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Supply chain coordination with stock-dependent demand rate and credit incentives

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ABSTRACT

In this paper, we consider a supply chain which consists of a single manufacturer and a single retailer with a single product type. Demand is assumed to be dependent on the retailer's stock level. Without coordination, the retailer determines its order quantity to maximize its own profit, which is usually smaller than the manufacturer's economic production quantity. Three coordination policies are presented to coordinate the manufacturer's and the retailer's decisions. First, the credit period policy and the quantity discount policy are developed and the total profits under the two policies are compared. Then we develop a centralized supply chain policy and show that there is a unique optimal order quantity to achieve a perfect coordination. The centralized supply chain can get higher or equal channel profit while the credit period policy and the quantity discount policy are easier to achieve. Numerical examples are provided to illustrate the proposed policies.

1. Introduction

In many real-life situations, the demand rate may be affected by the stock level, especially some perishable goods, such as vegetables, fruits, bread, etc. For a high level of inventory attracts more visibility and also may imply that the goods are popular and fresh. Since the perishable goods should be sold out in a short time and produced in small quantity, the manufacturer should often strike the balance between the production efficiency and market demand rate duo to the stock-dependent demand. In the literature, researchers usually used quantity discount policy to persuade retailers to order the quantity more than EOQ. In reality, offering a credit period (delay in payment) to retailers could be more effective for perishable goods. For, in credit period, retailers can earn revenue and save interests, which may alleviate pressure of the fund. There are several advantages of credit period mentioned by Shinn and Hwang (2003) and Sarmah et al. (2007). (i) Credit period can be seen as a means of competition to win over more orders. (ii) It can help to build a good long-term relationship with partners. (iii) Through credit, the manufacturer also shows a commitment of good quality to its customers. (iv) Credit period can also be seen as an important form of financing, especially in developing countries, where the financial service is not quite convenient. Therefore, the credit period policy may perform better than quantity discount policy sometimes. Although credit period is already used by many suppliers or manufacturers to promote market competition, it is still less talked about in the literature. This paper deals with the trade credit mechanism of supply chain with stock-dependent demand. In the following, we briefly review the relevant literature.

Many researchers have considered coordination issues such as replenishment policies and quantity discount schedule between manufacturers and retailers in supply chain management. First, Goyal (1977) considered an integrated inventory model with a single supplier and a single retailer. Rosenblatt and Lee (1985) determined the retailer's order quantity and the supplier's lot size when the supplier offers a linear quantity discount schedule. The above models assumed constant demand rate, which is not influenced by the selling price. In reality, the pricing strategy is also quite important in supply chain management. Weng (1995) considered the quantity discount policy to reduce the supplier's cost and increase the retailer's demand when the demand rate at the retailer's end is price-sensitive. Viswanathan and Wang (2003) considered quantity discounts and volume discounts as coordination mechanisms in distribution channels with a price-sensitive deterministic demand. Later on, many researchers have enriched literature on the problem of coordination issues of replenishment policies and pricing strategies, such as Munson and Rosenblatt (2001), Khouja (2003), Chen and Simchi-Levi (2006), Ouyang et al. (2009), etc.

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All the above works on supply chain coordination concerned the assumption that the demand rate is constant or price-sensitive. As mentioned earlier, the demand rate may also be influenced by the stock level. Whitin (1957) found that the sales and inventory are not independent from each other and high-level inventory may bring about more sales. Levin et al. (1972) and Silver and Peterson (1985) considered that the consumption rate is proportional to the inventory displayed. Baker and Urban (1988) developed a deterministic inventory model in which the demand rate is a polynomial function of instantaneous stock level. Later on, researchers considered more practical issues in the inventory model with stock-dependent demand, such as fixed life time, shortages and deterioration, etc. Mandal and Phaujdar (1989) presented an inventory model for deteriorating items, assuming the demand rate is a linear function of current stock level. Sarker et al. (1997) determined the optimal production cycle when the demand rate is stock-dependent and shortages are backordered. Zhou and Yang (2003) determined the optimal lot-size for the items with a stock-dependent demand rate and a fixed lifetime. Recently, Dye and Ouyang (2005), Wu et al. (2006), Yang et al. (2010) and Sajadieh et al. (2010) enriched literature of inventory model with stock-dependent demand. Zhou et al. (2008) considered quantity discount as the coordination mechanism with stock-dependent demand and showed that the quantity discount policy may also achieve full channel coordination. However, no papers considered trade credit as the coordination mechanism for perishable goods.

In this paper, a single-manufacturer and single-retailer supply chain is considered and the demand rate at the retailer's end is dependent on the instantaneous stock level. The credit period and quantity discount are used as incentives to coordinate the manufacturer's and the retailer's activities. The comparison of credit period and quantity discount policies is made for the manufacturer to choose. The division of surplus profit between the manufacturer and the retailer is also discussed. We also show that the centralized supply chain can always achieve equal or higher channel profit than both credit period and quantity discount policies. Nevertheless, the credit period and quantity discount policies are easier to achieve. The results are illustrated with some numerical data.

2. Model formulation for the supply chain coordination

The following assumptions and notations are used through the whole paper. Additional assumptions and notations are listed when needed.

Assumptions.

(1) The demand rate $D(t)$ at the retailer's end is dependent on the instantaneous stock level $q(t)$, $D(t)=aq(t)^b,\ \ a>0,\ \ 1>b>0$. a is the market scale parameter and b is the elasticity of the demand with respect to the stock level. There are several advantages of this demand pattern mentioned in Baker and Urban's (1988) paper: (i) the marginal increase in demand rate goes down for higher inventory levels, which has already been observed in reality. (ii) The elasticity parameter b can represent the ratio of the change in demand to the change in inventory. And we can expect this function to provide a good approximation with varying values of a and b. (iii) This function is simple and easy to use and the parameters can be easily estimated by regression.
(2) Shortages are not allowed.
(3) The lead time is zero.
(4) The manufacturer follows the lot-for-lot policy.
(5) The retailer replenishes the inventory when all the items are sold out.
(6) The manufacturer bears the transportation cost, which is $e+fQ$. e is the fixed cost per shipment and f is the unit transportation cost.

Notations

p	selling price per unit
w	wholesale price per unit
Q	order quantity (decision variable)
c	manufacturer's production cost per unit
$q(t)$	retailer's stock level at time t
R	manufacturer's production rate
μ	discount rate on the wholesale price
M	credit period that the supplier offers to the retailer
h_r	holding cost per unit per unit time for the retailer
h_m	holding cost per unit per unit time for the manufacturer
T	replenishment cycle length
T_m	manufacturer's production length per cycle
I_r	interest which can be earned per $ per year by the retailer
I_m	interest which can be earned per $ per year by the manufacturer
A_r	retailer's ordering cost
A_m	manufacturer's setup cost
π_r	retailer's average profit
π_m	manufacturer's average profit
π_c	channel's average profit

Consider a supply chain which consists of a single manufacturer and a single retailer. Within each replenishment cycle, the manufacturer produces items at a constant production rate R for T_m, $T_m \le T$, and dispatches them to the retailer at the end of each cycle. The retailer's inventory is depleting at a decreasing rate due to the stock-dependent demand until the inventory becomes zero (see Fig. 1).

Since the demand rate is equal to the decrease in the inventory level, we can describe the retailer's stock level $q(t)$ by the following differential equation:

$$\frac{dq(t)}{dt}=-aq(t)^b,\quad 0\le t\le T. \tag{1}$$

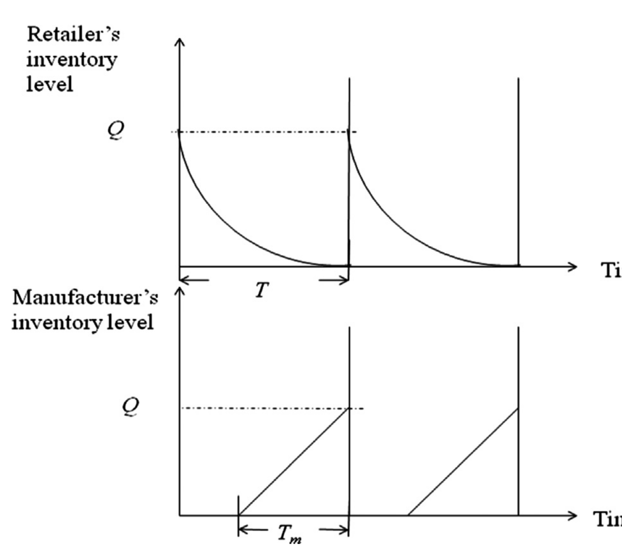

Fig. 1. Retailer's and manufacture's stock levels.

By integrating both sides of Eq. (1), we have

$$\int_0^t -\frac{1}{aq(t)^b}\, dq(t) = \int_0^t d(t). \tag{2}$$

Hence, we get the retailer's stock level at time t,

$$q(t) = [q(0)^{1-b} - a(1-b)t]^{1/1-b}. \tag{3}$$

Since $q(0) = Q$, we get

$$q(t) = [Q^{1-b} - a(1-b)t]^{1/1-b}. \tag{4}$$

At time T, the retailer's inventory level decreases to zero, and we get the replenishment cycle length

$$T = \frac{Q^{1-b}}{a(1-b)}. \tag{5}$$

2.1. Non-coordinated supply chain

Without coordination, the retailer determines order quantity Q to maximize its own average profit. The elements of the retailer's profit are as follows: sales revenue, purchasing cost, ordering cost and holding cost.

The retailer's objective function without coordination is

$$\begin{aligned}\pi_r^{nc} &= \frac{1}{T}\left[(p-w)Q - A_r - h_r \int_0^T q(t)\, dt\right] \\ &= \frac{1}{T}\left\{(p-w)Q - A_r - \frac{h_r}{a(2-b)}\left[Q^{2-b} - [-a(1-b)T + Q^{1-b}]^{(2-b)/(1-b)}\right]\right\}.\end{aligned} \tag{6}$$

Substituting Eq. (5) into Eq. (6), we get

$$\pi_r^{nc} = \frac{a(1-b)}{Q^{1-b}}\left[(p-w)Q - A_r - \frac{h_r}{a(2-b)}Q^{2-b}\right]. \tag{7}$$

Taking the second derivative of π_r^{nc} with respect to Q,

$$\frac{d^2\pi_r^{nc}}{dQ^2} = -a(1-b)^2\left[b(p-w)Q^{b-2} + (2-b)A_r Q^{b-3}\right] < 0. \tag{8}$$

Hence, π_r^{nc} is concave in Q. The optimal order quantity Q^* can be obtained when the first derivative equals zero.

$$\frac{d\pi_r^{nc}}{dQ} = a(1-b)\left[b(p-w)Q^{b-1} - (b-1)A_r Q^{b-2} - \frac{h_r}{a(2-b)}\right] = 0. \tag{9}$$

Substituting Q^* into Eq. (7), we can get the retailer' optimal average profit, π_r^{nc*}.

Without coordination, the manufacturer has to follow the retailer's decision. The manufacturer's profit consists of five elements: sales revenue, production cost, set up cost, transportation cost and holding cost. The manufacturer's objective function is

$$\pi_m^{nc} = \frac{1}{T}\left[(w-c)Q - A_m - (e+fQ) - \tfrac{1}{2}h_m Q T_m\right]. \tag{10}$$

In Eq. (10), $T_m = Q/R$. Substituting Q with Q^*, we can get the manufacturer's average profit, π_m^{nc*}. However, the retailer's optimal order quantity is usually different from the manufacturer's economic production quantity. In most cases, it is larger than the retailer's optimal order quantity.

Substituting Eq. (5) into Eq. (10) gives

$$\pi_m^{nc} = \frac{a(1-b)}{Q^{1-b}}\left[(w-c)Q - A_m - (e+fQ) - \frac{h_m Q^2}{2R}\right]. \tag{11}$$

Taking the second derivative of π_m^{nc} with respect to Q,

$$\frac{d^2\pi_m^{nc}}{dQ^2} = -a(1-b)\left[b(1-b)(w-c-f)Q^{b-2} + (1-b)(2-b)(A_m+e)Q^{b-3} + \frac{b(b+1)h_m}{2R}Q^{b-1}\right] < 0. \tag{12}$$

If π_m^{nc} is concave in Q, we can get the manufacturer's optimal production quantity at $d\pi_m^{nc}/dQ = 0$.

$$\frac{d\pi_m^{nc}}{dQ} = a(1-b)\left[b(w-c-f)Q^{b-1} - (A_m+e)(b-1)Q^{b-2} - \frac{(b-1)h_m}{2R}Q^b\right] = 0. \tag{13}$$

If the manufacturer's economic production quantity obtained by Eq. (13) is larger than the retailer's order quantity Q^*, the manufacturer can get higher profit when the retailer orders more.

2.2. Coordinated supply chain

In this part, two coordination policies are presented, credit period policy and centralized supply chain policy, to coordinate the manufacturer's and the retailer's activities.

2.2.1. Credit period policy

Under coordination, the manufacturer requires the retailer to increase its order quantity that the manufacturer can get higher profit because of lower set up cost and lower transportation cost. However, the retailer may not want to change its current order quantity because it is already optimal. Therefore, the manufacturer should compensate the retailer for its lost profit and probably provide extra savings. In this policy, the supplier offers the retailer an order quantity dependent credit period M, in which the retailer can save interest.

The manufacturer's objective function with offering credit period is

$$\pi_m^{cp} = \frac{1}{T}\left[(w-c)Q - A_m - (e+fQ) - \frac{1}{2R}h_m Q^2 - wQMI_m\right]$$

s.t.

$$\pi_r^{cp} - \pi_r^{nc*} = \frac{1}{T}\left[(p-w)Q - A_r - h_r\int_0^T q(t)\, dt + wQMI_r\right] - \pi_r^{nc*} \geq 0. \tag{14}$$

The constraint condition makes sure the retailer gets no less profit than no coordination case.

Proposition 1. When $\pi_r^{cp} - \pi_r^{nc*} = 0$ there exists a unique optimal solution Q^*, at which π_m^{cp} is maximized.

Proof. Obviously, $\pi_r^{cp} - \pi_r^{nc*} = 0$ is the lowest level that the manufacturer has to give to make the retailer accept the new policy.

Hence, we can get M, which is a function of Q, from $\pi_r^{cp} - \pi_r^{nc*} = 0$.

$$M(Q) = \frac{\pi_r^{nc*}}{wI_r}\frac{Q^{-b}}{a(1-b)} + \frac{A_r}{wQI_r} + \frac{h_r Q^{1-b}}{a(2-b)wI_r} - \frac{p-w}{wI_r}. \tag{15}$$

Substituting Eq. (15) into π_m^{cp}, we get

$$\begin{aligned}\pi_m^{cp} = a(1-b)\Bigg[&(w-c-f)Q^b - (A_m+e)Q^{b-1} - \frac{h_m}{2R}Q^{b+1} \\ &- \frac{\pi_r^{nc*} I_m}{a(1-b)I_r} - \frac{A_r I_m}{I_r}Q^{b-1} - \frac{h_r I_m}{a(2-b)I_r}Q + \frac{(p-w)I_m}{I_r}Q^b\Bigg].\end{aligned} \tag{16}$$

Taking the first and second derivative of π_m^{cp} with respect to Q, we can get the optimal Q^* by $d\pi_m^{cp}/dQ = 0$.

$$\begin{aligned}\frac{d\pi_m^{cp}}{dQ} = a(1-b)\Bigg[&b\left(w-c-+\frac{(p-w)I_m}{I_r}\right)Q^{b-1} \\ &-(b-1)\left(A_m + e + \frac{A_r I_m}{I_r}\right)Q^{b-2} - \frac{(1+b)h_m}{2R}Q^b - \frac{h_r I_m}{a(2-b)I_r}\Bigg] = 0\end{aligned} \tag{17}$$

$$\frac{d^2\pi_m^{cp}}{dQ^2} = -a(1-b)\left[b(1-b)\left(w-c-f+\frac{(p-w)I_m}{I_r}\right)Q^{b-2}\right.$$

$$+(1-b)(2-b)\left(A_m+e+\frac{A_rI_m}{I_r}\right)Q^{b-3}+\frac{b(1+b)h_m}{2R}Q^{b-1}\Bigg]<0. \quad (18)$$

Substituting Q^* into π_m^{cp}, we can get the manufacturer's optimal profit π_m^{cp*}.

Under coordination, the manufacturer can get higher profit if the retailer still gets the same profit as no coordination case. However, the retailer may not be satisfied. Therefore, the retailer requires having a negotiation with the manufacturer for higher profit. With different order quantities, the retailer's demand rates are different due to stock dependent demand. As a result, it is quite complicated to discuss both order quantity and credit period length. The average profit is the fundamental interests to both parties, so having a direct negotiation on the average profit is a good way. We assume the retailer and the manufacturer have come to an agreement that the retailer gets $\Delta\pi_r$ higher than its average profit without coordination.

$$\pi_r^{cp}-\pi_r^{nc*}=\frac{1}{T}\left[(p-w)Q-A_r-h_r\int_0^T q(t)\,dt+wQMI_r\right]-\pi_r^{nc*}=\Delta\pi_r. \quad (19)$$

Given $\Delta\pi_r$, we can get that M is a function of Q,

$$M(Q)=\frac{\pi_r^{nc*}+\Delta\pi_r}{wI_r}\frac{Q^{-b}}{a(1-b)}+\frac{A_r}{wQI_r}+\frac{h_rQ^{1-b}}{a(2-b)wI_r}-\frac{p-w}{wI_r}. \quad (20)$$

Proposition 2. The maximum $\Delta\pi_r$ that the retailer gets cannot exceed

$$\overline{\Delta\pi_r}=\frac{a(1-b)I_r}{I_m}\Bigg[(w-c-f)Q^{*b}-(A_m+e)Q^{*b-1}-\frac{h_m}{2R}Q^{*1+b}-\frac{A_rI_m}{I_r}Q^{*b-1}-\frac{h_rI_m}{a(2-b)I_r}Q^{*b}+\frac{(p-w)I_m}{I_r}Q^{*b}-\frac{\pi_m^{nc*}}{a(1-b)}\Bigg]$$

Proof. Substituting Eq. (20) into π_m^{cp}, we get the manufacturer's objective function

$$\pi_m^{cp}=a(1-b)\Bigg[(w-c-f)Q^b-(A_m+e)Q^{b-1}-\frac{h_m}{2R}Q^{b+1}-\frac{(\pi_r^{nc*}+\Delta\pi_r)I_m}{a(1-b)I_r}-\frac{A_rI_m}{I_r}Q^{b-1}-\frac{h_rI_m}{a(2-b)I_r}Q+\frac{(p-w)I_m}{I_r}Q^b\Bigg]. \quad (21)$$

Taking the first and second derivatives of π_m^{cp} with respect to Q, we can get the optimal Q^* by $d\pi_m^{cp}/dQ=0$.

$$\frac{d\pi_m^{cp}}{dQ}=a(1-b)\Bigg[b\left(w-c-f+\frac{(p-w)I_m}{I_r}\right)Q^{b-1}-(b-1)\left(A_m+e+\frac{A_rI_m}{I_r}\right)Q^{b-2}-\frac{(1+b)h_m}{2R}Q^b-\frac{h_rI_m}{a(2-b)I_r}\Bigg]=0 \quad (22)$$

$$\frac{d^2\pi_m^{cp}}{dQ^2}=-a(1-b)\Bigg[b(1-b)\left(w-c-f+\frac{(p-w)I_m}{I_r}\right)Q^{b-2}+(1-b)(2-b)\left(A_m+e+\frac{A_rI_m}{I_r}\right)Q^{b-3}+\frac{b(1+b)h_m}{2R}Q^{b-1}\Bigg]<0. \quad (23)$$

We can see that there is no $\Delta\pi_r$ in Eq. (22), so $\Delta\pi_r$ does not change the solution. The optimal production quantity is still Q^* obtained by Eq. (17).

Therefore, the minimal profit the retailer can get is $\underline{\Delta\pi_r}=\pi_r^{cp}-\pi_r^{nc*}=0$. And the maximum profit the retailer can get is $\overline{\Delta\pi_r}=\Delta\pi_r:\pi_m^{cp}-\pi_m^{nc*}=0$.

$$\pi_m^{cp}=a(1-b)\Bigg[(w-c-f)Q^{*b}-(A_m+e)Q^{*b-1}-\frac{h_m}{2R}Q^{*1+b}-\frac{(\pi_r^{nc*}+\Delta\pi_r)I_m}{a(1-b)I_r}-\frac{A_rI_m}{I_r}Q^{*b-1}-\frac{h_rI_m}{a(2-b)I_r}Q^{*b}+\frac{(p-w)I_m}{I_r}Q^{*b}\Bigg]=\pi_m^{nc*}. \quad (24)$$

From Eq. (24), we can get

$$\overline{\Delta\pi_r}=\frac{a(1-b)I_r}{I_m}\Bigg[(w-c-f)Q^{*b}-(A_m+e)Q^{*b-1}-\frac{h_m}{2R}Q^{*1+b}-\frac{A_rI_m}{I_r}Q^{*b-1}-\frac{h_rI_m}{a(2-b)I_r}Q^{*b}+\frac{(p-w)I_m}{I_r}Q^{*b}-\frac{\pi_m^{nc*}}{a(1-b)}\Bigg]. \quad (25)$$

2.2.2. Centralized supply chain

In this part, we assume the manufacturer and the retailer are willing to behave as an integrated firm. They determine the order quantity to maximize the channel profit together.

The objective function of the whole supply chain is

$$\pi_c^{cs}=\frac{1}{T}\Bigg[(p-c)Q-A_m-(e+fQ)-\frac{1}{2R}h_mQ^2-A_r-\frac{h_r}{a(2-b)}Q^{2-b}+wQM(I_r-I_m)\Bigg]. \quad (26)$$

Taking the second derivative of π_c^{cs} with respect to Q, we get

$$\frac{d^2\pi_c^{cs}}{dQ^2}=-a(1-b)\Bigg[b(1-b)(p-c-f)Q^{b-1}+(1-b)(2-b)(A_m+A_r+e)Q^{b-3}+\frac{b(b+1)}{2R}h_mQ^b+b(1-b)Q^{b-2}wM(I_r-I_m)\Bigg]. \quad (27)$$

In Eq. (26), M is an unlimited variable. When $I_r-I_m=0$, there is no M in the objective function, M can be any value. When $I_r-I_m>0$, the channel profit π_c^{cs} increases as M goes up, thus, the optimal M is more than zero, $M=M_{\max}$. When $I_r-I_m<0$, if we define M must be no less than zero, then $M=0$. If M can be less than zero, which means the retailer can make payment in advance for some time, then $M=M_{\min}$. Therefore, $d^2\pi_c^{cs}/dQ^2<0$, the channel profit is concave in Q once M is given. The optimal order quantity can be obtained when the first derivative of π_c^{cs} equals zero.

$$\frac{d\pi_c^{cs}}{dQ}=a(1-b)\Bigg[b(p-c-f)Q^{b-1}-(b-1)(A_m+A_r+e)Q^{b-2}-\frac{b+1}{2R}h_mQ^b-\frac{h_r}{a(2-b)}+bQ^{b-1}wM(I_m-I_r)\Bigg]=0. \quad (28)$$

3. Comparison of the credit period and quantity discount policies for coordinated supply chain

In this part, we introduce the quantity discount policy and make a comparison of the quantity discount and credit period policies.

3.1. Quantity discount policy

Quantity discount is a usually used policy for a manufacturer to encourage a retailer to increase its order quantity. When the retailer's order quantity is larger than EOQ, the manufacturer compensates the retailer's lost profit and possibly provides extra savings by offering the retailer a wholesale price discount.

The manufacturer's objective function with offering quantity discount is

$$\pi_m^{qd}=\frac{1}{T}\left[(w-c)Q-A_m-(e+fQ)-\frac{1}{2R}h_mQ^2-\mu wQ\right]$$

s.t.

$$\pi_r^{qd}-\pi_r^{nc*}=\frac{1}{T}\left[(p-w)Q-A_r-h_r\int_0^T q(t)\,dt+\mu wQ\right]-\pi_r^{nc*}=\Delta\pi_r. \quad (29)$$

In Eq. (29), $\Delta\pi_r \geq 0$, the constraint condition makes sure the retailer gets no less profit than no coordination case. From the constraint condition, we can get the discount rate μ, which is a function of Q.

$$\mu(Q)=\frac{1}{wQ}T(\pi_r^{nc*}+\Delta\pi_r)-\frac{1}{w}(p-w)+\frac{1}{wQ}A_r+\frac{1}{wQ}h_r\int_0^T q(t)\,dt. \quad (30)$$

Substituting Eq. (30) into π_m^{qd}, the manufacturer's objective function becomes

$$\pi_m^{qd}=a(1-b)\left[(p-c-f)Q^b-(A_m+e)Q^{b-1}-\frac{h_m}{2R}Q^{1+b}-\frac{\pi_r^{nc*}+\Delta\pi_r}{a(1-b)}-A_rQ^{b-1}-\frac{h_r}{a(2-b)}Q\right]. \quad (31)$$

Taking the first and second derivatives of π_m^{qd} with respect to Q, we can get the optimal Q^* by $d\pi_m^{qd}/dQ=0$.

$$\frac{d\pi_m^{qd}}{dQ}=a(1-b)\left[b(p-c-f)Q^{b-1}-(b-1)(A_m+A_r+e)Q^{b-2}-\frac{b+1}{2R}h_mQ^b-\frac{h_r}{a(2-b)}\right]=0 \quad (32)$$

$$\frac{d\pi_m^{qd}}{dQ}=-a(1-b)\left[b(1-b)(p-c-f)Q^{b-1}+(1-b)(2-b)(A_m+A_r+e)Q^{b-3}+\frac{b(b+1)}{2R}h_mQ^b\right]<0. \quad (33)$$

Substituting Q^* into Eq. (31), we can get the manufacturer's optimal average profit π_m^{qd*} when $\pi_r^{qd}-\pi_r^{nc*}=0$. From Eq. (32) we can see the increase in $\Delta\pi_r$ just equals the decrease in π_m^{qd}. Therefore, the surplus profit under the quantity discount policy (compared to no coordination case) equals $\overline{\Delta\pi_r}=\pi_m^{qd*}-\pi_m^{nc*}$.

3.2. Comparison of the credit period and quantity discount policies

From Eqs. (16) and (31), we can see that quantity discount and credit period policies are quite similar to each other. They just use different ways to compensate the retailer. Let $\Delta\pi_m$ equals the difference between the manufacturer's objective function of credit period policy and quantity discount policy, $\Delta\pi_m=$ Eqs. (16)–(31)$=\pi_m^{cp}-\pi_m^{qd}$, we get

$$\Delta\pi_m=a(1-b)\left[(p-w)Q^b-A_rQ^{b-1}-\frac{h_r}{a(2-b)}Q-\frac{\pi_r^{nc*}+\Delta\pi_r}{a(1-b)}\right]\left(\frac{I_m}{I_r}-1\right) \quad (34)$$

Obviously, $(p-w)Q^b-A_r-(h_r/a(2-b))-(\pi_r^{nc*}+\Delta\pi_r/a(1-b))<0$, because it equals $-wQ^bMI_r$.

Proposition 3. When $I_r>I_m$, the manufacturer prefers credit period policy to quantity discount policy. When $I_r<I_m$, the manufacturer prefers quantity discount policy. When $I_r=I_m$, the two policies are the same to the manufacturer.

Proof. When $I_r>I_m$, $\Delta\pi_m>0$. $\Delta\pi_m>0$ means the manufacturer can get higher profit under credit period policy with the assumption that the retailer gets the same profit under the two policies. When $I_r<I_m$, $\Delta\pi_m<0$, the manufacturer can get higher profit under quantity discount policy. When $I_r=I_m$, $\Delta\pi_m=0$, the manufacturer gets the same profit.

In real commerce, the manufacturers could use production facilities and plants as collateral to get cheap loans. On the other hand, the retailers are relatively difficult to get loans, but they usually have a higher profit margin than manufacturers, which makes the credit period policy possible in many situations.

4. Numerical experiments

In order to test how these policies perform with various parameter values, numerical experiments are provided in this section. Of all the parameters, the demand parameter and interest rate have a major impact on the order quantity and policy performance. Therefore, we emphasize on demand elasticity parameter b and retailer's interest rate I_r.

First, we consider a case with the following data to compare the non-coordinated supply chain and credit period policy. In credit period policy, we assume the manufacturer gets all the surplus profit to calculate the channel profit.

$a=120,\ p=30,\ w=22,\ c=15,\ h_r=8,\ h_m=5,$
$A_m=100,\ A_r=50,\ R=6000,\ I_r=0.3,$
$I_m=0.1,\ e=30,\ f=2$

Figs. 2 and 3 show that as b increases the order quantity and the manufacturer's and retailer's profits grow more swiftly, that is because as demand becomes more sensitive to the inventory level, the credit period policy can bring much more demand and more profit. The figures also show that the increase in profit is not as remarkable as the increase in order quantity, which is because the model is robust like EOQ and the profit is relatively not sensitive to order quantity. Although the increase in profit is smaller, the demand is enlarged. Therefore, the manufacturer can extend market share under credit period policy. When $b=0$, the demand

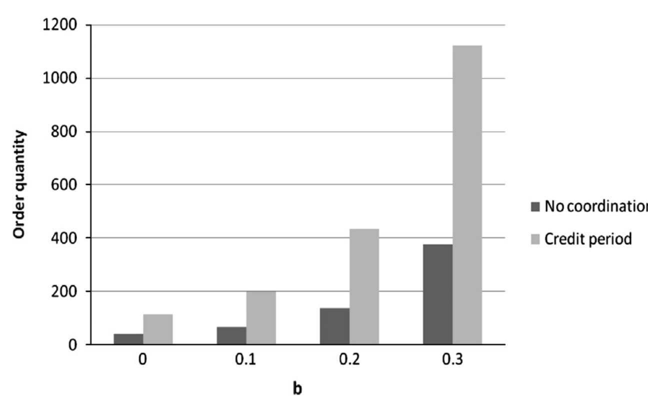

Fig. 2. Order quantity under credit period policy and without coordination with different b.

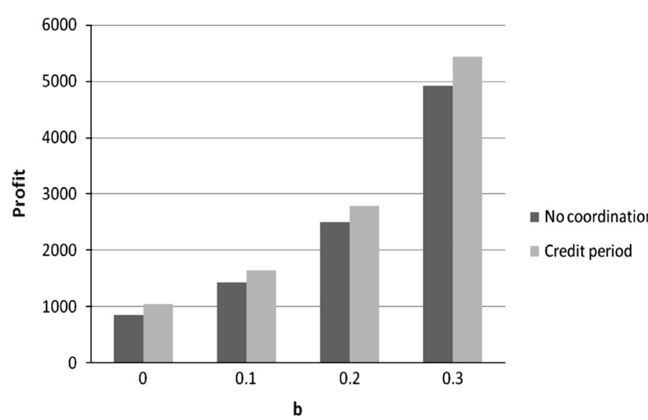

Fig. 3. Channel profit under credit period policy and without coordination with different b.

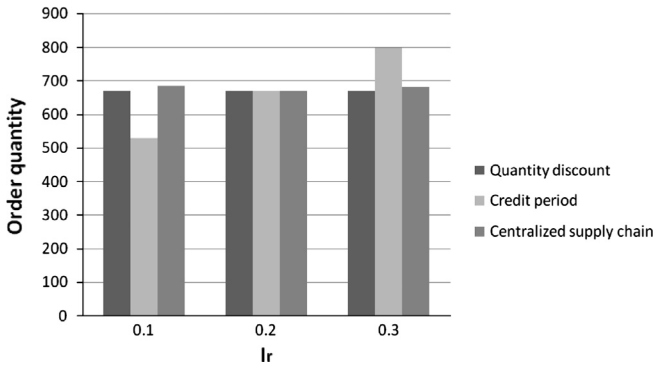

Fig. 4. Order quantity under quantity discount, credit period and centralized supply chain policy with different I_r.

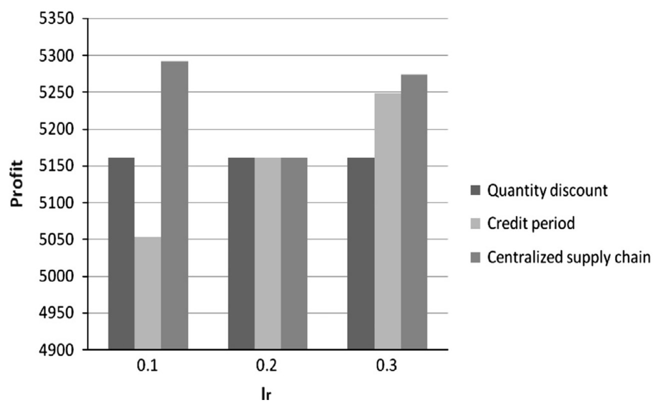

Fig. 5. Channel profit under quantity discount, credit period and centralized supply chain policy with different I_r.

is constant. Then we can see that the credit period policy still works when the demand is constant.

In the second case, we compare the quantity discount, credit period and centralized supply chain policies with different I_r. Assume the manufacturer gets all the surplus profit and the retailer gets the same profit as no coordination case in the quantity discount and credit period policies. For M is an unlimited variable in the centralized supply chain, when $I_r > I_m$, let M equals the value of M in credit period policy; when $I_r < I_m$, let $M = -0.1$. When $I_r = I_m$, there is no M in the channel's profit.

$$a = 120,\ \ b = 0.3,\ \ p = 30,\ \ w = 22,\ \ c = 15,\ \ h_r = 8,$$

$$h_m = 5,\ \ A_m = 100,\ \ A_r = 50,\ \ R = 6000,$$

$$e = 30,\ \ f = 2,\ \ I_m = 0.2$$

Fig. 4 shows that as I_r increases, the order quantity also goes up in credit period policy, which means more demands are generated (compared with quantity discount policy) when the credit period policy is applied (when $I_r > I_m$). That is because the retailer can get longer credit period when orders more and with more orders, the demand goes up duo to the stock-dependent demand. In Fig. 5, we can see that the centralized supply chain always gets no less channel profit than in credit period and quantity discount policies. And the channel profit goes up as $|I_r - I_m|$ increases, which is because the centralized supply chain can achieve a perfect coordination and the channel can benefit from the higher interest rate duo to coordination. When $I_r = I_m$, all three policies achieve the same channel profit. Therefore, if we do not consider the interest issue or $I_r = I_m$, the quantity discount policy can achieve a perfect coordination, too.

5. Conclusions

In this paper, three coordination policies: credit period, quantity discount and centralized supply chain are discussed. The final consumption rate is dependent on the retailer's stock level. Without coordination, the retailer makes policy to maximize its own profit. Usually, the retailer's economic order quantity is less than the manufacturer's optimal production quantity. Therefore, if the loss that the retailer suffers with increased order quantity is less than the manufacturer's increased profit, the manufacturer could offer a credit period/quantity discount as an incentive to make the retailer increase its current order quantity. When the retailer's interest rate is higher than the manufacturer's, the manufacturer prefers the credit period policy; otherwise the manufacturer prefers the quantity discount policy. No matter whose interest is higher, the centralized supply chain can always get equal or higher channel profit than credit period and quantity discount policies. While the centralized supply chain needs a close coordination that the manufacturer and retailer should behave as an integrated company. Therefore, if the centralized supply chain cannot be achieved, the credit period/quantity discount policy is also a good choice.

This paper can be extended in several ways. First, we could change the stock-dependent deterministic demand into stochastic demand. Second, more coordination policies, such as buy-back, quantity-flexibility and franchise policies could be talked about. Third, the current model could be extended to the case of one supplier and multiple retailers.

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References

Baker, R.C., Urban, T.L., 1988. A deterministic inventory system with an inventory-level-dependent demand rate. Journal of the Operational Research Society 39 (9), 823–831.

Chen, X., Simchi-Levi, D., 2006. Coordinating inventory control and pricing strategies: the continuous review model. Operations Research Letters 34 (3), 323–332.

Dye, C.Y., Ouyang, L.Y., 2005. An EOQ model for perishable items under stock-dependent selling rate and time-dependent partial backlogging. European Journal of Operational Research 163 (3), 776–783.

Goyal, S.K., 1977. An integrated inventory model for a single supplier single customer problem. International Journal of Production Research 15 (1), 107–111.

Khouja, M., 2003. Optimizing inventory decisions in a multistage multi customer supply chain. Transportation Research Part E 39 (3), 193–208.

Levin, R.I., McLaughlin, C.P., Lamone, R.P., Kottas, J.F., 1972. Production/Operations Management: Contemporary Policy for Managing Operation Systems. McGraw-Hill, New York p. 373.

Mandal, B.N., Phaujdar, S., 1989. An inventory model for deteriorating items and stock-dependent consumption rate. Journal of the Operational Research Society 40 (5), 483–488.

Munson, C.L., Rosenblatt, M.J., 2001. Coordinating a three level supply chain with quantity discounts. IIE Transactions 33 (4), 371–384.

Ouyang, L.Y., Ho, C.H., Su, C.H., 2009. An optimization approach for joint pricing and ordering problem in an integrated inventory system with order-size dependent trade credit. Computers and Industrial Engineering 57 (3), 920–930.

Rosenblatt, M.J., Lee, H.L., 1985. Improving profitability with quantity discounts under fixed demand. IIE Transactions 17 (4), 388–395.

Sajadieh, M.S., Thorstenson, A, Jokar, M.R.K., 2010. An integrated vendor–buyer model with stock-dependent demand. Transportation Research Part E 46 (6), 963–974.

Sarker, B.R., Mukherjee, S., Balan, C.V., 1997. An order-level lot size inventory model with inventory-level dependent demand and deterioration. International Journal of Production Economics 48 (3), 227–236.

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Sarmah, S.P., Acharya, D., Goyal, S.K., 2007. Coordination and profit sharing between a manufacturer and a buyer with target profit under credit option. European Journal of Operational Research 182 (3), 1469–1478.

Shinn, W.S., Hwang, H., 2003. Optimal pricing and ordering policies for retailers under order size dependent delay in payments. Computers and Operations Research 30 (1), 35–50.

Silver, E.A., Peterson, R., 1985. Decision systems for inventory management and production planning, Second ed.. John Wiley, NewYork.

Viswanathan, S., Wang, Q., 2003. Discount pricing decision in distribution channels with price-sensitive demand. European Journal of Operational Research 149 (3), 571–587.

Weng, Z.K., 1995. Modeling quantity discount under general price-sensitive demand functions: optimal policies and relations. European Journal of Operational Research 86 (2), 300–314.

Whitin, T.M., 1957. The Theory of Inventory Management. Princeton University Press, New Jersey.

Wu, K.S., Ouyang, L.Y., Yang, C.T., 2006. An optimal replenishment policy for non-instantaneous deteriorating items with stock-dependent demand and partial backlogging. International Journal of Production Economics 101 (2), 369–384.

Yang, H.L., Teng, J.T., Chern, M.S., 2010. An inventory model under inflation for deteriorating items with stock-dependent consumption rate and partial backlogging shortages. International Journal of Production Economics 123 (1), 8–19.

Zhou, Y.W., Min, J., Goyal, S.K., 2008. Supply chain coordination under an inventory-level-dependent demand rate. International Journal of Production Economics 113 (2), 518–527.

Zhou, Y.W., Yang, S.L., 2003. An optimal replenishment policy for items with inventory-level-dependent demand and fixed lifetime under the LIFO policy. Journal of the Operational Research Society 54 (6), 585–593.

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