Structural Safety 76 (2019) 68–80

Contents lists available at ScienceDirect

Structural Safety

journal homepage: www.elsevier.com/locate/strusafe

Risk-based optimal inspection strategies for structural systems using dynamic Bayesian networks

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ARTICLE INFO

Keywords:
Deterioration
Inspection planning
Reliability
Bayesian networks
Optimization

ABSTRACT

In most structural systems, it is neither possible nor optimal to inspect all system components regularly. An optimal inspection-repair strategy controls deterioration in structural systems efficiently with limited cost and acceptable reliability. At present, an integral risk-based optimization procedure for entire structural systems is not available; existing risk-based inspection methods are limited to optimizing inspections component by component. The challenges to an integral approach lie in the large number of optimization parameters in the inspection-repair process of a structural system, and the need to perform probabilistic inference for the entire system at once to address interdependencies among all components. In this paper, we outline a methodology for an integral risk-based optimization of inspections in structural systems, which utilizes a heuristic approach to formulating the optimization problem. It takes basis in a recently developed dynamic Bayesian network (DBN) framework for rapid computation of the system reliability conditional on inspection results. The optimization problem is solved by nesting the DBN inside a Monte-Carlo simulation for computing the expected cost associated with a system-wide inspection strategy. The proposed methodology is applied to a structural system subject to fatigue deterioration and it is demonstrated that it enables an integral risk-based inspection planning for structural systems.

1. Introduction

Deterioration processes in engineering structures lead to a reduction of service life and can affect the safety of the structures. Accurate modeling of deterioration remains a challenge today, due to the complexity of the processes and their inherent uncertainties. To address explicitly the prediction uncertainties, probabilistic approaches are suitable for deterioration modeling in an engineering context ([21,7,31,45,32,72]).

To reduce the uncertainty in deterioration processes, regular inspections are common practice for most engineering structures. An optimal inspection strategy balances the cost of inspections with the achieved risk reduction. An inspection strategy defines [8]: (a) what to inspect for (e.g., thickness diminution due to corrosion or erosion, fatigue cracks), (b) how to inspect (the inspection technique), (c) when to inspect, and (d) where to inspect (which components). Each combination of these factors defines an inspection strategy, among which the optimal one is sought.

Methods for risk-based optimization of inspections on structural systems have been developed during the past 40 years [68,69,63,27,52,57,58,38]. The scientific literature also documents industrial applications of inspection planning on offshore structures, aircrafts, bridges or ships [51,43,10,22,11,35,6]. The theory and the applications have focused almost exclusively on the optimization at the component level, with a simplified treatment of the system [57]. Only limited research efforts have been directed towards optimization procedures for entire systems, accounting for the statistical dependence among the deterioration states of individual structural details [56,57,54,42,33].

Risk-based optimization of inspection-repair strategies for large engineering systems is challenging in practice. Firstly, the interdependence among stochastic deterioration processes for all the system components must be modeled. The two common approaches to such an integral probabilistic deterioration modeling are random fields [16,66,53,70,18] and hierarchical models [28,29,44,2,23]. Secondly, Bayesian updating is required for computing the probability of failure of all components and the system conditional on a potentially large number of inspection results. This is a computationally challenging problem in itself [49]. In the context of inspection planning, these computations must be performed multiple times for the optimization of

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https://doi.org/10.1016/j.strusafe.2018.08.002
Received 28 June 2017; Received in revised form 19 August 2018; Accepted 19 August 2018
0167-4730/

the inspection strategies. Thirdly, the inspection optimization must consider system-wide strategies, which – in the general case – leads to a number of optimization parameters that is exponentially increasing with the number of components [57].

Bayesian methods enable incorporating information from inspections into probabilistic deterioration models to quantify the reduction in uncertainty and to update the reliability estimate [61,26,36,60]. Bayesian Networks (BNs) can facilitate such analyses. BNs have been applied to engineering risk analysis problems during the last two decades [64,13,30,9,15,36,12,67,3]. Conditional independence among model parameters encoded in the graphical structure of the BN can facilitate the Bayesian updating. In addition, if a process can be represented by discrete random variables (e.g. by discretizing all continuous random variables), exact inference algorithms can provide fast and robust solutions to the Bayesian updating. These properties have been exploited in Straub [54] and Luque and Straub [25], where dynamic Bayesian networks (DBNs) are utilized to evaluate deterioration at the component and system level. Bespoke exact inference algorithms ensure rapid computation of the conditional probability of system failure given all inspection results, which is essential for solving the optimal inspection problem.

In this paper, we propose a heuristic approach to finding the optimal inspection strategy in structural systems. In contrast to existing methods, the approach can simultaneously account for system effects arising from (a) the dependence among the deterioration at different components, (b) the joint effect of deterioration at multiple components on the system reliability, and (c) the interaction among inspection costs, i.e. the reduction in the marginal cost of an inspection if these are grouped in larger inspection campaigns. This is achieved with the proposed heuristic approach to the optimization, which enables the definition of a system-wide inspection plan with just a few parameters. The optimization criterion is the total expected life-cycle cost, whose computation is made feasible by a novel two-level approach, in which the system DBN algorithm of Luque and Straub [25] is nested within a Monte-Carlo simulation that addresses the uncertainty on the inspection outcomes. The DBN algorithm allows to compute the conditional probability of system failure given inspection outcomes.

The proposed methodology is demonstrated and investigated by application to a Daniels system, an idealized redundant structural system, whose components are subject to fatigue deterioration.

2. Methodology

2.1. The inspection optimization problem

An inspection strategy for a structural system defines when, where, what and how to inspect. In general, static inspection regimes are not optimal; instead, one should account for results from previous inspections and maintenance activities when deciding upon new inspections. For this reason, the optimal inspection-planning problem belongs to the class of sequential decision problems [1,19].

The sequential inspection planning problem is visualized in the decision tree of Fig. 1. Branches following a circular node represent random outcomes (e.g. the deterioration state of the system, or the inspection outcomes) and branches after a square node represent possible decision alternatives (e.g. if and where to inspect or repair). This decision tree is equally applicable to single components or entire systems. When considering systems, the outcome space of the random variables and the number of decision alternatives increase exponentially with the number of components. This is one of the main reasons why previous work on risk-based inspection planning has focused mainly on individual components.

Solutions to sequential decision problems can be found through the definition of policies. Here, a policy for a decision at time t defines where, what and how to inspect and repair, taking into account the full history of the structure up to t, i.e. past inspection outcomes and repair actions. The set of policies at all times t is the strategy $\mathscr{S}$. If the policies are the same for all t, the strategy is stationary [17].

For a structural system with N components subject to deterioration, the inspection optimization problem of Fig. 1 can be formalized as follows. The joint deterioration state $\mathbf{D}$ of all components is represented through a probabilistic system deterioration model with random parameters $\mathbf{X}_D$. Each component can be inspected and/or repaired at discrete times t from 0 to the end of service life T. The strategy $\mathscr{S}$ defines for each component at each time step if and how that component is inspected and repaired, based on all previous inspection outcomes $\mathbf{Z}$ and the repair history of the structure.

Inspections, repairs and system failure are associated with consequences. These are quantified by the present value of total life-cycle cost C_T in function of the strategy $\mathscr{S}$ and the inspection outcomes $\mathbf{Z}$. It is defined as the sum of the life-time inspection cost C_I, repair cost C_R, and failure risk R_F:

$$C_T(\mathscr{S},\mathbf{Z}) = C_I(\mathscr{S},\mathbf{Z}) + C_R(\mathscr{S},\mathbf{Z}) + R_F(\mathscr{S},\mathbf{Z}) \tag{1}$$

For a given strategy $\mathscr{S}$ and inspection outcomes $\mathbf{Z}$, the inspection and repair actions are fixed. Hence, $C_I(\mathscr{S},\mathbf{Z})$ and $C_R(\mathscr{S},\mathbf{Z})$ can be directly evaluated in function of the cost of individual inspections and repairs, and the relevant discount rate.

The failure risk R_F is defined as:

$$\begin{aligned} R_F(S,\mathbf{Z}) &= \sum_{t=1}^{T} c_F\cdot\gamma(t)\cdot\Pr(F_t\,|\mathbf{Z}_{0:t-1}) \\ &= c_F\cdot\sum_{t=1}^{T}\gamma(t)\cdot[\Pr(E_{S,t}=Fail\,|\mathbf{Z}_{0:t-1}) - \Pr(E_{S,t-1}=Fail\,|\mathbf{Z}_{0:t-1})] \end{aligned} \tag{2}$$

where c_F is the undiscounted cost of a system failure event, $\gamma(t)$ is a discount factor, F_t is the event of a system failure during time step t, and $E_{S,t}$ is the system condition at time step t.

The conditional probability $\Pr(E_{S,t}=Fail|\mathbf{Z}_{0:t-1})$ is the probability of a system failure up to time t for given inspection outcomes $\mathbf{Z}_{0:t-1}$. Its computation is a structural reliability problem, which can be formulated as an integral over all random variables $\mathbf{X}$ of the problem (which include the deterioration parameters $\mathbf{X}_D$, but also load parameters):

$$\Pr(E_{S,t}=Fail\,|Z_{0:t-1}) = \int_{\Omega_x} [g_{S,t}(x)\leqslant 0]\cdot f_{x|Z_{0:t-1}}(x)\mathrm{d}x \tag{3}$$

$g_{S,t}(\mathbf{x}) \le 0$ is the limit state function describing system failure up to t, $I[\bullet]$ is the indicator function and $f_{\mathbf{X}|\mathbf{Z}_{0:t-1}}$ is the conditional probability density function of $\mathbf{X}$ given inspection outcomes $\mathbf{Z}_{0:t-1}$.

The solution of Eq. (3) is non-trivial, in particular if the system size and the number of observations are large. First Order Reliability Method-(FORM) and sampling-based solutions to this problem are available [55,60,49]. In inspection planning, the conditional probability must be evaluated many times, and an efficient and robust solution of Eq. (3) is thus required. For this reason, we apply DBNs to solve Eq. (3) following Luque and Straub [25].

Because the inspection outcomes $\mathbf{Z}$ are random variables themselves and are not known in advance, the total cost is also a random variable. If $f_{\mathbf{Z}}$ is the probability distribution of the vector of inspection outcomes, whose support $\Omega_{\mathbf{Z}(\mathscr{S})}$ depends on the strategy $\mathscr{S}$, then the expected total life-cycle cost associated with the strategy $\mathscr{S}$ is obtained as

$$\mathrm{E}_{\mathbf{Z}}[C_T(S,\mathbf{Z})] = \int_{\Omega_{\mathbf{Z}(S)}} C_T(S,\mathbf{z})f_{\mathbf{Z}}(\mathbf{z})\mathrm{d}\mathbf{z} \tag{4}$$

The optimal strategy $\mathscr{S}^*$ is defined as the one that minimizes the expected total cost:

$$\mathscr{S}^* = \underset{\mathscr{S}}{\mathrm{argmin}}\,\mathrm{E}_{\mathbf{Z}}[C_T(\mathscr{S},\mathbf{Z})] \tag{5}$$

This optimization is commonly subject to constraints on the

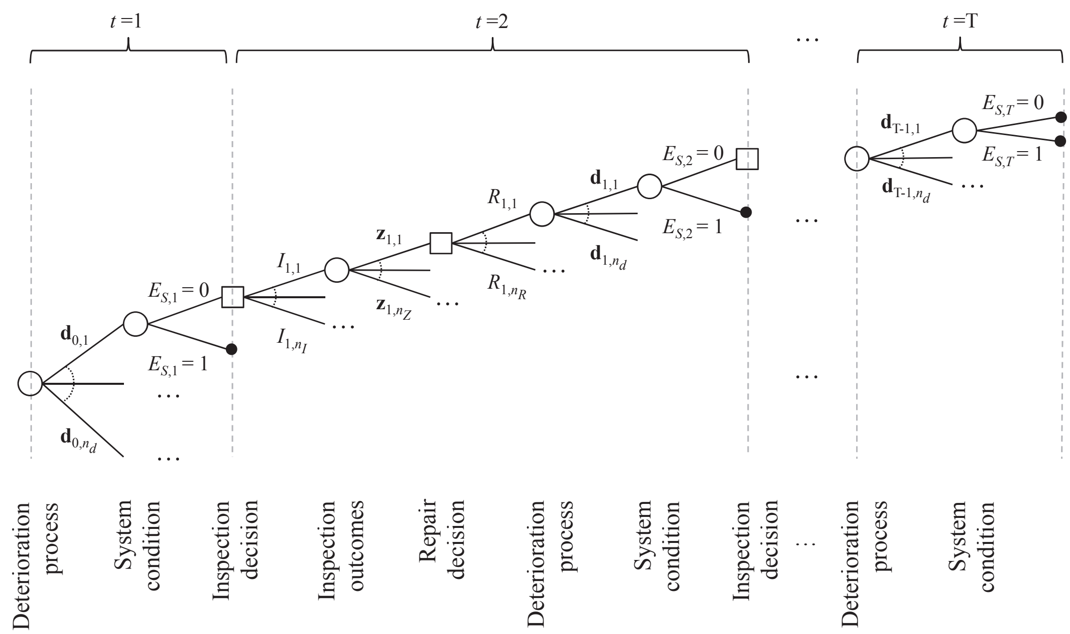

Fig. 1. Example of a decision tree with deterioration vector $\mathbf{D}_{t,i}$ for all system components, the system performance $E_{s,t}$ (0: safe, 1: fail), inspection and repair decisions at each time step t = 1, ⋯,T, and a set of observations $\mathbf{Z}_{t,i}$ after each inspection decision. A black dot marks the end of a branch, which corresponds to either a system failure or the end of service life.

minimum reliability and the maximum budget for inspections.

The two main challenges in finding the optimal strategy through Eq. (5) are (a) the large number of possible inspection strategies $\mathscr{S}$, which increases exponentially with time steps and number of components Fig. 1, and (b) the expectation operations in Eqs. (3) and (5). These challenges are already non-trivial for single components. For this reason, we first review existing approaches at the component level to solve the optimization problem defined by Eq. (5) in Section 2.2, before presenting a solution at the system level Section 2.3. The approach employs the DBN framework for computing the conditional probabilities $\Pr(E_{S,t}=Fail|\mathbf{Z}_{0:t-1})$, which is summarized in Section 2.4.

2.2. Optimization at the component level

Risk-based optimization of inspection planning for individual components has been studied extensively [58,38]. In the following, we briefly review the solutions based on influence diagrams, Markov decision processes, and stationary strategies.

2.2.1. Influence diagrams

An influence diagram (ID) is an extension of Bayesian networks, which includes decision and utility nodes [17]. An example ID is shown in Fig. 2.

The ID is a graphical representation of a decision problem, not a solution method. The classical ID is based on the no-forgetting principle, i.e. when making a decision, it is conditional on all previously available information. The solution of such a general ID therefore faces the same exponential complexity as described earlier for the decision tree of Fig. 1. A common approach for approximating the optimal solution is to consider only a subset of the past observations (e.g. the n most recent ones) at each decision step. This approach is known as limited memory influence diagram (LIMID) [20,17]. A widely-used algorithm to approximate the optimal solution is the single-policy-updating algorithm [20]. This algorithm considers a strategy $\mathscr{S}$ as (locally) optimal if changing only one its policies (i.e. the set of rules at only one decision node) does not lead to a better strategy in terms of the cost function. This approach has been used to estimate the optimal solution at the component level, e.g. in Nielsen and Sørensen [40] and Luque and Straub [24]. However, these applications were limited to

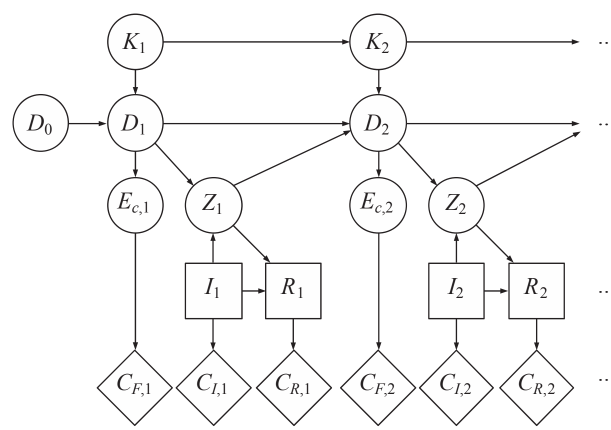

Fig. 2. Example influence diagram for inspection planning of a single component. Circular nodes are random variables, square nodes are decisions and diamond-shaped nodes are costs. Node D_t represents the component deterioration state at time step t as a function of the previous deterioration D_{t-1} and a time-dependent parameter K_t; $E_{c,t}$ represents the condition (e.g. safe or failed) of the component; Z_t is the inspection outcome; I_t and R_t are the inspection and repair decisions; $C_{F,t}$, $C_{I,t}$, $C_{R,t}$ are the failure, inspection and repair cost nodes.

simplified deterioration models at the component level, because the available optimization algorithms require many evaluations of the expectations in Eqs. (3) and (5), including the solution of conditional reliability problems.

2.2.2. Markov decision processes

If deterioration can be described by a Markov process, the optimal inspection problem can be solved by means of Markov decision processes (MDPs). Markov chains (discrete time Markov processes) have been frequently used for modeling deterioration processes in engineering applications [48,34]. Even non-Markovian processes can be translated into Markov chains by state-space augmentation [54]. MDPs have been proposed and applied by a series of authors for obtaining the optimal strategy of engineering components or systems described by simple deterioration models [62,47].

One distinguishes between fully and partially observable decision processes, depending on the type of available information. If all parameters of the deterioration process are directly observable, the process is fully observable. This is applicable only to simple, typically empirical deterioration models. In reality, the full deterioration process is observable only indirectly or incompletely; partially observable Markov decision processes (POMDPs) are then applied. The model of Fig. 2 is a POMDP, in which the state of the component is represented by the three random variables K_t, D_t and $E_{C,t}$ and Z_t is the (potential) inspection outcome The partial observability implies that not only Z_t, but all past inspection outcomes $Z_1, \cdots, Z_{t-1}$ have an effect on the probability distribution of K_t, D_t and $E_{C,t}$. For this reason, the POMDP is solved by introducing a so-called belief state, which represents the knowledge of the decision maker at each point in time, summarizing the past inspection history [19].

POMDPs have been used to find the optimal strategy at the component level [38,50] and at the system level [42,33], but their application to larger systems is still computationally challenging. In addition, a main limitation of these approaches for their application to structural systems as considered in this paper is that they cannot handle problems in which the costs at the system level are a non-linear function of the costs at the component level. This is however the case when the failure of the system is described by a structural model in function of component states.

2.2.3. Heuristic strategies

The most common approach to risk-based inspection planning for components consists of limiting the set of possible strategies $\mathscr{S}$ to a small number of parametrized stationary strategies, based on simple heuristics. The two most commonly applied heuristics are briefly summarized in the following.

2.2.3.1. Probability threshold. The stationary strategy is specified by a threshold on the probability of component failure p_{th}. An inspection is required in any time step before the probability of failure (conditional on previous inspection results) exceeds p_{th}, as illustrated in Fig. 3.

2.2.3.2. Fixed-interval (periodic) inspections. Inspections are performed at fixed regular intervals Δt_I, e.g. inspections every ten years (Fig. 4). This approach is commonly used in practice because it is easier to incorporate into the overall asset integrity management of a structure.

In both heuristics, the repair policy can be fixed in advance. In most applications, it is required that any identified damage is immediately repaired. In this case, the optimization of Eq. (5) reduces to finding either the optimal value of p_{th} or the optimal interval between inspections Δt_I. Alternatively, it is also possible to add a parameter for the repair criterion, in which case two optimization parameters have to be considered [41]. By assuming that a repaired component performs like a new component, it is possible to reduce the number of evaluations of the conditional probability of failure of Eq. (3), following Straub and Faber [58].

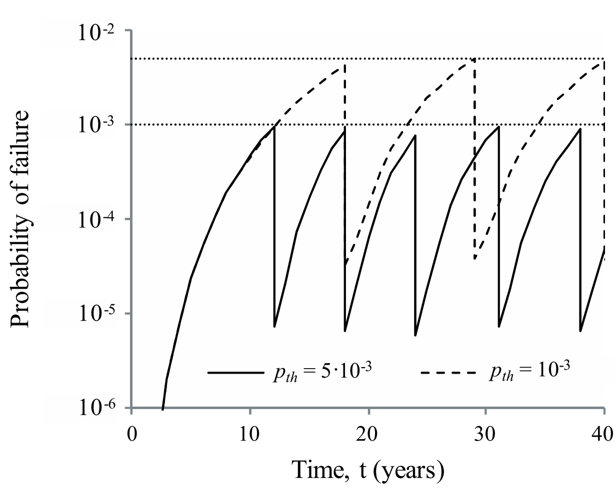

Fig. 3. Probability of failure of a structural element using the probability threshold heuristic with $p_{th} = 10^{-3}$ and $p_{th} = 5 \cdot 10^{-3}$. An inspection is performed prior to exceeding the threshold. The probability of failure shown here is conditional on not having identified any defect in past inspections.

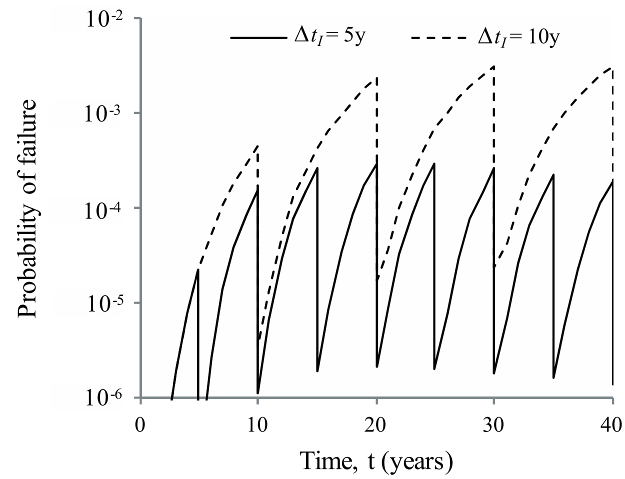

Fig. 4. Probability of failure of a structural element using periodic inspections every 5 and 10 years. The probability of failure shown here is conditional on not having identified any defect in past inspections.

It has been demonstrated that heuristic approaches give a good approximation of the optimal solution in risk-based inspection planning, with orders of magnitude less computation effort than other approaches like LIMIDs or POMDPs [40,24]. Fig. 5 shows a comparison between the optimal inspection strategy of a single component using LIMIDs and heuristic approaches (periodic inspections and probability threshold) from a theoretical example investigated in Luque and Straub [24]. The expected inspection, repair, and failure costs of the optimal solutions are compared in Fig. 6.

2.3. Optimization at the system level

The identification of the optimal inspection strategy is significantly more challenging for structural systems than for individual components. The number of possible inspection strategies $\mathscr{S}$ in Eq. (5) increases exponentially with the number of components, and the computation of the conditional reliabilities in Eq. (3) are much more demanding. This is because inspection results and deterioration failure events from the entire structure must be considered in a single integral computation. Given the difficulties one encounters already in solving the optimization problem at the component level, it appears that heuristics are the most promising, if not the only practically feasible, approach to optimizing inspections at the system level. In the following Section 2.3.1, we present such a heuristic for the system-level inspection planning.

Even with a heuristic approach to defining inspection-repair strategies, to solve the optimization problem at the system level necessitates a computationally efficient and robust algorithm for computing conditional probabilities of failure (Eq. (3)). The DBN framework developed in Luque and Straub [25] provides such an algorithm; it is presented in Section 2.4.

2.3.1. Heuristic strategy at the system level

At the system level, identifying heuristics is less straightforward than at the component level, mainly because it is not only necessary to identify the timing, but also the locations of inspections [57].

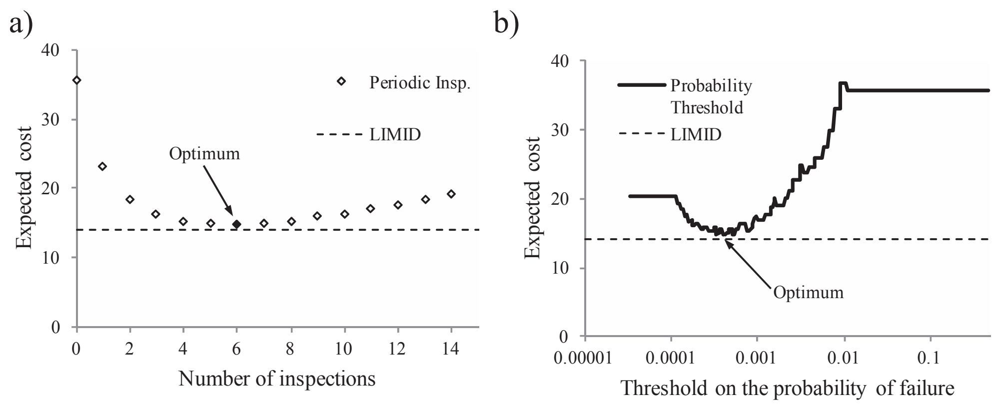

Fig. 5. Comparison between a) the LIMID solution and the Periodic Inspection heuristic; and b) the LIMID solution and the Probability Threshold heuristic [24].

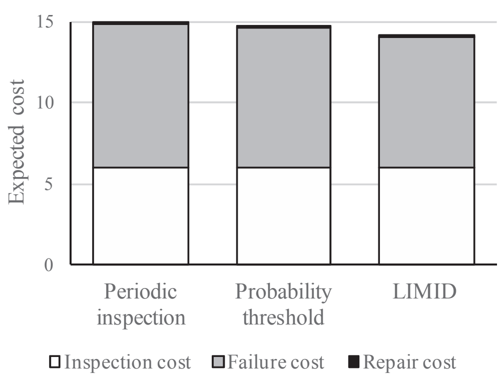

Fig. 6. Expected costs associated with the optimal strategies by periodic inspection, probability threshold, and LIMID approaches obtained at the component level [24]

Nevertheless, we find that the heuristics applied at the component level can be extended to the system level. Our proposed heuristic takes into account that it is typically cheaper to bundle component inspections in campaigns and that regular inspection intervals are preferred for organizational purposes.

The proposed heuristic distinguishes between inspection campaigns (when to inspect?) and individual inspections (where to inspect?). It can be summarized as follows:

1. Inspection campaigns are performed at regular time intervals, in analogy to the fixed-interval heuristic for single components. The time between regular campaigns is Δt_I.
2. The initial number of inspected components during each inspection campaign is fixed at n_I.
3. The components to inspect during a campaign are determined based on the value of information (VoI) [46] associated with the component inspection, following the idea of Straub and Faber [57]. Exact computation of the VoI is difficult, hence a proxy must be identified that provides a similar ranking than the VoI. This is further elaborated for the specific structural system considered in Section 3.4.
4. Whenever the updated system probability of failure exceeds a threshold value p_{th}, additional inspections must be carried out, either within the existing campaign or through an additional inspection campaign.
5. Repairs are performed according to a fixed repair criterion, e.g., any identified defect with a size larger than d_R is repaired.

Adjustments to these rules can and should be made according to the operational environment and constraints. In summary, the heuristic strategy $\mathscr{S}_k$ is a combination of the above stationary rules and is defined by the following parameters:

- the frequency of regular inspections Δt_I,
- the failure probability threshold p_{th},
- the number of components to inspect n_I,
- the repair criterion d_R.

The optimal combination of these parameters is found by solving Eq. (5). This requires the computation of the expected cost associated with a strategy $\mathscr{S}_k = (\Delta t_I, p_{th}, n_I, d_R)$.

2.3.2. Computation of the expected cost of a strategy

A Monte Carlo approach is employed to estimate the total expected life-cycle cost of a strategy $\mathscr{S}_k$ defined according to Eq. (4). The expected value is approximated as

$$E_{\mathbf{Z}}[C_T(\mathscr{S}_k, \mathbf{Z})] \approx \frac{1}{n_s}\sum_{j=1}^{n_s} C_T(\mathscr{S}_k, \mathbf{z}_{k,j}) \tag{6}$$

where $\{\mathbf{z}_{k,1}, \mathbf{z}_{k,2}, \cdots, \mathbf{z}_{k,n_s}\}$ are Monte Carlo samples of inspection outcomes, and n_s is the number of samples. To obtain these samples, one first generates random samples of the deterioration history of the entire structure, and then generates random inspection results conditional on these deterioration histories. The total cost C_T is computed according to Eq. (1).

The number of samples n_s required to ensure sufficient accuracy is a function of the coefficient of variation of the total cost δ_{C_T}. To ensure a relative error in the estimate of less than ε with a confidence of $1-\alpha$, the required number of samples is

$$n_s \geq \left[\frac{\Phi^{-1}\left(\frac{\alpha}{2}\right)}{\varepsilon}\delta_{C_T}\right]^2 \tag{7}$$

In all cases we investigated, the value of δ_{C_T} was around 1.5. If one requires a relative error less than 10% (i.e. $\varepsilon = 0.1$) with a confidence of 95% (i.e. $\alpha = 0.05$), the required number of samples is $n_s \geq 384\delta_{C_T}^2 = 864$. Typically, the requirements on the accuracy of the estimated total expected life-cycle cost are not as strict, and a number of samples in the order of 200 is expected to be sufficient for most practical applications. Note that the reason for this relatively small number lies in the fact that the conditional probability of failure is computed within each MC sample through the DBN.

2.4. DBN framework

A key element in the proposed procedure is an efficient computation of the updated probabilities of failure at the component and the system level given inspection results, i.e. a fast solution to Eq. (3). At the component level, Straub [54] developed a DBN framework for stochastically modeling deterioration processes and updating the failure probability, which was shown to be efficient and robust. In contrast to other Bayesian analysis methods, the DBN combined with exact inference algorithms has the advantage that its performance does not deteriorate with increasing amount of inspection data. Recently, Luque and Straub [25] extended this framework to the system level through a hierarchical definition of the deterioration model parameters. Its main characteristics are summarized in the following.

The framework developed in Straub [54] enables translating commonly employed probabilistic deterioration models into a DBN. The probability of failure conditional on inspection results is computed by an adaptation of general purpose inference algorithms for DBNs [37]. The approach requires a discretization of continuous random variables, but very good accuracy can be achieved for standard deterioration models [54]. Other researchers have implemented this framework at the component level [39,71].

The framework by Luque and Straub [25] extends the DBN to the structural system level, accounting for dependence among deterioration parameters of different components through a hierarchical approach. To model the correlation structure among the deterioration parameters, a set of hyperparameters α is included in the DBN model. These hyperparameters are the link among time-independent parameters θ (e.g. material properties), time-dependent parameters ω (e.g. temperature), and deterioration state **D** in all components Fig. 7). Through the hyperparameters, any inspection result **Z** at a component will affect the reliability estimates of the other components. The system reliability is evaluated through the binary nodes $E_{S,t}$ (with $E_{S,t} = fail$ representing system failure at time t), in function of the component conditions $E_{C,i,t}$.

In the DBN model of Fig. 7 there are no links from $E_{S,t}$ to $E_{S,t+1}$. This is an approximation, because a structure that has failed at time t ($E_{S,t} = Fail$) will also be in a failed state in year $t+1$ ($E_{S,t+1} = Fail$). The probability $\Pr(E_{S,t+1} = Fail)$ computed with the DBN of Fig. 7 without these links is therefore an underestimation of the true probability of failure. However, introducing this link would significantly increase computational costs of the DBN. The approximation error is small if the dominant contribution to the probability of system failure is from the deterioration. This must be checked for a specific application. Alternatively, an upper bound to the probability of failure can be obtained from the DBN by considering the failure events in different time steps as independent events; this upper bound has been used frequently in the literature, e.g. [65].

An exact inference algorithm for solving the hierarchical DBN is available from Luque and Straub [25]. Because of the hierarchical structure of the model, the computation time increases approximately linearly with the number of components. The algorithm also facilitates the use of parallel computation. The following is a short summary of the algorithm, for details on the method, the reader is referred to [25].

In a first step of the algorithm, for all components the joint probability distribution of component i is updated with inspection results from component i, for given hyperparameter values α. In the hierarchical DBN, the deterioration states are statistically independent among components for fixed hyperparameters, i.e.

$$p(d_{i_1,t}, d_{i_2,t}|\alpha) = p(d_{i_1,t}|\alpha)\cdot p(d_{i_2,t}|\alpha) \quad (8)$$

for all components $i_1, i_2 = 1, \cdots, N$ with $i_1 \neq i_2$. This independence property can be exploited to parallelize the computations of the conditional probabilities given inspection results for individual components in the first step.

In the second step, the joint distribution of the hyperparameters α is updated with the inspection results from all components. Finally, the results of the first and the second step are combined to obtain the probability distributions of all components i conditional on all inspection results in the system.

To speed up computation, it is possible to partly reuse updated probability distributions. If the sampled inspection results for a component i are identical among two samples j_1 and j_2, the first step in the computation can be avoided, and the previously computed probabilities can be utilized. Additionally, for time steps t prior to the first inspection campaign, the system reliability will be identical among all samples j and has to be calculated only once. Further computational savings may

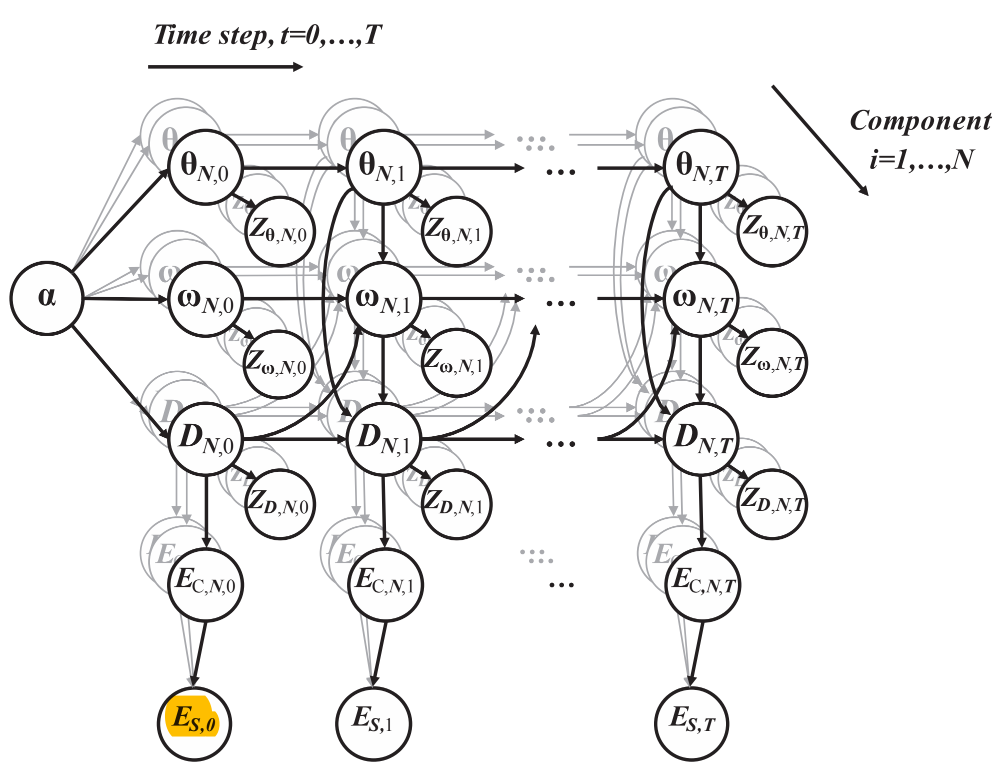

Fig. 7. Hierarchical DBN system deterioration model [25]. Node $D_{i,t}$ represents the deterioration state of the i-th component at time step t as function of the previous deterioration $D_{i,t-1}$, the time-independent parameter $\theta_{i,t}$, and the time-dependent parameter $\omega_{i,t}$; observations $Z_{\theta,i,t}$, $Z_{\omega,i,t}$, and $Z_{D,i,t}$; $E_{C,i,t}$ and $E_{S,t}$ represent the condition (e.g. safe or failed) of the component and the system; α is the set of hyperparameters that links all components.

be possible for specific cases, e.g. if components have the same deterioration model.

2.5. Summary of the proposed methodology

The proposed procedure for integral optimization of inspections in a structural system consists of the following steps:

1. Define system-wide inspection strategies $\mathscr{S}_k$ through the heuristic of Section 2.3.1, with optimization parameters Δt_I, p_{th}, n_I, and d_R. Alter the proposed heuristic if necessary to account for operational constraints.
2. Choose an optimization algorithm to identify the solution of Eq. (5). The algorithm must be able to handle numerical noise, because the objective function is evaluated with MCS. Perform steps 3–5 to determine the expected total cost associated with a strategy.
3. For every strategy $\mathscr{S}_k$ (i.e. combination of optimization parameter values), generate n_s Monte Carlo samples of inspection outcomes $\mathbf{z}_{kj}$, where $j = 1, \cdots, n_s$. These define the times and locations of inspections and their outcomes. n_S must be chosen sufficiently high (see Eq. (7).
4. For every strategy $\mathscr{S}_k$ and inspection sample $\mathbf{z}_{kj}$:
   a. Compute the conditional probabilities of system failure over the service life (Eq. (3)) by means of the DBN.
   b. Compute the failure risk (Eq. (2)).
   c. Compute the total cost (Eq. (1)).
5. Estimate the total expected life-cycle cost of each strategy $\mathscr{S}_k$ by means of Eq. (6).

The procedure deals with the two main challenges outlined in the last paragraph of Section 2.1 by (a) extending the heuristic approach from the component level to the system level, through the use of suitable system-wide heuristics, and (b) by computing the expected risk and cost through nesting a DBN computation insight a MCS that integrates over future inspection outcomes.

3. Numerical investigations

The proposed methodology is applied to optimize the inspection-repair strategy of a Daniels system subject to fatigue deterioration. This system facilitates the numerical investigation of the effect of system wide inspection strategies.

3.1. System definition

A Daniels system consists of a set of N load-sharing elements with independent and identically distributed random capacities R_i, $i = 1, \cdots, N$, and an external random load L [5]. The system is illustrated in Fig. 8 and its parameters are summarized in Table 1. The Daniels system facilitates studying the characteristics of load-sharing among the elements in redundant structural systems [14].

The examples presented in this paper are for a system of $N = 10$ components.

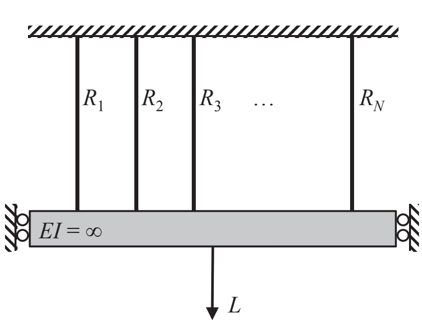

Fig. 8. Daniels system with N elements.

Table 1
Parameters of the Daniels system, following Luque and Straub [25].

Parameter	Type	Mean	Std. deviation	Correlation
L	Lognormal	μ_L	$\sigma_L = 0.25{\cdot}\mu_L$	
R_i	Normal	μ_{R_i}	$\sigma_{R_i} = 0.15{\cdot}\mu_{R_i}$	0
N	Deterministic	10		
Mean safety factor	Deterministic	$N{\cdot}\mu_{R_i}/\mu_L = 2.9$		

In this study, the components of the Daniel system are affected by fatigue deterioration (Section 3.2). At the system level, this deterioration is represented by a binary model, in which the component i either has its full capacity (prior to fatigue failure) or zero capacity (after fatigue failure). We neglect any interaction between the extreme load L and the fatigue deterioration.

Because of the exchangeability of components in the Daniels system, the system reliability is a function only of the number of components that have failed because of fatigue, $N_{f,N,t}$. The conditional system failure probability $\Pr(E_{S,t} = fail|N_{f,N,t} = n)$ is presented in [25].

3.2. Deterioration and inspection model

All components of the Daniels system are affected by a fatigue deterioration process D. Based on the case study presented in Straub [54] and Luque and Straub [25], a simple fracture mechanics based fatigue model is used to describe the crack depth D_i at component i of the Daniels system at time t:

$$\frac{\mathrm{d}D_i(t)}{\mathrm{d}t} = \nu C_i[\Delta S_{e,i}\sqrt{\pi D_i(t)}]^{M_i} \tag{9}$$

where ν is the stress cycle rate; $\Delta S_{e,i} = (\mathrm{E}[\Delta S_i^{M_i}])^{\frac{1}{M_i}}$ is the equivalent stress range per cycle with $\mathrm{E}[\bullet]$ being the expectation operator and ΔS_i the stress range per cycle, and C_i, M_i are material parameters.

With $D_{i,0}$ being the initial crack depth of component i at time $t = 0$, an analytical expression for the crack depth at time t is found from Eq. (9):

$$D_i(t) = \left[\left(1-\frac{M_i}{2}\right)C_i\Delta S_{e,i}^{M_i}\pi^{M_i/2}\nu t + D_{i,0}^{1-M_i/2}\right]^{(1-M_i/2)^{-1}} \tag{10}$$

The service life of the structure is discretized in $t = 0, 1, 2, \cdots, T$ time steps.

To represent dependence of the deterioration process among components, a set of hyperparameters $\alpha = \{\alpha_M, \alpha_K, \alpha_D\}$ is used to link material parameters, stress parameters, and initial crack depths. More details on the definition of the hyperparameters and their application in the DBN model can be found in Luque and Straub [25].

The component condition at each time step, $E_{C,i,t}$, is either safe or fail. A safe component has its full capacity, whereas a failed component has zero remaining capacity. The failure event is defined through a critical crack depth d_c as $\{E_{C,i,t} = fail\} = \{D_{i,t} \geq d_c\}$.

The inspection $Z_{D,i,t}$ of the i-th component at time step t has two possible outcomes: (a) detection, or (b) no detection (of a fatigue crack). The probability of each outcome is a function of the crack depth $D_{i,t}$ and is represented here with an exponential probability of detection (PoD) model with parameter ξ:

$$\Pr(Z_{D,i,t}{=}1|D_{i,t} = d) = \mathrm{PoD}(d) = 1{-}\exp\left(-\frac{d}{\xi}\right) \tag{11}$$

The deterioration state after the inspection, $D_{i,t}^*$, is a (stochastic) function of the deterioration state $D_{i,t}$ before inspection, the inspection outcome $Z_{D,i,t}$, and the repair policy. Here, we postulate that all detected cracks are repaired ($d_R = 0$), and the condition of a repaired component is as new. If no crack is found at the inspection, it is simply $D_{(i,t)}^* = D_{(i,t)}$

Table 2
Parameters of the DBN deterioration model, following Luque and Straub [25].

Parameter	Units	Type	Mean	Std. deviation	Correlation
N	–	Deterministic	10		
T	year	Deterministic	40		
ν	stress cycles per year	Deterministic	$5 \cdot 10^6$		
α_M, α_D, α_K	–	Normal	0	1	
$D_{i,0}$	mm	Exponential	1	1	0.5
$M_{i,0}$	–	Normal	3.5	0.3	0.6
$M_{i,t}$	–	Function	$M_{i,t} = M_{i,t-1}$		
$\ln C_{i,t}$	corresponding to N and mm	Function	$\ln C_{i,t} = -3.34 M_{i,t} - 15.84$		
$\Delta S_{i,t}$	N	Weibull	scale parameter $K_{i,t}$	shape parameter $\lambda = 0.8$	
$\Delta S_{e,i,t}$	N	Function	$\Delta S_{e,i,t} = K_{i,t}\Gamma\left(1 + \frac{M_{i,t}}{\lambda}\right)^{\frac{1}{M_{i,t}}}$		
$K_{i,0}$	N/mm^2	Lognormal	1.6	0.22	0.8
$K_{i,t}$	N/mm^2	Function	$K_{i,t} = K_{i,t-1}$		
d_C	mm	Deterministic	50		
ξ	mm	Deterministic	10		

$\Gamma(\bullet)$: Gamma function

Table 3
Discretization scheme.

Random variable	Number of states	Final interval boundaries
α_{D0}, α_M, α_K	5	Φ^{-1}(0: 0.2: 1)
D [mm]	80	0, exp{ln(0.01): [ln(50)–ln(0.01)]/78: ln(50)}, ∞
M [-]	20	0, ln{exp(2.2): [exp(4.8)–exp(2.2)]/18: exp(4.8)}, ∞
K [N/mm^2]	20	0, {0.86: (2.83–0.86)/18: 2.83}, ∞

The parameters and random variables of the hierarchical DBN and deterioration model, the discretization scheme, and the corresponding influence diagram of the DBN model are presented in Tables 2, 3, and Fig. 9. The analysis is performed for an anticipated service life time of $T = 40$ years.

The effect of the approximation made by omitting links between $E_{S,t}$ and $E_{S,t+1}$ in the DBN is investigated through a MCS analysis for the unconditional case. We find that the DBN underestimates the probability of failure by a factor of 2, hence the DBN results are adjusted by this factor.

3.3. Costs and failure risk

Inspection campaigns have a fixed cost c_C independent of the number of components to be inspected. This is the mobilization cost of personnel and equipment and the cost of interrupting operations. Individual inspections and repairs per component have costs c_I and c_R. The consequences of system failure are represented by the failure cost c_F. All costs in Eqs. (1) and (2) are discounted to their present value through the following discounting factor based on the real interest rate r (i.e. the interest rate after allowing for inflation):

$$\gamma(t) = \frac{1}{(1 + r)^t} \tag{12}$$

The ratio of inspection and repair to failure costs can vary significantly among different systems. Two cost cases are considered in this example, summarized in Table 4. The first case corresponds to a structure with high mobilization costs, such as an offshore structure, and potentially large consequences of failure. The second case corresponds to a case with lower failure costs relative to the inspection campaign, and is motivated by the situation of metallic bridge structures subject to fatigue.

3.4. Optimization

Following Section 2.3.1, the inspection strategy $\mathcal{S}_k$ is defined through the optimization parameters: Δt_I, the time between regular (i.e. fixed-interval) inspections; p_{th}, the failure probability threshold at which additional inspections are performed; n_I, the pre-defined number of components to inspect during a campaign; d_R, the repair criterion, which is here set to 0.

The optimization is performed through an exhaustive search among a discrete set of parameter values according to Table 5.

Following the heuristic, in each campaign the components with the largest VoI are inspected first. Because of the exchangeability of the components in a Daniels system, and because the dependence among all components is the identical (at least a-priori), the VoI is a direct function of the probability of failure (PoF) of the component. A component with a higher PoF has a larger impact on the system reliability; it also has larger uncertainty, hence the learning effect is higher for such a component. Therefore, components are selected for inspection according to their PoF. Because components are repaired upon detection of a damage, this implies that non-inspected components will be prioritized.

3.5. Results

Based on Eq. (7), the strategies $\mathcal{S}_k$ are evaluated with $n_s = 1000$ samples to compute the associated total expected life-cycle cost through Eq. (4). (As discussed in Section 2.3.2, a smaller number of samples would be sufficient for most practical purposes.) Each sample j corresponds to a possible future inspection history $\mathbf{z}_{kj}$. To illustrate the workings of the algorithm, results for single inspection histories are presented first, followed by the evaluation and optimization of the total expected cost. In Section 3.5.3, the results are compared to those obtained with classical component-based inspection planning.

3.5.1. Illustrative results for a single inspection sample

A sample inspection outcome $\mathbf{z}_{kj}$ for a strategy $\mathcal{S}_k$ defined by ($\Delta t_I = 10$yr, $p_T = 2 \cdot 10^{-5}$, $n_I = 3$) is summarized in Table 6. Fig. 10 presents the component and system failure probabilities associated with this inspection history.

The system PoF associated with all 1000 samples of inspection histories are shown in Fig. 11.

The total life-cycle cost associated with an inspection history is computed according to Eqs. (1)–(3). For the inspection history of Table 6 and cost case 1 Table 4, the present value (discounted to $t = 0$) of the costs are $C_I = 4.51 + 1.50 = 6.01$ for the inspection campaign and component inspections, $C_R = 0.63$ for component repairs and $R_F = 0.44$

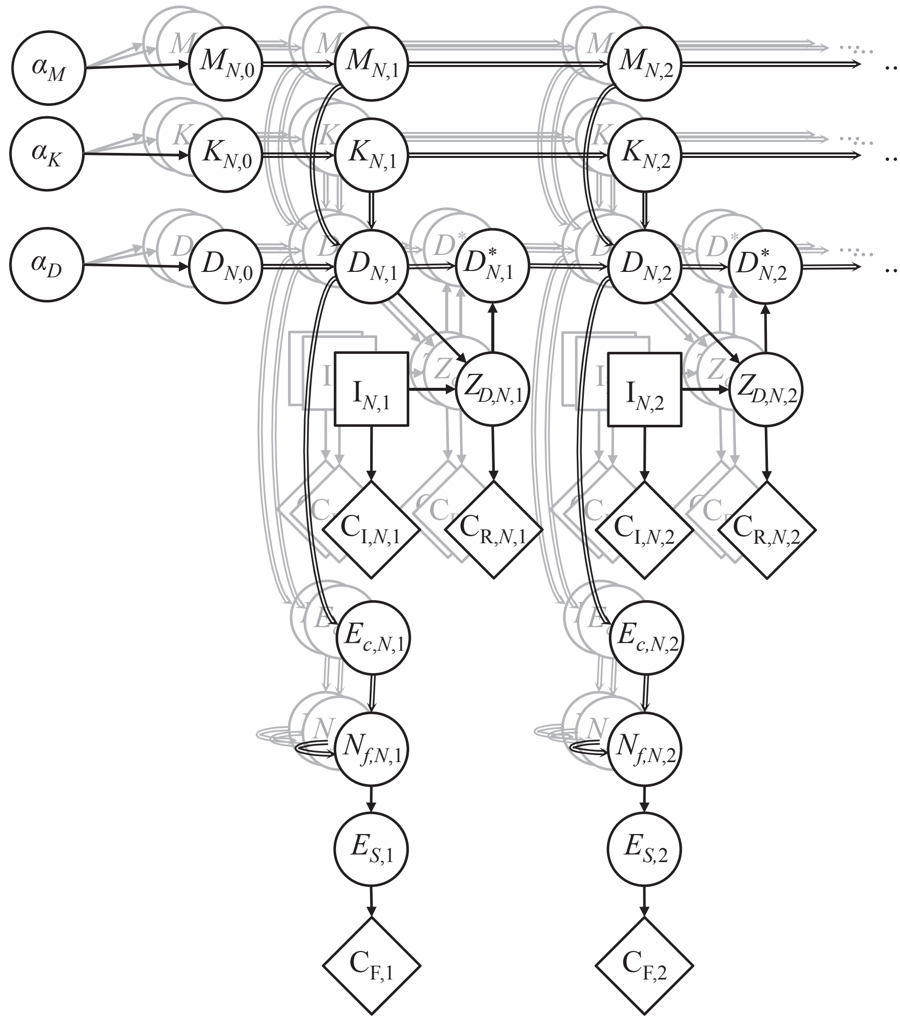

Fig. 9. Influence diagram of the Daniels system. Nodes M, K, and D are the material parameter, the scale parameter of the stress range distribution, and the fatigue crack depth with hyperparameters $\alpha = \{\alpha_M, \alpha_K, \alpha_D\}$. I is the inspection decision, Z_D is the inspection outcome, and D^* the crack depth after a possible repair. E_c, N_f, and E_S are the component condition, the number of failed components, and the system condition. C_I, C_R, and C_F are the inspection, repair, and system failure costs.

Table 4
Inspection, repair and failure cost for two different cases.

Cost	Case 1 (offshore structure)	Case 2 (bridge structure)
Inspection campaign, c_C	1	1
Component inspection, c_I	0.1	0.1
Component repair, c_R	0.3	1
System failure, c_F	$3{\cdot}10^4$	10^3
Discount rate, r	0.02	0.02

Table 5
Parameters defining the heuristic strategies.

Parameter	Values
Time between campaigns, Δt_I [year]	{5, 10}
PoF threshold, p_{th}	$\{2{\cdot}10^{-5}, 6{\cdot}10^{-5}, 2{\cdot}10^{-4}\}$
Number of inspected components, n_I	$\{1, 2, 3, \cdots, 10\}$
Repair criterion, d_R	0

for failure risk. This amounts to a total present value life-cycle cost of $C_T = 7.08$. The breakdown of these present values over the service life is shown in Fig. 12. This analysis is repeated for all $n_S = 1000$ samples, enabling the MCS estimation of the total expected life-cycle cost following Eq. (6).

Table 6
Sample inspection outcome, for a strategy with regular inspections interval $\Delta t_I = 10$yr, probability threshold $p_T = 2{\cdot}10^{-5}$, and $n_I = 3$ planned component inspections per campaign. Additional inspection campaigns at years 17, 19, 27 and 37 are necessary because of a threshold exceedance. In year 17, two additional component inspections are required during the campaign because of the identified defects.

Component	Time step of inspection						
	10	17*	19*	20	27*	30	37*
1	✓		✓			✓	
2	✓			✓		✓	
3	✓			✓			✓
4		✗					
5		✓		✗			
6		✗					
7		✓*			✓		✓
8		✓*			✓		✓
9			✓		✓		
10			✓			✓	

✓: Inspected without detection of a crack, ✗: Inspected with detection of a crack.
*: Extraordinary inspection campaign or additional inspected component due to a threshold exceedance.

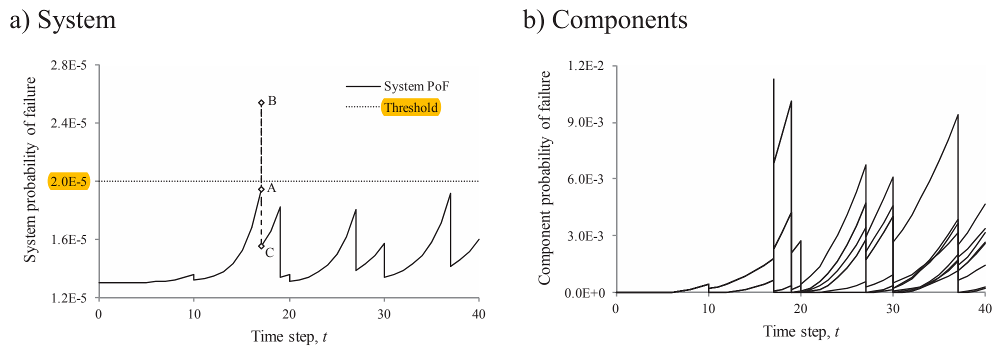

Fig. 10. Probability of failure of the (a) system and (b) individual components conditional on the sample inspection outcome from Table 6. Point A represents the PoF before the inspection at $t = 17$yr; B represents the PoF after inspecting the originally planned components (4, 5, and 6); C represents the PoF after inspecting two additional components (7 and 8) to comply with the threshold.

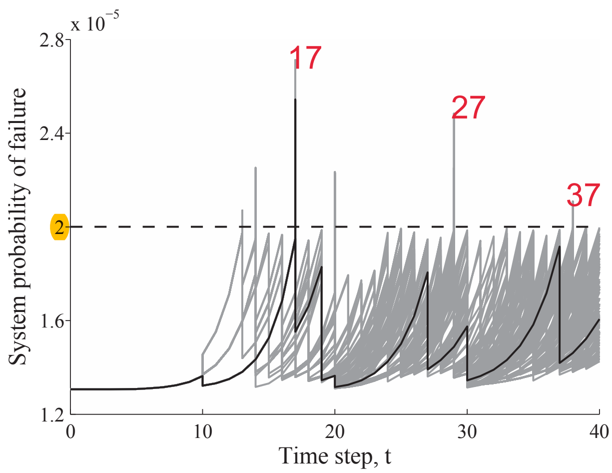

Fig. 11. System probability of failure for all sampled inspection histories. The dark curve corresponds to the outcome defined in Table 6 and plotted in Fig. 10a.

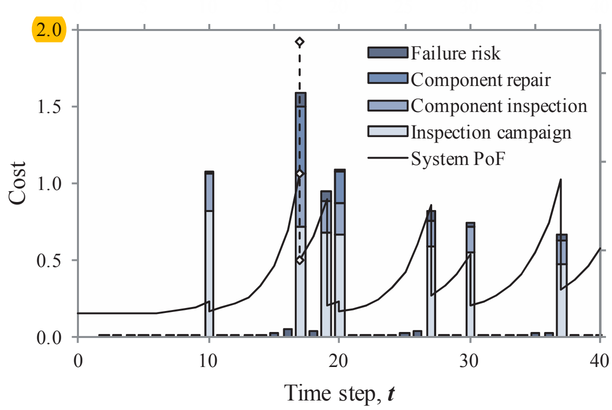

Fig. 12. Expected present values (discounted to $t = 0$) of costs for the inspection outcome from Table 6. The system PoF from Fig. 10a is included as a reference.

3.5.2. Expected costs and optimal inspection strategy

The expected costs of the strategies defined in Table 5 are shown in Fig. 13 for cost case 1 and Fig. 14 for cost case 2.

Among the strategies presented here, the optimal strategy for cost model 1 is to perform an inspection campaign every 10 years with inspection of 8 elements, following a probability threshold of $6{\cdot}10^{-5}$. For cost model 2, the optimal strategy is to perform an inspection campaign every 10 years with inspection of 3 elements, following a probability threshold of $2{\cdot}10^{-4}$.

3.5.3. Comparison to component-based inspection planning

For comparison we show the results of a classical component-based inspection planning for the considered Daniels system. The analysis follows the procedure outlined in Straub and Faber [58].

Because all components are identical in terms of their probabilistic deterioration model and their effect on the system integrity, the optimal inspection plan will be the same for all components a-priori. Fig. 3 shows the component probability of failure associated with different reliability thresholds and Fig. 4 shows the one associated with periodic inspections.

To estimate the cost of a component failure, one has to account for the effect of a component failure on the system reliability. In risk-based inspection planning, the system redundancy with respect to failure of component i is typically expressed by the change in the probability of system failure when removing component i [35,11]. This can be measured in terms of the single element importance measure [59], which is defined as

$$SEI_i = \Pr(\textit{System failure}|\textit{fatigue failure of component } i) \\ -\Pr(\textit{System failure}|\textit{ no fatigue failures}) \quad (13)$$

For the considered Daniels system, it is $SEI_i = 4.2{\cdot}10^{-5}–6.5{\cdot}10^{-6} = 3.6{\cdot}10^{-5}$.

When computing the component-based optimal inspection plan with these inputs, the resulting plan is not to perform any inspection. The reason lies in the underestimation of the consequence of a failure in the component-based approach. These are estimated as $C_F{\cdot}SEI_i$, which with cost model 1 results in a component failure cost of $3{\cdot}10^4{\cdot}3.6{\cdot}10^{-5} = 1.08$, and with cost model 2 in $10^3{\cdot}3.6{\cdot}10^{-5} = 0.036$. With such low failure consequences, inspections are not cost-effective. The problem with the SEI_i measure is that it neglects the possibility of two or more simultaneous component failures. For redundant system this underestimates the true risk, in some cases severely, as shown in [59].

Table 7 presents the comparison of the total expected life-cycle cost associated with not performing any inspection with the cost of the inspection plan obtained by the proposed system RBI planning. Clearly, the component-based approach is not suitable to determine optimal inspection plans for this structural system. It is noted that in practice such a plan would not be implemented and a minimum number of inspections would be performed, based on reliability constraints on the components. Furthermore, an improved redundancy measure (e.g. following [59]) could provide more realistic estimates of the consequences of component failures. Nevertheless, the integral system-based optimization will outperform any purely component-based optimization.

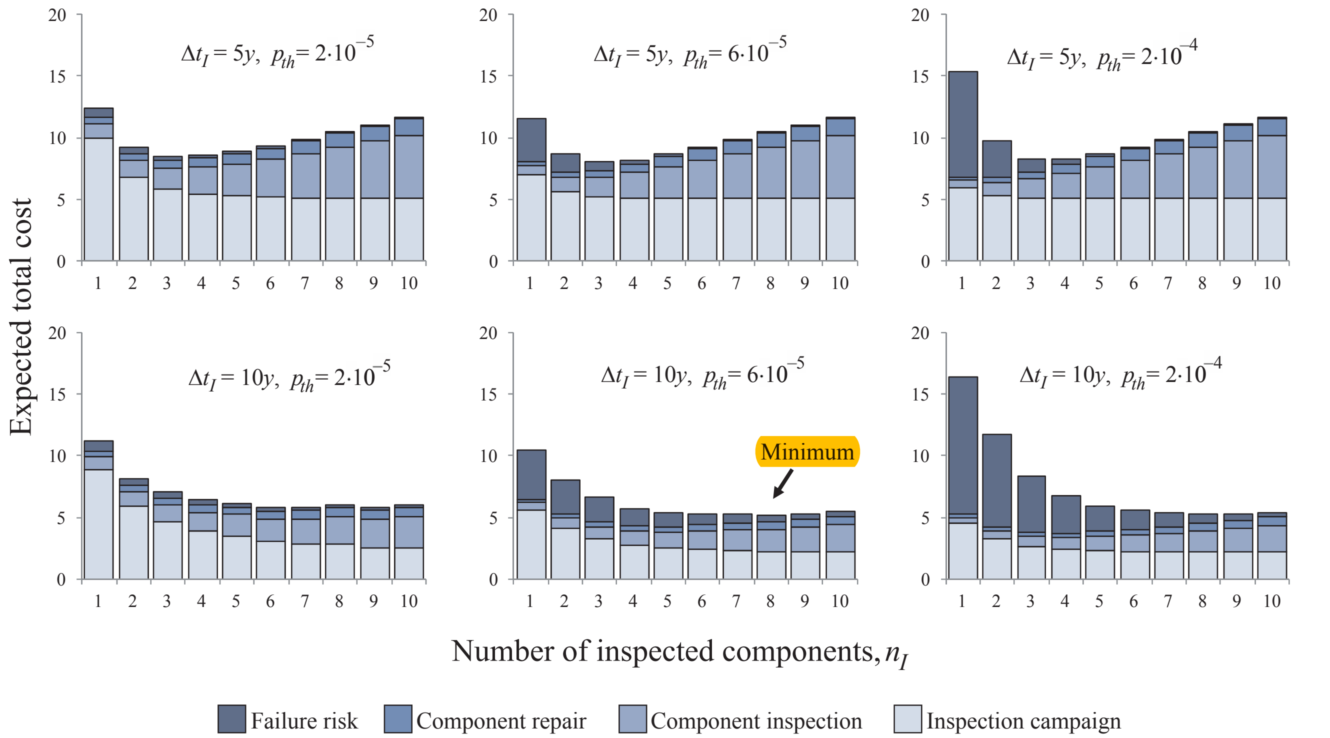

Fig. 13. Comparison of the expected total cost (case 1) varying the number of inspected components n_I, the probability threshold p_{th}, and the regular inspections periodicity Δt_I. The first row corresponds to $\Delta t_I = 5y$ and the second row to $\Delta t_I = 10y$. The columns correspond to the probability thresholds $2{\cdot}10^{-5}$, $6{\cdot}10^{-5}$, and $2{\cdot}10^{-4}$.

4. Discussion

The proposed framework determines optimal inspection-repair strategies for structural systems in an integral manner considering interdependences among component deterioration states and among the information from inspections. It also explicitly includes the interaction between the reliability of components and the structural system. By applying the DBN framework to computing the conditional reliability given inspection results, the method has a computational cost that is suitable for applications in practice.

To manage the complexity of the decision problem underlying the optimal inspection planning, the approach employs heuristics for

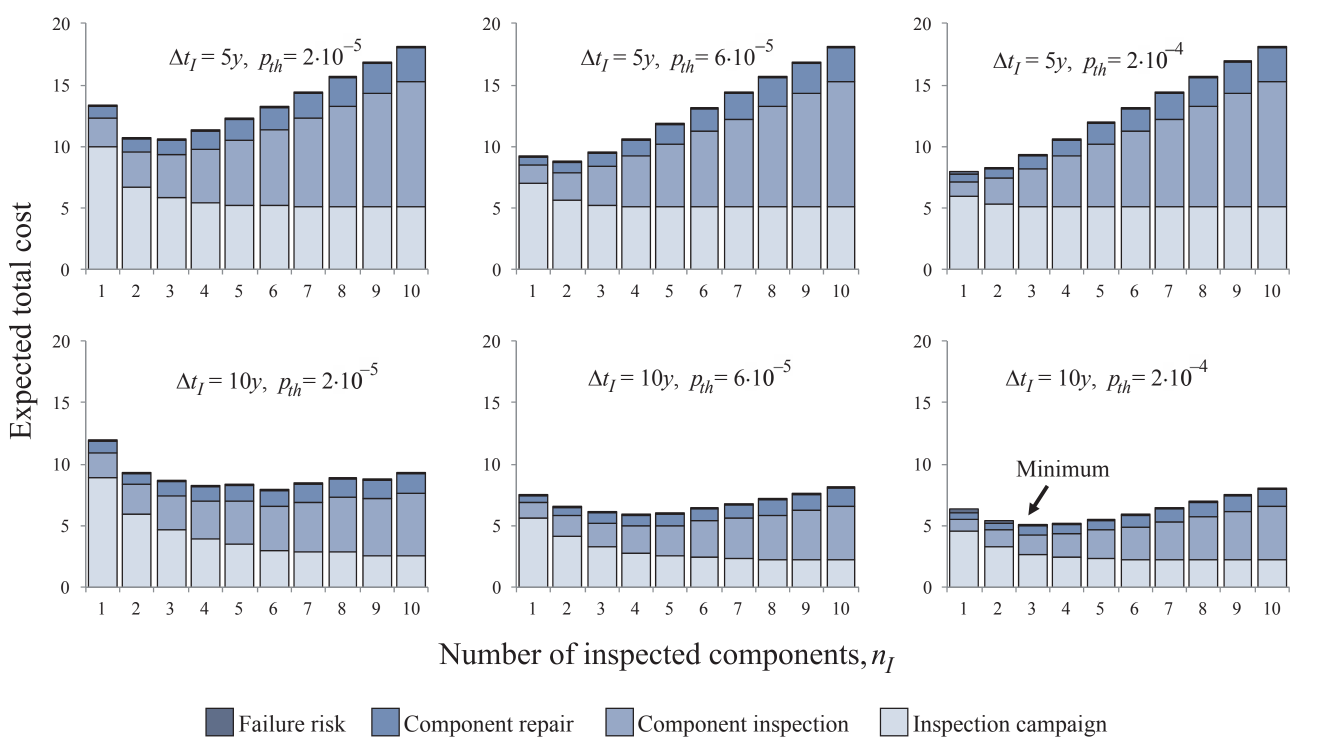

Fig. 14. Comparison of the expected total cost (case 2) varying the number of inspected components n_I, the probability threshold p_{th}, and the regular inspections periodicity Δt_I. The first row corresponds to $\Delta t_I = 5y$ and the second row to $\Delta t_I = 10y$. The columns correspond to the probability thresholds $2{\cdot}10^{-5}$, $6{\cdot}10^{-5}$, and $2{\cdot}10^{-4}$.

Table 7
Expected costs associated with the optimal strategies identified with component-based and the proposed system-wide optimization approach, for cost case 1. Note that the system-wide solution is not the global optimum; it is the optimum among the investigated strategies following Table 5 and Fig. 13.

Optimization algorithm	Component-based	Proposed system-wide
Expected cost for inspections C_I	0	3.56
Expected cost for repairs C_R	0	0.47
Risk R	117.3	1.54
Total expected cost C_T	117.3	5.57

defining possible system-wide inspection strategies. The heuristic approach results in inspection plans that are likely close to but not identical to the globally optimal plans. Because it is not actually possible to compute the optimal plan even for simple structures such as the Daniels system studied in this paper, there is no reference against which to evaluate goodness of the results. However, the computation of the expected cost for a fixed strategy is accurate (bare the Monte Carlo error and approximations in the deterioration model); it is therefore possible to compare the proposed inspection strategies against any other proposal.

The results also show that – in analogy to the optimization of inspections for components – the total expected life-cycle cost of different inspection strategies is rather flat around the optimum (see Figs. 13 and 14). For this reason, it is sufficient to restrict the optimization to investigating a discrete set of values of the optimization parameters, taking into account operational constraints.

In practice, inspection planning is commonly performed following a reliability-based rather than risk-based approach, i.e. instead of optimizing the total expected life-cycle cost one aims at minimizing inspection and repair cost while ensuring a minimum level of reliability. The proposed framework is also applicable in this context, by fixing the probability threshold at the system reliability level, and then optimizing the number of inspection campaigns and the number of inspections per campaign. For example, if the minimum reliability is associated with a probability of structural failure of 10^{-5} per year, then the optimum strategies change following Figs. 13 and 14. The advantage of the proposed approach is also that it correctly computes the system reliability, which component-based inspection planning can get wrong completely (see Section 3.5.3).

In the numerical investigation presented in this paper, we apply the framework to an idealized structure and an idealized deterioration model, which are chosen for demonstration purposes. In real-life applications, these models will be more sophisticated, which leads to additional challenges, but does not affect the main findings of this paper. It is rather straightforward to include more sophisticated deterioration models into the DBN, as long as the number of random variables in each time step is limited. This can be achieved in most cases by grouping random variables in the DBN (see [54] for an example). The extension of the DBN model to more complex structural systems is discussed in [25] and an application of the system RBI framework to such a system is presented in [4].

With respect to the optimization procedure, the presented Daniels system is simplified in that all elements have the same structural importance and the fatigue performance is assumed to be the same for all elements. This allows using a simple proxy for the VoI, because the learning potential is a function of the element probability of failure only. It remains to be investigated what is a good proxy for the VoI in structural systems whose elements have different degrees of importance. The optimization problem presented in this paper is also further complicated when deciding among multiple types of inspection techniques, or when considering structural health monitoring to complement inspections. These aspects are left for future investigation.

5. Conclusions

A framework to determine optimal inspection-repair strategies for deteriorating structural systems subject to reliability constraints is proposed. The framework – for the first time – enables a system-wide optimization, which accounts for (a) the interaction among element deterioration states, (b) the relation between the reliability of the structural elements and the structural system, and (c) the effect of information obtained on one element of the structure on the remaining elements and the overall system. The framework also enables the use of state-of-the-art deterioration models for the individual elements. To tackle the computational challenges associated with this complex pre-posterior optimization problem, we propose heuristics for planning inspections, which are informed by practical constraints commonly encountered in the asset integrity management of engineering structures. To compute the expected cost of a system-wide inspection strategy, we nest a dynamic Bayesian network (DBN) algorithm inside a Monte Carlo analysis that accounts for uncertain inspection outcomes. The numerical investigation demonstrates the effectiveness of the proposed framework.

Acknowledgement

We thank Elizabeth Bismut for her valuable comments on an earlier version of this manuscript. This work was supported by the Deutsche Forschungsgemeinschaft (DFG) through Grants STR 1140/3-1 and STR 1140/3-2 and the Consejo Nacional de Ciencia y Tecnología (CONACYT) through Grant No. 311700.

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