Hochschule Bremen
City University of Applied Sciences

MATLAB® SIMULINK® Exercise – Filling a tapered Container

Modelling and Simulation 2022/2023 | MEAM 19

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Problem Definition

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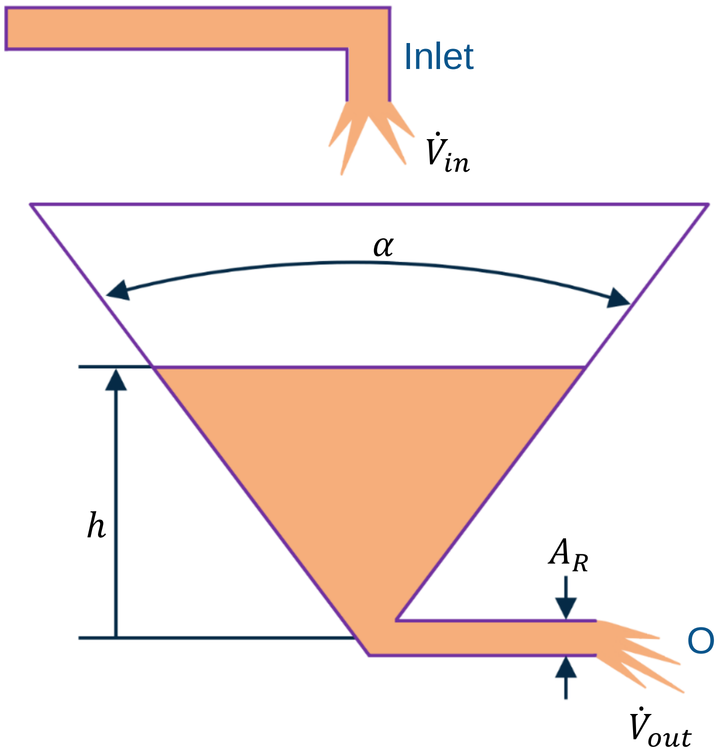
- A tapered container with a cone angle α is filled with a liquid at a flow rate $\dot{V}_{in}$
- The filling level h rises until a balanced state is given between $\dot{V}_{in}$ and $\dot{V}_{out}$
- Simplification: no friction

1. What is the value of $\dot{V}_{out}$ at $t \rightarrow \infty$?
2. State the differential equation
3. Build the block diagram
4. Plot $h(t)$ and $\dot{V}_{out}(t)$
5. Task list



Answer: 1.: $\dot{V}_{out} = 20 \left[\frac{m^3}{h}\right]$

Given:

$g = 9{,}81 \left[\frac{m}{s^2}\right]$ $\dot{V}_{in} = 20 \left[\frac{m^3}{h}\right]$

$\alpha = 60\ [°]$

$A_R = 12\ [cm^2]$

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Solution

2. State the differential equations

- The difference in in-and outgoing volume leads to a time rate of change of the volume in the container :

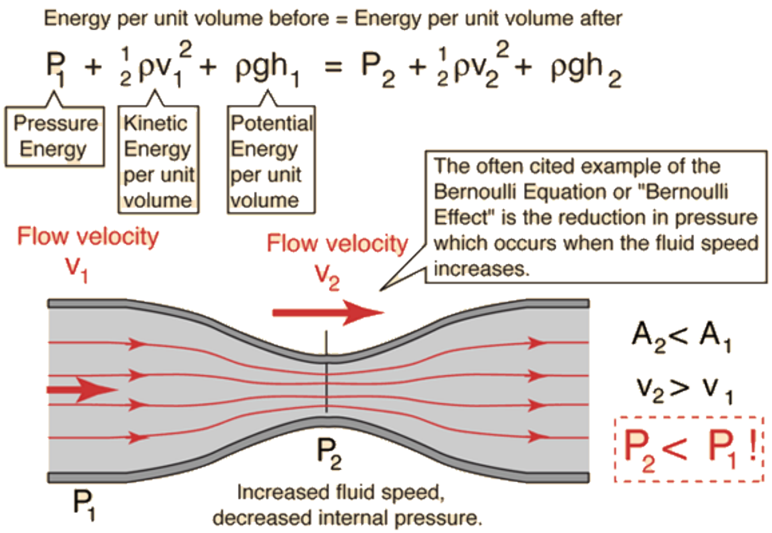
$$\dot{V}_{in} - \dot{V}_{out} = \frac{dV}{dt}$$

- The outgoing flow rate equals the average flow velocity v times the opening cross section A_R:

$$\dot{V}_{out} = v \cdot A_R$$

?

-> Bernoulli!

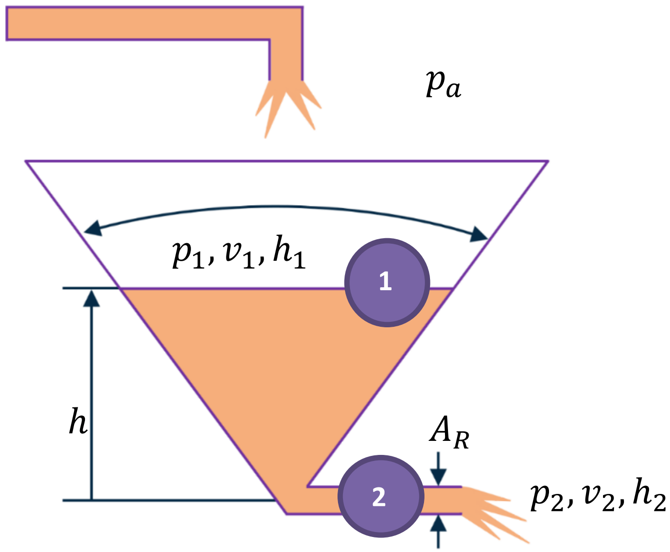


2. State the differential equations

- Bernoulli!

$$p_1 + \rho \cdot g \cdot h_1 + \frac{\rho}{2} v_1^2 = p_2 + \rho \cdot g \cdot h_2 + \frac{\rho}{2} v_2^2$$

- Any simplifications?
- Assuming: $p_1 = p_a$, $h_1 = h$, $v_1 = 0$, $p_2 = p_a$, $h_2 = 0$



- -> $g \cdot h_1 = \frac{1}{2} v_2^2$
- -> $v = v_2 = \sqrt{2 \cdot g \cdot h_1}$ (Torricelli´s Theorem)
- -> $\dot{V}_{out} = \sqrt{2 \cdot g \cdot h_1} \cdot A_R$

$$\underbrace{\dot{V}_{in} - \dot{V}_{out}}_{\text{known}} = \underbrace{\frac{dV}{dt}}_{\text{unknown}}$$

2. State the differential equations

- Volume V of a cone/ tapered container: $V = \frac{1}{3} \cdot \pi \cdot r^2 \cdot h$
- Radius r of the container: $r = tan\frac{\alpha}{2} \cdot h = \frac{h}{\sqrt{3}}$

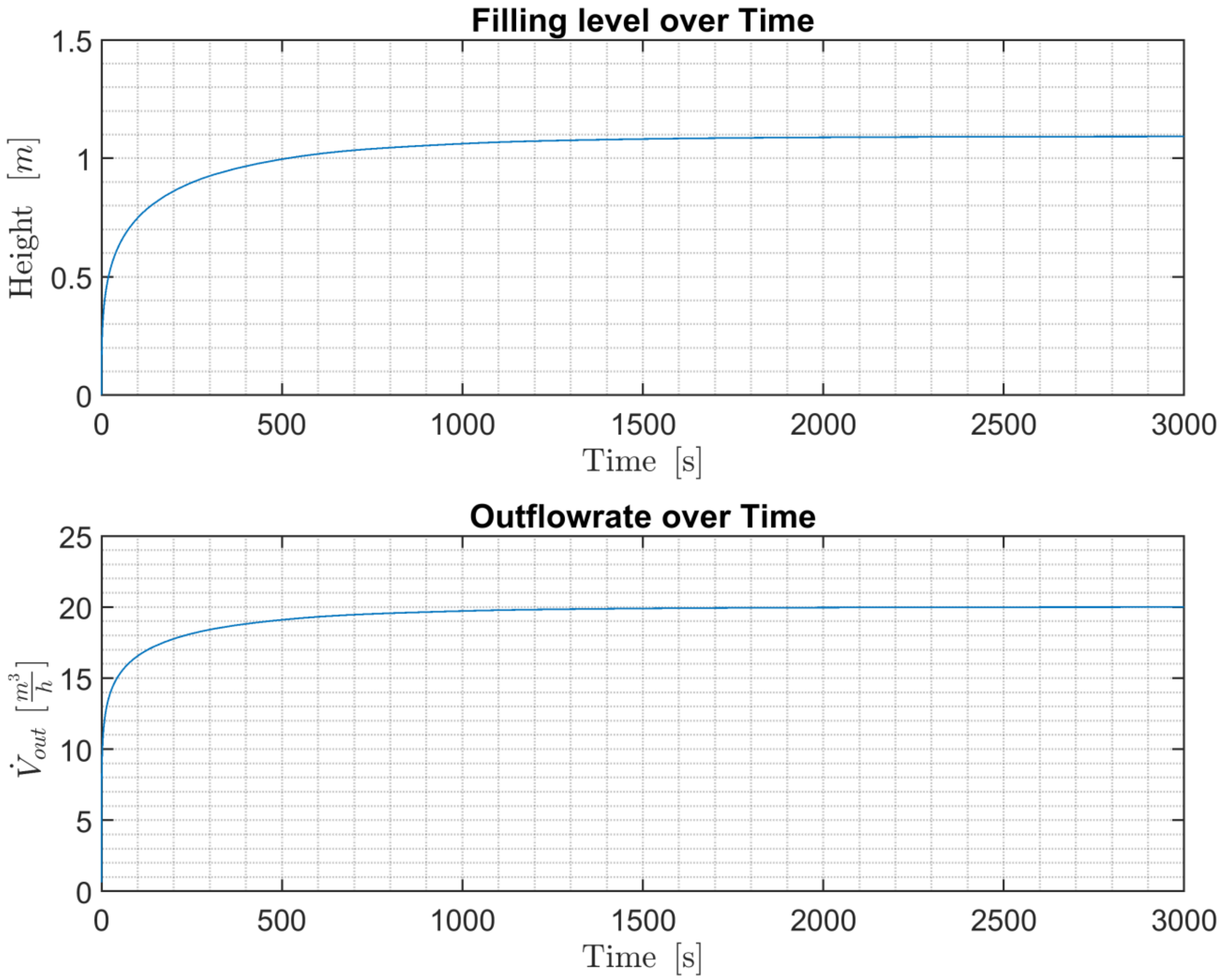
-> $V = \frac{1}{3} \cdot \pi \cdot \left(\frac{h}{\sqrt{3}}\right)^2 \cdot h = \frac{1}{9} \cdot \pi \cdot h^3$

-> $\frac{dV}{dt} = \left(\frac{\pi}{9} \cdot h^3\right) dt = \frac{\pi}{3} \cdot h^2 \cdot \dot{h}$

-> $\dot{V}_{in} - \sqrt{2 \cdot g \cdot h_1} \cdot A_R = \frac{\pi}{3} \cdot h^2 \cdot \dot{h}$, $h_{stat} = \frac{\dot{V}_{in}{}^2}{2 \cdot g \cdot A_R^2} = 1{,}1m$

-> $\dot{h} = \frac{\dot{V}_{in} - \sqrt{2 \cdot g \cdot h_1} \cdot A_R}{\frac{\pi}{3} \cdot h^2}$

4. Plot $h(t)$ and $\dot{V}_{out}(t)$



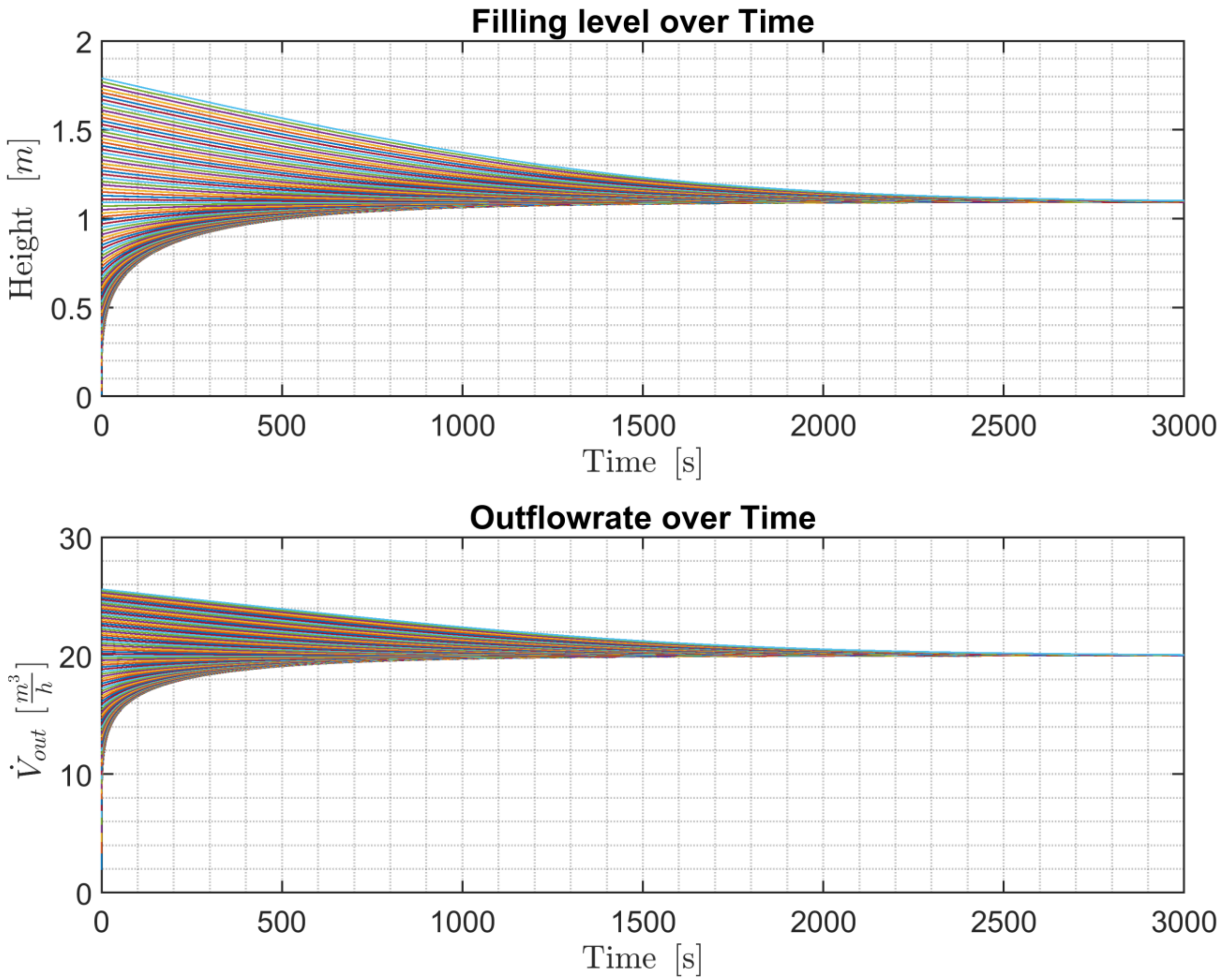
5. Task list | Option 1

- Create a Simulink model with the name *"P07_S_Filling_a_tapered_container_"+YourLastName+".slx* to achieve the presented results on slide 8

- Create a m-File with the name "*P07_M_Filling_a_tapered_container__"+YourLastName+".m*". This file should contain:
  - An Init part
  - A part to load all necessary variables
  - A part to run or sim your Model
  - A part that saves your result of the filling level h and the $\dot{V}_{out}$ over the time in one plot (*subplot* in 2019a or older, *tiledlayout* in 2019b or newer) with the name "*P07_Filling_a_tapered_container_Results1_O1_"+YourLastName+".png*". This should also implement a proper title, proper labels (with Latex) and of course the correct results.
  - Create an array or vector with at least ten different initial conditions for the integrator, simulate the modified model and create a second plot (*"P07…Results2_O1_… .png"*) as shown on slide 10
  - Use the *print* command for saving both figures as .png´s with a resolution of 600 dpi

  - Just send the .m and the .slx files to denis.zimmer@lba.hs-bremen.de

5. Plot $h(t)$ and $\dot{V}_{out}(t)$ | Option 1



5. Task list | Option 2

- Use Simscape ☺.
- The only rated result is a scope to see the result of a volume after a successful simulation. No .m file needed.

- Just send the .m (not necessary) and the .slx ((*“P07_S_...Simscape_... .slx”*) file to denis.zimmer@lba.hs-bremen.de

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Thank you for your Attention!

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