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Observer-based fractional-order adaptive type-2 fuzzy backstepping control of uncertain nonlinear MIMO systems with unknown dead-zone

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Abstract A new problem of observer-based fractional adaptive type-2 fuzzy backstepping control for a class of fractional-order MIMO nonlinear dynamic systems with dead-zone input nonlinearity is considered in the presence of model uncertainties and external disturbances where the control scheme is constructed by combining the backstepping dynamic surface control (DSC) and fractional adaptive type-2 fuzzy technique. First, a linear state observer estimates immeasurable states. Second, the unknown nonlinear functions of the uncertain system are approximated with interval type-2 fuzzy logic systems. Third, to avoid the complication of backstepping design process, the DSC is used. Fourth, by using the fractional adaptive backstepping, fractional adaptive laws are constructed, the proposed method is applied to a class of uncertain fractional-order nonlinear MIMO system. In order to have a better control performance in reducing tracking error, the controller parameters are tuned by using the PSO algorithm. Stability of the system is proven by the Mittag-Leffler method. It is presented that the proposed design guarantees the boundedness property for the system and also the tracking error can converge to a small neighborhood of the zero. The simulation examples are given to show the efficiency of the proposed controller.

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1 Introduction

For many years, fractional calculus was used as a theoretical subject, but in the last two decades, with the development of complex engineering applications, fractional calculus has been an effective branch of research in control and engineering due to its unusual properties [1–5]. Fractional calculus provides a powerful tool for describing inherent features and system memory. Fractional-order nonlinear systems constitute a new class of nonlinear systems. Two factors led to the development of fractional-order controllers:

1. Many of the plants and processes can be described accurately by fractional calculus.
2. Freedom of design and robustness are the factors of the superiority of fractional-order controllers compared to integer-order controllers [6].

Oustaloup firstly proposed a robust fractional-order controller (CRONE) in 1988 [7]. It was the starting point for entering fractional calculus in control. Since then, many types of researches with the viewpoint of

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control such as fractional-order adaptive control [8–16] and synchronization of nonlinear fractional-order systems [17–19] are investigated.

In most practical applications, the controlled plants are nonlinear and uncertain. Adaptive control is used to control uncertain systems. Most of the adaptive approaches have such a limitation that the nonlinear systems must be of the feedback linearization construction, which means that the unknown nonlinear functions need to satisfy the matching conditions. The problem of satisfying matching condition was completely eliminated by developing the backstepping-based adaptive controllers for a class of uncertain nonlinear systems [20–26], where the precise tracking performance and global stability for the resulting nonlinear systems were attained.

However, adaptive backstepping methods have limitations that they cannot be used when the system nonlinearities are unknown. Fuzzy logic systems (FLS) due to the global approximation property are utilized when the knowledge of the system dynamics does not exist. By applying the FLS, various adaptive backstepping strategies have been used to control nonlinear systems that are uncertain [27–31]. In [30], a new observer-based adaptive fuzzy control method for a class of nonstrict-feedback nonlinear time-delay systems has been developed. FLS have been applied to approximate the unknown nonlinear functions. Adaptive method and backstepping have been used to construct a controller. In [31], an adaptive fuzzy backstepping control method for a class of fractional-order nonlinear systems with triangle structures has been presented. In each step, a fuzzy system has been applied to approximate an unknown nonlinear function. Finally, a robust adaptive fuzzy controller has been constructed based on a fractional Lyapunov stability criterion in order to guarantee the convergence of the tracking error.

Although the adaptive fuzzy backstepping technique can be applied to a wide class of uncertain nonlinear systems, they suffer from the explosion of complexity problem. Therefore, the DSC technique was introduced to handle such a problem by using a first-order low-pass filter at each step of the backstepping method [32]. In [33], a fuzzy adaptive DSC backstepping control is designed for a type of uncertain SISO nonlinear system with input delay to prevail the difficulty of explosive growth of complexity intrinsic in backstepping method. Also, in [34], a DSC is used in adaptive neural network to control a type of uncertain SISO nonlinear system with unknown dead-zone. In the paper, semiglobally uniformly bounded stability for all closed-loop signals was achieved.

It should be mentioned that all the applications that are presented in [27–31] use the ordinary type-1 fuzzy logic system (T1FLS). Fuzzy rules in fuzzy systems are based on linguistic terms. The linguistic knowledge has a different meaning for different experts, so the fuzzy rules have some uncertainties. Other sources of uncertainties are the noisy training data and noisy measurements. The T1FLSs cannot handle such uncertainties due to the membership functions of a T1FLS are crisp; to overcome this problem, the type-2 FLS (T2FLS) as an extension of conventional T1FLSs is considered. The membership functions of T2FLSs are type-2 fuzzy sets that can deal with rule uncertainties. Thus, compared to T1FLS, T2FLS have the following advantages: (i) T2FLS can better deal with the uncertainty in linguistic knowledge as well as the inherent uncertainties in the system [35–37]. (ii) T2FLS are more suitable when the circumstances are too hard to specify the precise membership function for a fuzzy set [38].

In recent years, some type-2 fuzzy adaptive control techniques have been extended for a type of nonlinear uncertain system. Adaptive type-2 fuzzy backstepping control is proposed for SISO and MIMO nonlinear systems in [35] and [38], respectively. A sliding mode control based on type-2 fuzzy-neural was proposed for a type of nonlinear uncertain SISO system in [35], where IT2FNN is used to approximate system uncertainties. In [35], a new direct adaptive IT2FLS is developed to handle the rule uncertainties or noisy training data for nonlinear multivariable systems with external disturbances. The adaptive IT2FLS are used in [38] to estimate the unknown nonlinear functions, and also to handle the uncertainties in membership functions. Notice that all the above-mentioned adaptive type-2 fuzzy controllers are of integer order, but up to now according to our best knowledge, there is not a work on fractional type-2 fuzzy adaptive backstepping to control fractional nonlinear systems.

The drawback of the results in [33–37] is that all the systems states must be measured directly. It should be noted that in control methods based on state feedback, it is assumed that the all the states are present. Such assumption limits the use of control methods in practical engineering systems such as thermal processes, aerospace and power industry because practical systems states are often immeasurable or difficult to measure [33]. By designing state observers, two output feedback fuzzy adaptive control techniques were proposed in [31,38] for a class of nonlinear SISO system with unknown control directions.

The above-mentioned works in [33,34] were developed for uncertain nonlinear SISO systems. In practice, many electrical, mechanical and chemical systems, for example, telecommunications systems, ships engine room automation systems are MIMO nonlinear systems [39–44]. Thus, it is important to develop adaptive fuzzy backstepping approaches for uncertain nonlinear MIMO systems. In comparison with a large number of researches on SISO nonlinear systems, the less researches are carried out for MIMO nonlinear systems, due to existing uncertainties in the coupling matrices and unknown nonlinear functions in the MIMO where they are very challenging issues.

On the other hand, notice that all nonlinear systems in aforementioned investigations have smooth inputs. Because of actuator limitations in practical applications, the control input is usually affected by some nonlinearity such as saturation, backlash, hysteresis, and dead-zone [38,45–47]. The most significant non-smooth input nonlinearity is dead-zone, and it usually restricts system efficiency and even causes instability. Thus, in order to handle systems with the dead-zone nonlinearity, many adaptive fuzzy methods have been designed in recent years that consider the dead-zone nonlinearity for integer-order systems [26,34,45,48].

Fractional-order systems have high sensitivity. Therefore, existing input nonlinearities such as dead-zone may lead to undesirable movements. Several researchers have addressed this subject for the fractional-order SISO systems [49]. In [46], switching adaptive control technique is investigated for a type of fractional-order uncertain chaotic SISO nonlinear system, which contains dead-zone in control input. It should be emphasized that up to now according to the best of our knowledge, most of the previous works in the literature, which have been proposed to control fractional nonlinear systems, have not considered the uncertain MIMO systems with unknown dead-zone and with immeasurable states, which are important and more practical; therefore, it motivates us for this study.

Motivated by the above observation, the proposed controller design process in current manuscript investigates the fractional approximation-based adaptive backstepping DSC design for fractional-order uncertain nonlinear MIMO systems with unknown dead-zone and with immeasurable states.

In designing process, first, the immeasurable states are estimated by a linear observer. Then, by using the IT2FLS, the uncertain terms are approximated and by combining fractional adaptive controller with backstepping and DSC method, the fractional output feedback adaptive type-2 fuzzy controller is designed recursively, and a new observer-based fractional adaptive type-2 fuzzy control scheme is developed.

The stability of the controller is guaranteed by using the direct fractional-order Lyapunov (Mittag-Leffler). Furthermore, it guarantees that all closed-loop signals are bounded, and also the tracking errors can converge to origin.

Compared with the previous literature [6,9–12,14, 31,35,50,51], the major contributions of the literature are summarized as follows:

1. In different with [9,10,14,31], the proposed control method does not need that all the states of the controlled system are existent for measurement. In this article, a linear state observer is applied for estimating the full unknown states. Our previous control method in [50] assumed the fractional adaptive type-2 fuzzy control problem based on observer for fractional-order systems with immeasurable states. However, it is only concentrated on the SISO uncertain nonlinear systems, not on the MIMO nonlinear systems.
2. In different with [6,51], a DSC technique is incorporated with backstepping in order to avoid the explosive growth of complexity which exists in the backstepping method. So, this proposed method will be more simpler than the traditional backstepping design method.
3. Unlike the closely related work [31], that uses type-1 FLS as an approximation, in this research an IT2FLS is utilized as a universal approximation to model the uncertain terms; also for updating the type-2 fuzzy parameters online, the fractional-order adaptive laws are given. Compared to T1FLC, IT2FLC are more powerful in handling the uncertainties [35].
4. Unlike the previous works [11,12], the uncertain nonlinear fractional-order systems with dead-zone nonlinearity are considered in this research.

5. In this research, in order to attain the desired control performance, the parameters of the proposed controller are tuned by using the PSO algorithm.
6. Unlike previous schemes and especially [6], in the proposed method, for any subsystem that is of the order of n and $n \geq 2$, only two adaptive parameters are needed to be updated.

The rest of this article is formed as follows. In Sect. 2, some preliminaries in the basis of fractional calculus and fuzzy logic are introduced. In Sect. 3, the output feedback and type-2 fuzzy adaptive control method are systematically designed. In Sect. 4, some examples are given to show the validity of the proposed controller. Finally, the conclusion of this study is drawn in Sect. 5.

2 Preliminaries

2.1 Fractional calculus

Fractional calculus is the generalized definition of the integral and derivative of integer orders to real orders. Some definitions for fractional derivative $D^\beta (\beta > 0)$ are commonly used: GrunwaldLetnikov, RiemannLiouville, and Caputo definition. In most of the engineering applications, the definition of Caputo is used due to integer-order derivatives of $f(t)$ (i.e., $f(t)$; $f^{(1)}$; $f^{(2)}$, $\ldots$, $f^{(n)}$) are utilized as the initial conditions in its definition.

Definition 1 [1] The Caputo fractional-order derivative of a smooth function $f(t)$ with fractional-order is determined as

$$\begin{cases} D^\beta f(t) = \frac{1}{\Gamma(m-\beta)} \int_0^t \frac{f^m(\tau)}{(t-\tau)^{\beta-m+1}} & (m-1 \leq \beta < m) \\ f^m(t) & \beta = m \end{cases} \tag{1}$$

where $m \in N$, and $\Gamma(\beta) = \int_0^\infty x^{\beta-1} e^{-x}$ is the Gamma function.

Property 1 [2] It follows from (1) that for arbitrary $0 < \beta_1, \beta_2 < 1$ and zero initial conditions, the additive law of exponents for Caputo fractional-order derivative maintains as

$$D^{\beta_1}\big(D^{\beta_2} f(t)\big) = D^{\beta_2}\big(D^{\beta_1} f(t)\big) = D^{\beta_1+\beta_2} f(t) \tag{2}$$

Definition 2 [2] The Mittag-Leffler function with two parameters is determined as:

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\alpha k + \beta)}, \ \alpha, \beta > 0 \tag{3}$$

The Laplace transform of (3) is as follows

$$\mathcal{L}\Big\{t^{\beta-1} E_{\alpha,\beta}\big(-at^\alpha\big)\Big\} = \frac{s^{\alpha-\beta}}{s^\alpha + a}, \ \big(Re(s) > |a|^{1/\alpha}\big) \tag{4}$$

Now, we give four useful lemmas that will be used for proving the stability Theorem 1.

Lemma 1 [1] *Let β be a complex number. If $0 < \alpha < 2$ and $\frac{\pi\alpha}{2} < t < \min\{\pi, \pi\alpha\}$, then the following term establishes for an arbitrary integer $p \geq 0$*

$$E_{\alpha,\beta}(z) = -\sum_{k=1}^{p} \frac{z^{-k}}{\Gamma(\beta - \alpha k)} + o\Big(|z|^{-1-p}\Big), \tag{5}$$

where $|z| \to \infty$ and $t \leq |arg(z)| \leq \pi$.

Lemma 2 [1] *Let $0 < \alpha < 2$ and $\beta \in \Re$. If $\frac{\pi\alpha}{2} < t < \min\{\pi, \pi\alpha\}$, and $C > 0$ is a real constant, then*

$$|E_{\alpha,\beta}(z)| \leq \frac{C}{1+|z|}, \ t \leq |\arg(z)| \leq \pi \text{ and } |z| \geq 0 \tag{6}$$

Lemma 3 [3] *Assume that $x(t) = 0$ is the following system equilibrium point*

$$D_t^\beta x(t) f(t, x(t)) \tag{7}$$

If a Lyapunov function $V\big(t, x(t)\big)$ and class-K functions g_i, $i = 1, 2, 3$, exist such that

$$g_1\big(||x(t)||\big) \leq V\big(t, x(t)\big) \leq g_2\big(||x(t)||\big) \tag{8}$$

$$D_t^\beta V\big(t, x(t)\big) \leq -g_3\big(||x(t)||\big) \tag{9}$$

next asymptotically stability of the equilibrium point of the system (7) is guaranteed.

Lemma 4 [3] *suppose that $x(t) \in \Re^n$ is a smooth function. Next, for any $t > 0$*

$$\frac{1}{2} D_t^\beta x^{\mathrm{T}}(t) \leq x^{\mathrm{T}}(t) D_t^\beta x(t) \tag{10}$$

2.2 System explanation

Consider a class of uncertain MIMO nonlinear commensurate fractional-order system with external disturbances and dead-zone as follows,

Plant:

$$\begin{aligned}
D^{\beta}x_{i,1} &= x_{i,2}+f_{i,1}(x_{i,1})+\omega_{i,1}(t,x_i),\\
D^{\beta}x_{i,2} &= x_{i,3}+f_{i,2}(x_{i,1},x_{i,2})+\omega_{i,2}(t,x_i),\\
&\vdots\\
D^{\beta}x_{i,m_i-1} &= x_{i,m_i}+f_{i,m_i-1}(x_{i,1},\ldots,x_{i,m_i-1})\\
&\quad+\omega_{i,m_{i-1}}(t,x_i),\\
D^{\beta}x_{i,m_i} &= u_i+f_{i,m_i}(x,u_1,\ldots,u_{i-1})\\
&\quad+\omega_{i,m_i}(t,x_i),\\
y_i &= x_{i,1}
\end{aligned} \tag{11}$$

Dead-zone:

$$u_i = D_i(\upsilon_i) = \begin{cases} g_{i,r}(\upsilon_i-b_{i,r}) & \upsilon_i \ge b_{i,r}\\ 0 & -b_{i,l}<\upsilon_i<b_{i,r}\\ g_{i,l}(\upsilon_i+b_{i,l}) & \upsilon_i\le -b_{i,l}\end{cases} \tag{12}$$

where $x_i = [x_{i,1},x_{i,2},\ldots,x_{i,j_i}]^{\mathrm{T}} \in \Re^{j_i}$ $\big(i = 1,2,\ldots,n;\ j_i = 1,2,\ldots,m_i\big)$ denotes the state variables of the system, and $x=[x_1^{\mathrm{T}},x_2^{\mathrm{T}},\ldots,x_n^{\mathrm{T}}]^{\mathrm{T}}$. $u_i(.)$ is ith dead-zone output, $f_{i,j_i(.)}$ are unknown smooth nonlinear functions. $\omega_{i,j_i}(.)$ are the unknown bounded disturbances of the system; $\upsilon_i(t)\in\Re$ is ith dead-zone input, $b_{i,r}>0$ and $b_{i,l}<0$ are dead-zone breakpoints, which are unknown bounded parameters and $g_{i,r}(.)$ and $g_{i,l}(.)$ are right and left slopes; and also $\beta\in\big(0,1\big)$ is the fractional-order of the system.

Let us make the following assumptions for the control system design.

Assumption 1 [52] There exist smooth positive unknown functions ϖ_{i,j_i}, such that $\big|\omega_{i,j_i}(x,t)\big| \le \varpi_{i,j_i}$.

Assumption 2 [26] The strictly positive constants $g_{i,r}, g_{i,l}, b_{i,r}$ and $b_{i,l}$ are unknown.

So dead-zone (12) will be described as a time-varying function as follows,

$$u_i = D_i(\upsilon_i) = g_i(t)\upsilon_i(t)+d_i(t) \tag{13}$$

where

$$g_i(t) = \begin{cases} g_{i,l} & \upsilon_i\le 0\\ g_{i,r} & \upsilon_i>0\end{cases} \tag{14}$$

and

$$d_i(t) = \begin{cases} -g_{i,r}b_{i,r} & \upsilon_i\ge b_{i,r}\\ -g_i(t)\upsilon_i(t) & -b_{i,l}<\upsilon_i<b_{i,r}\\ g_{i,l}b_{i,l} & \upsilon_i\le -b_{i,l}\end{cases} \tag{15}$$

Also, from (15) there exists an unknown constant p_i^* so that

$$|d_i(t)|\le p_i^*,\quad p_i^* = \max\{g_{i,l}b_{i,l},\, g_{i,r}b_{i,r}\} \tag{16}$$

Designing a controller so that the system outputs y_i can follow the desired signals $y_{i,d}$ is the control purpose.

Assumption 3 It is supposed that the reference signals $y_{i,d}$ and the time derivatives of it up to the nth order $D^n y_{i,d}$ are available, continuous and bounded.

2.3 Interval type-2 fuzzy logic systems

An IT2FLS consists of five parts: a fuzzifier, a fuzzy rule base, an inference engine, a type reducer and a defuzzifier. The configuration of type-2 fuzzy logic system is similar to type-1 fuzzy logic system. The main difference is a type-reduction step which is added before defuzzification in interval type-2 FLS [35].

A summary of the method is presented below.

Fuzzification Each member of a crisp input vector $x = \big(x_1,\ldots,x_n\big)$ is mapped into a set of IT2FLS $\tilde{A}_x$ by the fuzzifier. Singleton fuzzifier is the most extensively used fuzzifier, whose input fuzzy set $\tilde{A}_x$ has just a single point of nonzero membership as follows,

$$\mu_{\tilde{A}_x}(x_i) = \begin{cases} 1 & x_i = x_i'\\ 0 & x_i\ne x_i'\end{cases} \tag{17}$$

where $x_i'\in U\subset R^n$.

Rule base Assume the rule base of an IT2FLS contains N rules, in which the nth rule can be explained as

R^n : IF x_1 is $\tilde{X}_1^n$ and … and x_I is $\tilde{X}_I^n$ THEN y is $\tilde{Y}^n$,

where $1\le n\le N$, and N is the total rules number. $x_i(i = 1,2,\ldots,I)$ and y are the inputs and output variables of the IT2FLS, respectively. $\tilde{X}_i^n$ and $\tilde{Y}^n = [\underline{y}^n,\overline{y}^n]$ are antecedent and consequent type-2 sets, respectively. Here $\underline{y}^n$ and $\overline{y}^n$ are crisp consequences.

The output steps of the IT2FLS for a crisp input $x' = \left(x_1', x_2', \ldots, x_I'\right)$ are as follows,

Fuzzy inference engine A mapping from the IT2FLS input to the IT2FLS output is provided by the fuzzy inference engine, which combines fuzzy IF-THEN rules.

The IT2FLS inputs can be defined in terms of membership interval of every x_i' on each $\tilde{X}_i^n$, i.e., $[\underline{\mu}_{\tilde{X}_i^n}, \overline{\mu}_{\tilde{X}_i^n}]$ $i = 1, 2, \ldots, I, n = 1, 2, \ldots, N$.

The total firing interval of the nth rule is

$$F^n(x') = [\underline{f}^n(x'), \overline{f}^n(x')] = [\underline{f}^n, \overline{f}^n], \quad n = 1, \ldots, N \tag{18}$$

where

$$\underline{f}^n = \underline{\mu}_{\tilde{X}_1^n}(x_1') * \cdots * \underline{\mu}_{\tilde{X}_I^n}(x_I') \tag{19}$$

$$\overline{f}^n = \overline{\mu}_{\tilde{X}_1^n}(x_1') * \cdots * \overline{\mu}_{\tilde{X}_I^n}(x_I') \tag{20}$$

the production operation or minimum t-norm is denoted by $*$.

Type-reduction The type reducer provides an interval type-1 fuzzy set $[y_l, y_r]$, and then this fuzzy set is defuzzified to obtain a crisp output. The center of sets (cos) type-reduction is used, $Y_{\cos}(x')$ is used in this research, which is explained as

$$Y_{\cos} = [y_l, y_r] = \bigcup_{\substack{f^n \in F^n(x') \\ y^n \in \tilde{Y}^n}} \frac{\sum_{n=1}^{N} y^n f^n}{\sum_{n=1}^{N} y^n f^n} \tag{21}$$

in which $Y_{\cos}$ is the interval set defined by left and right centroid endpoints y_l and y_r that are expressed as

$$y_l = \frac{\sum_{n=1}^{L} \underline{y}^n \overline{f}^n + \sum_{n=L+1}^{N} \underline{y}^n \underline{f}^n}{\sum_{n=1}^{L} \overline{f}^n + \sum_{n=L+1}^{N} \underline{f}^n} \tag{22}$$

$$y_r = \frac{\sum_{n=1}^{R} \overline{y}^n \underline{f}^n + \sum_{n=R+1}^{N} \overline{y}^n \overline{f}^n}{\sum_{n=1}^{R} \underline{f}^n + \sum_{n=R+1}^{N} \overline{f}^n} \tag{23}$$

Here L and R are calculated by the Karnik–Mendel (KM) algorithm [53].

Defuzzification The defuzzified crisp output is given as

$$y = \frac{y_l + y_r}{2} = \frac{W_l^{\mathrm{T}} S_l + W_r^{\mathrm{T}} S_r}{2} = \frac{1}{2}\left[W_l^{\mathrm{T}}\ W_r^{\mathrm{T}}\right]\begin{bmatrix} S_l \\ S_r \end{bmatrix} = W^{\mathrm{T}} S \tag{24}$$

where $W^{\mathrm{T}} = \left[W_l^{\mathrm{T}}\ W_r^{\mathrm{T}}\right]$, $\ S^{\mathrm{T}} = \frac{1}{2}\left[S_l\ S_r\right]$.

Lemma 5 [36] *$f(x)$ is a continuous function which is defined on a compact set Ω. Then, there exists a FLS (24) for any constant $\varepsilon > 0$, such that*

$$\sup_{x \in \Omega} \left| f(x) - W^{\mathrm{T}} S(x) \right| \le \varepsilon.$$

Remark 1 In approximation-based adaptive controller designing, two adaptive techniques are used to estimate the optimal weight vectors W_i, for $i = 1, 2, \ldots, n$, which are unknown parameters. The first technique approximates each element of W_i [26], and the second one approximates its norm [30]. The second method is considered in this manuscript.
The parameters number which must be approximated is augmented significantly when the fuzzy rules are increased. So, the online learning time will increase considerably. To solve this problem, in type-2 FLS, norm of the weighting vector is approximated instead of the weighting vector elements. Therefore, the number of adaptive laws will decrease significantly.

Remark 2 It is known that direct Lyapunov method is a basic gadget for analyzing the stability of nonlinear systems. For analyzing the fuzzy adaptive control stability of integer-order systems, square-type Lyapunov functions are usually used. According to references [54,55], the use of square Lyapunov functions for analyzing the stability of fractional-order nonlinear systems is very difficult because in fractional-order square quadratic Lyapunov function, unrestricted series are generated. Therefore, in related papers, stability analyze was performed by means of the integer-order Lyapunov stability methods. In recent years, the fractional-order Lyapunov stability has been introduced and extended because of the fact that in stable fractional-order nonlinear systems, the generalized energy does not diminish exponentially [56]. In the current paper, we solve the stability problem of fractional-order nonlinear systems by using the Mittag-Leffler stability theorem and the lemmas associated with it.

3 Observer design and adaptive fuzzy control

In the adaptive interval type-2 fuzzy design, when system states are not measurable, output feedback, which will be mentioned in Sect. 3.1, can be used. In this section, it is assumed that the states of the system (11) are not available, so an observer should be determined for estimating the states, and then output feedback type-2 fuzzy fractional adaptive control structure via backstepping is considered.

3.1 Observer design

None of the states x_{i,j_i} existed for feedback control scheme in the system (11); so, a full-order observer is needed to estimate $x_{i,1},\ldots,x_{i,m_i}$ and generate some signals for controller design. The linear observer [30] is considered for the system (11),

$$\begin{aligned}D^{\beta}\hat{x}_{i,j_i} &= \hat{x}_{i,j_i+1} - k_{i,j_i}\hat{x}_{i,1},\ \ 1\le j_i\le m_i-1\\ D^{\beta}\hat{x}_{i,m_i} &= u_i - k_{i,m_i}\hat{x}_{i,1}\end{aligned} \tag{25}$$

where $\hat{x}_{i,j_i}$ is the estimation of x_{i,j_i}. The design parameter k_{i,j_i} is selected so that the matrix

$$\mathbf{A} = \begin{bmatrix} -k_{i,1} & I \\ \vdots & \\ -k_{i,m_i} & \cdots\ 0\end{bmatrix}$$

is strict Hurwitz matrix. Thus, there exists a P_i for, a given Q_i, satisfying

$$A_i^{\mathrm{T}}P_i + P_iA_i = -2Q_i \tag{26}$$

Define the state error as $e_i = x_{i,j_i} - \hat{x}_{i,j_i},\ \ for 1 \le j_i \le m_i$. Then one obtains

$$\begin{aligned}D^{\beta}e_i &= e_{i+1} - k_{i,j_i}e_1 + f_{i,j_i}(\bar{x}_{i,j_i})\\ &\quad + \omega_{i,j_i}(t,x) + k_{i,j_i}y_i,\ \ for\ 1\le j_i\le m_i-1\\ D^{\beta}e_n &= -k_{i,m_i}e_1 + f_{i,m_i}(\bar{x}_{i,m_i})\\ &\quad + \omega_{i,m_i}(t,x) + k_{i,m_i}y_i\end{aligned}$$

and the error dynamics are as follows,

$$D^{\beta}e_i = A_ie_i + F_i(\bar{x}_i) + \Delta_i(t,x) \tag{27}$$

with

$$\begin{aligned}e_i &= (e_1,e_2,\ldots,e_{m_i})^{\mathrm{T}}\\ \Delta_i(t,x) &= \big(\omega_{i,1}(t,x),\ldots,\omega_{i,m_i}(t,x)\big)\\ F_i(\bar{x}) &= \big(f_{i,1}(\bar{x}_{i,1})\\ &\quad + k_{i,1}y_i,\ldots,f_{i,m_i}(\bar{x}_{i,m_i}) + k_{i,m_i}y_i\big)^{\mathrm{T}}\\ &= \big(F_{i,1}(\bar{x}),\ldots,F_{i,m_i}(\bar{x})\big)^{\mathrm{T}}\end{aligned}$$

3.2 Type-2 fuzzy adaptive control method and stabilization

In this part, the output feedback type-2 fuzzy fractional adaptive control based on backstepping and stability procedure will be developed. DSC technique which was proposed in [33] will be used into the controller for the system (11). In DSC technique, the recursive design method has m_i steps like the backstepping design method. In step j_i, a virtual control function α_{i,j_i} is suggested, and the true control law υ_i is designed at the final step. The virtual control signals and the real control functions will be designed as,

$$\begin{aligned}\alpha_{i,j_i}(Z_{i,j_i}) &= -\frac{1}{2\rho_{i,j_i}^2}\chi_{i,j_i}\hat{\theta}_i - (\lambda_{i,j_i}+0.5)\chi_{i,j_i},\\ &\quad i=1,\ldots,n;\ j_i=1,\ldots,m_i-1\end{aligned} \tag{28}$$

$$\upsilon_i = -\frac{1}{2\rho_{i,m_i}^2}\chi_{i,m_i}\hat{\theta}_{i,m_i} - (\lambda_{i,m_i}+0.5)\chi_{i,m_i} \tag{29}$$

with

$$\begin{aligned}Z_{i,1} &= \big(x_{i,1},y_{i,d},D^{\beta}y_{i,d}\big)^{\mathrm{T}},\\ Z_{i,j_i} &= \big(\bar{\hat{x}}_{i,j_i},\alpha_{i,j_if},D^{\beta}\alpha_{i,j_if}\big)^{\mathrm{T}},\\ j_i &= 2,\ldots,m_i-1\\ \bar{\hat{x}}^{\mathrm{T}}_{i,j_i} &= (\hat{x}_{i,1},\hat{x}_{i,2},\ldots,\hat{x}_{i,m_i})\end{aligned}$$

where $\lambda_{i,j_i},\rho_{i,j_i},j_i = 1,\ldots,m_i$ are positive design parameters.

The suggested control scheme is simplified by applying the DSC method, and the repeated differentiation of α_{i,j_i} is avoided. Under the coordinate transformation, one gets

$$\begin{aligned}\chi_{i,1} &= y_i - y_{i,d},\\ \chi_{i,j_i} &= \hat{x}_{i,j_i} - \alpha_{i,j_if},\ \ i=1,\ldots,n;\ 2,\ldots,m_i\end{aligned}$$

where α_{i,j_i-1} and α_{i,j_if} are the input and output of the first-order filter, respectively.

$\hat{\theta}_i$ and $\hat{\theta}_{i,m_i}$ are the estimation of θ_i and θ_i,m_i, respectively. They are unknown constants which specify as,

$$\theta_i = \max\left\{\|W_{i,j_i}\|^2 : i = 1, 2, \ldots, n;\ j_i = 1, \ldots, m_i - 1\right\}$$
$$\theta_{i,m_i} = \frac{1}{h_i}\|W_{i,m_i}\|^2$$

that h_i are positive constants. Also fractional-order adaptive laws are defined as

$$D^\beta\hat{\theta}_i = \sum_{k=1}^{m_i-1}\frac{\gamma_i}{2\rho_{i,k}^2}\chi_{i,k}^2 - \sigma_i\hat{\theta}_i$$
$$D^\beta\hat{\theta}_{i,m_i} = \frac{\gamma_{i,m_i}}{2\rho_{i,m_i}^2}\chi_{i,m_i}^2 - \sigma_{i,m_i}\hat{\theta}_{i,m_i} \tag{30}$$

where $\rho_{i,j_i}(i = 1, \ldots, n;\ j_i = 1, \ldots, m_i)$, γ_i, γ_{i,m_i}, σ_i and σ_{i,m_i} are positive design parameters.

Theorem 1 *Consider the nonlinear fractional-order system (11) and construct linear observer (25) with the intermediate virtual control function α_{i,j_i} (28) for $1 \le j_i \le m_i - 1$, and the control law and adaptive laws defined as (29) and (30), respectively. If suitable design parameters are selected, then tracking error will converge to a neighborhood of zero, and all the system signals are bounded.*

Proof **Step i, 1:**
Specify the tracking error of the system (11) as

$$\chi_{i,1} = y_i - y_{i,d}$$

Then by expressing the estimate of $x_{i,2}$, one obtains

$$\begin{aligned} D^\beta\chi_{i,1} &= D^\beta y_i - D^\beta y_{i,d} \\ &= x_{i,2} + f_{i,1}(x_{i,1}) + \omega_{i,1} - D^\beta y_{i,d} \\ &= \hat{x}_{i,2} + e_{i,2} + f_{i,1}(x_{i,1}) + \omega_{i,1} - D^\beta y_{i,d} \end{aligned} \tag{31}$$

By applying the β-order filter, $\alpha_{i,2f}$ is introduced to prevent repeated differentiation of $\alpha_{i,1}$.

$$\tau_{i,2}D^\beta\alpha_{i,2f} + \alpha_{i,2f} = \alpha_{i,1},\ \alpha_{i,2f}(0) = \alpha_{i,1}(0).$$

which $\tau_{i,2}$ is defined as a time constant and $\alpha_{i,2f}(0) = \alpha_{i,1}(0)$.

Define the filter output error as $\psi_{i,2} = \alpha_{i,2f} - \alpha_{i,1}$; then, one obtains $D^\beta\alpha_{i,2f} = -\frac{\psi_{i,2}}{\tau_{i,2}}$, and

$$D^\beta\psi_{i,2} = D^\beta\alpha_{i,2f} - D^\beta\alpha_{i,1} = -\frac{\psi_{i,2}}{\tau_{i,2}} + C_{i,2}(Z_{i,1})$$

and $\hat{x}_{i,2} = \chi_{i,2} + \alpha_{i,2f}$ will be considered.

Substituting $\alpha_{i,2f}$ into the above relation yields

$$\hat{x}_{i,2} = \chi_{i,2} + \psi_{i,2} + \alpha_{i,1} \tag{32}$$

Now assume the following Lyapunov function,

$$V_{i,1} = e_i^{\mathrm{T}}P_ie_i + \frac{1}{2}\chi_{i,1}^2 \tag{33}$$

From (26), (31) and (32), the fractional derivative of $V_{i,1}$ is obtained as

$$\begin{aligned} D^\beta V_{i,1} =& \frac{1}{2}D^\beta e_i^{\mathrm{T}}P_ie_i + \frac{1}{2}e_i^{\mathrm{T}}P_iD^\beta e_i + \chi_{i,1}D^\beta\chi_{i,1} \\ =& \frac{1}{2}e_i^{\mathrm{T}}\left[P_iA_i^{\mathrm{T}} + A_iP_i\right]e_i + e_i^{\mathrm{T}}P_i\left(F_i + \Delta_i\right) \\ &+ \chi_{i,1}\left(\hat{x}_{i,2} + e_{i,2} + f_{i,1} + \omega_{i,1} - D^\beta y_{i,d}\right) \\ =& -e_i^{\mathrm{T}}Q_ie_i + e_i^{\mathrm{T}}P_i\left(F_i + \Delta_i\right) \\ &+ \chi_{i,1}\left(\chi_{i,2} + \psi_{i,2} + \alpha_{i,1} + e_{i,2}\right. \\ &\left. + f_{i,1} + \omega_{i,1} - D^\beta y_{i,d}\right) \end{aligned} \tag{34}$$

According to Lemma 5, the unknown function $F_i(\bar{x}_i)$ via a FLS $W_{i,0}^{\mathrm{T}}S_{i,0}(\bar{x}_i)$ is approximated for any given $\varepsilon_{i,0}$, so that

$$F_i(\bar{x}_i) = W_{i,0}^TS_{i,0}(\bar{x}_i) + \xi_{i,0}(\bar{x}_i),$$
$$|\xi_{i,0}(\bar{x}_i)| \le \varepsilon_{i,0} \tag{35}$$

As $S_{i,0}^{\mathrm{T}}S_{i,0} \le 1$, and due to the definition of θ_i, $\|W_{i,0}\|^2 \le \theta_i$ holds. So, the following inequality can be obtained:

$$\begin{aligned} e_i^TP_iF_i &= e_i^{\mathrm{T}}P_i\left(W_{i,0}^{\mathrm{T}}S_0(\bar{x}_i) + \xi_{i,0}(\bar{x}_i)\right) \\ &\le \frac{1}{2}\|e_i\|^2 + \frac{1}{2}\|P_i\|^2\theta_i + \frac{1}{2}\|P_i\|^2\varepsilon_{i,0}^2 \end{aligned} \tag{36}$$

In addition, Δ_i can be written as

$$e_i^TP_i\Delta_i \le \frac{1}{2}\|e_i\|^2 + \frac{1}{2}\|P_i\|^2\|\bar{\Delta}_i\|^2 \tag{37}$$

where $\bar{\Delta}_i = (\bar{\omega}_{i,1}, \bar{\omega}_{i,2}, \ldots, \bar{\omega}_{i,m_i})$.

Using the inequality, $2ab \le a^2 + b^2$ yields

$$e_{i,2}\chi_{i,1} \le \frac{1}{2}|e_{i,2}|^2 + \frac{1}{2}\chi_{i,1}^2 \le \frac{1}{2}\|e_i\|^2 + \frac{1}{2}\chi_{i,1}^2 \tag{38}$$

Applying Young's inequality, the following inequality holds

$$\chi_{i,1}\omega_{i,1} \leq \frac{1}{2}\zeta_i^{-2}\chi_{i,1}^2 + \frac{1}{2}\zeta_i^2\bar{\omega}_{i,1}^2 \tag{39}$$

where ζ_i is a positive constant.

Replacing the inequalities (36)–(39) into (34), the fractional derivative of $V_{i,1}$ is obtained as follows,

$$\begin{aligned} D^\beta V_{i,1} \leq & -\left[\lambda_{\min}(Q_i) - 3/2\right]\|e_i\|^2 \\ & + \frac{1}{2}\|P_i\|^2\left(\theta_i + \varepsilon_{i,0}^2 + \|\bar{\Delta}_i^2\|\right) + \frac{1}{2}\zeta_i^2\bar{\omega}_{i,1}^2 \\ & - \frac{1}{2}\chi_{i,1}^2 + \chi_{i,1}(\chi_{i,2} + \psi_{i,2} + \alpha_{i,1} + \bar{f}_{i,1}) \end{aligned} \tag{40}$$

where

$$\bar{f}_{i,1}(Z_{i,1}) = f_{i,1} + \frac{\chi_{i,1}}{2}\left(1 + \zeta_i^{-2}\right) - D^\beta y_{i,d} + \frac{1}{2}\chi_{i,1}$$

However, $\bar{f}_{i,1}(Z_{i,1})$ is an uncertain nonlinear function which includes $f_{i,1}(x_{i,1})$, which cannot be performed in practice. So, according to Lemma 5, the fuzzy logic system $W_{i,1}^{\mathrm{T}} S_{i,1}(Z_{i,1})$ is applied to approximate the unknown continuous nonlinear function $\bar{f}_{i,1}(Z_{i,1})$, for any given constant $\bar{\varepsilon}_{i,1} > 0$,

$$\bar{f}_{i,1}(Z_{i,1}) = W_{i,1}^T S_{i,1}(Z_{i,1}) + \xi_{i,1}(Z_{i,1})$$

where $\xi_{i,1}(Z_{i,1})$ represents the approximation error and satisfies $|\xi_{i,1}(Z_{i,1})| \leq \bar{\varepsilon}_{i,1}$.

As $S_{i,1}^{\mathrm{T}} S_{i,1} \leq 1$, and according to the definition of θ_i, $\|W_{i,1}\|^2 \leq \theta_i$ holds. So, the following inequality will be obtained

$$\begin{aligned} \chi_{i,1}\bar{f}_{i,1} &= \chi_{i,1}\frac{W_{i,1}^{\mathrm{T}}}{\|W_{i,1}\|}S_{i,1}\|W_{i,1}\| + \chi_{i,1}\xi_{i,1} \\ &\leq \frac{1}{2\rho_{i,1}^2}\chi_{i,1}^2\theta_i + \frac{1}{2}\rho_{i,1}^2 + \frac{1}{2}\chi_{i,1}^2 + \frac{1}{2}\bar{\varepsilon}_{i,1}^2 \end{aligned} \tag{41}$$

Also from the definition of $\alpha_{i,1}$, one gets

$$\chi_{i,1}\alpha_{i,1} = -\frac{1}{2\rho_{i,1}^2}\chi_{i,1}^2\hat{\theta}_i - (\lambda_{i,1} + 0.5)\chi_{i,1}^2 \tag{42}$$

We use the inequality $S_{i,1}^{\mathrm{T}}(Z_{i,1})S_{i,1}(Z_{i,1}) \leq 1$.

Then substituting (41) and (42) into (40) gives

$$\begin{aligned} D^\beta V_{i,1} \leq & -\left[\lambda_{\min}(Q_i) - 3/2\right]\|e_i\|^2 \\ & + \chi_{i,1}(\chi_{i,2} + \psi_{i,2}) - \lambda_{i,1}\chi_{i,1}^2 \\ & + \frac{1}{2\rho_{i,1}^2}\chi_{i,1}^2\tilde{\theta}_i + \Upsilon_{i,1} \end{aligned} \tag{43}$$

with $\tilde{\theta}_i = \theta_i - \hat{\theta}_i$,where

$$\begin{aligned} \Upsilon_{i,1} = & \frac{1}{2}\|P_i\|^2\left(\theta_i + \varepsilon_{i,0}^2 + \|\bar{\Delta}_i^2\|\right) + \frac{1}{2}\zeta_i^2\bar{\omega}_{i,1}^2 \\ & + \frac{1}{2}\rho_{i,1}^2 + \frac{1}{2}\bar{\varepsilon}_{i,1}^2. \end{aligned}$$

Step $i, 2$**:**
Define the following tracking error

$$\chi_{i,2} = \hat{x}_{i,2} - \alpha_{i,2f}$$

and then differentiate it as

$$\begin{aligned} D^\beta \chi_{i,2} &= D^\beta \hat{x}_{i,2} - D^\beta \alpha_{i,2f} \\ &= \hat{x}_{i,3} - k_{i,2}\hat{x}_{i,1} - D^\beta \alpha_{i,2f} \end{aligned} \tag{44}$$

One obtains

$$D^\beta \psi_{i,2} = D^\beta \alpha_{i,2f} - D^\beta \alpha_{i,1} = -\frac{\psi_{i,2}}{\tau_{i,2}} + C_{i,2}(Z_{i,1})$$

Then, to obtain $\alpha_{i,3f}$, $\alpha_{i,2}$ is passed via a β-order filter as follows,

$$\tau_{i,3}D^\beta \alpha_{i,3f} + \alpha_{i,3f} = \alpha_{i,2},\ \ \alpha_{i,3f}(0) = \alpha_{i,2}(0).$$

which $\tau_{i,3}$ is defined as a time constant.

Next, define the filter output error as $\psi_{i,3} = \alpha_{i,3f} - \alpha_{i,2}$; then, $D^\beta \alpha_{i,3f} = -\frac{\psi_{i,3}}{\tau_{i,3}}$ will be obtained and

$$D^\beta \psi_{i,3} = D^\beta \alpha_{i,3f} - D^\beta \alpha_{i,2} = -\frac{\psi_{i,3}}{\tau_{i,3}} + C_{i,3}(Z_{i,2}) \tag{45}$$

and $\hat{x}_{i,3} = \chi_{i,3} + \alpha_{i,3f}$ will be defined.

Substituting $\alpha_{i,3f}$ in (45) yields

$$\hat{x}_{i,3} = \chi_{i,3} + \psi_{i,3} + \alpha_{i,2} \tag{46}$$

By replacing (46) in (44), one gets

$$D^\beta \chi_{i,2} = \chi_{i,3} + \psi_{i,3} + \alpha_{i,2} + k_{i,2}e_{i,1} - k_{i,2}y_i - D^\beta \alpha_{i,2f}$$

Now assume the following Lyapunov function:

$$V_{i,2} = V_{i,1} + \frac{1}{2}\chi_{i,2}^2 + \frac{1}{2}\psi_{i,2}^2 \tag{47}$$

The fractional derivative of $V_{i,2}$ is as follows,

$$\begin{aligned} D^\beta V_{i,2} =& D^\beta V_{i,1} + \chi_{i,2} D^\beta \chi_{i,2} \\ \le& -\left[\lambda_{\min}(Q_i) - 3/2\right]\|e_i\|^2 \\ &+ \chi_{i,1}\psi_{i,2} - \lambda_{i,1}\chi_{i,1}^2 + \frac{1}{2\rho_{i,1}^2}\chi_{i,1}^2\tilde{\theta}_i \\ &+ \chi_{i,2}\left(\chi_{i,3} + \psi_{i,3} + \alpha_{i,2} + \bar{f}_{i,2}\right) \\ &- \frac{\psi_{i,2}^2}{\tau_{i,2}} + \psi_{i,2}C_{i,2}(Z_{i,1}) + \Upsilon_{i,1} \end{aligned} \tag{48}$$

where

$$\bar{f}_{i,2}(Z_{i,2}) = k_{i,2}e_{i,1} - k_{i,2}y_i - D^\beta \alpha_{i,2f} + \chi_{i,1}$$

Since $\bar{f}_{i,2}(Z_{i,2})$ is an unknown continuous function, the type-2 FLS $W_{i,2}^{\mathrm{T}} S_{i,2}(Z_{i,2})$ is handled to estimate it. For any determined constant $\bar{\varepsilon}_{i,2} > 0$, there is a type-2 FLS $W_{i,2}^{\mathrm{T}} S_{i,2}(Z_{i,2})$ so that

$$\bar{f}_{i,2}(Z_{i,2}) = W_{i,2}^{\mathrm{T}} S_{i,2}(Z_{i,2}) + \xi_{i,2}(Z_{i,2}) \tag{49}$$

where approximation error is expressed by $\xi_{i,2}(Z_{i,2})$ and satisfies $|\xi_{i,2}(Z_{i,2})| \le \bar{\varepsilon}_{i,2}$.

As a similar procedure in (41) and (42), the following inequalities hold

$$\chi_{i,2}\bar{f}_{i,2} \le \frac{1}{2\rho_{i,2}^2}\chi_{i,2}^2\theta_i + \frac{1}{2}\rho_{i,2}^2 + \frac{1}{2}\chi_{i,2}^2 + \frac{1}{2}\bar{\varepsilon}_{i,2}^2 \tag{50}$$

$$\chi_{i,2}\alpha_{i,2} = -\frac{1}{2\rho_{i,2}^2}\chi_{i,2}^2\hat{\theta}_i - (\lambda_{i,2} + 0.5)\chi_{i,2}^2 \tag{51}$$

Then, replacing (50) and (51) into (48) yields

$$\begin{aligned} D^\beta V_{i,2} \le& -\left[\lambda_{\min}(Q_i) - 3/2\right]\|e_i\|^2 \\ &+ \chi_{i,1}\psi_{i,2} + \chi_{i,2}(\chi_{i,3} + \psi_{i,3}) \\ &- \sum_{j_i=1}^{2}\lambda_{i,j_i}\chi_{i,j_i}^2 + \sum_{j_i=1}^{2}\frac{1}{2\rho_{i,j_i}^2}\chi_{i,j_i}^2\tilde{\theta}_i \\ &+ \psi_{i,2}C_{i,2}(Z_{i,1}) + \Upsilon_{i,2} \end{aligned} \tag{52}$$

where

$$\begin{aligned} \Upsilon_{i,2} =& \frac{1}{2}\|P_i\|^2\left(\theta_i + \varepsilon_{i,0}^2 + \|\bar{\Delta}_i^2\|\right) + \frac{1}{2}\zeta_i^2\bar{\omega}_{i,1}^2 \\ &+ \frac{1}{2}\sum_{j_i=1}^{2}\rho_{i,j_i}^2 + \frac{1}{2}\sum_{j_i=1}^{2}\bar{\varepsilon}_{i,j_i}^2. \end{aligned}$$

Step $i, j_i \quad (2 \le j_i \le m_i - 1)$**:**
Define the following tracking error

$$\chi_{i,j_i} = \hat{x}_{i,j_i} - \alpha_{i,j_i f}$$

and then differentiate it as

$$\begin{aligned} D^\beta \chi_{i,j_i} &= D^\beta \hat{x}_{i,j_i} - D^\beta \alpha_{i,j_i f} \\ &= \hat{x}_{i,j_i+1} - k_{i,j_i}\hat{x}_{i,1} - D^\beta \alpha_{i,j_i f} \end{aligned} \tag{53}$$

One obtains

$$\begin{aligned} D^\beta \psi_{i,j_i} &= D^\beta \alpha_{i,j_i f} - D^\beta \alpha_{i,j_i-1} \\ &= -\frac{\psi_{i,j_i}}{\tau_{i,j_i}} + C_{i,j_i}(Z_{i,j_i-1}) \end{aligned}$$

By applying the β-order filter, α_{i,j_i+1f} is introduced to prevent repeated differentiation of α_{i,j_i}.

$$\begin{aligned} \tau_{i,j_i+1}D^\beta \alpha_{i,j_i+1f} + \alpha_{i,j_i+1f} &= \alpha_{i,j_i},\ \alpha_{i,j_i+1f}(0) \\ &= \alpha_{i,j_i}(0). \end{aligned}$$

which τ_{i,j_i+1} is defined as a time constant.

Define the filter output error as $\psi_{i,j_i+1} = \alpha_{i,j_i+1f} - \alpha_{i,j_i}$; then $D^\beta \alpha_{i,j_i+1f} = -\frac{\psi_{i,j_i+1}}{\tau_{i,j_i+1}}$ will be obtained and

$$\begin{aligned} D^\beta \psi_{i,j_i+1} &= D^\beta \alpha_{i,j_i+1f} - D^\beta \alpha_{i,j_i} \\ &= -\frac{\psi_{i,j_i+1}}{\tau_{i,j_i+1}} + C_{i,j_i+1}(Z_{i,j_i}) \end{aligned} \tag{54}$$

and $\hat{x}_{i,j_i+1} = \chi_{i,j_i+1} + \alpha_{i,j_i+1f}$ will be defined.

Substituting α_{i,j_i+1f} in (54) yields

$$\hat{x}_{i,j_i+1} = \chi_{i,j_i+1} + \psi_{i,j_i+1} + \alpha_{i,j_i} \tag{55}$$

By substituting (55) in (53), we have

$$\begin{aligned} D^\beta \chi_{i,j_i} =& \chi_{i,j_i+1} + \psi_{i,j_i+1} + \alpha_{i,j_i} \\ &+ k_{i,j_i}e_{i,1} - k_{i,j_i}y_i - D^\beta \alpha_{i,j_i f} \end{aligned}$$

In this step, Lyapunov function is considered as

$$V_{i,j_i} = V_{i,j_i-1} + \frac{1}{2}\chi_{i,j_i}^2 + \frac{1}{2}\psi_{i,j_i}^2 \tag{56}$$

The fractional derivative of V_{i,j_i} will be as

$$\begin{aligned}D^{\beta}V_{i,j_i} =& D^{\beta}V_{i,j_i-1} + \chi_{i,j_i}\big(\hat{x}_{i,j_i+1} + k_{i,j_i}e_{i,1} \\ &- k_{i,j_i}y_i - D^{\beta}\alpha_{i,j_if}\big) + \psi_{i,j_i}D^{\beta}\psi_{i,j_i} \\ \le& -\big[\lambda_{\min}(Q_i) - 3/2\big]\|e_i\|^2 \\ &+ \sum_{k=1}^{j_i-1}\chi_{i,k}\psi_{i,k+1} - \sum_{k=1}^{j_i-1}\lambda_{i,k}\chi_{i,k}^2 \\ &+ \sum_{k=1}^{j_i-1}\frac{1}{2\rho_{i,k}^2}\chi_{i,k}^2\tilde{\theta}_i \\ &- \sum_{k=1}^{j_i-1}\left(\frac{\psi_{i,k+1}^2}{\tau_{i,k+1}} - \psi_{i,k+1}C_{i,k+1}(Z_{i,k})\right) \\ &+ \chi_{i,j_i}\big(\chi_{i,j_i+1} + \psi_{i,j_i+1} + \alpha_{i,j_i} + \bar{f}_{i,j_i}\big) \\ &+ \Upsilon_{i,j_i-1} \end{aligned} \tag{57}$$

where

$$\bar{f}_{i,j_i}(Z_{i,j_i}) = k_{i,j_i}e_{i,1} - k_{i,j_i}y_i - D^{\beta}\alpha_{i,j_if} + \chi_{i,j_i-1}$$

Similarly, the fuzzy logic system $W_{i,j_i}^{\mathrm{T}}S_{i,j_i}(Z_(i,j_i))$ can approximate $\bar{f}_{i,j_i}(Z_{i,j_i})$ as

$$\bar{f}_{i,j_i}(Z_{i,j_i}) = W_{i,j_i}^{\mathrm{T}}S_{i,j_i}(Z_{i,j_i}) + \xi_{i,j_i}(Z_{i,j_i}) \tag{58}$$

where approximation error is defined by $\xi_{i,j_i}(Z_{i,j_i})$ and satisfies $|\xi_{i,j_i}(Z_{i,j_i})| \le \bar{\varepsilon}_{i,j_i}$; then, the following inequalities hold

$$\chi_{i,j_i}\bar{f}_{i,j_i} \le \frac{1}{2\rho_{i,j_i}^2}\chi_{i,j_i}^2\theta_i + \frac{1}{2}\rho_{i,j_i}^2 + \frac{1}{2}\chi_{i,j_i}^2 + \frac{1}{2}\bar{\varepsilon}_{i,j_i}^2 \tag{59}$$

$$\chi_{i,j_i}\alpha_{i,j_i} = -\frac{1}{2\rho_{i,j_i}^2}\chi_{i,j_i}^2\hat{\theta}_i - (\lambda_{i,j_i} + 0.5)\chi_{i,j_i}^2 \tag{60}$$

Next, replacing (59) and (60) into (57) yields

$$\begin{aligned}D^{\beta}V_{i,j_i} \le& -\big[\lambda_{\min}(Q_i) - 3/2\big]\|e_i\|^2 \\ &+ \sum_{k=1}^{j_i-1}\chi_{i,k}\psi_{i,k+1} + \chi_{i,j_i}(\chi_{i,j_i+1} + \psi_{i,j_i+1}) \\ &- \sum_{k=1}^{j_i-1}\lambda_{i,k}\chi_{i,k}^2 - \sum_{k=1}^{j_i-1}\left(\frac{\psi_{i,k+1}^2}{\tau_{i,k+1}} - \psi_{i,k+1}C_{i,k+1}(Z_{i,k})\right) \\ &+ \Upsilon_{i,j_i} + \sum_{k=1}^{j_i}\frac{1}{2\rho_{i,k}^2}\chi_{i,k}^2\tilde{\theta}_i \end{aligned} \tag{61}$$

where

$$\begin{aligned}\Upsilon_{i,j_i} =& \frac{1}{2}\|P_i\|^2\big(\theta_i + \varepsilon_{i,0}^2 + \|\bar{\Delta}_i^2\|\big) + \frac{1}{2}\zeta_i^2\bar{\omega}_{i,1}^2 \\ &+ \frac{1}{2}\sum_{k=1}^{j_i}\rho_{i,k}^2 + \frac{1}{2}\sum_{k=1}^{j_i}\bar{\varepsilon}_{i,k}^2.\end{aligned}$$

Step i, m_i**:**
Define the following tracking error

$$\chi_{i,m_i} = \hat{x}_{i,m_i} - \alpha_{i,m_if}$$

and then derive it as

$$\begin{aligned}D^{\beta}\chi_{i,m_i} &= D^{\beta}\hat{x}_{i,m_i} - D^{\beta}\alpha_{i,m_if} \\ &= u_i + k_{i,m_i}y_i - k_{i,m_i}\hat{x}_{i,1} - D^{\beta}\alpha_{i,m_if}\end{aligned} \tag{62}$$

One obtains

$$\begin{aligned}D^{\beta}\psi_{i,m_i} &= D^{\beta}\alpha_{i,m_if} - D^{\beta}\alpha_{i,m_i-1} \\ &= -\frac{\psi_{i,m_i}}{\tau_{i,m_i}} + C_{i,m_i}(Z_{i,m_i-1})\end{aligned} \tag{63}$$

In the last step, Lyapunov function is selected as:

$$\begin{aligned}V_{i,m_i} &= V_{i,m_i1} + V_{i,m_i2} \\ V_{i,m_i1} &= V_{i,m_i-1} + \frac{1}{2}\chi_{i,m_i}^2 + \frac{1}{2}\psi_{i,m_i}^2 \\ V_{i,m_i2} &= \frac{1}{2\gamma_i}\tilde{\theta}_i^2 + \frac{h_i}{2\gamma_{i,m_i}}\tilde{\theta}_{i,m_i}^2\end{aligned}$$

where h_i are positive constants.

The fractional derivative of V_{i,m_i1} is

$$\begin{aligned}D^{\beta}V_{i,m_i1} =& D^{\beta}V_{i,m_i-1} \\ &+ \chi_{i,m_i}\big(u_i(\upsilon_i) - k_{i,m_i}\hat{x}_{i,1} - D^{\beta}\alpha_{i,m_if}\big) \\ &+ \psi_{i,m_i}D^{\beta}\psi_{i,m_i} \\ \le& -\big[\lambda_{\min}(Q_i) - 3/2\big]\|e_i\|^2 \\ &+ \sum_{k=1}^{m_i-1}\chi_{i,k}\psi_{i,k+1} \\ &- \sum_{k=1}^{m_i-1}\lambda_{i,k}\chi_{i,k}^2 \\ &+ \sum_{k=1}^{m_i-1}\frac{1}{2\rho_{i,k}^2}\chi_{i,k}^2\tilde{\theta}_i \\ &- \sum_{k=1}^{m_i-1}\left(\frac{\psi_{i,k+1}^2}{\tau_{i,k+1}} - \psi_{i,k+1}C_{i,k+1}(Z_{i,k})\right) \\ &+ \chi_{i,m_i}\big(u_i(\upsilon_i) + \bar{f}_{i,m_i}\big) + \Upsilon_{i,m_i-1}\end{aligned} \tag{64}$$

where

$$\bar{f}_{i,m_i}(Z_{i,m_i}) = k_{i,m_i}e_{i,1} - k_{i,m_i}y_i - D^{\beta}\alpha_{i,m_i f} + \chi_{i,m_i-1}$$

Similarly, the fuzzy logic system $W_{i,m_i}^{\mathrm{T}} S_{i,m_i}(Z_(i, m_i))$ can approximate $\bar{f}_{i,m_i}(Z_{i,m_i})$ as

$$\bar{f}_{i,m_i}(Z_{i,m_i}) = W_{i,m_i}^{\mathrm{T}} S_{i,m_i}(Z_{i,m_i}) + \xi_{i,m_i}(Z_{i,m_i}) \quad (65)$$

where the approximation error is denoted by $\xi_{i,m_i}(Z_{i,m_i})$ and satisfies $|\xi_{i,m_i}(Z_{i,m_i})| \leq \bar{\varepsilon}_{i,m_i}$. Also the following inequalities hold

$$\begin{aligned} \chi_{i,m_i}\bar{f}_{i,m_i} &\leq \frac{h_i}{2\rho_{i,m_i}^2}\chi_{i,m_i}^2\theta_{m_i} + \frac{1}{2}\rho_{i,m_i}^2 \\ &\quad + \frac{1}{2}\chi_{i,m_i}^2 + \frac{1}{2}\bar{\varepsilon}_{i,m_i}^2 \quad (66) \\ \chi_{i,m_i}u_i(\upsilon_i) &= \chi_{i,m_i}\Big(K_i^{\mathrm{T}}(t)\Phi_i(t)\upsilon_i + d_i(\upsilon_i)\Big) \\ &\leq -\frac{h_i}{2\rho_{i,m_i}^2}\chi_{i,m_i}^2\hat{\theta}_{m_i} - h_i\big(\lambda_{i,m_i}+0.5\big)\chi_{i,m_i}^2 \\ &\quad + \chi_{i,m_i}^2 + \frac{1}{4}p_i^{*2} \quad (67) \end{aligned}$$

Then, replacing (66) and (67) into (64) yields

$$\begin{aligned} D^{\beta}V_{i,m_i} \leq &-\big[\lambda_{\min}(Q_i) - 3/2\big]\|e_i\|^2 \\ &+ \sum_{k=1}^{m_i-1}\chi_{i,k}\psi_{i,k+1} - \sum_{k=1}^{m_i-1}\lambda_{i,k}\chi_{i,k}^2 \\ &+ \frac{h_i}{2\rho_{i,m_i}^2}\chi_{i,m_i}^2\tilde{\theta}_{i,m_i} \\ &+ \Big(-h_i\big(\lambda_{i,m_i}+0.5\big)+1\Big)\chi_{i,m_i}^2 \\ &+ \frac{1}{4}p_i^{*2} + \sum_{k=1}^{m_i-1}\frac{1}{2\rho_{i,k}^2}\chi_{i,k}^2\tilde{\theta}_i \\ &- \sum_{k=1}^{m_i-1}\left(\frac{\psi_{i,k+1}^2}{\tau_{i,k+1}} - \psi_{i,k+1}C_{i,k+1}(Z_{i,k})\right) \\ &+ \Upsilon_{i,m_i} \quad (68) \end{aligned}$$

with $\tilde{\theta}_{i,m_i} = \theta_{i,m_i} - \hat{\theta}_{i,m_i}$, where

$$\begin{aligned} \Upsilon_{i,m_i} = &\frac{1}{2}\|P_i\|^2\big(\theta_i + \varepsilon_{i,0}^2 + \|\bar{\Delta}_i^2\|\big) + \frac{1}{2}\zeta_i^2\bar{\omega}_{i,1}^2 \\ &+ \frac{1}{2}\sum_{k=1}^{m_i}\rho_{i,k}^2 + \frac{1}{2}\sum_{k=1}^{m_i}\bar{\varepsilon}_{i,k}^2. \end{aligned}$$

For the second part of Lyapunov, one gets

$$D^{\beta}V_{i,m_i2} = -\frac{1}{\gamma_i}\tilde{\theta}_i D^{\beta}\hat{\theta}_i - \frac{h_i}{\gamma_{i,m_i}}\tilde{\theta}_{i,m_i}D^{\beta}\hat{\theta}_{i,m_i} \quad (69)$$

Substituting (30) into (69) gives

$$\begin{aligned} D^{\beta}V_{i,m_i2} \leq &\sum_{k=1}^{m_i-1}\frac{1}{2\rho_{i,k}^2}\chi_{i,k}^2\tilde{\theta}_i + \frac{1}{\gamma_i}\sigma_i\tilde{\theta}_i\hat{\theta}_i \\ &- \frac{h_i}{2\rho_{i,m_i}}\chi_{i,m_i}^2\tilde{\theta}_{i,m_i} \\ &+ \frac{h_i}{\gamma_{i,m_i}}\sigma_{i,m_i}\tilde{\theta}_{i,m_i}\hat{\theta}_{i,m_i} \quad (70) \end{aligned}$$

Then

$$D^{\beta}V_{i,m_i} = D^{\beta}V_{i,m_i1} + D^{\beta}V_{i,m_i2}$$

Then

$$\begin{aligned} D^{\beta}V_{i,m_i} \leq &-\big[\lambda_{\min}(Q_i) - 3/2\big]\|e_i\|^2 \\ &+ \sum_{k=1}^{m_i-1}\chi_{i,k}\psi_{i,k+1} \\ &- \sum_{k=1}^{m_i}\lambda_{i,k}\chi_{i,k}^2 + \frac{1}{\gamma_i}\sigma_i\tilde{\theta}_i\hat{\theta}_i \\ &+ \frac{h_i}{\gamma_{i,m_i}}\sigma_{i,m_i}\tilde{\theta}_{i,m_i}\hat{\theta}_{i,m_i} \\ &+ \Big(-h_i\big(\lambda_{i,m_i}+0.5\big)+1\Big)\chi_{i,m_i}^2 \\ &+ \frac{1}{4}p_i^{*2} \\ &- \sum_{k=1}^{m_i-1}\left(\frac{\psi_{i,k+1}^2}{\tau_{i,k+1}} - \psi_{i,k+1}C_{i,k+1}(Z_{i,k})\right) \\ &+ \Upsilon_{i,m_i} \quad (71) \end{aligned}$$

Consider $D^{\beta}V_i = D^{\beta}V_{i,m_i}$.

By using Young's inequality, the following inequalities will be obtained

$$\begin{aligned} &\chi_{i,j_i}\psi_{i,j_i+1} \leq \frac{1}{2}\chi_{i,j_i}^2 + \frac{1}{2}\psi_{i,j_i+1}^2 \\ &|\psi_{i,j_i+1}C_{i,j_i+1}| \leq \frac{\psi_{i,j_i+1}^2 C_{i,j_i+1}^2}{2\kappa_i} + 2\kappa_i \\ &\tilde{\theta}_i\hat{\theta}_i = \tilde{\theta}_i\big(\theta_i - \tilde{\theta}_i\big) \leq -\frac{1}{2}\tilde{\theta}_i^2 + \frac{1}{2}\theta_i^2 \\ &\tilde{\theta}_{i,m_i}\hat{\theta}_{i,m_i} = \tilde{\theta}_{i,m_i}\big(\theta_{i,m_i} - \tilde{\theta}_{i,m_i}\big) \leq -\frac{1}{2}\tilde{\theta}_{i,m_i}^2 + \frac{1}{2}\theta_{i,m_i}^2 \end{aligned} \quad (72)$$

where κ_i is a design constant.

By substituting (72) into (71), time derivative of V_i is given as

$$D^{\beta}V_i \leq -\left[\lambda_{\min}(Q_i)-3/2\right]\|e_i\|^2 - \sum_{k=1}^{m_i-1}\left(\lambda_{i,k}-\frac{1}{2}\right)\chi_{i,k}^2 + \left(-h_i\left(\lambda_{i,m_i}+0.5\right)+1\right)\chi_{i,m_i}^2 - \frac{\sigma_i}{2\gamma_i}\tilde{\theta}_i^2 + \frac{\sigma_i}{2\gamma_i}\theta_i^2 - \frac{h_i\sigma_{i,m_i}}{2\gamma_{i,m_i}}\tilde{\theta}_{i,m_i}^2 + \frac{h_i\sigma_{i,m_i}}{2\gamma_{i,m_i}}\theta_{i,m_i}^2 - \sum_{k=1}^{m_i-1}\left(\frac{1}{\tau_{i,k+1}}-\frac{1}{2}+\frac{C_{i,k+1}^2}{2\kappa_i}\right)\psi_{i,k+1}^2 + \bar{\Upsilon}_i$$

$$\bar{\Upsilon}_i = \Upsilon_{i,m_i} - 2\kappa_i + \frac{1}{4}p_i^{*2} \tag{73}$$

Finally, the Lyapunov function candidate is considered for the total system as

$$V = \sum_{i=1}^{n} V_i \tag{74}$$

The fractional-order time derivative is

$$D^{\beta}V = \sum_{i=1}^{n} D^{\beta}V_i \tag{75}$$

Thus, we have

$$D^{\beta}V \leq \sum_{i=1}^{n}\Bigg\{-\left[\lambda_{\min}(Q_i)-3/2\right]\|e_i\|^2 - \sum_{k=1}^{m_i-1}\left(\lambda_{i,k}-\frac{1}{2}\right)\chi_{i,k}^2 + \left(-h_i\left(\lambda_{i,m_i}+0.5\right)+1\right)\chi_{i,m_i}^2 - \frac{\sigma_i}{2\gamma_i}\tilde{\theta}_i^2 + \frac{\sigma_i}{2\gamma_i}\theta_i^2 - \frac{h_i\sigma_{i,m_i}}{2\gamma_{i,m_i}}\tilde{\theta}_{i,m_i}^2 + \frac{h_i\sigma_{i,m_i}}{2\gamma_{i,m_i}}\theta_{i,m_i}^2 - \sum_{k=1}^{m_i-1}\left(\frac{1}{\tau_{i,k+1}}-\frac{1}{2}-\frac{C_{i,k+1}^2}{2\pi_i}\right)\psi_{i,k+1}^2\Bigg\} + \Upsilon \tag{76}$$

where $\Upsilon = \sum_{i=1}^{n}\bar{\Upsilon}_i$.

Let $\lambda_{\min}(Q_i) - 3/2 > 0$, and denote

$$c = \min\left\{\frac{\lambda_{\min}(Q_i)-3/2}{\lambda_{\max}(P_i)}, 2\left(\lambda_{i,j_i}-\frac{1}{2}\right), \sigma_i, \sigma_{i,m_i}, 2\left(h_i\left(\lambda_{i,m_i}+0.5\right)-1\right), 2\left(\frac{1}{\tau_{i,j_i+1}}-\frac{1}{2}+\frac{C_{i,j_i+1}^2}{2\kappa_i}\right)\right\}, j_i = 1, \ldots, m_i.$$

To ensure $c_i > 0$, the control gains λ_{i,j_i} and τ_{i,j_i+1} are chosen to satisfy the following conditions, $\lambda_{i,j_i} - \frac{1}{2} > 0$ and $\frac{1}{\tau_{i,j_i+1}} - \frac{1}{2} + \frac{C_{i,j_i+1}^2}{2\kappa_i} > 0$.

Therefore,

$$D^{\beta}V \leq -cV + \Upsilon \tag{77}$$

where

$$c = \min\{c_1, c_2, \ldots, c_n\}$$

There is a positive function $N(t)$, which satisfies the following equation

$$D^{\beta}V(t) + N(t) = -cV(t) + \Upsilon \tag{78}$$

Taking the Laplace transform of (78) gives

$$s^{\beta}V(s) - s^{\beta-1}V(0) + N(s) = -cV(s) + \frac{\Upsilon}{s} \tag{79}$$

where $V(0)$ is a nonnegative constant and $V(s) = L\{V(t)\}$. Next, it is obtained

$$V(s) = \frac{s^{\beta-1}V(0) - N(s) + \frac{\Upsilon}{s}}{s^{\beta}+c} \tag{80}$$

If $x(0) = 0$, then $V(0) = 0$ and if $x(0) \neq 0$, then $V(0) > 0$. Since $V(t,x)$ is locally Lipschitz in x, it is obtained by the inverse Laplace transform and the fractional existence and uniqueness theorem [1], and Eq. (80) has a unique solution as

$$V(t) = V(0)E_{\beta}\left(-ct^{\beta}\right) - N(t) * t^{\beta-1}E_{\beta,\beta}\left(-ct^{\beta}\right) + \Upsilon t^{\beta}E_{\beta,\beta+1}\left(-ct^{\beta}\right), \ t \geq 0 \tag{81}$$

where $*$ is convolution operator. On the one hand, $t^{\beta-1}$ and $E_{\beta,\beta}\left(-ct^{\beta}\right)$ are nonnegative [46], thus

$$N(t) * t^{\beta-1}E_{\beta,\beta}\left(-ct^{\beta}\right), \ t \geq 0 \tag{82}$$

then one gets

$$V(t) \leq V(0)E_\beta\big(-ct^\beta\big)+\Upsilon t^\beta E_{\beta,\beta+1}\big(-ct^\beta\big),\ t\geq 0 \tag{83}$$

Noting that $arg\big(-ct^\beta\big)=-\kappa_i,\ |-ct^\beta|\geq 0$ for all $t\geq 0$ and $t\leq|-\kappa_i|\leq\kappa_i$, then by using lemma 2, a positive constant C exists such that

$$|E_\beta\big(-ct^\beta\big)|\leq\frac{C}{1+ct^\beta} \tag{84}$$

It concludes from (84) that

$$\lim_{t\to\infty}|V(0)|E_\beta\big(-ct^\beta\big)=0 \tag{85}$$

Hence, for every $\delta>0$, a constant $t_1>0$ exists such that $t>t_1$, which implies

$$|V(0)|E_\beta\big(-ct^\beta\big)\leq\delta \tag{86}$$

On the other hand, by using Lemma 1 and choosing $n=1$ one has

$$E_{\beta,\beta+1}\big(-ct^\beta\big)=\frac{t^{-\beta}}{c\Gamma(1)}+o\Big(|-ct^\beta|^{-1-1}\Big) \tag{87}$$

From (87), for every $\delta>0$, a positive constant t_2 exists such that

$$\Upsilon t^\beta E_{\beta,\beta+1}\big(-ct^\beta\big)\leq\frac{\Upsilon}{c}+\delta \tag{88}$$

For all $t>t_2$.

The design parameters can be adjusted such that it concludes from (83), (86) and (88) that

$$|V(t)|\leq 3\delta \tag{89}$$

It concludes from (89) that $V(t)$ is limited by 3δ. Therefore, all system signals are bounded. In addition, when the controller parameter c is increased, i.e., λ_{i,j_i} and τ_{i,j_i+1} are regulated, the errors in the controlled system can converge to neighborhood of zero. This concludes the proof. □

3.3 Optimal settings for controller parameters

Particle swarm optimization (PSO) is utilized as an optimization technique in order to tune the adaptive controller parameters. Solving complex optimization problems with high nonlinearity is very effective by using the PSO[57].

In a conventional adaptive type-2 fuzzy backstepping method, the controllers parameters are chosen by a trial-and-error approach; Even if the controller works well with these parameters, it is not guaranteed that optimal parameters are selected. Therefore, PSO algorithm is applied to find the optimal parameters. By using the optimal parameters, the stability of systems is guaranteed.

The adaptive type-2 fuzzy backstepping controller is optimized by minimizing the following objective functions,

$$\begin{aligned}I_1 =& w_1\int_0^\infty t\big|\chi_{1,1}\big|\mathrm{d}t\\ &+w_2\int_0^\infty t\big|\chi_{2,1}\big|\mathrm{d}t+\cdots+w_n\int_0^\infty t\big|\chi_{n,1}\big|\mathrm{d}t\\ I_2 =& w_{n+1}\int_0^\infty \upsilon_1^2\mathrm{d}t\\ &+w_{n+2}\int_0^\infty \upsilon_2^2\mathrm{d}t+\cdots+w_{2n}\int_0^\infty \upsilon_n^2\mathrm{d}t\\ I =& I_1+I_2\end{aligned} \tag{90}$$

The above objective function minimizes the control energy (effort) and the tracking error, simultaneously, where $\upsilon_i^2, i=1,2$ are the control signals, $w_k, k=1,2,3,4$ are the weighting coefficients, and $\chi_{i,1}, i=1,2$ are the tracking errors.

The outcome of PSO Algorithm verifies that fractional controller achieves less objective function.

4 Simulation examples

Example 1 We will examine this example in two parts:

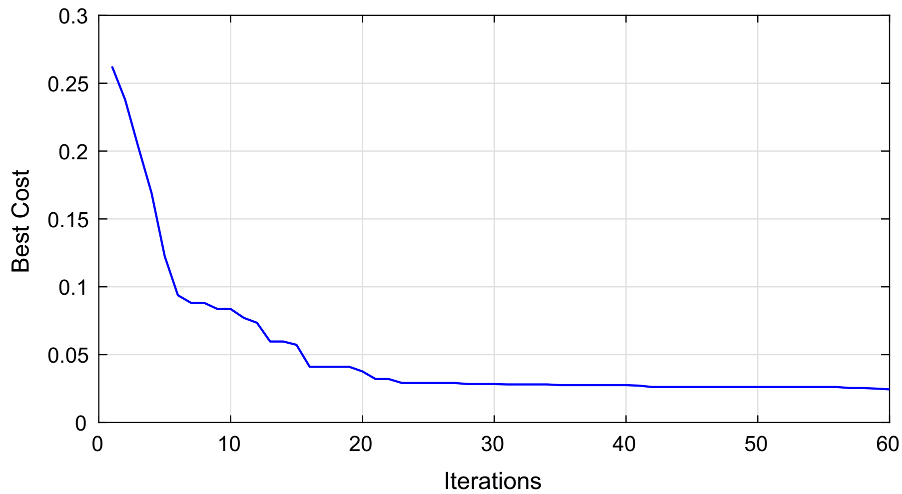


Fig. 1 Cost-function value versus iterations averaged over 60 random runs for PSO

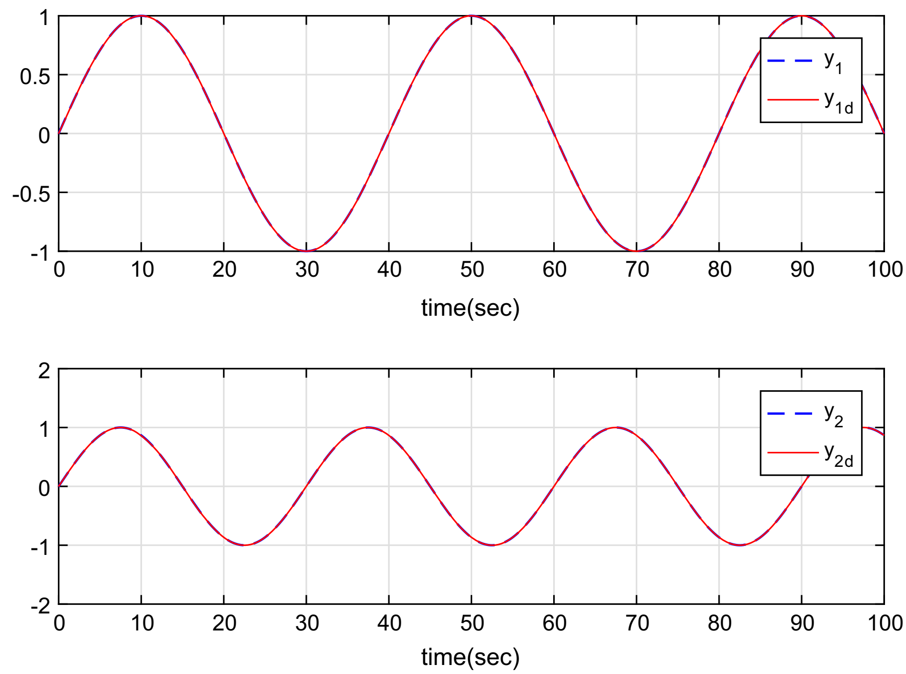


Fig. 2 System outputs $y_1 = x_{1,1}$ and $y_2 = x_{2,1}$, and reference signals y_{1d} and y_{2d}

Part 1. The fractional-order MIMO nonlinear system with dead-zone is considered as follows [58]:

$$\begin{aligned} D^{0.8}x_{1,1} &= x_{1,2} \\ D^{0.8}x_{1,2} &= -x_{1,1}^3 - x_{1,2} \\ &\quad + \left(1 + e^{-x_{1,1}+x_{1,2}^2}\right)u_1 + \omega_{1,2}(x,t) \\ y_1 &= x_{1,1} \\ D^{0.8}x_{2,1} &= x_{2,2} \\ D^{0.8}x_{2,2} &= -x_{2,1} - x_{2,2}^2 \\ &\quad + \left(2 + \sin(x_{2,1})\right)u_2 + \omega_{2,2}(x,t) \\ y_2 &= x_{2,1} \end{aligned} \tag{91}$$

where the disturbance terms are as follows,

$$\begin{aligned} \omega_{1,2}(x,t) &= 0.6\cos(t) + 0.4x_{1,2}\sin(0.5t) \\ &\quad + 0.2x_{2,1}\sin(3t) + 0.5x_{2,2}\cos(10t) \\ \omega_{2,2}(x,t) &= 0.5\sin(5t) + 2x_{1,1}\sin(x_{2,1}) \\ &\quad + 0.4x_{2,2}\sin(0.2t) \end{aligned}$$

The dead-zone parameters in (12) are chosen as: $m_{1r} = 2, m_{1l} = 1, b_{1r} = 1, b_{1l} = 1, m_{2r} =$

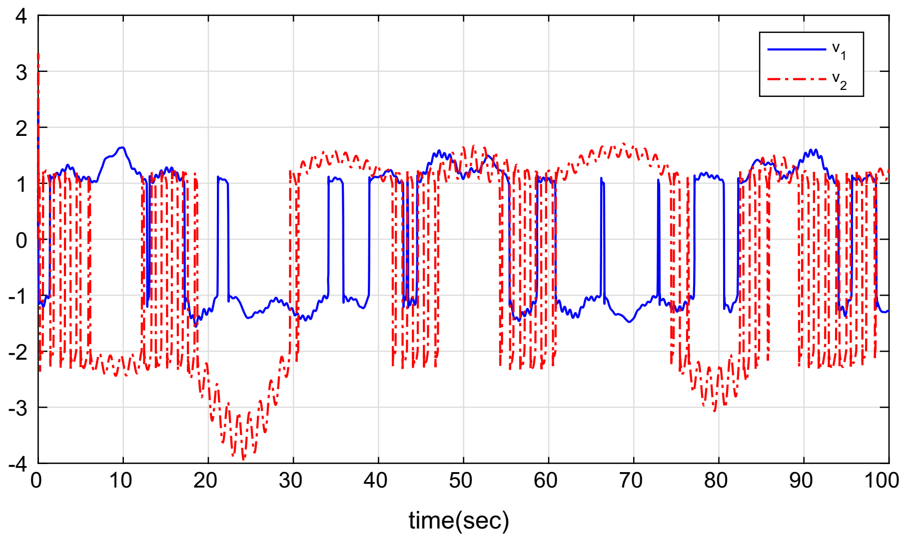


Fig. 3 The dead-zone inputs υ_1 and υ_2

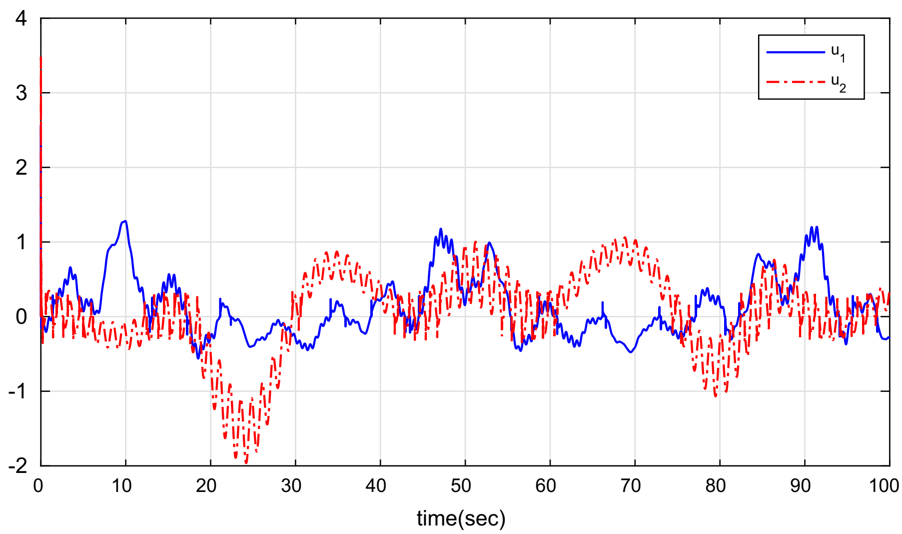


Fig. 4 The control inputs u_1 and u_2

1.5, $m_{2l} = 1$, $b_{2r} = 1$ and $b_{2l} = 2$; also the reference signals are $y_{1d} = \sin\left((\pi/20)t\right)$ and $y_{2d} = \sin\left((\pi/15)t\right)$.

The goal is to design a fractional adaptive type-2 fuzzy DSC control based on observer, in order for the output y_i to follow y_{id} for $i = 1, 2$ under the assumption that the state variables $x_{1,2}$ and $x_{2,2}$ in (91) are immeasurable.

Based on Theorem 1, the virtual control functions, actual control laws and the adaptive laws are constructed as follows,

$$\alpha_{1,1} = -\frac{1}{2\rho_{1,1}^2}\chi_{1,1}\hat{\theta}_1 - (\lambda_{1,1} + 0.5)\chi_{1,1}$$

$$\alpha_{2,1} = -\frac{1}{2\rho_{2,1}^2}\chi_{2,1}\hat{\theta}_2 - (\lambda_{2,1} + 0.5)\chi_{2,1} \tag{92}$$

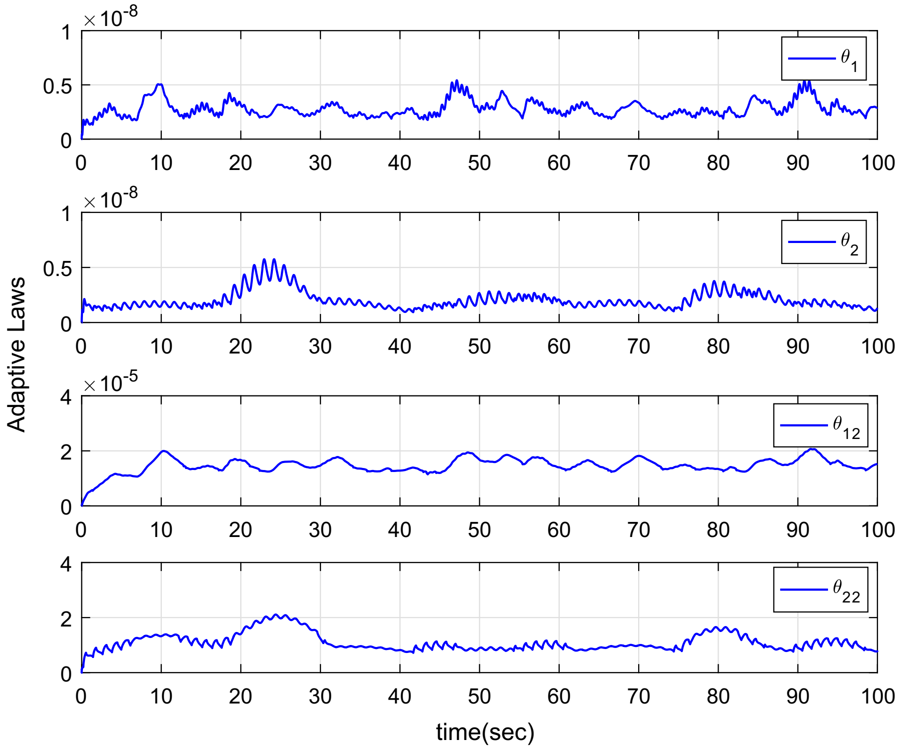
$$\upsilon_1 = -\frac{1}{2\rho_{1,2}^2}\chi_{1,2}\hat{\theta}_{1,2} - (\lambda_{1,2} + 0.5)\chi_{1,2}$$

$$\upsilon_2 = -\frac{1}{2\rho_{2,2}^2}\chi_{2,2}\hat{\theta}_{2,2} - (\lambda_{2,2} + 0.5)\chi_{2,2} \tag{93}$$

$$D^\beta\hat{\theta}_1 = \frac{\gamma_1}{2\rho_{1,1}^2}\chi_{1,1}^2 - \sigma_1\hat{\theta}_1$$

$$D^\beta\hat{\theta}_{1,2} = \frac{\gamma_{1,2}}{2\rho_{1,2}^2}\chi_{1,2}^2 - \sigma_{1,2}\hat{\theta}_{1,2}$$

Fig. 5 Adaptive laws θ_1, θ_2, $\theta_{1,1}$ and $\theta_{2,1}$



$$D^{\beta}\hat{\theta}_2 = \frac{\gamma_2}{2\rho_{2,1}^2}\chi_{2,1}^2 - \sigma_2\hat{\theta}_2$$
$$D^{\beta}\hat{\theta}_{2,2} = \frac{\gamma_{2,2}}{2\rho_{2,2}^2}\chi_{2,2}^2 - \sigma_{2,2}\hat{\theta}_{2,2} \quad (94)$$

The PSO weighting coefficients are selected as $w_1 = 1, w_2 = 1, w_3 = 0, w_4 = 0$, and the optimal design parameters are achieved as follows:

$\gamma_1 = 85.6, \gamma_{1,2} = 63.25, \gamma_2 = 44.1, \gamma_{2,2} = 52.73, \rho_{1,1} = 35.12, \rho_{1,2} = 35.22, \rho_{2,1} = 38.59, \rho_{2,2} = 0.1, \sigma_1 = 0.74, \sigma_{1,2} = 0.5, \sigma_2 = 0.57, \sigma_{2,2} = 0.59, \lambda_{1,1} = 76.92, \lambda_{1,2} = 69.13, \lambda_{2,1} = 80.59, \lambda_{2,2} = 55.2$.

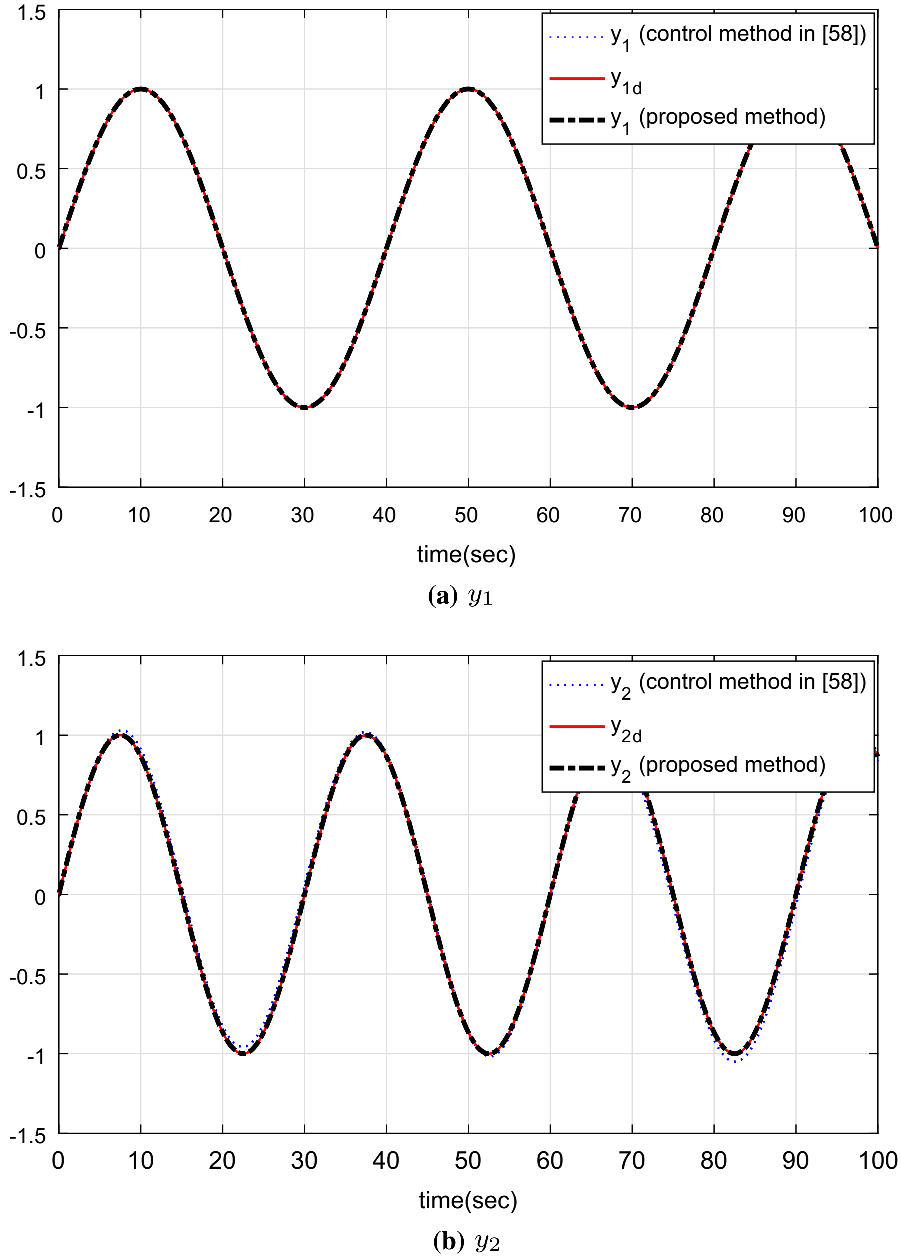
In the simulation, initial conditions are chosen as $x(0) = [0, 0, 0, 0]$ and $\theta(0) = [0, 0, 0, 0]$.

The control scheme on MIMO fractional-order system is shown in Figs. 1, 2, 3, 4 and 5. Figure 1 represents the objective function. As it is seen from Fig. 1, PSO has reached global minimum at the 23rd iteration. The best cost value is about 0.03. Tracking performance of the control system is shown in Fig. 2, as it is observed, the reference signals could be tracked well by the output signals. Figures 3 and 4 display the dead-zone inputs and the control inputs (dead-zone outputs), respectively. Figure 5 shows the adaptive parameter curves. From Figs. 3, 4 and 5, it is clear that the signals $v_1, v_2, u_1, u_2, \theta_1, \theta_2, \theta_{1,2}$ and $\theta_{2,2}$ are bounded. The robustness and effectiveness of our proposed adaptive fuzzy controller are verified by the simulation results because its tracking performance is good in the presence of perturbation and all the closed-loop signals are bounded.

Part 2. In this part, the efficiency of the proposed control design without dead-zone is specified by comparing the proposed method with the fractional sliding mode controller in [58].

The fractional-order MIMO nonlinear system is considered as (91). The disturbance terms and the reference signals are the same as in Part 1. The goal is to show the effectiveness of the proposed method in comparison with the control method in [58]. The optimal design parameters and initial conditions are chosen as the same as in Part 1.

Comparative figures between the proposed method and the method in [58] are shown in Figs. 6,7 and 8. From a comparison among Figs. 6, 7 and 8, it shows that the proposed control approach, which uses the adaptive backstepping control method, can obtain improved per-



(a) y_1

(b) y_2

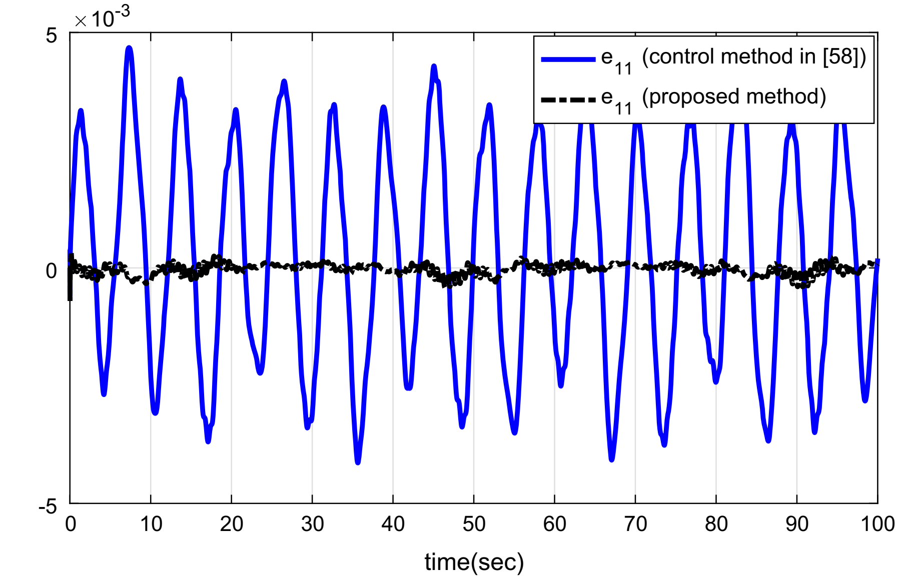
Fig. 6 Trajectories of y_1 and y_2 in [58] and in this paper

formance in the trajectory tracking responses, tracking errors and the control signal. As can be seen in Fig. 7, the tracking error of signals in the proposed method is lower than the control method in [58]. Also, Fig. 8 shows that the control signal in the proposed method has a less overshoot in subsystem 1 and less undershoot in both subsystems than the control signals in the control method in [58].

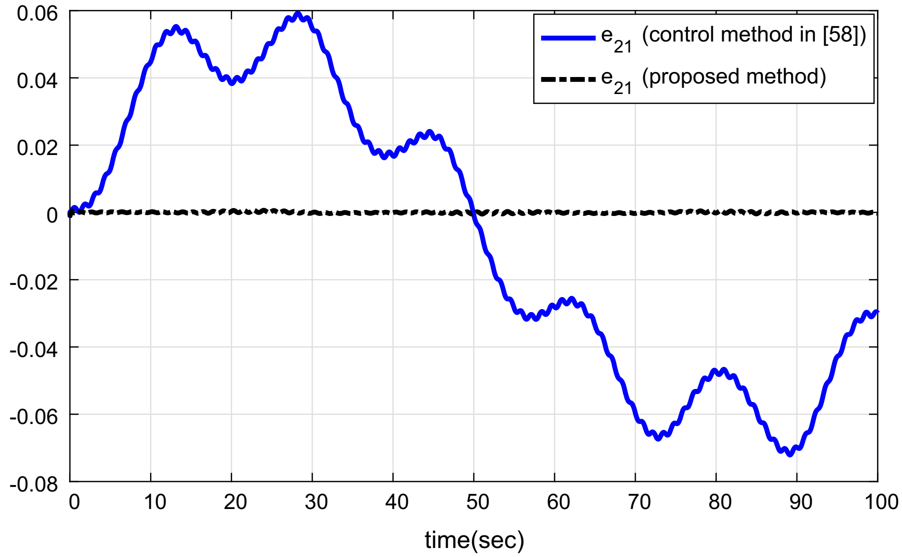
Remark 3 In the system of [58], the interconnection terms $L_1(x)$ and $L_2(x)$ are uncertain and fuzzy logic is used to approximate them. The uncertain functions of our proposed model are more than the uncertain terms in [58]. (The following functions: $f_{1,1}(x_{1,1})$, $f_{1,2}(x_{1,1}, x_{1,2})$, $f_{2,1}(x_{2,1})$, $f_{2,2}(x_{2,1}, x_{2,2})$, $\omega_{1,2}(t, x_2)$ and $\omega_{2,2}(t, x_2)$, are unknown in our model.)

Remark 4 In the system of [58], the fuzzy sets of input variable are defined according to the membership

Fig. 7 Tracking error signals e_{11} and e_{21} in [58] and in this paper



(a) e_{11}



(b) e_{21}

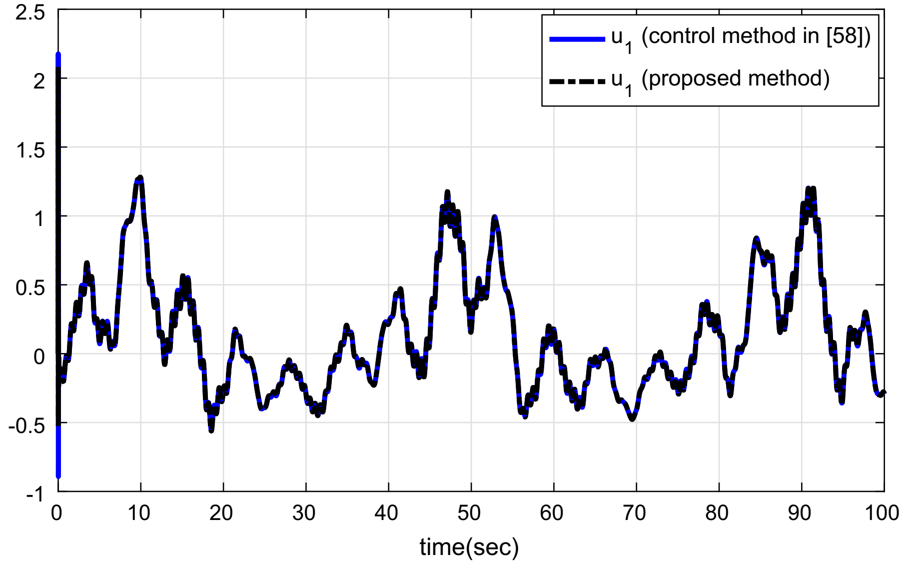
functions in which the number of fuzzy rules will be 16; while in our proposed method, the controller with respect to the approximation $S^{\mathrm{T}}S \leq 1$ is not dependent on the definition of Gaussian functions and fuzzy rules.

Remark 5 In the system of [58], all states are measurable and the prior knowledge of the system model is needed, while in our proposed method all states except one state are immeasurable and the system model can be fully unknown.

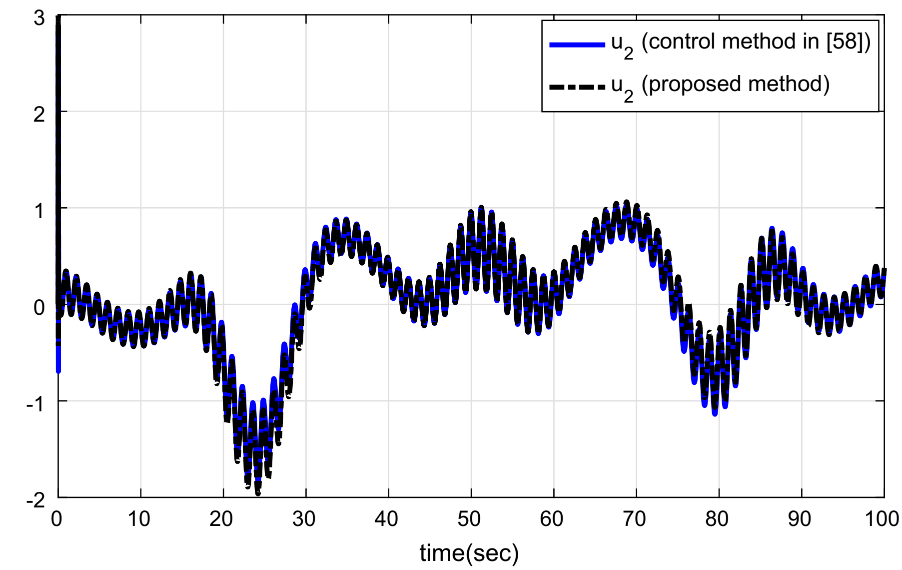
Remark 6 In the proposed method, the hard nonlinearity dead-zone is considered in the path of the control signal, which is an instability factor in practical systems, while in the reference [58], this problem is not considered.

Remark 7 The number of adaptive rules in the method of [58] is much more than our proposed method. The proposed control has a simple structure in such a man-

Fig. 8 The control inputs u_1 and u_2 in [58] and in this paper



(a) u_1



(b) u_2

ner that the number of tuning variables is reduced. To give the best visibility and show the effectiveness of the proposed control strategy compared to [58], a comparison study is performed in Table 1.

Example 2 To show more results of the proposed method, we consider the double inverted pendulum with an unknown dead-zone by substituting integer-order derivatives with fractional-order ones [26]. Denot-

Table 1 Comparison between our control strategy and the control strategy in [58]

Comparison tasks	Proposed method	Method in Majidabad et al. [58]
$\text{ITAE} = \int_0^t t\lvert e_{11}(t)\rvert \mathrm{d}t$	0.4423	10.6069
$\text{ITAE} = \int_0^t t\lvert e_{21}(t)\rvert \mathrm{d}t$	0.8258	215.2971
$\text{IAE} = \int_0^t \lvert e_{11}(t)\rvert \mathrm{d}t$	0.008919	0.2132
$\text{IAE} = \int_0^t \lvert e_{21}(t)\rvert \mathrm{d}t$	0.0165	3.8908
$\text{ISE} = \int_0^t u_1^2(t)\mathrm{d}t$	18.19	18.1189
$\text{ISE} = \int_0^t u_2^2(t)\mathrm{d}t$	31.16	29.3446
Overshoot u_1	2.08	2.176
Overshoot u_2	2.985	2.825
Undershoot u_1	– 0.5305	– 0.8917
Undershoot u_2	– 0.4317	– 0.6945

Table 2 System parameters

Parameter	m_1	m_2	J_1	J_2	r	k	l	b	g
Value	2	2.5	0.5	0.625	0.5	100	0.5	0.4	9.81

ing $x_{1,1} = \Theta_1$ (angular position), $x_{1,2} = D^{0.85}\Theta_1$ (angular rate), $x_{2,1} = \Theta_2$ and $x_{2,2} = D^{0.85}\Theta_2$. Angular rates $x_{1,2} = D^{0.85}\Theta_1$ and $x_{2,2} = D^{0.85}\Theta_2$ are unavailable.

The dynamic equations of the inverted pendulum can be described as

$$
\begin{aligned}
D^{0.85}x_{1,1} &= x_{1,2}\\
D^{0.85}x_{1,2} &= \left(\frac{m_1 gr}{J_1} - \frac{kr^2}{4J_1}\right)\sin(x_{1,1})\\
&\quad + \frac{kr}{2J_1}(l-b) + \frac{1}{J_1}u_1 + \omega_{1,2}(x)\\
y_1 &= x_{1,1}\\
D^{0.85}x_{2,1} &= x_{2,2}\\
D^{0.85}x_{2,2} &= \left(\frac{m_2 gr}{J_2} - \frac{kr^2}{4J_2}\right)\sin(x_{2,1})\\
&\quad - \frac{kr}{2J_2}(l-b) + \frac{1}{J_2}u_2 + \omega_{1,2}(x)\\
y_2 &= x_{2,1}
\end{aligned}
\tag{95}
$$

where $\omega_{1,2}(x) = \left(kr^2/4J_1\right)\sin(x_{2,1})$ and $\omega_{2,2}(x) = \left(kr^2/4J_2\right)\sin(x_{1,2})$ are disturbance terms, m_1 and m_2 are the pendulum end masses, J_1 and J_2 are the moments of inertia, r is the pendulum height, k is the spring constant, l is the spring natural length, b is the interval among the pendulum hinges, and g is the gravitational acceleration.

These parameters values are considered in Table 2; also the dead-zone parameters are selected as: $m_{1r} = 1.5, m_{1l} = 1, b_{1r} = 2, b_{1l} = 1, m_{2r} = 3, m_{2l} = 4, b_{2r} = 3$ and $b_{2l} = 2$.

The reference signals are $y_{1d} = \sin\left((\pi/20)t\right)$ and $y_{2d} = \sin\left((\pi/15)t\right)$.

An observer-based fractional adaptive type-2 fuzzy DSC scheme is designed so that the output y_i follows y_{id} for $i = 1, 2$ under the assumption that the state variables $x_{1,2}$ and $x_{2,2}$ in (91) are immeasurable.

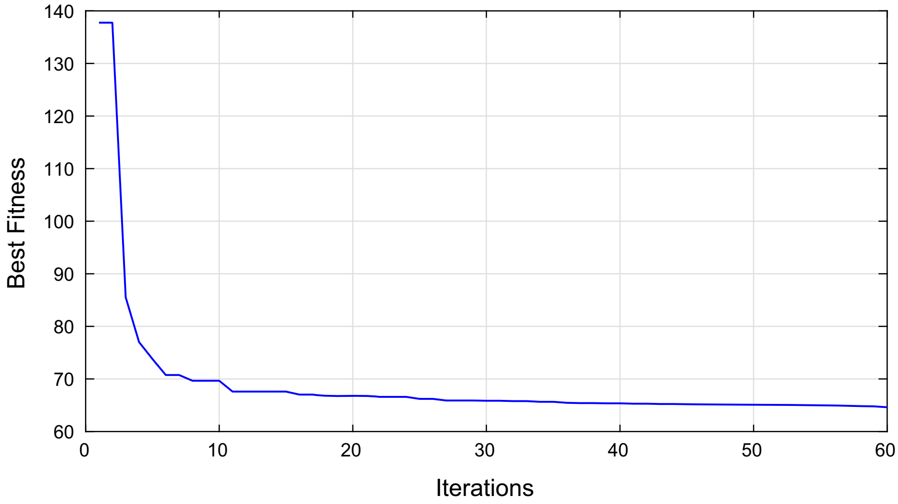


Fig. 9 Cost-function value versus iterations averaged over 60 random runs for PSO

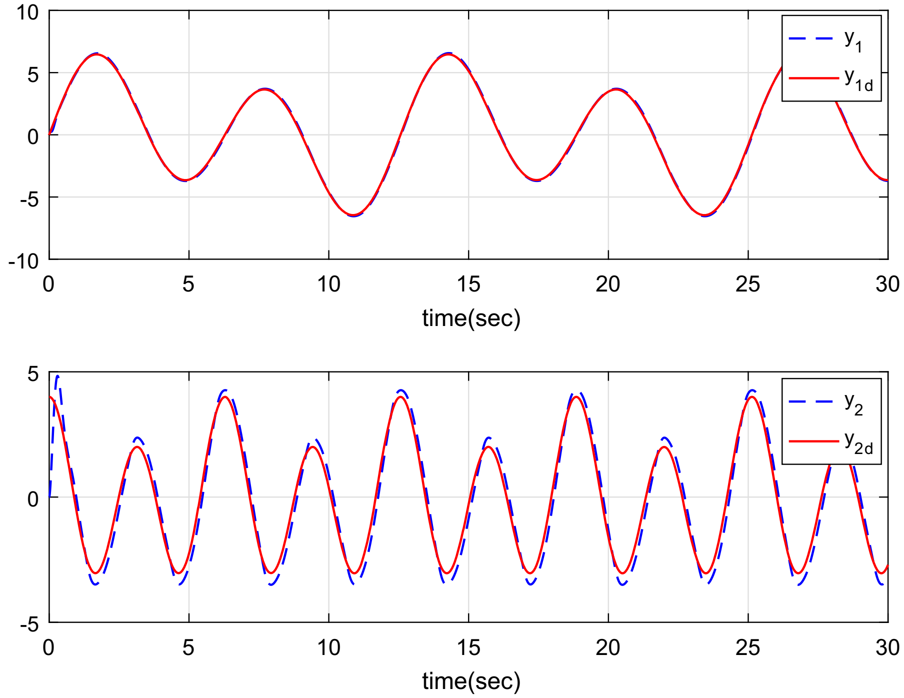


Fig. 10 System outputs $y_1 = x_{1,1}$ and $y_2 = x_{2,1}$ and references signals $y_{1,d}$ and $y_{2,d}$

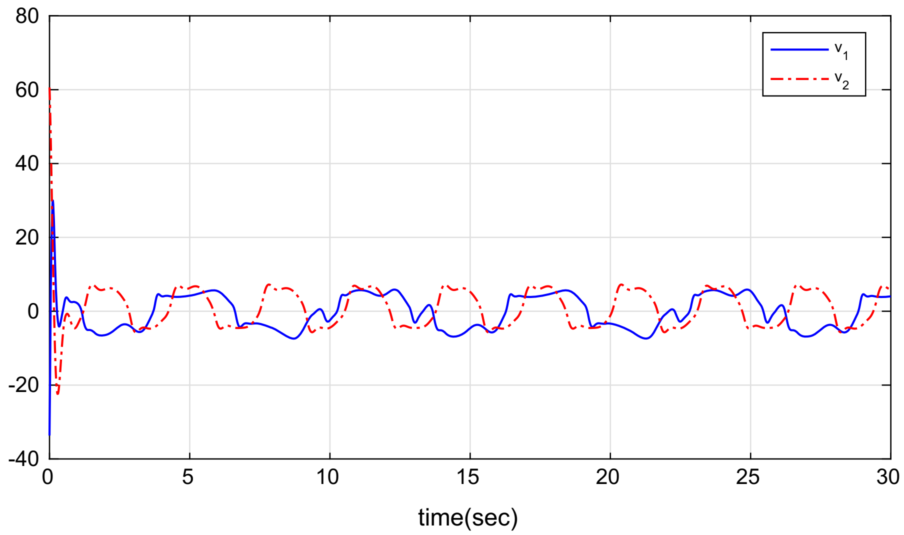


Fig. 11 The dead-zone inputs υ_1 and υ_2

The virtual control functions (92), actual control laws (93) and the adaptive laws (94) are constructed according to Theorem 1.

The PSO weighting coefficients of objective function are selected as $w_1 = 1$, $w_2 = 1$, $w_3 = 0.05$, $w_4 = 0.05$, and the optimal design parameters are achieved as follows,

$\gamma_1 = 39.08, \gamma_{1,2} = 87.94, \gamma_2 = 64.25, \gamma_{2,2} = 45.56, \rho_{1,1} = 96.95, \rho_{1,2} = 85.73, \rho_{2,1} = 53.03, \rho_{2,2} = 52.69, \sigma_1 = 0.44, \sigma_{1,2} = 0.41, \sigma_2 = 0.34, \sigma_{2,2} = 0.43, \lambda_{1,1} = 36.53, \lambda_{1,2} = 2.39, \lambda_{2,1} = 24.75, \lambda_{2,2} = 0.1$, and the initial conditions are chosen as $x(0) = [\pi/10, 0, 0, 0]$ and $\theta = [0, 0, 0, 0]$.

The results of the proposed controller on MIMO fractional-order system are represented in Figs. 9, 10, 11, 12 and 13. Figure 9 shows the objective function. As it is seen from Fig. 9, PSO has reached global min-

Fig. 12 The control inputs u_1 and u_2

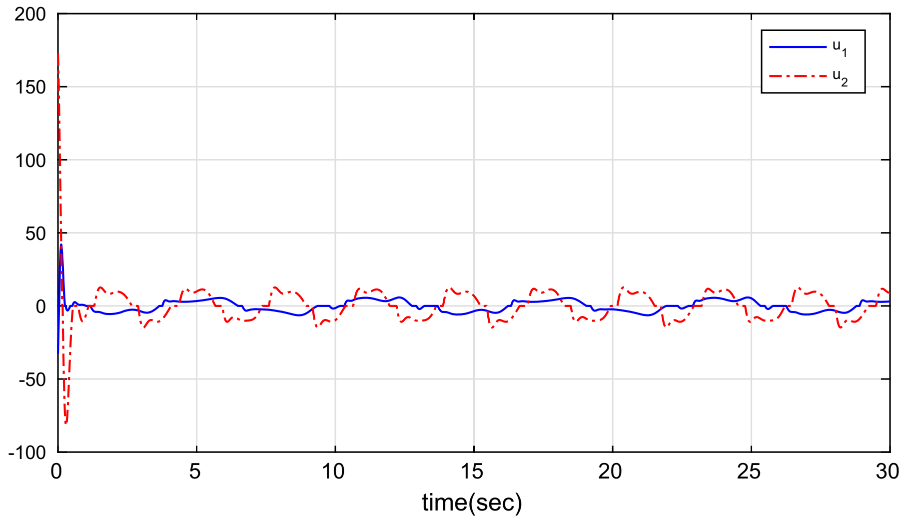
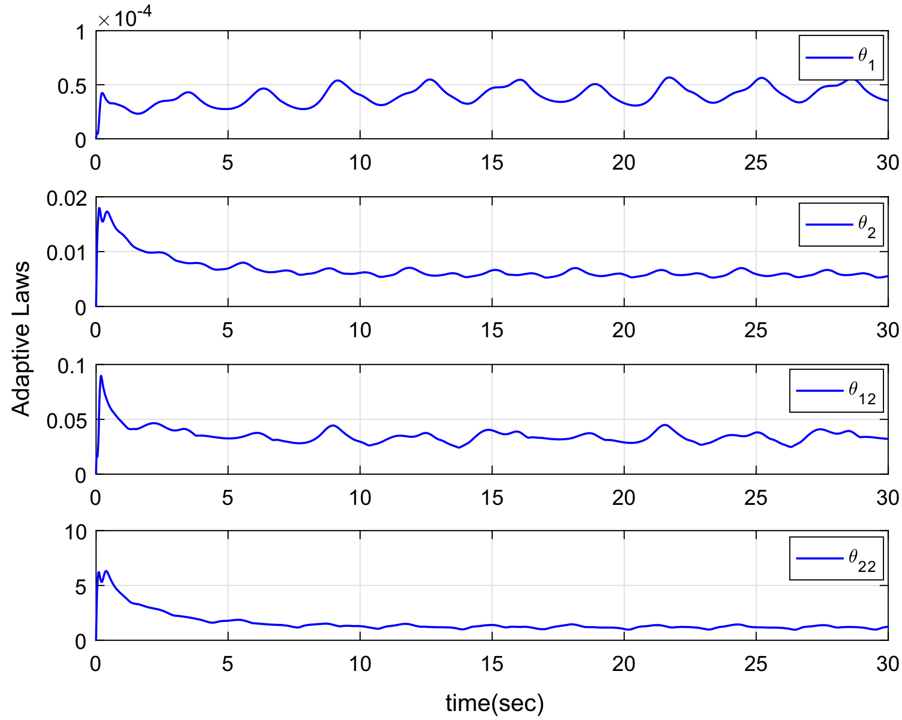


Fig. 13 Adaptive laws θ_1, θ_2, $\theta_{1,1}$ and $\theta_{2,1}$



imum at the 23rd iteration. The best cost value is about 0.03.

Figure 10 represents the output tracking performance. Figures 11 and 12 display the dead-zone input and the control input (dead-zone output), respectively. Figure 13 shows the adaptive fuzzy parameters. From Figs. 11,12 and 13, it is clear that the signals $v_1, v_2, u_1, u_2, \theta_1, \theta_2, \theta_{12}$ and θ_{22} are bounded. The simulation results are representative that although the accurate information on the nonlinear system functions is not available, the adaptive type-2 fuzzy scheme can guarantee the good convergence and good tracking performance; also all the closed-loop signals are bounded.

5 Conclusion

An output feedback fractional interval type-2 fuzzy adaptive based on backstepping DSC with fractional adaptive laws has been considered for a type of fractional-order nonlinear MIMO system with external disturbances, system uncertainties and unknown dead-zone as input nonlinearity for solving the problem of Mittag-Leffler stabilization. Fractional adaptive type-2 fuzzy design was combined with the backstepping DSC approach to construct the desired controller. The DSC method was applied to prevent the explosive growth of complexity inherent in the backstepping method. Also, unknown nonlinear functions in uncertain MIMO system are approximated by IT2FLS. In order to better control performance in reducing tracking error, the PSO algorithm was utilized for tuning the controller parameters. It was demonstrated that the proposed scheme guarantees the boundedness property for all the system signals, and also the tracking error will converge to a small neighborhood of origin. Finally, two simulation examples have been presented in this paper to illustrate the efficiency of the proposed scheme. In future research works, a generalization of the proposed methods to nonlinear systems with stochastic disturbances, convergence in finite time, unmatched disturbance, unknown control direction, and utilizing a supervisory controller can be considered.

Compliance with ethical standards

Conflict of interest The authors declare that they have no conflict of interest concerning the publication of this manuscript.

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