

Notes:

- You have to write your code for the practical problems in python.
- You are free to use any libraries unless said otherwise, i.e., said to implement.
- Delivering a Jupyter notebook is enough, if you want to deliver your codes in python code, then you also need to write a document as report for the plots and explanations requested.
- For each day of delay, a penalty of 25% is imposed.

If you have any questions, *please feel free to contact me*

Problem 1:

In this problem, we will take a practical approach toward the subject of Fourier transform. What we have discussed so far is the Fourier transform of continuous-time signals, which is only available in analog devices, and when it comes to computers, there is no way to truly define continuous-time signals as everything in computers is digitized (considering you are not using symbolic programming, which is another thing and its computational power is limited). In light of this, python and other libraries in other languages implement something called the Discrete Fourier Transform (DFT). Consider a vector $x[n]$ with length N , then its DFT is another vector, $X[k]$ with the same length N and is defined as:

$$X[k] = \sum_{n=0}^{N-1} x[n] e^{-j\left(\frac{2\pi}{N}\right)kn}$$

Now, answer/implement the following questions:

1. With respect to the given equation for DFT, find the equation for IDFT (Inverse DFT).
2. Implement a function called **fft** that takes a vector as input and produces its DFT vector as output.
3. Implement the function for IDFT called **ifft**.
4. Consider the signal $x(t) = \cos(\pi t)$. Construct the vector $x[n] = x(nT)$ with length N . Plot this vector for $T = \frac{1}{2}$ and $N = 1000$ for n ranging from -500 to 500.
 - Obviously, when trying to reference the vector's elements, $x[n]$, one should use a valid index ranging from 0 to $N - 1$ (for python).
5. Compute Fourier transform of $x(t)$.
6. Use **fft** to compute DFT of $x[n]$, $X[k]$. Plot $X[k]$. Compare the result with 5.
7. Use **ifft** on $X[k]$. Is the resulting vector equal to $x[n]$?
8. Show the effects of changing N and T on the DFT of $x[n]$, in different plots.

Problem 2: In this question, we want to cancel unwanted effect of a system on a sound. This action is called *channel equalization*. For such purpose, we have *clean1.wav* and *distorted1.wav* as input and output signals of this system. Clean1 is a signal without any distortion and distorted1 is the output of the system which has been distorted by the system. *distorted2.wav* is another signal that has been the output of this system. The problem is that we do not have access to the clean2 signal. Using *clean1.wav* and *distorted1.wav* signals, first calculate the frequency response of the system. Then try to recover *clean2* signal using system's inverse response. Discuss your methodology and attach the recovered signal named *recovered2.wav*

Problem 3:

In this question we want to recover a noisy signal by applying a suitable filter (*signal de-noising*). Consider the *noisy1.wav* signal which is made from a clear sound added with some noise. You should design an appropriate filter and apply it to the noisy signal. In order to find the frequency of the noise and attenuate it in the filter, you should first examine the signal's spectrum and detect the speech gaps. In these gaps you can find the noise frequency. Then, design a filter which attenuates these frequencies. Explain your methodology and attach the final de-noised signal named *denoise1.wav* to your report.

سوال ۴: تبدیل Z

۱. با استفاده از خواص تبدیل Z ، $X(z)$ را برای سیگنال زیر به صورت دستی حساب کنید :

$$x[n] = (0.5)^n \cos\left(\frac{\pi n}{3}\right) u[n]$$

حال هدف این است که چک کنیم $X(z)$ ایی که به دست آوردید، درست است یا نه. برای این کار گام های زیر را طی کنید :

- (a) ۸ نمونه اول سیگنال $x[n]$ را محاسبه کنید و در یک بردار ذخیره کنید.
- (b) یک سیگنال ضربه با طول ۸ و در مبدا صفر ایجاد کنید.
- (c) یک فیلتر با ضرایب $X(z)$ ایجاد کنید.
- (d) سیگنال ضربه ایی که قبلا ایجاد کردید را وارد فیلتر کنید. حال اگر سیگنال خروجی همان سیگنال قسمت (a) بشود، $X(z)$ را درست حساب کرده اید!
- (e) توضیح دهید چرا این اتفاق می افتد؟

۲. یک سیستم علی با معادله تفاضلی زیر داریم :

$$y[n] - 0.9y[n-1] - x[n] = 0$$

(a) $H(z)$ را محاسبه کنید و نمودار صفر-قطب آن را رسم کنید.

(b) نمودارهای $|H(e^{j\omega})|$ و $\angle H(e^{j\omega})$ را رسم کنید.