Bi-Objective Supply Chain Network Design Model

1. Problem Description

This model addresses a **bi-objective supply chain network design problem** aiming to simultaneously:

1. **Minimize total logistics cost**
2. **Minimize total delivery time**

The supply chain consists of **suppliers**, **distribution centers (warehouses)**, and **customers**.

2. Sets

- $i \in I$: set of suppliers
- $j \in J$: set of distribution centers
- $k \in K$: set of customers

3. Parameters

- c_{ij}: unit transportation cost from supplier i to distribution center j
- c_{jk}: unit transportation cost from distribution center j to customer k
- f_j: fixed opening cost of distribution center j
- t_{ij}: transportation time from supplier i to distribution center j
- t_{jk}: transportation time from distribution center j to customer k
- s_i: supply capacity of supplier i
- d_k: demand of customer k
- M: a sufficiently large positive number (Big-M)

4. Decision Variables

- x_{ij}: quantity shipped from supplier i to distribution center j
- y_{jk}: quantity shipped from distribution center j to customer k
- $z_j \in \{0, 1\}$: binary variable indicating whether distribution center j is opened

$$z_j = \begin{cases} 1 & \text{if distribution center } j \text{ is opened} \\ 0 & \text{otherwise} \end{cases}$$

5. Objective Functions

Objective 1: Minimize Total Cost

$$\min Z_1 = \sum_{i \in I} \sum_{j \in J} c_{ij} x_{ij} + \sum_{j \in J} \sum_{k \in K} c_{jk} y_{jk} + \sum_{j \in J} f_j z_j$$

Objective 2: Minimize Total Delivery Time

$$\min Z_2 = \sum_{i \in I} \sum_{j \in J} t_{ij} x_{ij} + \sum_{j \in J} \sum_{k \in K} t_{jk} y_{jk}$$

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6. Constraints

6.1 Supplier Capacity Constraints

$$\sum_{j \in J} x_{ij} \leq s_i \quad \forall i \in I$$

6.2 Customer Demand Satisfaction

$$\sum_{j \in J} y_{jk} = d_k \quad \forall k \in K$$

6.3 Flow Balance at Distribution Centers

$$\sum_{i \in I} x_{ij} = \sum_{k \in K} y_{jk} \quad \forall j \in J$$

6.4 Distribution Center Activation Constraint

$$\sum_{i \in I} x_{ij} \leq M z_j \quad \forall j \in J$$

6.5 Variable Domains

$$x_{ij}, y_{jk} \geq 0 \quad \forall i \in I, j \in J, k \in K$$

$$z_j \in \{0, 1\} \quad \forall j \in J$$

I = {1, 2, ..., 10} Suppliers

J = {1, 2, ..., 6} Warehouses

K = {1, 2, ..., 20} Customers

Supplier (i)	Capacity (s_i)
1	400
2	450
3	500
4	450
5	480
6	520
7	610
8	470
9	530
10	460

Customer (k)	Demand (d_k)
1	120
2	90
3	110
4	80
5	100
6	130
7	95
8	105
9	115
10	85
11	125
12	90
13	110
14	100
15	95
16	120
17	85
18	105
19	115
20	90

Warehouse (j)	Fixed Cost (f_j)
1	18,000
2	20,000
3	17,500
4	19,000
5	16,500
6	21,000

c_{ij}

(i \ j)	1	2	3	4	5	6
1	6	8	7	9	6	10
2	7	6	8	7	9	8
3	5	7	6	8	7	9
4	8	9	7	6	8	7
5	6	8	9	7	6	8
6	7	6	8	9	7	6
7	5	7	6	8	9	7
8	9	8	7	6	8	7
9	6	7	8	9	6	8
10	7	6	9	8	7	6

c_{jk}

(j \ k)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	6	8	7	5	9	10	6	8	7	6	9	8	7	6	8	9	5	7	8	6
2	7	6	9	8	6	7	8	6	9	7	6	8	9	7	6	8	7	9	6	8
3	5	7	6	9	7	8	6	7	8	6	7	6	8	9	7	6	8	7	6	9
4	8	9	7	6	8	9	7	6	8	7	9	7	6	8	9	7	6	8	7	6
5	6	8	9	7	5	6	8	9	7	6	8	9	7	6	8	7	9	6	8	7
6	7	6	8	9	6	7	9	8	6	7	6	8	9	7	6	8	7	6	9	8

t_{ij}

(i \ j)	1	2	3	4	5	6
1	5	7	6	8	5	9
2	6	5	7	6	8	7
3	4	6	5	7	6	8
4	7	8	6	5	7	6
5	5	7	8	6	5	7
6	6	5	7	8	6	5
7	4	6	5	7	8	6
8	8	7	6	5	7	6
9	5	6	7	8	5	7
10	6	5	8	7	6	5

t_{jk}

(j \ k)	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
1	2	3	2	1	3	4	2	3	2	2	3	3	2	2	3	4	1	2	3	2
2	3	2	4	3	2	3	3	2	4	3	2	3	4	3	2	3	3	4	2	3
3	1	2	2	3	2	3	2	2	3	2	2	2	3	3	2	2	3	2	2	3
4	3	4	3	2	3	4	3	2	3	3	4	3	2	3	4	3	2	3	3	2
5	2	3	4	3	1	2	3	4	3	2	3	4	3	2	3	3	4	2	3	3
6	3	2	3	4	2	3	4	3	2	3	2	3	4	3	2	3	3	2	4	3

۱. کد گمز مدل فوق با روش محدودیت اپسیلون.

2. مرز پارتوی بدست آمده بصورت نموداری.

3. خلاصه جواب ها شامل مقدار توابع هدف و انبارهای فعال شده برای جواب های پارتو.