Project Planning and Control
Guest Lecture

Monte Carlo Simulation

Application to Scheduling in Project Management

Dr. Markus Dann

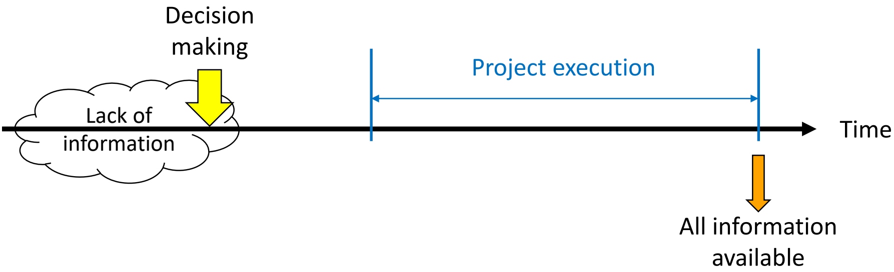
Program evaluation and review technique (PERT)

Some quantities of interest:

- Probability that the project will finish in 61 days?
- Probability that the project will finish in 58 days?
- Probability that the project will be 3 days longer than expected?
- Probability that the project will finish at least 4 days earlier than expected?
- Completion date to finish with a 95% confidence level?

Why are projects under / over budget and ahead / behind schedule?

Decision making under uncertainty



Motivation to study numerical simulation techniques

Real-world problems usually lead to complex probabilistic models for their analysis

Closed-form solutions are limited to (extremely) simple cases

Numerical simulation:

- Commonly referred to as Monte Carlo (MC) simulation
- Approach: Simulate n possible scenarios of the considered problem
- Based on the frequentist interpretation of probability ($n \to \infty$)
- Versatile approach to estimate probabilities

Frequentist interpretation of probability

Probability $\Pr(A)$ is interpreted as the relative frequency of occurrence of event A as observed in an experiment with $n_{exp} \to \infty$ trials:

$$\Pr(A) = \lim_{n_{exp} \to \infty} (n_A / n_{exp})$$

where n_A is the number of experiments where event A occurred.

Idea behind numerical simulation:

- Generate a large number n of possible realizations of the problem
- Count the number $n_A \leq n$ of realizations of event A that is of interest
- Determine the probability $\Pr(A) \approx n_A / n$

Generate random samples

To make numerical sampling applicable for a variety of problems it is necessary to obtain samples for any type of probability distribution.

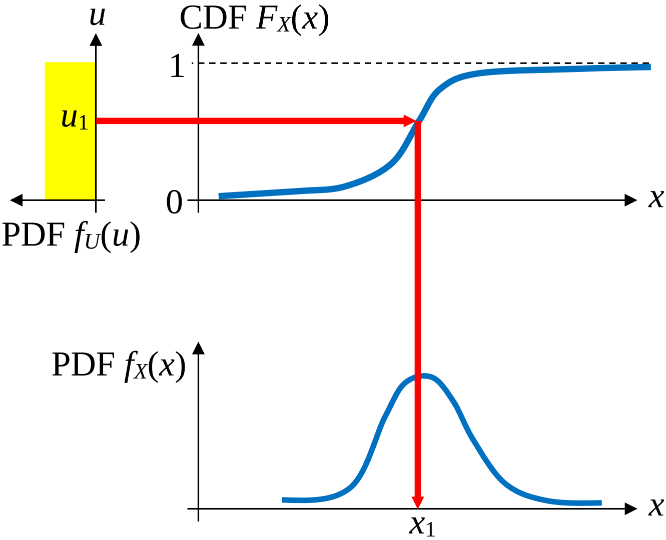
Standard random number generators provide samples from a uniform distribution in the interval [0, 1].

Many software tools have built-in random number generators for a variety of probability distributions.

Methods to generate random samples:

- Inverse transformation algorithm
- Acceptance-rejection method
- Polar method

Inverse transformation algorithm



Example 1: Introduction

$1 \to 2 \to 3 \to 4 \to 5 \to 6 \to 7 \to 8 \to 9$

Activity	Mean value	Variance
1	3.0	0.44
2	6.0	1.00
3	13.5	2.25
4	8.5	2.25
5	3.5	1.36
6	10.0	1.23
7	7.5	2.25
8	6.0	1.00
9	3.5	1.36

PERT: mean value $= 61.5$, variance $= 13.14$

Example 1: MC analysis & results

Assumptions:

1. All events are independent
2. Duration of each activity follows a normal distribution

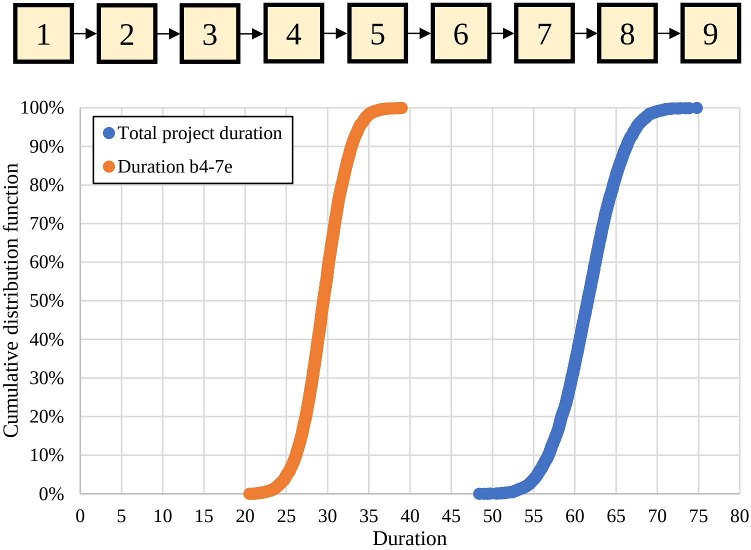
Sample size: $n = 5,000$

Analysis: see Excel file

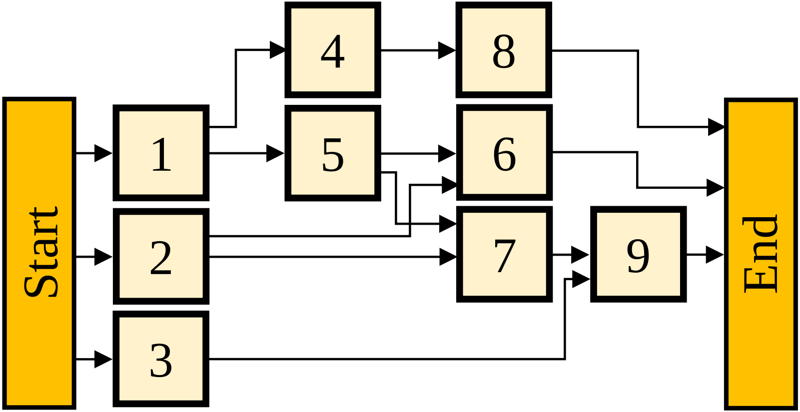
Comparison of PERT and MC results:

Statistic	PERT	MC
Mean value	61.5	61.6
Variance	13.1	13.2

Example 1: Results



Example 2: Introduction



PERT results:

- Mean value = 17.5
- Variance = 3.69 and 5.41

Example 2: MC analysis & results

Assumptions:

1. All events are independent
2. Duration of each activity follows a normal distribution

Sample size: $n = 10,000$

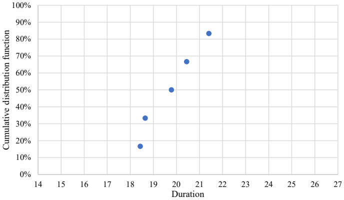
Analysis: see Excel file

Comparison of PERT and MC results:

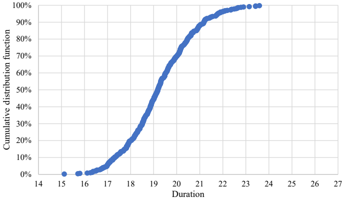
Statistic	PERT	MC
Mean value	17.5	19.2
Variance	3.7 and 5.4	2.4

Example 2: Total project duration

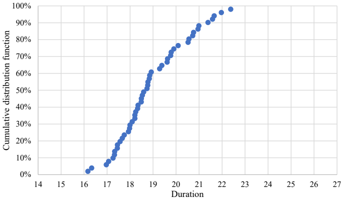
$n = 5$



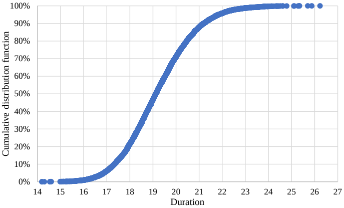
$n = 500$



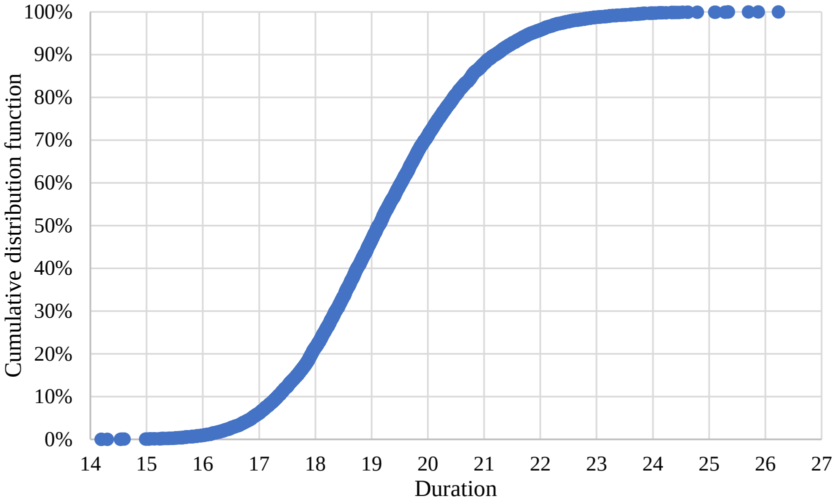
$n = 50$



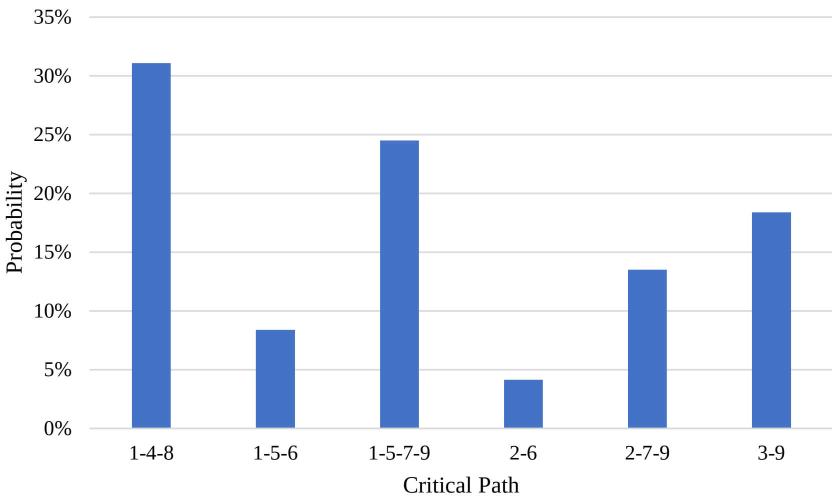
$n = 10,000$



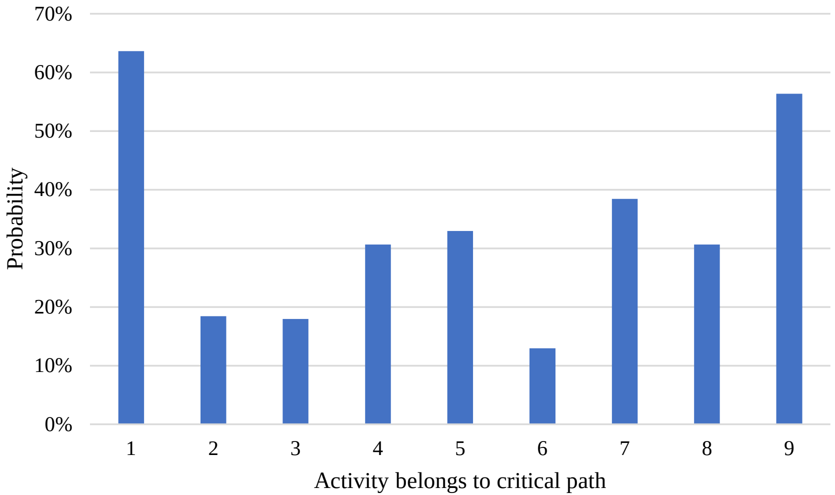
Example 2: Total project duration



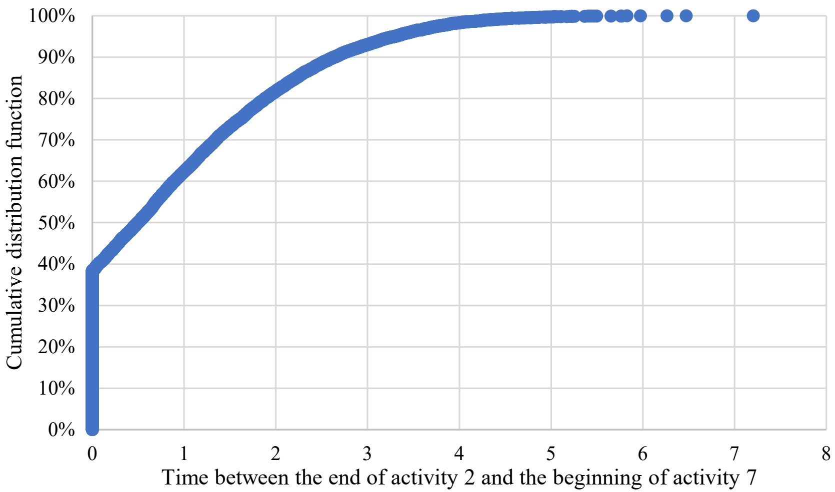
Example 2: Critical path



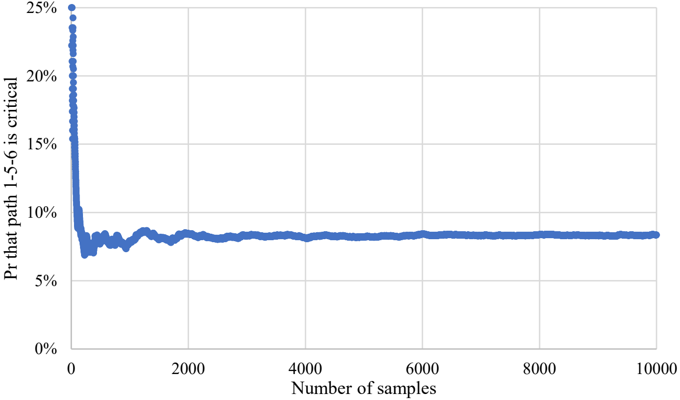
Example 2: Critical activity



Example 2: Time between the end of activity 2 and the beginning of activity 7



How do we know that there are enough samples?



Required sample size depends on the complexity of the problem:

- Monitor the results of interest as a function of sample size
- Results need to be "stable"

MC simulation: Summary

MC simulation is a very versatile tool to solve (complex) probabilistic models / schedules.

The accuracy of the MC results increases with the sample size.

If possible, use built-in random number generators; check the parameterization of the distribution function.

There is no such a thing as a purely random sample. Set the seed to avoid sampling variations between runs.

References

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