

Minimization of the fuel consumption of a gasoline engine using dynamic optimization

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ABSTRACT

This paper discusses the minimization of the fuel consumption of a gasoline engine through dynamic optimization. The minimization uses a mean value model of the powertrain and vehicle. This model has two state variables: the pressure in the engine intake manifold and the engine speed. The control input is the throttle valve angle. The model is identified on a universal engine dynamometer. Optimal state and control trajectories are calculated using Bock's direct multiple shooting method, implemented in the software *MUSCOD-II*. The developed approach is illustrated both in simulation and experimentally for a generic test case where a vehicle accelerates from 1100 rpm to 3700 rpm in 30 s. The optimized trajectories yield minimal fuel consumption. The experiments show that a linear engine speed trajectory yields an extra fuel consumption of 13% when compared to the optimal trajectory. It is shown that, with a simple model, a significant amount of fuel can be saved without loss of the fun-to-drive.

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1. Introduction

Transport is known to be the fastest rising source of CO₂ emissions. Environmental issues thus demand cars with a high fuel efficiency. Also the dependency on oil and the rising fuel prices trigger the interest of car manufacturers to produce vehicles with a low fuel consumption. Therefore several new systems and technologies have been introduced in combustion engines and powertrains that have greatly improved the fuel economy of vehicles. The problem is that the room for technology improvements is constantly shrinking and it is believed that cost-effective solutions will come to an end in a foreseeable future. Apart from technical measures, a high potential to reduce fuel consumption lies in the adaptation of the driving style [1], especially in speed transients. These accelerations and decelerations frequently occur in city traffic and significantly increase the fuel consumption. Manzie et al. [2] smoothed these transients by a look-ahead capability. They showed in simulation that with a 7 s look-ahead, fuel economy can be improved by up to 33%. Unfortunately, constrained by the surrounding traffic, this method is not always applicable and jeopardizes the driveability and fun-to-drive. A better strategy optimizes the speed trajectory to minimize the fuel consumption, resulting in fuel-optimal trajectories as often used in aerospace applications. This strategy is

hardly investigated. Stoicescu [3] minimized fuel consumption for a given vehicle acceleration using Pontryagin's Maximum Principle for dynamic optimization. This indirect and purely analytical method seems to be more of theoretical rather than practical use. This paper minimizes fuel consumption for a given acceleration and uses direct methods for dynamic optimization that are more suitable for use in practical implementations such as autonomous cruise control [4] and predictive cruise control [5].

The next section gives a brief overview of the current methods used for vehicle speed control. Section 3 gives an introduction to dynamic optimization. Section 4 takes a look at the models used for the optimization. A practical case for vehicle acceleration with fuel-optimal speed trajectories is presented in Section 5. Both numerical simulation and experiments on a dynamometer are considered. Section 6 concludes this paper.

2. Vehicle speed control

The applications of vehicle control where a certain desired speed of the driver is adapted to improve the fuel economy are rather limited. Autonomous cruise control (ACC) is a system that keeps a certain speed and can overrule this set speed to keep a safe distance from vehicles driving ahead [4]. ACC systems are widely used nowadays, although all these systems are for safety improvement only. Intelligent speed adaptation (ISA) is the name for a system where the vehicle has information on the speed limit and gives

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Nomenclature

A	frontal flow area of the throttle body (m^2)	r	specific gas constant of air ($\text{J}/(\text{kg K})$)
AFR	air-fuel ratio ($-$)	S	frontal vehicle surface (m^2)
AFR_{st}	stoichiometric air-fuel ratio ($-$)	t_i	injection duration (s)
c_f	fuel calibration factor ($-$)	T_a	atmospheric temperature (K)
C_d	discharge coefficient of the throttle body ($-$)	T_e	effective torque (Nm)
C_x	vehicle drag coefficient ($-$)	T_l	load torque (Nm)
f_r	friction coefficient ($-$)	T_m	manifold temperature (K)
F_v	force on the vehicle wheels (N)	u	throttle valve position ($-$)
g	gravity constant (m/s^2)	v_v	vehicle velocity (m/s)
i_D	reduction ratio of the differential ($-$)	V_m	manifold volume (m^3)
i_G	reduction ratio of the gearbox ($-$)	V_{tot}	total cylinder volume (m^3)
I_m	rotating inertia (kg m^2)	Q_v	heating value (J/kg)
$\dot{m}_{\text{ai}}$	air mass flow into the manifold (kg/s)	R_w	wheel radius (m)
$\dot{m}_{\text{ao}}$	air mass flow into the cylinders (kg/s)	α	throttle valve angle (rad)
$\dot{m}_f$	fuel mass flow (kg/s)	η_e	effective efficiency ($-$)
M_v	vehicle mass (kg)	η_m	power transmission efficiency ($-$)
n	engine speed (rad/s)	η_v	volumetric efficiency ($-$)
p_a	atmospheric pressure (Pa)	κ	isentropic exponent ($-$)
p_m	manifold pressure (Pa)	ϕ	ratio of equivalence ($-$)
p_r	manifold pressure ratio ($-$)	ρ_a	air density (kg/m^3)
$p_{r,\text{crit}}$	critical manifold pressure ratio ($-$)	θ	road slope (rad)

feedback to the driver or limits the maximum speed [6]. This system is intended for safety improvement, but can also have a positive effect on the fuel economy. Predictive cruise control (PCC) is a system that improves conventional or autonomous cruise control by using positioning systems and 3D map information of the road. PCC allows the vehicle speed to vary around the cruise control set speed in order to reduce fuel consumption [7,8]. Minimum fuel controllers that optimize single engine transients currently do not exist.

3. Dynamic optimization

Dynamic optimization [9–11] uses dynamic system models to minimize an objective function, with multiple constraints taken into account. It is used for several off-line tasks such as parameter estimation, model development, optimal control and trajectory optimization. A possible online task is model predictive control (MPC) [12]. Dynamic optimization can solve an optimization problem of the following form:

$$\min_{x(\cdot), u(\cdot)} E(x(T)) + \int_0^T L(x(t), u(t)) dt, \quad (1a)$$

subject to

$$\dot{x}(t) - f(x(t), u(t)) = 0, \quad t \in [0, T], \quad (1b)$$

$$h(x(t), u(t)) \geq 0, \quad t \in [0, T], \quad (1c)$$

$$r_s(x(0)) = 0, \quad (1d)$$

$$r_e(x(T)) = 0. \quad (1e)$$

The optimization determines the optimal control input $u(t)$ and corresponding state variables $x(t)$ for the interval $[0, T]$ that minimize the objective function (1a). The model differential equations are denoted by Eq. (1b). Eq. (1c) is a general nonlinear inequality constraint and Eqs. (1d), (1e) describe start and end point constraints. The final time T may be fixed or subject to optimization. There are three basic approaches to solve an optimization problem of the form (1):

- Hamilton–Jacobi–Carathéodory–Bellman partial differential equation and Dynamic Programming

- Indirect methods: Calculus of Variations, Euler–Lagrange differential equations and the Maximum Principle
- Direct methods, based on a finite dimensional parametrization of the controls

The direct methods are typically easier to use than the other methods and are widely used in practical applications. Most popular are the following three direct methods: single shooting, multiple shooting and collocation. Bock's direct multiple shooting method [13] is used in this paper, because of its favorable treatment of terminal constraints. The software *MUSCOD-II* [14] is used.

4. Powertrain and vehicle model

A system model is necessary for dynamic optimization. This section describes the model of the powertrain and vehicle. The powertrain consists of a gasoline engine, transmission and driveline. Since this paper focusses on the fuel consumption during speed transients, there is no need to model dynamics that are much faster than the longitudinal dynamics of the vehicle.

4.1. Gasoline engine model

The engine model is a mean value phenomenological model [15–19]. This type of model has been designed and successfully used for control applications. The considered engine is a Toyota 1.6l 3ZZ-FE gasoline engine, which is mounted on a fully equipped universal dynamometer. Possible measurements on the test bench are engine speed, effective torque, intake air flow mass, intake manifold pressure and temperature, engine temperature, fuel consumption and ratio of equivalence. The throttle valve of the engine can be controlled. Certain model parameters are determined from a set of steady-state experiments at various engine speeds and throttle valve positions. Table 1 gives the specifications of the gasoline engine.

The engine model has three subsystems: the intake manifold, the fuel delivery and the torque production. Since this paper focusses only on fuel consumption, the exhaust system is not modeled.

Table 1
Specifications of the Toyota 3ZZ-FE gasoline engine.

No. of cylinders and arrangement	4 In-line
Valve mechanism	16-Valve DOHC, chain drive
Displacement	1598 cm ³
Bore × stroke	79.0 mm × 81.5 mm
Compression ratio	10.5
Max. power	81 kW @ 6000 rpm
Max. torque	150 Nm @ 4800 rpm

4.1.1. Intake manifold

The air intake into the manifold is controlled with the throttle valve:

$$\dot{m}_{ai} = \frac{p_a}{\sqrt{rT_a}} A(\alpha) C_d f_n(p_r), \quad (2)$$

where $\dot{m}_{ai}$ (kg/s) is the mass flow of the air intake, r (J/(kg K)) the specific gas constant of air, p_a (Pa) and T_a (K) the atmospheric pressure and temperature, $A(\alpha)$ (m²) the frontal flow area of the throttle body, α (rad) the throttle valve angle, C_d (–) the throttle body discharge coefficient and $p_r = \frac{p_m}{p_a}$ the manifold pressure ratio, where p_m (Pa) is the manifold pressure. $f_n(p_r)$ (Fig. 1) is a function that takes into account possible sonic choking of the air flow through the throttle body:

$$f_n = \begin{cases} \sqrt{\frac{2\kappa}{\kappa-1} \left(p_r^{\frac{2}{\kappa}} - p_r^{\frac{\kappa+1}{\kappa}} \right)}, & \text{if } p_r > p_{r,crit}, \\ \sqrt{\kappa} \left(\frac{2}{\kappa+1} \right)^{\frac{\kappa+1}{2(\kappa-1)}}, & \text{if } p_r \leq p_{r,crit}, \end{cases} \quad (3)$$

where κ (–) is the isentropic exponent and $p_{r,crit} = \left(\frac{2}{\kappa+1} \right)^{\frac{\kappa}{\kappa-1}}$. p_a and T_a are considered constant. C_d is determined experimentally as a function of the throttle valve angle α and the manifold pressure p_m . The air flow $\dot{m}_{ao}$ (kg/s) out of the manifold into the cylinders is determined by the engine speed and the manifold pressure:

$$\dot{m}_{ao} = \eta_v \frac{p_m}{rT_m} V_{tot} \frac{n}{2}, \quad (4)$$

where η_v (–) is the volumetric efficiency, T_m (K) the manifold temperature, V_{tot} (m³) the total cylinder volume and n (rad/s) the engine speed. η_v is determined experimentally as a function of p_m and n . It is considered that $T_m = T_a$. The ideal gas law and conservation of mass yield:

$$\frac{dp_m}{dt} = \frac{rT_m}{V_m} (\dot{m}_{ai} - \dot{m}_{ao}), \quad (5)$$

where V_m (m³) is the volume of the manifold. Eqs. (1b)–(1e) yield:

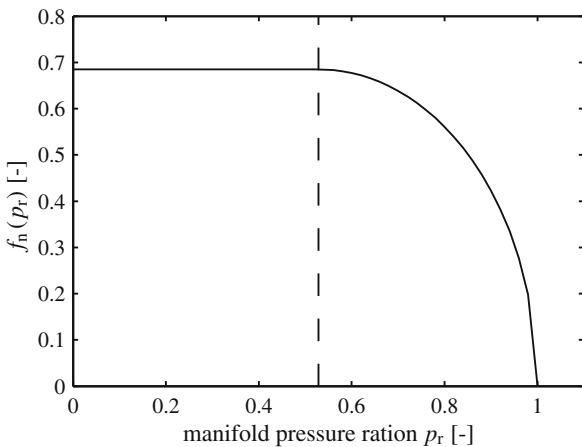


Fig. 1. Function that takes into account sonic choking.

$$\dot{p}_m = \frac{rT_m}{V_m} \frac{p_a}{\sqrt{rT_a}} A(\alpha) C_d f_n \left(\frac{p_m}{p_a} \right) - \frac{\eta_v V_{tot}}{2V_m} n p_m. \quad (6)$$

This equation represents the dynamics of the air in the intake manifold.

4.1.2. Fuel delivery

The amount of injected fuel is normalized by the ratio of equivalence ϕ (–):

$$\phi = \frac{AFR}{AFR_{st}}, \quad (7)$$

where AFR is the real air-fuel ratio and AFR_{st} the stoichiometric air-fuel ratio. ϕ is determined by the engine control unit (ECU). There are two dynamic delays in the fuel intake system: a delay due to the control action of the ECU and a delay due to wall wetting. Experiments on the test bench show that these fuel dynamics are negligible. Although wall wetting is known to introduce significant temperature-dependent dynamics in the fuel delivery [15,17], the ECU control algorithms are clearly well designed and counteract these dynamics. Thus, the fuel mass flow $\dot{m}_f$ (kg/s) into the cylinder is given by:

$$\dot{m}_f = \frac{\dot{m}_{ao}}{\phi AFR_{st}}, \quad (8)$$

where $\dot{m}_f$ and ϕ are determined experimentally as a function of p_m and n . The fuel consumption is measured by timing of the control signal from the ECU to the injectors. The amount of fuel m_f (kg) injected, is determined by the duration t_i (s) of the injection:

$$m_f = t_i c_f(t_i), \quad (9)$$

where c_f (–) is the fuel calibration factor. $c_f(t_i)$ is determined by means of fitting balance fuel measurements to a second order dynamic fuel injection model [20]. This approach gives a statistical probability of 99% that the measured fuel consumption is within a deviation of $\pm 1\%$ of the real consumption.

4.1.3. Torque production

The combustion and torque production is modeled with the effective efficiency η_e (–):

$$T_e = \eta_e \frac{\dot{m}_f Q_v}{2\pi n}, \quad (10)$$

where T_e (Nm) is the effective torque (the torque measured on the engine shaft), Q_v (J/kg) the heating value of gasoline and η_e is determined with experimental tests as a function of p_m and n .

4.2. Transmission and driveline model

The driveline converts the force F_v (N) on the wheels of the vehicle into a load torque T_l (Nm) on the engine shaft. This is modeled by means of algebraic relations:

$$F_v v_v = 2\pi n T_l \eta_m, \quad (11)$$

where v_v (m/s) is the vehicle velocity and η_m (–) the power transmission efficiency of the drivetrain which is considered constant. Further:

$$v_v = R_w \frac{2\pi n}{i_d i_g}, \quad (12)$$

where R_w (m) is the radius of the wheels, i_d (–) the reduction ratio of the differential and i_g (–) the reduction ratio of the gearbox.

4.3. Vehicle model

For evaluating the fuel consumption of vehicles, usually only the longitudinal dynamics are taken into account [21–23]:

Table 2

Parameters of the engine, drivetrain and vehicle model.

$A(\alpha) = 0.00158 - 0.001595 \cos(\alpha)$	$k = 0.002 \frac{s^2}{m^2}$	$V_{tot} = 1.598 \times 10^{-3} m^3$
$AFR_{st} = 14.7$	$M_v = 1365 kg$	$Q_v = 43,700 \frac{kJ}{kg}$
$f_{r0} = 0.015$	$p_a = 101,325 Pa$	$R_w = 0.316 m$
$g = 9.81 \frac{m}{s^2}$	$p_{r,crit} = 0.528$	
$I_D = 4.31$	$r = 287 \frac{J}{kgK}$	$\eta_m = 1$
$i_G = 3.32/2.00/1.36/1.01/0.82$	$SC_x = 1 m^2$	$\kappa = 1.4$
$I_m = 1.0256 kg m^2$	$T_a = 293 K$	$\rho_a = 1.2 \frac{kg}{m^3}$
	$V_m = 2.04 \times 10^{-3} m^3$	$\theta = 0 rad$

$$F_v = \underbrace{M_v \frac{dv_v}{dt}}_{\text{inertia}} + \underbrace{SC_x \frac{\rho_a v_v^2}{2}}_{\text{wind resistance}} + \underbrace{f_r M_v g \cos \theta}_{\text{rolling resistance}} + \underbrace{M_v g \sin \theta}_{\text{road slope}} \quad (13)$$

where M_v (kg) is the mass of the vehicle, S (m^2) the frontal surface of the vehicle, C_x (–) the drag coefficient of the vehicle, ρ_a (kg/m^3) the density of air, $f_r = f_{r0}(1 + kv_v^2)$ (–) the friction coefficient and θ (rad) the slope of the road. Model parameters for the driveline, transmission and vehicle model are based on a Toyota Corolla Verso. Values for these parameters are shown in Table 2.

4.4. Linking of the models

The link between the engine model and the models of the transmission, driveline and vehicle is given by:

$$I_m \frac{dn}{dt} = T_e - T_l, \quad (14)$$

where I_m ($kg m^2$) is the moment of inertia of the powertrain.

To summarize, Eqs. (5), (8)–(14) represent the system as a non-linear state space model. The normalized throttle valve angle u is the control input, with u a linear scaling of the throttle valve angle α : $u = \frac{2\alpha}{\pi}$, thus $0 \leq u \leq 1$ for $0 \leq \alpha \leq \frac{\pi}{2}$. The state variables are the manifold pressure p_m and the engine speed n . Both state variables are measurable. The engine speed n and the fuel mass flow rate $\dot{m}_f$ are the outputs of the system. A validation of the model is given in the next section.

5. Optimization: a case study

This section describes a case study for minimizing the fuel consumption using dynamic optimization. The goal in this case study is to accelerate a vehicle in a given time using the least possible amount of fuel. Dynamic optimization, described in Section 3, is used for optimal control and trajectory optimization; it is used to calculate the trajectory of the state variables of the model and the required control input that result in the minimum fuel consumption. The following equations give the optimization problem of the case study in the standard form (1):

$$\min_{p_m(\cdot), n(\cdot), u(\cdot)} \int_0^T \dot{m}_f(p_m(t), n(t)) dt, \quad (15a)$$

subject to

$$n(0) - n_s = 0, \quad (15b)$$

$$\dot{n}(0) = 0, \quad (15c)$$

$$n(T) - n_e = 0, \quad (15d)$$

$$\dot{n}(T) = 0, \quad (15e)$$

$$n(t) - n_s \geq 0, \quad t \in [0, T], \quad (15f)$$

$$1 \geq u(t) \geq 0, \quad t \in [0, T], \quad (15g)$$

$$\dot{p}_m(t) - f_1(p_m(t), n(t), u(t)) = 0, \quad t \in [0, T], \quad (15h)$$

$$\dot{n}(t) - f_2(p_m(t), n(t)) = 0, \quad t \in [0, T]. \quad (15i)$$

Eq. (15a) requires that the state variables $p_m(t)$, $n(t)$ and the corresponding control input $u(t)$ for $t \in [0, T]$ should be chosen in order to

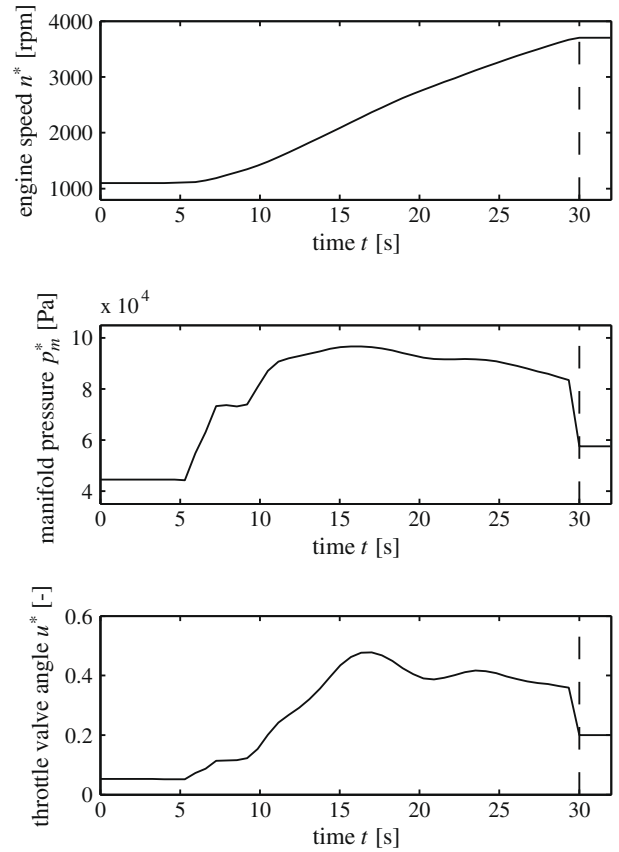


Fig. 2. Solution of the optimal control problem: optimal control and state trajectories.

minimize the integral of the fuel flow rate $\dot{m}_f$. Eqs. (15b)–(15e) specify start and end constraints. Eqs. (15f) and (15g) bound the state variables and control input. Eqs. (15h) and (15i) are the differential equations of the model.

A specific case is presented where $n_s = 1100$ rpm, $n_e = 3700$ rpm and $T = 30$ s. This test case is generic. It is used for showing the capabilities and the potential of dynamic optimization rather than to represent a real-time driving manoeuvre. Fig. 2 shows the solution of the optimization problem as solved by MUSCOD-II. Optimally, the vehicle will start to accelerate after 5 s. At low speeds, the engine consumes less fuel. If the vehicle

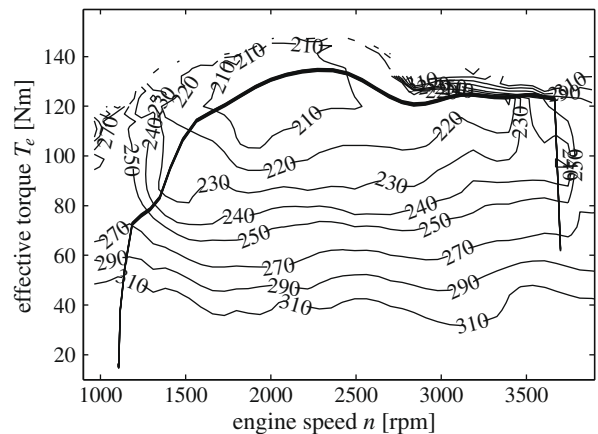


Fig. 3. Solution of the optimal control problem in the specific fuel consumption map.

waits too long to accelerate, the engine has to generate more power to overcome the inertia. This power is produced at lower efficiencies. An interpretation of the shape of the optimal trajectories is given in Fig. 3. This figure shows the solution in the specific fuel consumption (sfc) map, also called performance map. This map is determined with measurements on the dynamometer. In the beginning of the acceleration, the optimal trajectory is almost perpendicular to the iso curves of the specific fuel consumption. At the end of the acceleration, the optimal trajectory avoids rich burning (at high engine speed and torque) and the corresponding high specific fuel consumption.

5.1. Simulation

A sensitivity analysis validates the optimal trajectories. This analysis is done by altering the shape and the start time of the optimal engine speed trajectory. Fig. 4 shows the optimal engine speed trajectory, two trajectories with a slightly different shape and a linear trajectory. A simulation determines the fuel consumption required by these trajectories in the interval between the dashed lines. The fuel consumptions are given in Table 3. The optimal trajectory leads to a fuel consumption of 46.38 g, alternative 1 and 2, respectively to 48.08 g (103.7%) and 48.80 g (105.2%). A linear, more intuitive trajectory, leads to a fuel consumption of 48.67 g (104.9%). Since the optimal engine speed trajectory realizes the same acceleration as the linear trajectory, there is no loss of fun-to-drive. A shape change is considered. A negative shape change stretches the trajectory to higher engine speeds. A positive shape change stretches to lower engine speeds. Fig. 5 shows the relative fuel consumption in function of this shape change.

Fig. 6 shows the optimal engine speed trajectory, two trajectories with a slightly different start time and a wide open throttle trajectory. The fuel consumptions are given in Table 4. The optimal trajectory leads to a fuel consumption of 48.01 g, alternative 3 and 4, respectively to 48.64 g (101.1%) and 51.07 g (106.2%). A wide open throttle trajectory leads to a fuel consumption of 51.74 g

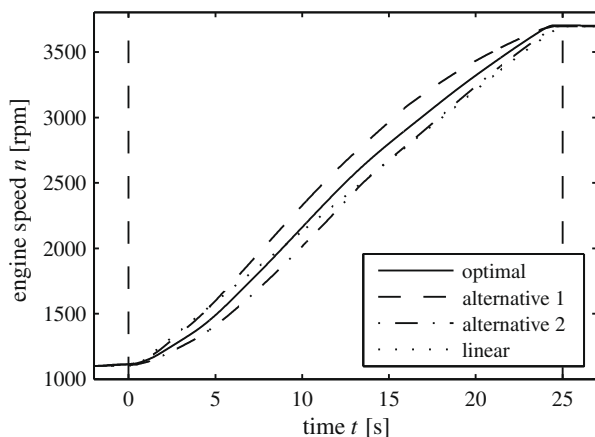


Fig. 4. Optimal engine speed trajectory and trajectories with altered shape in simulation.

Table 3
Fuel consumption in simulations during acceleration with the optimal engine speed trajectory and trajectories with different shape.

Engine speed trajectory	Fuel consumption (g)	Relative fuel consumption (%)
Optimal	46.38	100
Alternative 1	48.08	103.7
Alternative 2	48.80	105.2
Linear	48.67	104.9

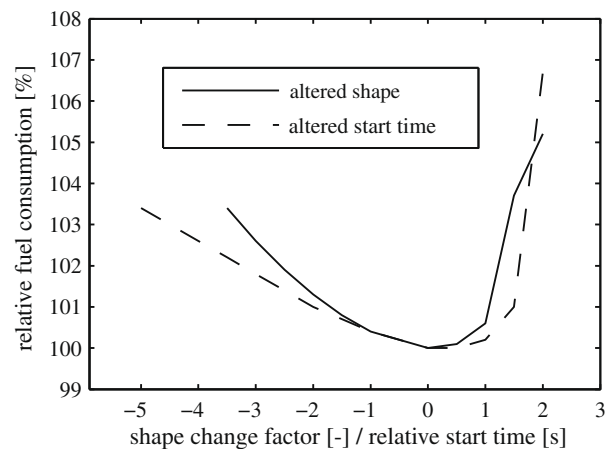


Fig. 5. Influence of the shape and the start time of the engine speed trajectory on the fuel consumption.

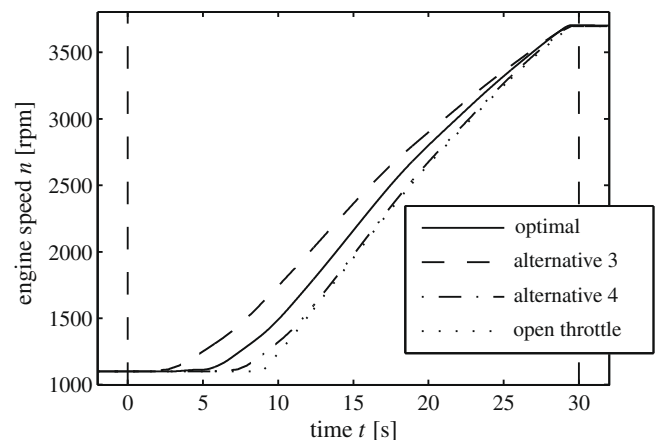


Fig. 6. Optimal engine speed trajectory and trajectories with altered start time in simulation.

Table 4

Fuel consumption in simulations during acceleration with the optimal engine speed trajectory and trajectories with different start time.

Engine speed trajectory	Fuel consumption (g)	Relative fuel consumption (%)
Optimal	48.01	100
Alternative 3	48.64	101.1
Alternative 4	51.07	106.2
Wide open throttle	51.74	107.6

(107.6%). Fig. 5 shows the relative fuel consumption when the start time is altered as in alternative 3 and 4.

5.2. Experiments on the test bench

Fig. 7 shows the speed trajectory both in simulation and in experiment, when the optimal throttle valve angle of the considered control problem is applied open loop. Although there is a fairly good resemblance between the two trajectories, the experimental trajectory is not sufficient since the desired end speed is not attained. Due to this plant-model mismatch, a feedback controller is necessary to ensure good tracking of the optimal speed trajectory in experiments on the engine test bench. Fig. 8 shows the control configuration. It consists of a feedforward of the calculated optimal throttle valve angle $u^*(t)$ and a feedback of

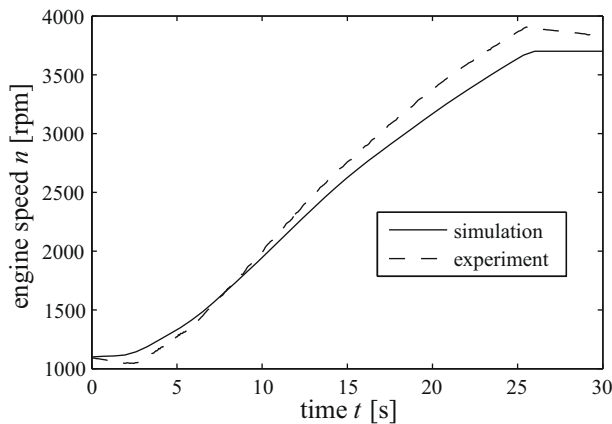


Fig. 7. Validation of the model.

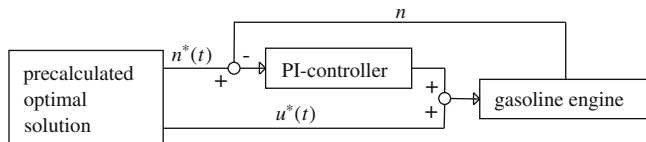


Fig. 8. Control scheme of the experiments.

the difference between the calculated optimal engine speed $n^*(t)$ and the actual engine speed $n(t)$. This feedback controller is a PI controller, experimentally tuned on the dynamometer. It will be shown that this feedback approach is sufficient for fuel consumption minimization. Thus, it is not necessary to use a more complex engine model, which would lead to larger computational burden.

Fig. 9 shows several speed trajectories realized on the dynamometer: the optimal engine speed trajectory and trajectories with an altered shape. The fuel consumptions are given in Table 5. The optimal trajectory leads to a fuel consumption of 44.80 g, alternative 1 and 2, respectively to 46.03 g (102.7%) and 48.31 g (107.8%). A linear, more intuitive driving trajectory, leads to a fuel consumption of 50.72 g (113.2%).

Fig. 10 shows the optimal engine speed trajectory, two trajectories with a slightly different start time and a wide open throttle trajectory. The fuel consumptions are given in Table 6. The optimal trajectory leads to a fuel consumption of 45.12 g, alternative 3 and 4, respectively to 45.59 g (101.0%) and 45.73 g (101.3%). A wide

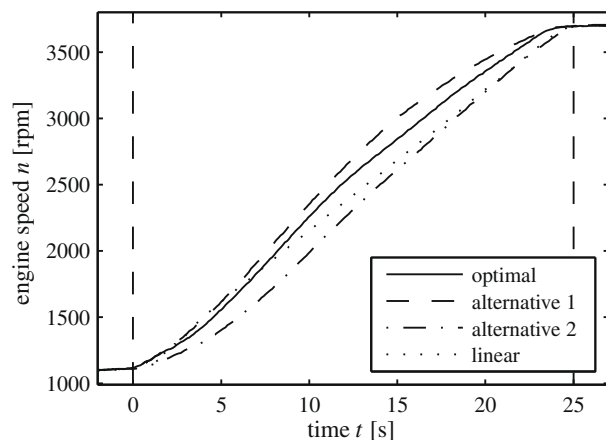


Fig. 9. Optimal engine speed trajectory and trajectories with altered shape in experiment.

Table 5

Fuel consumption in experiments during acceleration with the optimal engine speed trajectory and trajectories with different shape.

Engine speed trajectory	Fuel consumption (g)	Relative fuel consumption (%)
Optimal	44.80	100
Alternative 1	46.03	102.7
Alternative 2	48.31	107.8
Linear	50.72	113.2

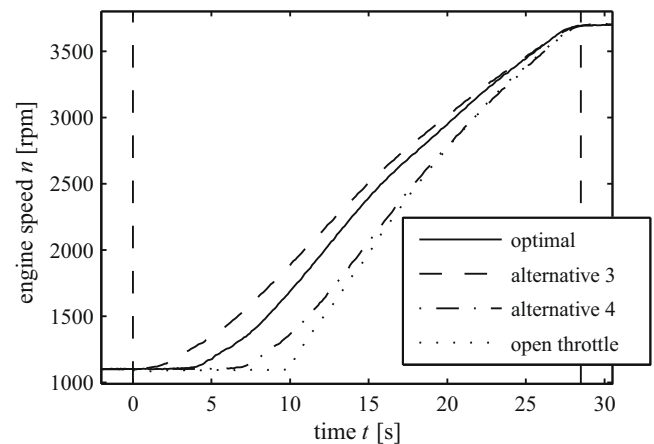


Fig. 10. Optimal engine speed trajectory and trajectories with altered start time in experiment.

Table 6

Fuel consumption in experiments during acceleration with the optimal engine speed trajectory and trajectories with different start time.

Engine speed trajectory	Fuel consumption (g)	Relative fuel consumption (%)
Optimal	45.12	100
Alternative 3	45.59	101.0
Alternative 4	45.73	101.3
Linear	59.01	130.8

open throttle trajectory leads to a fuel consumption of 59.01 g (130.8%).

6. Conclusions

Fuel consumption can be minimized through dynamic optimization, using a simple model for engine, drivetrain and vehicle. This paper presented such a model consisting of two state variables: the intake manifold pressure and the engine speed. The control input is the throttle valve angle. The model is identified on a universal engine dynamometer. Bock's direct multiple shooting method is used to determine optimal trajectories for engine speed and throttle valve angle. Simulations and experimental tests on a dynamometer show that the simple two state model is appropriate for optimization and that the approach used, has great potential for reducing fuel consumption. The achieved fuel savings might seem small, but on a small modern engine, these savings are of the same order that can be achieved with start/stop and regenerative braking and are therefore significant. This paper is the first to describe fuel-optimal trajectories for vehicle accelerations both in computational simulation as on a experimental setup. The direct multiple shooting method for dynamic optimization suggests opportunities for practical applications. This paper has used this method in an off-line approach. The next step is online optimization (e.g. model predictive control), in combination with technologies that are

currently deployed in autonomous cruise control, predictive cruise control and autonomous driving.

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