



Practice article

Model-free adaptive integral terminal sliding mode predictive control for a class of discrete-time nonlinear systems[☆]

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HIGHLIGHTS

- The proposed control approach is easy to implement because it merely uses the input and output data model of the system.
- A new model-free adaptive digital integral terminal sliding mode predictive control scheme without depending on any information of mathematical model.

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ABSTRACT

In this paper, a new model-free adaptive digital integral terminal sliding mode predictive control scheme is proposed for a class of nonlinear discrete-time systems with disturbances. The characteristic of the proposed control approach is easy to be implemented because it merely adopts the input and output data model of the system based on compact form dynamic linearization (CFDL) data-driven technique, while the technique of perturbation estimation is applied to estimate the disturbance term of the system. Moreover, by means of combining model predictive control and CFDL digital integral terminal sliding mode control (CFDL-DITSMC), the CFDL digital integral terminal sliding mode predictive control (CFDL-DITSMPC) method is proposed, which can further improve the tracking accuracy and disturbance rejection performance in comparison with the CFDL model-free adaptive control, neural network quasi-sliding mode control and the CFDL-DITSMC scheme. Meanwhile, the stability of the proposed approach is guaranteed by theoretical analysis, and the effectiveness of the proposed method is also illustrated by numerical simulations and the experiment on the two-tank water level control system.

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1. Introduction

In recent years, due to the continuous improvement of the control precision requirements, the control problem of nonlinear discrete time systems with perturbation and uncertainties has received increasing attention. Sliding mode control (SMC), as a nonlinear control strategy, is easy to be implemented and robust in the presence of interference, and has been successfully adopted in a large variety of areas [1,2]. However, the fine characteristics of continuous-time control systems disappear in the discrete-time control systems when the control sampling time increases.

To solve the problem, in [3] the idea of discrete-time sliding mode control (DSMC), which has gradually received attention, is presented.

So far, numerous approaches have displayed the DSMC schemes on various papers [4,5]. The existing literature has illustrated that SMC with a conventional proportional type of sliding function is able to possess a worse control performance than the integral one [6,7]. Based on a sliding mode state observer, the integral sliding function was used to design the controller, and the control method can be easily applied to a micro/nanopositioning piezostage [8]. These methods mentioned above require the model of the plant, that is to say, the control techniques based on the model are usually implemented under the assumption concerning excellent understanding of process dynamics and their operational environment. However, it is actually extremely difficult to obtain an accurate mathematical model.

As we know, the typical modern control methods ordinarily include adaptive control [9], model predictive control (MPC) [10] Backstepping control [11], and so on. Since the performance

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valuation, stability analysis, controller designing, etc., are all dependent on the assumptions, structure, and dynamics imposed on the mathematical model of the controlled process, it is also called model based control theory. At present, data-driven has been applied to decision-making, fault diagnosis, and fault-tolerant control, etc. In [12,13], a model-free adaptive control (MFAC) method based on the compact form dynamic linearization (CFDL) data-driven modeling is proposed, when controller design and the closed-loop system analysis only used the measured input and output data of the control system, without any model dynamics involved. In practical industries, more than 500 scholars took the MFAC approach to solve a variety of control issues, such as the PWM control system [14,15], liquid-level control systems [16–18], controller design for wind turbines and solar power generation [19–21]. The literature [22] has first introduced the model-free adaptive sliding mode control about the combination of MFAC with DSMC. However, it fails to take the effects of disturbance and system uncertainty into account. Based on [22], a neural network adaptive quasi-sliding mode control law (NN-QSMC) based on CFDL is proposed in [23], in which radial basis function neural network (RBFNN) is used to estimate the uncertainty and disturbance of the control system. Subsequently, many scholars have combined DSMC and MFAC to solve practical problems. By combining MFAC and the sliding mode exponential reaching law, a speed controller for all the joints of the robot is developed to implement the target speed tracking [24,25]. A method that combines advantages of SMC and MFAC is put forward for a pressure control system [26]. The combination of second-order SMC with MFAC has been adopted in the three-tank level control system [27]. A model-free adaptive integral SMC method with control input saturation is presented for autonomous four-wheeled mobile vehicle parking systems [28].

Recently, the literature [29] has employed the MPC to present digital integral terminal sliding mode predictive control (DITSMPC) algorithm. However, combining the method of DITSMPC and CFDL has not been realized before, and how to yield an improved performance through combining them together is unknown. In this work, a CFDL-DITSMPC scheme designed for a class of nonlinear discrete-time system with disturbances is proposed.

The main contribution of this paper is to propose a new CFDL-DITSMPC scheme without depending on any information of the mathematic model, which utilizes the CFDL technique and the technique of perturbation estimation. Numerical simulation and experimental verification show that the CFDL-DITSMPC algorithm is much better than CPDL-MFAC, NN-QSMC and CFDL-DITSMC schemes in reducing the tracking error and disturbance rejection performance. In the next section, a class of nonlinear systems and notations is introduced. The design procedure of CFDL-DITSMC and CFDL-DITSMPC algorithms is explained in details in Sections 3 and 4. Numerical simulation examples and experimental investigation are shown in Section 5, to explain the different performances of the control system. Section 6 has given a brief summary of the research approach.

2. Problem formulation

The proposed control algorithm is designed based on CFDL technique. Therefore, we first simply introduce the equivalent FDL data model with a concept called pseudo partial derivative (PPD) for single input and single output (SISO) discrete-time nonlinear system in this section. Details of the CFDL can be found in [12,13,16].

A SISO discrete-time nonlinear system is described as

$$y_m(k+1) = f(y(k), y(k-1), \dots, y(k-n_y), u(k), u(k-1), \dots, u(k-n_u)) + f_p(k) \quad (1)$$

where $y_m(k)$ and $u(k)$ are output and input of the system, respectively; n_y and n_u are the unknown positive integers; $f_p(k)$ represents the bounded disturbances with $|f_p(k)| \leq D$, $D > 0$. $f(\cdot)$ stands for the discrete-time nonlinear system $y(k+1)$ as in

$$y_m(k+1) = y(k+1) + f_p(k) \quad (2)$$

Assumption 1. System (1) is controllable and observable, that is, if the desired signal $y_r(k+1)$ is bounded, then, the control input signal $u(k)$ is bounded, and $y_m(k+1)$ can track the desired signal $y_r(k+1)$.

Assumption 2. The partial derivative of $f(\cdot)$ with respect to control input $u(k)$ is continuous.

Assumption 3. The nonlinear system (1) is generalized Lipschitz, that is, $|\Delta y_m(k+1)| \leq b |\Delta u(k)|$ under the condition of any k and $|\Delta u(k)| \neq 0$, where $b > 0$, $\Delta y_m(k+1) = y_m(k+1) - y_m(k)$, and $\Delta u(k) = u(k) - u(k-1)$.

Remark 1. Assumption 1 is a basic requirement for the system to be controlled. To control such a system is impossible if Assumption 1 is not satisfied. Assumption 2 is satisfied for a general nonlinear system. Assumption 3 imposes an upper bound limitation on $\Delta y_m(k+1)$, which is a common condition in many nonlinear systems.

Lemma 1. If $f(\cdot)$ satisfies Assumptions 1 and 2, when $|\Delta u(k)| \neq 0$, $\phi(k)$ called the PPD definitely exists. The CFDL data model can be described as

$$\Delta y(k+1) = \phi(k) \Delta u(k) \quad (3)$$

where $|\phi(k)| \leq b$ is bounded for constant k at any time.

Rewrite (3) as

$$y(k+1) = y(k) + \phi(k) \Delta u(k) \quad (4)$$

From (2) and (4), we have

$$\Delta y_m(k+1) = \Delta y(k+1) + \Delta f_p(k) \quad (5)$$

where $\Delta y_m(k+1) = y_m(k+1) - y_m(k)$, $\Delta y(k+1) = y(k+1) - y(k)$, and $\Delta f_p(k) = f_p(k) - f_p(k-1)$.

Considering (3), then (5) can be further written as

$$y_m(k+1) = y_m(k) + \phi(k) \Delta u(k) + \Delta f_p(k) \quad (6)$$

Let $\phi(k) = \vartheta(k) + \delta$, and then it follows

$$y_m(k+1) = y_m(k) + (\vartheta(k) + \delta) \Delta u(k) + \Delta f_p(k) \quad (7)$$

where δ is the nonnegative constant.

Remark 2. It should be mentioned that the value of $\phi(k)$ in (3) may not be consistent with the variation range of $\phi(k)$, or in other words, $\phi(k)$ may satisfy

$$\max \left| \frac{\phi(k) - \phi(k-1)}{h} \right| \leq \bar{\varepsilon} \quad (8)$$

where $\bar{\varepsilon}$ is a positive constant and h represents the sampling interval. Therefore, $\phi(k)$ can be divided into the variable term $\vartheta(k)$ and the constant term δ . It is notable that $\phi(k)$ may be extremely small at some sampling times, and then the parameter δ is selected to ensure that $\phi(k)$ is maintained at an appropriate range.

3. CFDL-DITSMC design

The design process of the CFDL-DITSMC scheme is described in this section. First, the disturbance is estimated by adopting the technique of perturbation estimation. Then, a model free adaptive controller is obtained based on the CFDL data model and the DITSMC scheme. The stability of the proposed control method is also proved.

3.1. Disturbance estimation

The disturbance term $\Delta f_p(k)$ of the nonlinear system (6) can be estimated as

$$\Delta \hat{f}_p(k) = \Delta f_p(k-1) = y_m(k) - y_m(k-1) - \phi(k-1) \Delta u(k-1) \quad (9)$$

Then (6) can be written as

$$y_m(k+1) = y_m(k) + \phi(k) \Delta u(k) + \Delta \hat{f}_p(k) - \Delta \tilde{f}_p(k) \quad (10)$$

where $\Delta \tilde{f}_p(k) = \Delta \hat{f}_p(k) - \Delta f_p(k)$ is the disturbance estimation error, and we have

$$\begin{aligned} \Delta \tilde{f}_p(k) &= \Delta f_p(k-1) - \Delta f_p(k) \\ &= ((y_m(k) - y_m(k-1)) - (y_m(k+1) - y_m(k)) \\ &\quad - \phi(k-1) \Delta u(k-1) + \phi(k) \Delta u(k)) \end{aligned} \quad (11)$$

From Assumption 1 and Lemma 1, we know that $y_m(k)$, $u(k)$, and $\phi(k)$ are bounded. Thus, we can assume that $\Delta \tilde{f}_p(k)$ is bounded.

3.2. PPD estimation

Define $\hat{\phi}(k) = \hat{\vartheta}(k) + \delta$, and then the index function given in [13] for the unknown PPD estimation is used in this paper as

$$\begin{aligned} J(\hat{\phi}(k)) &= \left| y(k) - y(k-1) - \hat{\phi}(k) \Delta u(k-1) \right|^2 \\ &\quad + \mu \left| \hat{\phi}(k) - \hat{\phi}(k-1) \right|^2 \end{aligned} \quad (12)$$

where $\hat{\phi}(k)$ is the estimation value of $\phi(k)$. Using $\partial J / \partial \hat{\phi}(k) = 0$, we have

$$\begin{aligned} \hat{\phi}(k) &= \hat{\phi}(k-1) + \frac{\eta \Delta u(k-1)}{\mu + \Delta u(k-1)^2} \\ &\quad \times \left[\Delta y_m(k) - \Delta \hat{f}_p(k) - \hat{\phi}(k-1) \Delta u(k-1) \right] \end{aligned} \quad (13)$$

$$\hat{\vartheta}(k) = \hat{\vartheta}(1), \text{ if } \left| \hat{\vartheta}(k) \right| \leq \varepsilon \text{ or } \left| \Delta u(k-1) \right| \leq \varepsilon \quad (14)$$

where $\mu > 0$; $\eta \in (0, 1]$; ε is a significantly small positive constant; the nonnegative constant $\hat{\vartheta}(1)$ is the initial value of $\hat{\vartheta}(k)$.

Then, (10) can be further written as

$$y_m(k+1) = y_m(k) + (\hat{\vartheta}(k) + \delta) \Delta u(k) + \Delta \hat{f}_p(k) - \Delta \tilde{f}_p(k) \quad (15)$$

where $\hat{\vartheta}(k)$ is the estimation value of $\vartheta(k)$, which can be obtained by Eq. (13).

Remark 3. In order to ensure the feasibility of the adaptive law (13), the choice of μ should be less than the value of $|\Delta u(k)|$.

Remark 4. The updating law (13) is designed merely by input and output measurement data of the controlled system, without any relations with model structural and dynamics information of the controlled system.

Remark 5. For the parameter estimation algorithm (13) have stronger ability in tracking time-varying parameter, a reset mechanism has been given in (14). In addition, Eq. (14) also has been utilized in stability analysis.

3.3. Design of the CFDL-DITSMC scheme

Define the tracking error as

$$e(k) = y_m(k) - y_r(k) \quad (16)$$

where $y_m(k)$ is the system output signal and $y_r(k)$ is the bounded desired input signal.

Define the PI type of the discrete terminal sliding function as follows [29]

$$s(k) = \lambda_1 e(k) + \lambda_2 E(k-1) \quad (17)$$

where $\lambda_1 > 0$ and $\lambda_2 > 0$, and the integral error item is

$$E(k) = \sum_{i=0}^k e_i^\alpha = E(k-1) + e_k^\alpha \quad (18)$$

where $0 < \alpha < 1$, which is the ratio of two odd numbers.

The CFDL-DITSMC control strategy can be derived from the discrete reaching law as follows

$$\Delta s(k) = s(k+1) - s(k) = 0 \quad (19)$$

From (19), the following equations can be obtained

$$s(k+1) = s(k) \quad (20)$$

$$\Rightarrow \lambda_1 e(k+1) + \lambda_2 E(k) = s(k) \quad (21)$$

$$\Rightarrow \lambda_1 (y_m(k+1) - y_r(k+1)) + \lambda_2 E(k) = s(k) \quad (22)$$

Substituting (15) into (22) yields

$$\begin{aligned} s(k) &= \lambda_1 \left[y_m(k) + (\hat{\vartheta}(k) + \delta) \Delta u(k) \right. \\ &\quad \left. + \Delta \hat{f}_p(k) - \Delta \tilde{f}_p(k) - y_r(k+1) \right] + \lambda_2 E(k) \end{aligned} \quad (23)$$

The equivalent control $\Delta u^{eq}(k)$ can be obtained by Eq. (23); the disturbance estimation error $\Delta \tilde{f}_p(k)$ is not taken into account

$$\begin{aligned} \Delta u^{eq}(k) &= \frac{1}{(\hat{\vartheta}(k) + \delta)} \left(\lambda_1^{-1} s(k) - \lambda_1^{-1} \lambda_2 E(k) \right. \\ &\quad \left. - \Delta \hat{f}_p(k) + y_r(k+1) - y_m(k) \right) \end{aligned} \quad (24)$$

The implementation of the equivalent controller in (24) requires a future reference position $y_r(k+1)$. In practice, the desired trajectory is usually predefined in the control system, so $y_r(k+1)$ is known in (24).

It is worth noticing that the control law (24) can be obtained by assuming that $\Delta \tilde{f}_p(k)$ is accurately estimated, i.e., $\Delta \tilde{f}_p(k) = 0$. However, $\Delta \tilde{f}_p(k) \neq 0$. To guarantee the robustness of $\Delta u^{eq}(k)$, consider a switching control action as follows

$$\Delta u^{sw}(k) = \frac{-\lambda_s}{(\hat{\vartheta}(k) + \delta)} \text{sign}(s(k)) \quad (25)$$

where $\text{sign}(\cdot)$ denotes the signum function; λ_s satisfies the following condition

$$\lambda_s > |\Delta \tilde{f}_p(k)| \quad (26)$$

where $\Delta \tilde{f}_p(k)$ is described by (11). The control action of $\Delta u^{sw}(k)$ is to tolerate the error of disturbance estimation so that the final CFDL-DITSMC control law is shown as follows:

$$\Delta u(k) = \Delta u^{eq}(k) + \Delta u^{sw}(k) \quad (27)$$

That is

$$\Delta u(k) = \frac{1}{(\hat{\vartheta}(k) + \delta)} \left[\lambda_1^{-1} s(k) - \lambda_1^{-1} \lambda_2 E(k) - \Delta \hat{f}_p(k) + y_r(k+1) - y_m(k) - \lambda_s \text{sgn}(s(k)) \right] \quad (28)$$

4. Stability analysis

Theorem 1. The nonlinear system (1), satisfying Assumptions 1–3, is controlled by the CFDL-DITSMC algorithms (13), (14), and (28), and the desired signal $y_r(k+1)$ is bounded, and then the estimation value of $\hat{\phi}(k)$ is bounded.

Proof. When $|\hat{\vartheta}(k)| \leq \varepsilon$, the reset algorithm (14) ensures that $\hat{\vartheta}(k)$ is bounded.

When $|\hat{\vartheta}(k)| > \varepsilon$, subtracting $\phi(k)$ on both sides of (13) yields

$$\tilde{\phi}(k) = \Delta \phi(k) + \left(1 - \frac{\eta \Delta u(k-1)^2}{\mu + \Delta u(k-1)^2} \right) \tilde{\phi}(k-1) \quad (29)$$

where $\tilde{\phi}(k) = \phi(k) - \hat{\phi}(k)$, $\Delta \phi(k) = \phi(k) - \phi(k-1)$.

From Lemma 1, b is a positive constant and $|\phi(k)| \leq b$, it is obvious that

$$|\tilde{\phi}(k)| < \left| \left(1 - \frac{\eta \Delta u(k-1)^2}{\mu + \Delta u(k-1)^2} \right) \right| |\tilde{\phi}(k-1)| + 2b \quad (30)$$

Since $\mu > 0$ and $\eta \in (0, 1]$, then

$$\Delta u(k-1)^2 < \Delta u(k-1)^2 < \mu + \Delta u(k-1)^2 \quad (31)$$

Therefore, there is a positive constant $d \in (0, 1]$, which leads to

$$1 - \frac{\eta \Delta u(k-1)^2}{\mu + \Delta u(k-1)^2} \leq d < 1 \quad (32)$$

Hence

$$\begin{aligned} |\tilde{\phi}(k)| &\leq d |\tilde{\phi}(k-1)| + 2b \\ &\leq d^2 |\tilde{\phi}(k-2)| + 2db + 2b \\ &\leq \dots \leq d^k |\tilde{\phi}(0)| + \frac{2b(1-d^k)}{1-d} \end{aligned} \quad (33)$$

In view of (33), $\tilde{\phi}(k)$ is bounded. In this case, $\hat{\phi}(k)$ is bounded.

Theorem 2. Assuming that Assumptions 1–3 are held for the system (1), if the controller (28) with (26) is employed, and the time-varying parameters $\Delta \hat{f}_p(k)$ and $\hat{\phi}(k)$ can be updated by (9), (13) and (14), the quasi-sliding mode will achieve convergence in a finite number of steps.

Proof. From (14), the sliding mode function can be expressed as

$$s(k+1) = \lambda_1 e(k+1) + \lambda_2 E(k) \quad (34)$$

Substituting (16) into (34) and noting (15) as well as (28) gives

$$\begin{aligned} s(k+1) &= \lambda_1 (y_m(k+1) - y_r(k+1)) + \lambda_2 E(k) \\ &= \lambda_1 \left[y_m(k) + (\hat{\vartheta}(k) + \delta) \Delta u(k) + \Delta \hat{f}_p(k) - y_r(k+1) \right] + \lambda_2 E(k) \\ &= \lambda_1 \left[\lambda_1^{-1} s(k) - \lambda_1^{-1} \lambda_2 E(k) + \Delta \hat{f}_p(k) - \Delta \hat{f}_p(k-1) - \lambda_s \text{sgn}(s(k)) \right] + \lambda_2 E(k) \\ &= s(k) + \lambda_1 (\Delta \hat{f}_p(k) - \Delta \hat{f}_p(k-1)) - \lambda_1 \lambda_s \text{sgn}(s(k)) \end{aligned} \quad (35)$$

Substituting (11) into (35) yields

$$s(k+1) - s(k) = -\lambda_1 \lambda_s \text{sgn}(s(k)) - \lambda_1 \Delta \tilde{f}_p(k) \quad (36)$$

From (26), we have

$$\lambda_s > \Delta \tilde{f}_p(k) > -\lambda_s \quad (37)$$

Hence, when $s(k) > 0$, from (36) and (37), the following equation can be obtained

$$\begin{aligned} s(k+1) - s(k) &= -\lambda_1 \lambda_s - \lambda_1 \Delta \tilde{f}_p(k) \\ \Rightarrow -2\lambda_1 \lambda_s &< s(k+1) - s(k) < 0 \end{aligned} \quad (38)$$

When $s(k) < 0$, we have

$$\begin{aligned} s(k+1) - s(k) &= \lambda_1 \lambda_s - \lambda_1 \Delta \tilde{f}_p(k) \\ \Rightarrow 0 &< s(k+1) - s(k) < 2\lambda_1 \lambda_s \end{aligned} \quad (39)$$

In view of (38) and (39), then we can get

$$|s(k+1) - s(k)| < 2\lambda_1 \lambda_s \quad (40)$$

$$|s(k+1)| < |s(k)| \quad (41)$$

Eq. (41) satisfies the existing conditions of the discrete sliding mode [30]

$$\begin{aligned} [s(k+1) - s(k)] \text{sgn}(s(k)) &< 0 \\ [s(k+1) + s(k)] \text{sgn}(s(k)) &> 0 \end{aligned} \quad (42)$$

Thus, $s(k)$ is monotonically decreased, the quasi-sliding mode state will be reached in a limited number of steps, or in other words, the control system is stable and the tracking error converges to zero neighborhoods.

Remark 6. The chattering phenomenon may occur due to the sign function in (28). Therefore, in order to reduce the chattering phenomenon, the saturation function (43) is employed in the control law (28) to replace the sign function

$$\text{sat}(k) = \begin{cases} \text{sign}(s(k)), & \text{if } |s(k)| > \Delta \\ s(k)/\Delta, & \text{if } |s(k)| < \Delta \end{cases} \quad (43)$$

where $\Delta > 0$, and the parameter Δ is usually tuned to make a trade-off between robustness of the controller and the chattering effect.

4. CFDL-DITSMPC design

In this section, a new strategy known as CFDL-DITSMPC for further improving the control system's tracking accuracy is developed.

4.1. The CFDL-DITSMPC scheme

A control action $\Delta u^{mp}(k)$ is generated by the MPC to drive the output of the system to the sliding surface. In view of (27), the total control action of CFDL-DITSMPC is

$$\Delta u^{sm}(k) = \Delta u(k) + \Delta u^{mp}(k) \quad (44)$$

Substituting the control algorithm (44) into (34) gets

$$\begin{aligned} s(k+1) &= \lambda_1 (y_m(k+1) - y_r(k+1)) + \lambda_2 E(k) \\ &= \lambda_1 \left[y_m(k) + (\hat{\vartheta}(k) + \delta) \Delta u^{sm}(k) + \Delta \hat{f}_p(k) - y_r(k+1) \right] + \lambda_2 E(k) \\ &= \lambda_1 \left[\lambda_1^{-1} s(k) - \lambda_1^{-1} \lambda_2 E(k) + \Delta \hat{f}_p(k) - \Delta \hat{f}_p(k-1) - \lambda_s \text{sat}(s(k)) + (\hat{\vartheta}(k) + \delta) \Delta u^{mp}(k) \right] + \lambda_2 E(k) \\ &= s(k) + \lambda_1 (\hat{\vartheta}(k) + \delta) \Delta u^{mp}(k) - \lambda_1 \Delta \tilde{f}_p(k) - \lambda_1 \lambda_s \text{sat}(s(k)) \end{aligned} \quad (45)$$

From (45), we can obtain the N-step forward sliding mode function

$$\begin{aligned} s(k+N) = & s(k) + \lambda_1 \left[\left(\hat{\vartheta}(k) + \delta \right) \Delta u^{mp}(k) + \left(\hat{\vartheta}(k+1) + \delta \right) \right. \\ & * \Delta u^{mp}(k+1) + \cdots \\ & + \left(\hat{\vartheta}(k+N-1) + \delta \right) \Delta u^{mp}(k+N-1) \left. \right] \\ & - \lambda_1 \left[\Delta \tilde{f}_p(k) + \Delta \tilde{f}_p(k+1) + \cdots + \Delta \tilde{f}_p(k+N-1) \right] \\ & - \lambda_1 \lambda_s [\text{sat}(s(k)) + \text{sat}(s(k+1)) + \cdots \\ & + \text{sat}(s(k+N-1))] \end{aligned} \quad (46)$$

where N is the number of the predictive horizon, let

$$S(k) = [s(k+1) s(k+2) \cdots s(k+N)]^T \quad (47)$$

$$U(k-1) = [\Delta u^{mp}(k) \Delta u^{mp}(k+1) \cdots \Delta u^{mp}(k+N-1)]^T \quad (48)$$

$$F(k-1) = [\Delta \tilde{f}_p(k) \Delta \tilde{f}_p(k+1) \cdots \Delta \tilde{f}_p(k+N-1)]^T \quad (49)$$

$$T(k-1) = [\text{sat}(s(k)) \text{sat}(s(k+1)) \cdots \text{sat}(s(k+N-1))]^T \quad (50)$$

$$\Phi(k) = \begin{bmatrix} \hat{\vartheta}(k) + \delta & 0 & \cdots & 0 \\ \hat{\vartheta}(k) + \delta & \hat{\vartheta}(k+1) + \delta & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ \hat{\vartheta}(k) + \delta & \hat{\vartheta}(k+1) + \delta & \cdots & \hat{\vartheta}(k+N-1) + \delta \end{bmatrix} \quad (51)$$

Then rewrite (46) as

$$S(k) = \Lambda s(k) + \Phi U(k-1) - \Gamma F(k-1) - K T(k-1) \quad (52)$$

where Λ represents the identity matrix; Γ and K are lower triangular matrices consisting of λ_1 and $\lambda_1 \lambda_s$, respectively.

Define the performance function as follows

$$J = S^T(k) S(k) + \omega U^T(k-1) U(k-1) \quad (53)$$

where the value of ω determines the magnitude of the MPC control action. The smaller ω is, the greater the control action is, and vice versa.

Substituting (52) into (53), under the following optimization condition [31]

$$\frac{\partial J}{\partial U(k-1)} = 0 \quad (54)$$

Yields

$$U(k-1) = -(\Phi^T \Phi + \omega I)^{-1} \Phi^T (\Lambda s(k) - \Gamma F(k-1) - K T(k-1)) \quad (55)$$

Due to the future disturbance value $F(k-1)$, the saturation function value $T(k-1)$ and the PPD matrix $\Phi(k)$ are unknown, and they are estimated as

$$\hat{F}(k-1) = [\Delta \tilde{f}_p(k-1) \Delta \tilde{f}_p(k-1) \cdots \Delta \tilde{f}_p(k-1)]^T \quad (56)$$

$$\hat{T}(k-1) = [\text{sat}(s(k)) \text{sat}(s(k)) \cdots \text{sat}(s(k))]^T \quad (57)$$

$$\hat{\Phi}(k) = \begin{bmatrix} \hat{\vartheta}(k) + \delta & 0 & \cdots & 0 \\ \hat{\vartheta}(k) + \delta & \hat{\vartheta}(k) + \delta & \cdots & 0 \\ \vdots & \vdots & \ddots & 0 \\ \hat{\vartheta}(k) + \delta & \hat{\vartheta}(k) + \delta & \cdots & \hat{\vartheta}(k) + \delta \end{bmatrix} \quad (58)$$

In view of (55), the MPC control action for the k time is obtained as

$$\Delta u^{mp}(k)$$

$$= -X \left(\hat{\Phi}^T \hat{\Phi} + \omega I \right)^{-1} \hat{\Phi}^T \left(\Lambda s(k) - \Gamma \hat{F}(k-1) - K \hat{T}(k-1) \right) \quad (59)$$

where $X = [\lambda_1^{-1} 0 \cdots 0]$.

Therefore, substituting (24) and (59) into (44), then (44) can be expressed as

$$\begin{aligned} \Delta u^{sm}(k) = & \frac{1}{(\hat{\vartheta}(k) + \delta)} \left[\lambda_1^{-1} s(k) - \lambda_1^{-1} \lambda_2 E(k) - \Delta \tilde{f}_p(k) \right. \\ & + y_r(k+1) - y_m(k) \left. \right] - X \left(\Phi^T \Phi + \omega I \right)^{-1} \Phi^T (\Lambda s(k) \\ & - \Gamma \hat{F}(k-1) - K \hat{T}(k-1)) \end{aligned} \quad (60)$$

4.2. Stability analysis

Theorem 3. The nonlinear system (1), satisfying Assumptions 1–3, is controlled by the CFDL-DITSMPC scheme (60), and the desired signal $y_r(k+1)$ is bounded. Then, the quasi-sliding mode will achieve convergence within a limited number of steps, and $e(k)$ is bounded

Proof. The proof of boundedness of $\hat{\vartheta}(k)$ is the similar as Theorem 1.

In order to facilitate the proof of stability, the attributes shown in [32] are taken into consideration, when assuming that the disturbance $\Delta f_p(k)$ has the following properties.

Property 1. $\Delta f_p(k) = O(T)$, $\Delta f_p(k) - \Delta f_p(k-1) = O(T^2)$, and $\Delta f_p(k) - 2\Delta f_p(k-1) + \Delta f_p(k-2) = O(T^3)$.

Property 2. Assuming $e(k+1) = \Theta e(k) + \gamma(k)$, where the magnitude of $\gamma(k)$ is of the order $O(T^3)$, and Θ is asymptotically stable ($\|\Theta\| < 1$), the magnitude of $e(k)$ is of the order $O(T^2)$ when $k \rightarrow \infty$.

Substituting (55)–(58) into (52) yields

$$\begin{aligned} S(k) = & \Lambda s(k) - \Phi (\Phi^T \Phi + \omega I)^{-1} \Phi^T (\Lambda s(k) - \Gamma \hat{F}(k-1) \\ & - K \hat{T}(k-1)) - \Gamma F(k-1) - K T(k-1) \end{aligned} \quad (61)$$

where $K T(k-1)$ is the switching control action, which is taking effect in the initially reaching phase [29]. Therefore, for the steady-state analysis, it is not necessary to consider the transient switching control action.

Furthermore, we assumed that there is no penalty for the $\Delta u^{mp}(k)$, namely $\omega = 0$. Then rewrite (61) as

$$S(k) = \Gamma (\hat{F}(k-1) - F(k-1)) \quad (62)$$

Next, taking (10), (49), (52) and (56) into account, we have

$$\begin{aligned} s(k+1) = & \lambda_1 (\Delta \tilde{f}_p(k-1) - \Delta \tilde{f}_p(k)) \\ = & \lambda_1 (\Delta f_p(k) - 2\Delta f_p(k-1) + \Delta f_p(k-2)) \end{aligned} \quad (63)$$

Hence, the magnitude of $s(k+1)$ is of the order $O(T^3)$ according to Property 1. $s(k+1)$ has a finite extreme value as follows

$$|s(k+1)| \leq \lambda_p \quad (64)$$

where λ_p is the quasi-sliding mode bandwidth.

Therefore, it can be concluded that the nonlinear system (1) controlled by (60) satisfies the reaching condition of the QSMC in a limited number of steps.

3. Error bound analysis

Define the error $e(k+1)$ as

$$(k+1) = y_m(k+1) - y_r(k+1) \quad (65)$$

Substituting (6) into Eq. (65), in view of (15) and (60), the predictive control action weight $\omega = 0$ is assumed. For the convenience of analyzing the ultimate tracking error, the transient switching control action $\hat{T}(k-1)$ is excluded [26]. Hence, the following expression is obtained

$$\begin{aligned} e(k+1) &= y_m(k) + \left[\lambda_1^{-1} s(k) - \lambda_1^{-1} \lambda_2 E(k) - \Delta \hat{f}_p(k) \right. \\ &\quad \left. + y_r(k+1) - y_m(k) \right] - \left(\hat{\vartheta}(k) + \delta \right) X \left(\Phi^T \Phi + \omega I \right)^{-1} \Phi^T \\ &\quad \times \left(\Delta s(k) - \Gamma \hat{F}(k-1) - K \hat{T}(k-1) \right) + \Delta \hat{f}_p(k) - y_r(k+1) \\ &= \lambda_1^{-1} s(k) - \lambda_1^{-1} \lambda_2 E(k) - \Delta \hat{f}_p(k) - \lambda_1^{-1} s(k) + \Delta \hat{f}_p(k-1) \end{aligned} \quad (66)$$

Rewrite (66) as

$$e(k+1) = (1 - \lambda_1^{-1} \lambda_2 e^{\alpha-1}(k)) e(k) - \lambda_1^{-1} s(k) + \Delta \hat{f}_p(k) - 2\Delta \hat{f}_p(k-1) + \Delta \hat{f}_p(k-2) \quad (67)$$

We know that $\Delta \hat{f}_p(k) - 2\Delta \hat{f}_p(k-1) + \Delta \hat{f}_p(k-2) = O(T^3)$ and $s(k) = O(T^3)$ by Property 1. In view of Property 2, if $|1 - \lambda_1^{-1} \lambda_2 e^{\alpha-1}(k)| < 1$, the output tracking error $e(k)$ converges to $O(T^2)$.

4. Simulation and experimental studies

To verify the effectiveness of the CFDL-DITSMC and CFDL-DITSMPC design schemes, simulation examples and an experiment are presented in this section. The experiment is implemented on the two-tank system YC-SX manufactured by unchuang Technology Company of China. Moreover, to show the characteristics of the proposed method, CPLD-MFAC and IN-QSMC are adopted for comparisons in simulation and experiments.

4.1. Simulation

Consider the SISO nonlinear system as

$$\begin{aligned} (k+1) &= \frac{y(k)y(k-1)y(k-2)u(k-1)(y(k-2)-1) + u(k)}{1 + y(k-1)^2 + y(k-2)^2} \\ &\quad + d(k) \end{aligned} \quad (68)$$

where $d(k)$ is the external disturbance, and it is given by

$$(k) = [0.5, 0.15 \sin(k/30)] [y(k), y(k-1)]^T \quad (69)$$

The initial states are assumed as $u(1:2) = 0$, $y(1:3) = 0$, $\hat{\vartheta}(1:2) = 2$, and $\varepsilon = 10^{-5}$. The resetting value $\hat{\vartheta}(1)$ is 0.5, and the CFDL-DITSMC parameters are set as $\mu = 0.5$, $\eta = 0.3$, $s = 0.0002$, $\lambda_1 = 0.3$, $\lambda_2 = 0.2$, $\alpha = 5/7$, $\omega = 0.01$, $\Delta = 0.002$, $\sigma = 3$, and $\varepsilon = 10^{-5}$. The coefficient λ_1 and λ_2 are selected $s \lambda_1 > 0$ and $\lambda_2 > 0$. Especially, in the case of selecting small λ_1 or large λ_2 , the control law (28) will become larger and it may even lead to extreme chattering phenomenon. Meanwhile, in the opposite case of selecting large λ_1 or small λ_2 , the control law (28) may be inadequate to guarantee the tracking convergence of the closed-loop system. The CFDL-DITSMPC controller parameters have been chosen according to CFDL-DITSMC, but with extra $N = 0$ and $\omega = 0.01$.

1) Effectiveness of the proposed methods

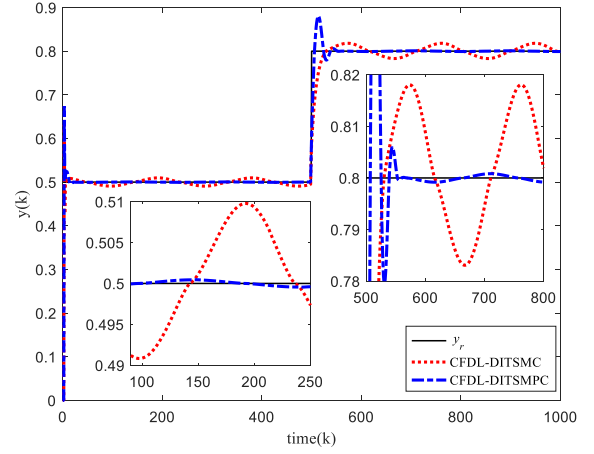


Fig. 1. Control performances of two proposed algorithms.

For the comparison purpose, simulation results of CFDL-DITSMC and CFDL-DITSMPC are given in Fig. 1. It can be observed that CFDL-DITSMPC achieves effective disturbance attenuation ability. After the 500th step, the given signal changes from 0.5 to 0.8, the system output can quickly adapt to desired signal changes and successfully converge to 0.8. Notably, the tracking error caused by the disturbance of the CFDL-DITSMPC scheme is much smaller than that of the CFDL-DITSMC scheme. Therefore, the main highlight of the CFDL-DITSMPC method compared with the CFDL-DITSMC method is that, under the drawback of the overshoot presence, the tracking error is clearly better in the steady state.

The control action and the output tracking error of CFDL-DITSMPC are demonstrated in Fig. 2(a) and (b) respectively. It is clear that there is no chattering phenomenon from Fig. 2(a). Fig. 2(c) shows that the disturbance estimation error $\Delta \hat{f}_p(k) = \Delta \hat{f}_p(k-1) - \Delta \hat{f}_p(k)$ is bounded.

In CFDL-DITSMPC, the magnitude of the MPC control action is adjusting by the weight value ω . The control system outputs with the different ω are illustrated in Fig. 3. Then it can be clearly observed that there is a smaller tracking error but a larger overshoot with a smaller ω from the red dashed line in Fig. 3. Meanwhile, a larger ω will present opposite results shown in Fig. 3 with the green dashed line. When the desired signal changes, the conclusion is the same. Consequently, it is necessary to choose a suitable ω to make a tradeoff between the disturbance rejection and the overshoot.

(2) Comparison results with MFAC and NN-QSMC methods

The CFDL-MFAC [12] scheme is

$$\Delta u(k) = \rho (y_r(k+1) - y(k)) \hat{\phi}(k) / \left(\lambda + |\hat{\phi}(k)|^2 \right) \quad (70)$$

where $\lambda > 0$, $\rho \in (0, 1]$, the estimate algorithm of $\hat{\phi}(k)$ is similar to (13)–(14). The parameters of the CFDL-MFAC scheme are $\mu = 0.5$, $\eta = 0.3$, $\rho = 0.6$, $\lambda = 0.3$, and $\varepsilon = 10^{-5}$.

For NN-QSMC [23], the controller structure is

$$\begin{aligned} \Delta u(k) &= (y_r(k+1) - y(k) - \bar{c}^T E(k) - \Delta \hat{f}_p(k) \\ &\quad + \varepsilon \operatorname{sgn}(\bar{c}^T E(k))) \hat{\phi}(k) / (\sigma + \hat{\phi}(k)) \end{aligned} \quad (71)$$

where $\bar{c}^T = [1 - q - c_0, (1 - q)c_0]$, $E^T(k) = [e(k), e(k-1)]$, the $\Delta \hat{f}_p(k)$ of the system disturbance part is estimated by employing a RBFNN-based predictor. The controller parameters are set as $\mu = 0.5$, $\eta = 0.3$, $c_0 = 2$, $q = 0.9$, and $\sigma = 3$.

In Fig. 4, it is obvious that the output of NN-QSMC has a large overshoot in the initial state and there is a tracking error caused

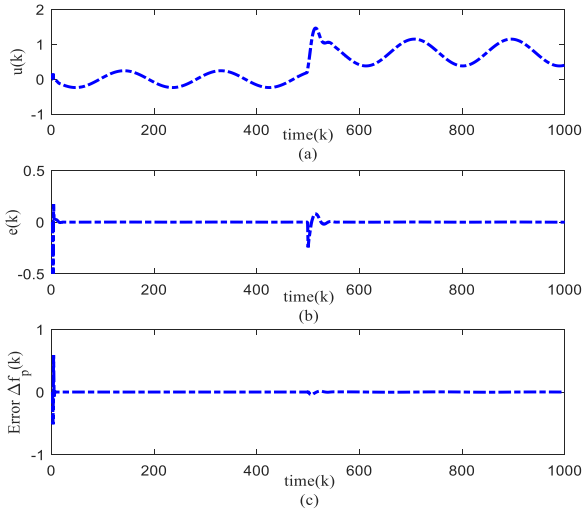


Fig. 2. Results of CFDL-DITSMPC scheme. (a) Control action. (b) CFDL-DITSMPC tracking error. (c) Disturbance estimation error $\Delta f_p(k)$.

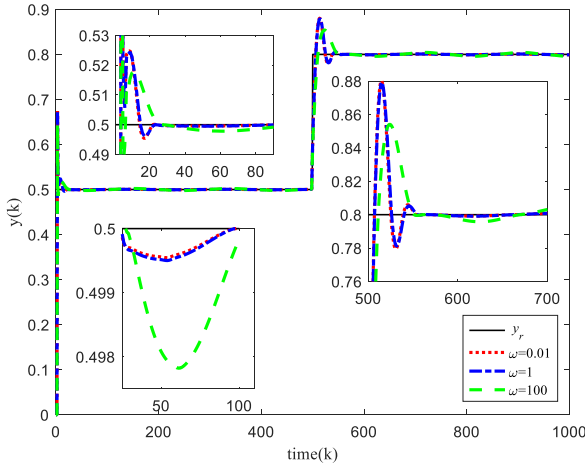


Fig. 3. The CFDL-DITSMPC outputs with a different ω .

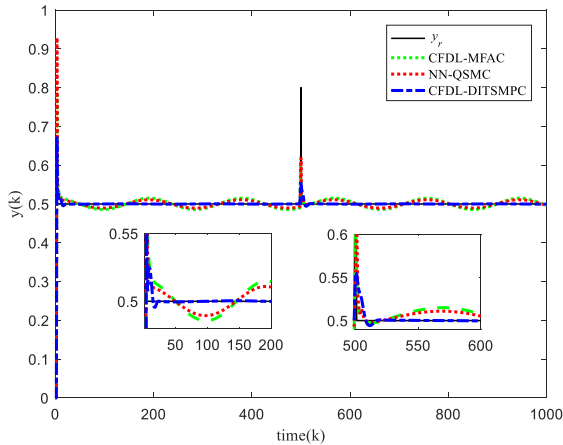


Fig. 4. Control performances comparison for three different algorithms.

by the disturbance. However, by the RBFNN-based predictor, the tracking error of NN-QSMC is smaller than that of CPDL-MFAC. Meanwhile, the output performance of CFDL-DITSMPC is superior to that of NN-QSMC. The output of CFDL-DITSMPC has

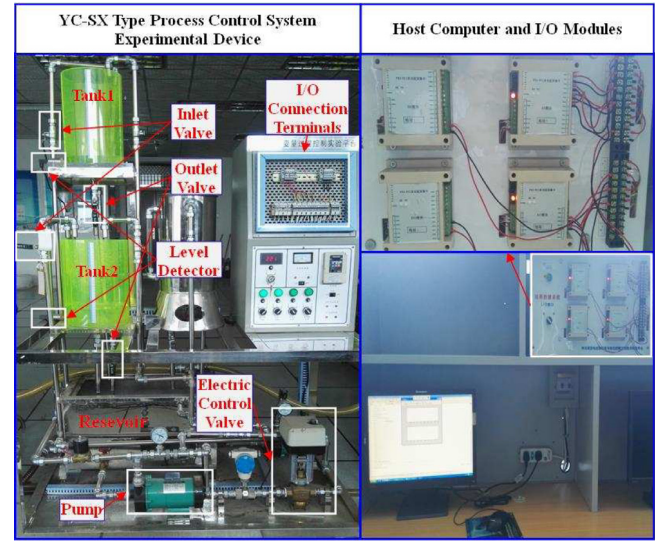


Fig. 5. The equipment of two-tank liquid level control system.

a small overshoot when adding a constant disturbance with a amplitude of 0.3 at the 500th step. When compared with CPDL-MFAC and NN-QSMC, the output of CFDL-DITSMPC adapts to this change quickly, and reduces the tracking error effectively. It is notable that the perturbation estimation of CFDL-DITSMPC is more concise with disturbance estimation technique, and avoid system instability in the NN-SMC method which is caused by the random initial weights of the RBF neural network. Consequently the CFDL-DITSMPC method achieves a satisfactory performance compared with the CPDL-MFAC and the NN-QSMC methods.

5.2. Experiments

The equipment of the two-tank process control system and the schematic are depicted in Figs. 5 and 6, respectively. Tank 1 and Tank 2 are series connection structures. A manual inlet valve and a manual outlet valve appear in each tank. A pressure sensor with the accuracy of 0.25 cm is adopted to detect the tank level. Meanwhile, the type of the electric control valve is ML7420A808 produced by Honeywell Company, and it is used to regulate the flow of the pump.

I/O modules are employed to obtain the analog voltage signal 0–5 V from the pressure sensor of Tank 2, and output the analog current signal 4–20 mA to the electric control valve, and communicate with the host computer via RS485. The sampling period is 1 s. The host computer software program is implemented in MATLAB and includes analog-to-digital conversion, control algorithm, the communication program, etc.

In this experiment, the control target is adjusting the control input signal of the electric control valve, so that tank 2 can maintain the desired liquid level (set as 4 cm). Valves V1, V3 and V4 are opened about 75%, and the valve V2 is closed.

The compared methods are the CFDL-MFAC and the NN-QSMC. For CFDL-MFAC, the parameters are set as $\mu = 1$, $\eta = 1$, $\rho = 0.6$, $\lambda = 0.3$, and $\varepsilon = 10^{-5}$. For NN-QSMC, the main parameters are set as $\mu = 1$, $\eta = 1$, $c_0 = 3$, $q = 0.6$, and $\sigma = 3$. The RBF neural network weights take random numbers. The parameters of the CFDL-DITSMPC scheme are $\mu = 1$, $\eta = 1$, $\lambda_s = 0.002$, $\lambda_1 = 0.15$, $\lambda_2 = 0.98$, $\alpha = 5/7$, $\omega = 0.01$, $N = 10$, $\delta = 3$, and $\varepsilon = 10^{-5}$. The tracking performances and the control input currents of the electric valve of these three approaches are shown in Fig. 7(a) and (b). It can be observed from the experimental results before the 450 s that the rising times of the three methods

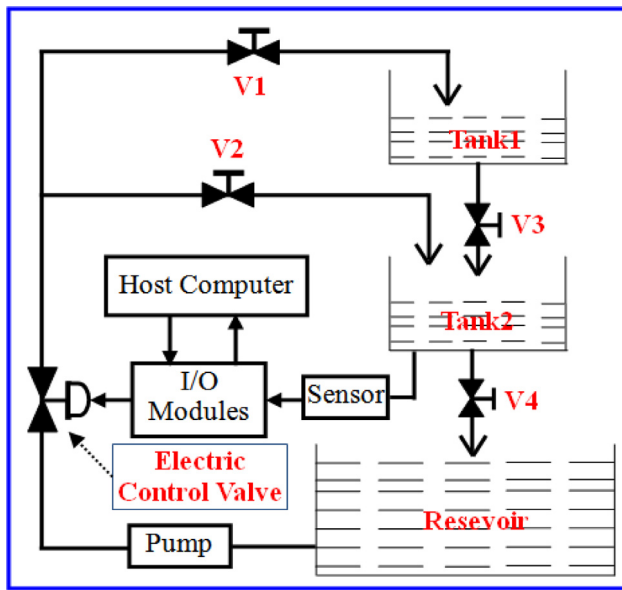


Fig. 6. The schematic of two-tank liquid level control system.

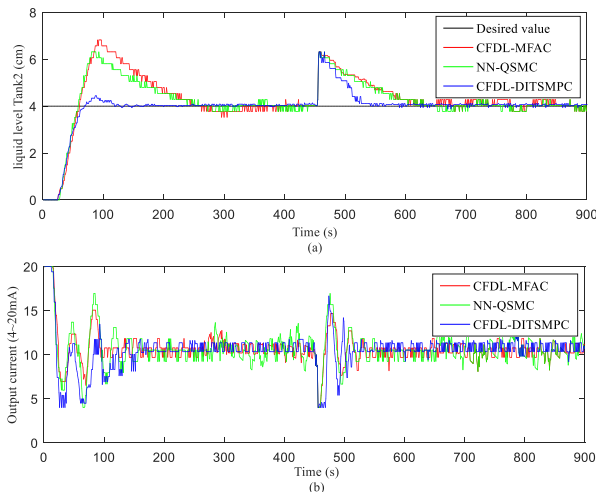


Fig. 7. The schematic of two-tank liquid level control system.

re basically the same. However, the CFDL-DITSMPMC method has the smaller overshoot and the faster settling time than those of the other two methods.

In order to contrast the robustness of the three algorithms, we pour 1000 ml water into Tank 2 at the 450 s after the system arrives at the steady state, which causes the level of Tank2 to rise by about 2 cm immediately. After the 450 s, the experimental results are shown in Fig. 7(a), when the CFDL-MFAC and NN-QSMC make the system return to the steady state at about the 600 s, and CFDL-DITSMPMC at about the 520 s. Therefore, it can be concluded that all of these three algorithms worked well for the two-tank liquid level control system. Furthermore, it can be illuminated that the robust performance of the CFDL-DITSMPMC method is superior to CFDL-MFAC and NN-QSMC.

Conclusion

In this paper, based on the CFDL technique and the DITSMPMC method, a novel model-free adaptive control scheme, including the CFDL-DITSMPMC and the CFDL-DITSMPMC, is presented for a class

of discrete-time nonlinear systems with disturbances. With the advantage of the CFDL technique, CFDL-DITSMPMC with the disturbance estimate technique is proposed, when the controller design does not require model information. Moreover, the CFDL-DITSMPMC method combines the MPC technique and CFDL-DITSMPMC to enhance the tracking accuracy. Furthermore, the effectiveness of CFDL-DITSMPMC is verified by numerical simulations and the liquid level control system. Considering that the CFDL-DITSMPMC control strategy possesses more control parameters, future research could focus on the control parameter adaptive method for the CFDL-DITSMPMC.

Acknowledgment

Conflict of interest

There is no conflict of Interest.

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