 < r < 1$, $v > 0$ is a small constant, and a_{11} and a_{22} satisfy $a_{11} = (2 - r)v^{r-1}$, $a_{22} = (r - 1)v^{r-2}$.

Then, base on the (1) and (3), the time derivative of (27) is obtained as

$$\begin{aligned}
J_0 \dot{s} &= -\omega^\times(J_0\omega + N^T\dot{\chi} + M^T\dot{\eta}) + k \cdot J_0\dot{\beta}(q_v) \\
&\quad - N^T\ddot{\chi} - M^T\ddot{\eta} + u + \Delta u + d - \Delta J\dot{\omega} - \omega^\times\Delta J\omega \\
&= g(t) + u + \Xi
\end{aligned} \tag{29}$$

where Ξ denotes the lumped uncertainty of the system, $g(t)$ denotes the normal part of the system, which satisfies the following form

$$g(t) = -\omega^\times J_0\omega + k \cdot J_0\dot{\beta}(q_v) \tag{30}$$

$$\dot{\beta}(q_i) = \begin{cases} r|q_i|^{r-1}\dot{q}_i, & \text{if } \bar{s}_i = 0 \text{ or } \bar{s}_i \neq 0, \quad |q_i| > \nu \\ a_{11}\dot{q}_i + 2a_{12}|q_i|\dot{q}_i, & \text{if } \bar{s}_i \neq 0, \quad |q_i| \leq \nu \end{cases} \tag{31}$$

Thus, the u is designed as:

$$u = -g(t) - \delta_1 s - \delta_2 sig^r(s) - \hat{\theta}^T\xi \tag{32}$$

where δ_1, δ_2 are positive designed constants.

Lemma 1 [51]: Consider nonlinear system $\dot{x} = f(x, u)$, where x is the state vector and u is the control vector. Assuming that there are continuous differentiable positive definite functions $V(x)$, scalar $\lambda > 0$, $0 < a < 1$, $0 < \vartheta < \infty$, which makes inequality $\dot{V}(x) \leq -\lambda V^\alpha(x) + \vartheta$ valid, then the system $\dot{x} = f(x, u)$ is actually stable in finite time, and the convergence time is

$$T_{reach} \leq \frac{V^{1-\alpha}(x_0)}{\lambda\phi(1-\alpha)}, 0 < \phi < 1 \tag{33}$$

where $V(x_0)$ is the initial value of $V(x)$.

B. STABILITY ANALYSIS

Theorem 2:Consider the nonlinear sapcecraft system (1)-(2) with control saturation (6). If designing the terminal sliding mode controller (32) augmented by the fuzzy disturbance observer (13) and adaptive law (18) under $\kappa_0, \gamma_0 > 0$, $0 < r < 1$, and $\delta_1, \delta_2 > 0$, then the attitude quaternion q_v and the attitude angular velotity ω will converge to small neighborhoods of origion by selecting suitable controller parameters.

Proof: Define Lyapunov function candidate as

$$V = V_1 + V_2 + V_3 \tag{34}$$

where

$$V_1 = \frac{1}{2}s^T J_0 s \tag{35}$$

$$V_2 = \frac{1}{2}q_v^2 \tag{36}$$

V_3 is (21) which verifies the stability of the FDO. The mathematical proof has been shown in section III-B.

Taking the time derivative of (35), it follows that

$$\begin{aligned} \dot{V}_1 &= s^T J_0\dot{s} \\ &\leq s^T[g(t) + u + \Omega] \\ &= s^T[-\delta_1 s - \delta_2 sig^r(s) - \hat{\theta}^T\xi + \theta^{*T}\xi + \varepsilon] \\ &= -\delta_1 s^2 - \delta_2\sum_{i=1}^{3}|s_i|^{r+1} - s^T\tilde{\theta}^T\xi + s^T\varepsilon \\ &= -\delta_1 s^2 - \delta_2\sum_{i=1}^{3}|s_i|^{r+1} + s^T m + s^T\varepsilon \end{aligned} \tag{37}$$

By applying the following inequalities

$$s^T m \leq \frac{1}{2}\sigma s^2 + \frac{1}{2\sigma}m^2$$

$$s^T\varepsilon \leq \frac{1}{2}\sigma s^2 + \frac{1}{2\sigma}\bar{\varepsilon}^2 \tag{38}$$

Thus, the derivative $\dot{V}_1$ can be derived as

$$\begin{aligned} \dot{V}_1 &\leq -\delta_1 s^2 - \delta_2\sum_{i=1}^{3}|s_i|^{r+1} + \frac{1}{2}\sigma s^2 + \frac{1}{2\sigma}m^2 + \frac{1}{2}\sigma s^2 \\ &\quad + \frac{1}{2\sigma}\bar{\varepsilon}^2 \\ &\leq -(\delta_1 - \sigma)s^2 - \delta_2\sum_{i=1}^{3}|s_i|^{r+1} + \frac{1}{2\sigma}m^2 + \frac{1}{2\sigma}\bar{\varepsilon}^2 \\ &\leq -(\delta_1 - \sigma)s^2 - \frac{2^{(r+1)/2}\delta_2}{\lambda_{max}^{(r+1)/2}}V_1^{(r+1)/2} + \frac{1}{2\sigma}m^2 + \frac{1}{2\sigma}\bar{\varepsilon}^2 \end{aligned} \tag{39}$$

When the parameters are chosen as $\delta_1 > \sigma > 0$, the inequality (39) becomes

$$\dot{V}_1 \leq -\frac{2^{(r+1)/2}\delta_2}{\lambda_{max}^{(r+1)/2}}V_1^{(r+1)/2} + \frac{1}{2\sigma}m^2 + \frac{1}{2\sigma}\bar{\varepsilon}^2 \tag{40}$$

According to the Lemma1, it means that the system is actually stable in finite time and the system states will reach the sliding surface $s = 0$ (27) in finite time, which implies that

$$2(q_0 I_3 + q_v^\times)^{-1}\dot{q}_v = -k\beta(q_v) \tag{41}$$

thus we can draw the conclusion that $\dot{V}_2 = q_v\dot{q}_v < 0$, the q_v is exponentially stable. This means that the system states will reach to the desired equilibrium point asymptotically under the designed control law. Therefore, the proof is completed.

V. SIMULATION RESULTS AND ANALYSIS

In this section, the effectiveness and performance of the FDO (31) and TSMC in (32) for LFS-FS (1)(2)(6) will be verified by simulation.

A. PARAMETER SETTING

It is assumed that the inertia matrix and nominal inertia matrix for the coupled spacecraft are selected as

$$J = [360 \ 3 \ 4; 3 \ 279 \ 10; 4 \ 10 \ 198] \tag{42}$$

$$J_0 = [350 \ 0 \ 0; 0 \ 270 \ 0; 0 \ 0 \ 190] \tag{43}$$

The physical parameters are chosen as [47], [48], [52]:

$$N = \begin{bmatrix} 6.45637 & 1.27814 & 2.15629 \\ -1.25819 & 0.91756 & -1.67264 \\ 1.11687 & 2.48901 & -0.83674 \\ 1.23637 & -2.6581 & -1.12503 \end{bmatrix}$$

$$C_1 = diag\{0.0086, \ 0.0190, \ 0.0487, \ 0.1275\}$$

$$K_1 = diag\{0.5900, \ 1.2184, \ 3.5093, \ 6.5005\}$$

$$C_2 = diag\{3.334, \ 3.334, \ 0.237, \ 0.237\}$$

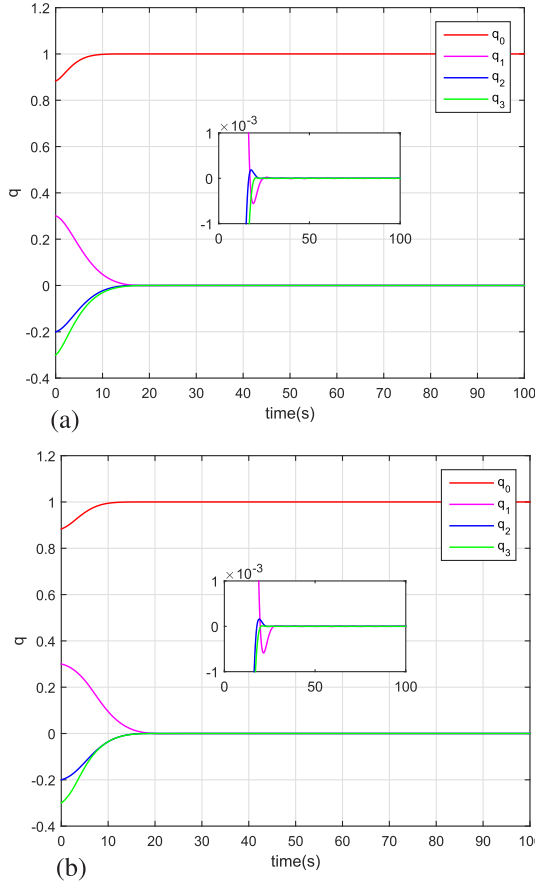$$K_2 = diag\{55.21, \ 55.21, \ 7.27, \ 7.27\}$$

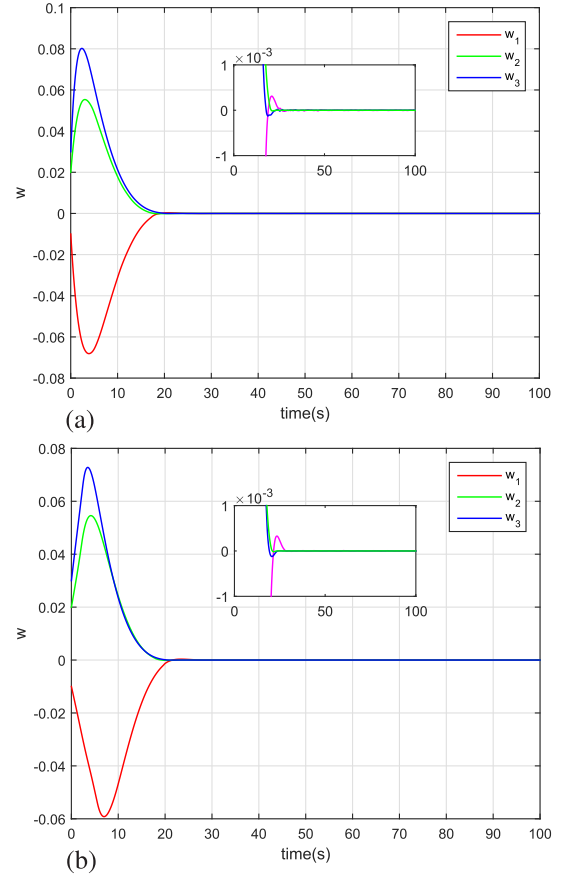**FIGURE 2.** Time responses of quaternions in case 1. (a) Without control saturation.(b) With control saturation.

FIGURE 3. Time responses of angular velocity in case 1. (a) Without control saturation. (b) With control saturation.

$m_1 = 20kg$, $m_2 = 0.8kg$, $b_1 = 1.127m$, $b_2 = 0.994m$. The controller parameters of controller design are set to be: $\delta_1 = 100$, $\delta_2 = 2$, $k = 0.2$, $r = 0.8$, $\sigma = 1.2$, $\kappa_0 = \gamma_0 = 1$. The initial values of angular velocity and attitude for coupled spacecraft are $\omega = \begin{bmatrix} -0.01 & 0.02 & 0.03 \end{bmatrix}$ and $q = [0.8832 \quad 0.3 \quad -0.2 \quad -0.3]$. The external disturbance added to the system are designed as $d = (\|\omega\|^2 + 0.05)[\sin 0.8t \ \cos 0.5t \ \cos 0.3t]^T$.

B. SIMULATION ANALYSIS

To further illustrate the effectiveness of the proposed method, simulation of two cases are made for the comparison. In the first case, the robustness of the proposed control scheme is verified by applying it to the conditions with input saturation and without input saturation respectively. In the second case, to prove the necessity of studying LFS-FS under control saturation, the adaptive variable structure control presented in [21] is applied to the control input saturation problem considered above, the simulation results show the advantages of the controller in this paper.

Case 1: In this case, we verify that the proposed algorithm can effectively overcome the drawback caused by the control saturation. Firstly, the angular velocity and attitude quaternion trajectories are shown in (a) of Figure 2 and Figure 3.

As we can see, the coupled spacecraft system achieves stabilization smoothly in less than 20 s with a high accuracy of 10^{-3} in static error. The trajectories of the control input is shown in Figure 4 (a), which demonstrates the upper bounder of required torque is aboud $10Nm$. Besides, Figure 5 (a) shows the effectiveness of the FDO. The reconstructed lumped uncertainty Ω is caught almost immediately.

Then, to testify the robustness of the proposed controller when faced with input constraint conditions. The restrictions on the actuator output torque are set as $u_{max} = 2Nm$ and $u_{min} = -2Nm$, which are much smaller than the required ones. Comparing the (a) and (b) of Figure 2 and Figure 3, it can be seen that the state trajectory and the convergence time for and are substantially unchanged, and the static error and the vibration are kept within a small scale, indicating that the controller is robust to the input limitation and can effectively overcome the lag of control effect caused by it. Time responses of the demand control torques are depicted in Figure 4 (b). Note that the control torques are strictly limited to $[-2, 2]Nm$. Figure 5 shows the result of FDO which reflects that with the consideration of the constrained part of the control input, the reconstructed lumped uncertainties became larger in the initial stage, but when the system is gradually stable under the control, the observer's value would be restored to a small one.
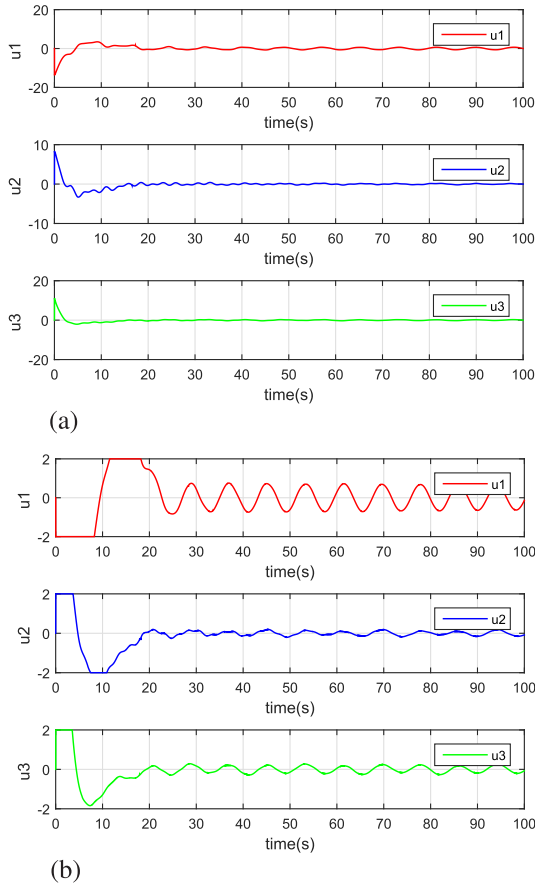
FIGURE 4. Time responses of control torques in case 1. (a) Without control saturation. (b) With control saturation.

FIGURE 5. Time responses of FDO in case 1. (a) Without control saturation. (b) With control saturation.

Case 2: In this case, the fuzzy disturbance observer based terminal sliding mode controller (FDOTSC) proposed in (32) is compared to the adaptive variable structure controller(AVSC) [21]. The red solid line and blue dash line in Figure 6-Figure 10 denote the control performance by (32) and AVSC in [21] respectively. For the AVSC in [21], the control parameters are chosen as $\delta = 0.02$, r $= 0.05$. The parameters in (32) are consistent with case 1. Figure 6 and Figure 7 show time responses of LFS-FS attitude quaternion and the angular velocity. As we can see, with the controller in (32), the system achieves stabilization smoothly with the time less than 25 s and with a high accuracy of 10^{-3} in static error. Although with the AVSC more time is required to achieve the attitude stabilization and the static error exists even after 100 s. Accordingly, the case of FDOTSC in (32) achieves the better performance than the AVSC in [21], which further illustrates the advantages and feasibility of the FDOTSC method.

Time responses of control torques is pictured in Figure 8. The demand control is strictly limited in [-2Nm,2Nm] for both of the controller. Furthermore, note that when the spacecraft system stabilize, the control torque in FDOTC is equal to the lumped uncertainties observed by the FDO, which compensates for the system and makes the performance better.
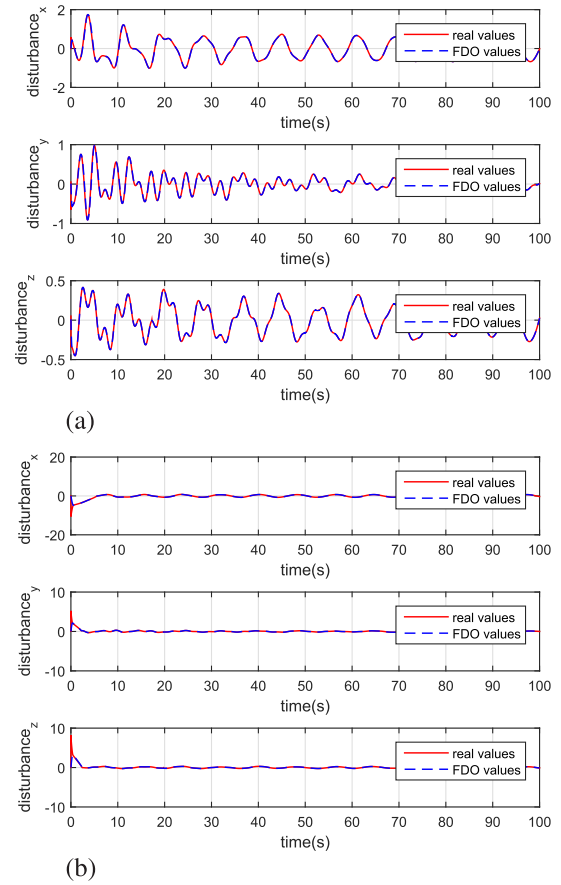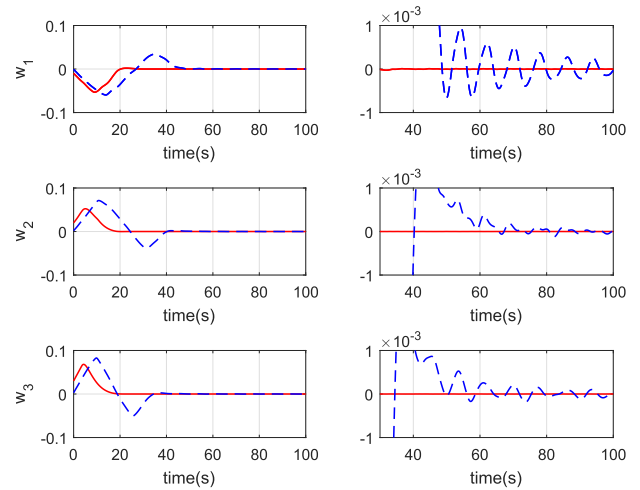

FIGURE 6. Comparison of angular velocity.

Because the first two orders of vibration and sloshing modes have the greatest impact on system stability, we just list the time responses of the sloshing liquid η_1, η_2 and flexible structure χ_1, χ_2, which is shown in Figure 9 and Figure 10. Although, sloshing liquid and flexible structure can cause tracking error and instability during spacecraft maneuvering
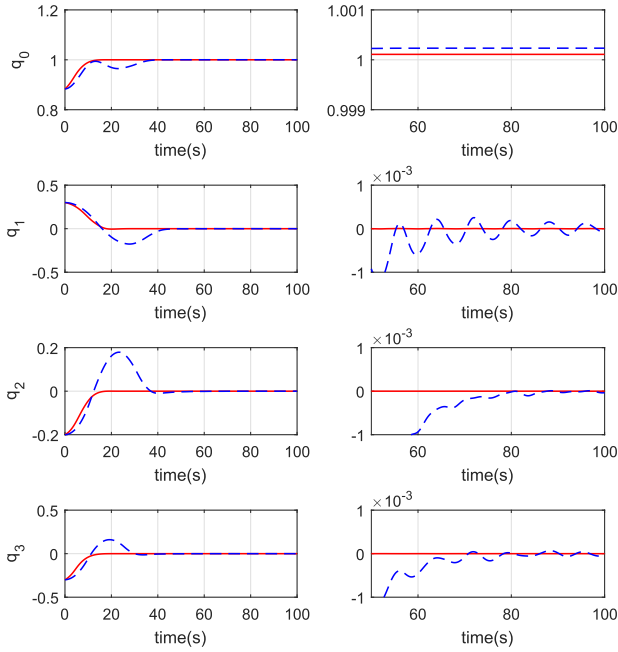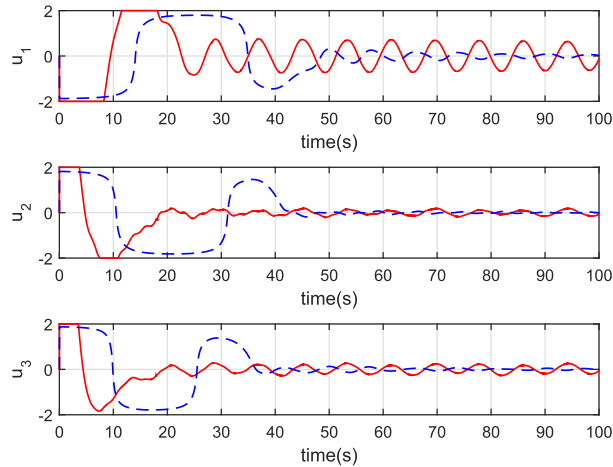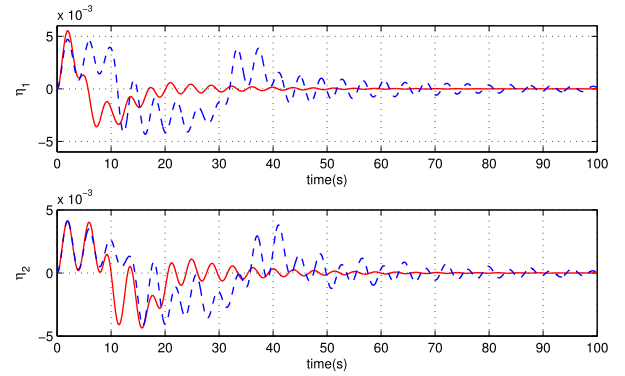
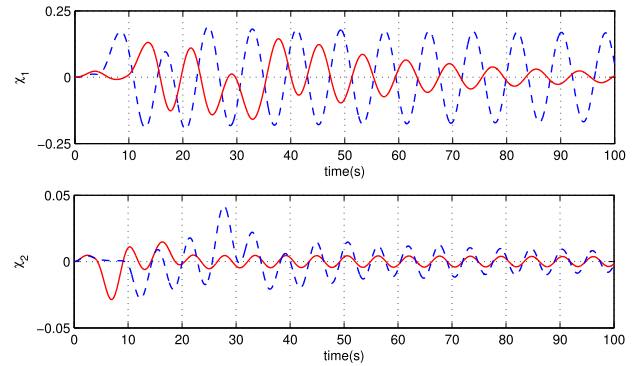**FIGURE 7.** Comparison of quaternion.

FIGURE 8. Comparison of control torque.

FIGURE 9. Comparison of sloshing liquid η.

FIGURE 10. Comparison of flexible structure χ.

process, the FDOTSC compared to AVSC in [21] can effectively track and observe the effects of flexible vibration and liquid sloshing, and achieve fast and stable attitude tracking. From the Figure 9 and Figure 10, as we can see, compared with [21], the FDOTSC proposed by this paper brings less liquid sloshing and flexible vibration. One reason is because the FDO with universal approximation can observe the overall disturbance including the inertia uncertainty, external disturbance and the coupling of LF, another reason is that the FDO-based TSMC controller designed in this work improves system convergence speed and control performance. And the figures also verifies the effectiveness of the proposed method in this work for the problem of liquid slosh and flexible vibration.

Summarizing all the cases above, the controller (FDOTSC) proposed is robust to the control saturation, besides, the flexibility in the selection of control parameters can be used to achieve better performance while satisfying the constraints of control torque and uncertainty. In addition,the control method AVSC designed for spacecraft with small rigid body cannot control the spacecraft with as well. Thus, the studying of control method for FLS under control saturation is of strong utility value. This control method provides a theoretical basis for the practical application of advanced control theory in the design of spacecraft control system.

VI. CONCLUSION

In this paper, the integration of TSMC and FDO for the LFS-FS under control saturation, uncertainties and disturbances is proposed. The mathematical expressions for the LFS-FS under control saturation constrain is firstly introduced and the FDO is designed to estimate the lumped uncertainties caused not only by the external disturbance but also by the fuel slash, flexible structure, inertia uncertainties and control saturation. With the estimated value by the FDO, the TSMC is presented, which makes up for the shortcomings of traditional sliding mode control and ensures that the system converges in a finite time. The stability of the closed-loop system is proved by Lyapunov theory. Simulations of the two cases are carried out to testify the performance of the proposed control scheme. However, the directly inhibition of

the flexible vibration and the fluid shaking is not considered in this paper, which is one of subjects for the improvement of control methods for LFS-FS.

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MIAOMIAO DU received the B.S. degree in Electrical and Information Engineering from Zhengzhou University, in 2017. She is currently pursuing the master's degree with the School of Electrical and Electronic Engineering, Tianjin University. Her main research interests include nonlinear control and flight control.
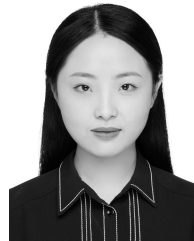
XIUYUN ZHANG received the B.S. degree in electrical engineering and automation from Qingdao University, in 2014. She is currently pursuing the Ph.D. degree with the School of Electrical and Information Engineering, Tianjin University. Her main research interests include fault diagnosis and fault tolerant control for spacecraft systems.

HUI DU received the M.S. degree from the School of Electrical and Information Engineering, Tianjin University, in 2019. Her main research interests include fault tolerant control and flight control.

LIQIAN DOU received the B.S., M.S., and Ph.D. degrees in automatic control from Tianjin University, Tianjin, China, in 1999, 2005, and 2008, respectively. He was an Academic Visitor with the School of Electrical and Electronic Engineering, University of Manchester, Manchester, U.K., from June 2015 to June 2016. He is currently an Associate Professor with the School of Electrical and Information Engineering, Tianjin University. His main research interest includes coordinate control of multi-UAVs.

WENJING LIU received the Ph.D. degree in automatic control from Tianjin University, Tianjin, China, in 2009. She is currently a Senior Engineer with the Beijing Institute of Control Engineering. Her main research interests include fault diagnosis and fault-tolerant control for satellite control systems.

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