Computers & Operations Research 133 (2021) 105361

Contents lists available at ScienceDirect

Computers and Operations Research

journal homepage: www.elsevier.com/locate/cor

Vehicle routing with private and shared delivery locations

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ARTICLE INFO

Keywords:
Last-mile delivery
Sharing
Routing
Locker boxes
Matheuristics

ABSTRACT

The rapid growth of e-commerce has led to an increase of home delivery requests. Providing efficient distribution systems for services on the last mile has become a challenging issue for logistics companies, where a trade-off between the classical approaches, attended home delivery (AHD) and usage of shared delivery locations (SDLs) has been identified. AHD provides a higher quality of service but implies very high costs for the company, while usage of SDL requires customers to perform the very last mile by themselves. For companies, this bears the risk of a decrease in the perceived service quality. However, due to consolidation effects, transportation costs can be considerably mitigated. We propose a mixed delivery approach, which combines AHD and SDL usage in an innovative way. Customers can either be served at home during their preferred time window, or they can be asked to pick up their parcel at one of the SDLs. For each customer served using an SDL, the company pays a compensation price in order to reduce the perceived decrease in service quality. The newly introduced decision problem is formulated mathematically. We propose two matheuristics to efficiently solve large instances. In an extensive numerical study, we show that the new approach clearly outperforms standard ones. We observe an increase in solution quality of up to 40%, while customers' perceived quality of service is not affected. Additionally, the results reveal that the obtained improvements are robust for different customer-specific time-window preferences, accepted travel times needed to reach an SDL, and different compensation schemes.

1. Introduction

In the last decade, the advent of e-commerce has radically changed customers' shopping habits. Nowadays, customers can compare, in just a few minutes, a huge number of alternatives and offers using electronic devices such as tablets, smartphones or even smartwatches. Thus, e-commerce is on the rise and logistics companies have to cope with an increase of parcel delivery, especially in urban areas. Impressive numbers on e-commerce growth have been presented, while it has been discussed that distribution, in particular on the last mile, could cost up to 40% of the price of a product (Barenji et al., 2019).

Attended home delivery (AHD) has established new standards in terms of the quality of service. The large increment of home-delivery requests has begun to have a crucial impact on last-mile delivery. In fact, given the large amount of requests, companies cannot guarantee to perform the delivery within the customers' preferred time windows – which are mainly at the start or end of the day – but have to increase the length of the delivery windows by several hours. This might result in a decrease in the perceived service quality and, consequently, decreased customer satisfaction and loyalty. Moreover, logistics' providers are facing efficiency losses by having to visit locations several times due to customers' absence. Obviously, this negatively affects transportation costs as well as ecological objectives due to a system-wide increase of traffic in urban areas.

To overcome this issue, in recent years, a new delivery system, named unattended delivery, has been established. This requires that deliveries be performed using shared delivery locations (SDLs) such as digital locker terminals. These facilities are generally located in supermarkets, train stations or other places with very long opening hours.

The advantage of this system is two-fold. On the one hand, customers do not have to attend the delivery at home but can individually pick up their goods according to their convenience. On the other hand, transport companies may perform the delivery at any time, which enables consolidation of parcels destined for different customers but assigned to the same SDL. This reduces the number of delivery locations, leading to a decrease in delivery cost and traffic congestion.

The aim of this paper is to propose a new delivery system that combines AHD with SDL deliveries. It is innovative, since the company,

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https://doi.org/10.1016/j.cor.2021.105361
Received 8 June 2020; Received in revised form 29 April 2021; Accepted 30 April 2021
Available online 3 May 2021

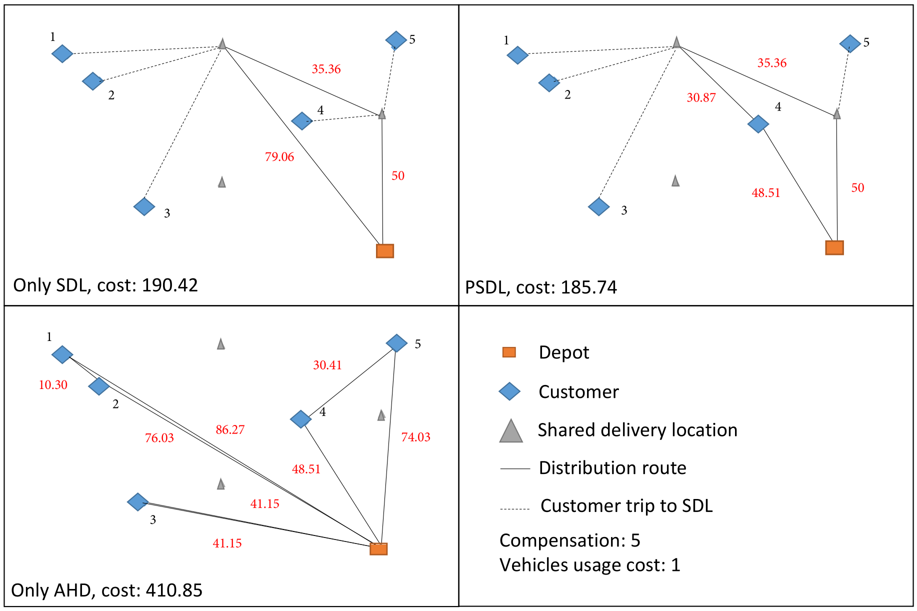

Fig. 1. Comparison of optimal solutions for different distribution strategies: AHD, SDL distribution and the mixed strategy (PSDL).

and not the customer, decides on the delivery location. In our study, we compare the following three options for the delivery:

1. Option 1: AHD, where customers can select a specific time window for their deliveries;
2. Option 2: SDL service, where customers can indicate the location at which they are willing to pick up their parcel; if customers select this option, a small compensation for the discomfort of not receiving the parcel at home is provided.
3. Option 3: (when the customer is fine with both Option 1 and Option 2) private and shared delivery locations (PSDL), where companies decide whether to deliver a parcel to the receiver's private address within a customer-selected time window, or to an SDL. The latter case implies that a compensation payment is charged.

In this combined system, customers who have to rely on home delivery will choose Option 1. For customers who wish to profit from the compensation payment and prefer to be free to pick up the parcel from SDLs at any time, Option 2 is more attractive. For the set of indifferent customers, the company can choose the most beneficial option according to their daily routing plans. Both customer satisfaction and cost minimisation are tackled simultaneously. Customers with a strict preference for a delivery option are satisfied, while flexible customers who do not have a clear preference may help to make the delivery operations more sustainable. One of our primary goals is to show the potential benefit of customer flexibility. Therefore, even if we model a general case in which customers may insist on a preferred delivery option (AHD or SDL), in the computational study, we address the case in which all customers are flexible. This way, we are able to quantify the maximum cost reduction obtainable compared to a single-option delivery strategy. Furthermore, from a computational point of view, the more challenging case is the one in which all customers choose Option 3. This is based on the fact that the more the customer preferences are fixed, the smaller the solution space and the easier the problem is to solve.

In Fig. 1, we depict optimal routing solutions for (i) pure AHD, (ii) pure SDL distribution and (iii) the proposed mixed strategy (PSDL) for a small instance with 5 customers and 3 SDLs. Travel costs are reported in red. Vehicle usage cost and compensation for being served using an SDL are set to 1 and 5, respectively.

Our contribution to existing literature is as follows:

1. The newly introduced problem is formulated mathematically.
2. We propose two matheuristic-based methods in order to generate solutions in a short amount of time. Both methods are bench-marked against an exact solution approach.
3. In an extensive computational study, we show that the newly proposed delivery system outperforms standard approaches and is robust to customer-specific preferences (e.g. time windows and accepted travel times needed to reach an SDL). By letting the company decide on the delivery location while compensating the customers for potential inconvenience, total cost can be considerably reduced.
4. Experimental data is made publicly available in order to enable and encourage future research on the topical problem of deliveries on the last mile.

The remainder of the paper is structured as follows. Related literature is discussed in Section 2. The newly introduced problem is defined and formulated in Section 3. The proposed solution approaches are described in Section 4. The numerical experiments are presented and managerial insights identified in Section 5. Lastly, our work is summarised in Section 6.

2. Literature review

With the advent of e-commerce, an increasing share of companies have adopted AHD service models, where customers are invited to purchase goods online and have them delivered directly to their front door or any customer-defined location (e.g. the workplace). Despite the evident increase in customers' perceived service quality, these delivery options bear potential losses of transport efficiency for logistics companies.

In classical supply chains, logistics companies deliver goods to stores, where customers perform their purchasing. This way, last-mile distribution can be carried out at low cost, since the number of stores to be visited is relatively small and the very last mile is performed by the customers themselves. In a home-delivery system, however, the cost related to the last mile is considerably higher, since several places have to be visited. While major players in the field (e.g. Amazon) successfully cope by developing adaptive distribution systems, many small companies struggle to stay in business (Agatz et al., 2008). Another crucial issue in home delivery is related to the high percentage of unsuccessful delivery attempts (Morganti et al., 2014), which occur if AHD customers are not available to receive their deliveries. Typically, drivers have to make additional attempts in order to deliver the goods. These returns are costly and, since they are not easy to predict, can hardly be efficiently planned. In order to decrease the probability of customer absence, time window assignments can be offered. Alternatively, goods are shipped to the SDL, from where receivers can pick them up individually. Although AHD reduces the probability of unsuccessful delivery attempts, operational costs related to this system are relatively high. In fact, many customers have identical time window preferences (e.g. early morning or late afternoon), which requires companies to use large fleets of vehicles in order to perform all deliveries at the confirmed point in time. To overcome this issue, time window pricing techniques have been proposed. Different criteria, such as the desirability of a given time window, are taken into account such that customer preferences can be matched with minimal-cost routing. An early contribution has been presented by Campbell and Savelsbergh (2006), who propose the usage of incentive schemes for influencing customer behaviour. Klein et al. (2019) construct customer behaviour as a rank-based choice model in which delivery costs are computed by explicitly considering routing constraints. Mackert (2019) proposes a logit choice model to integrate customer behaviour in time slot management. The optimal assignment of time windows to geographical areas is addressed by Yang and Strauss (2017). The authors explicitly model customers' behaviour based on time window prices, as well as companies' decision making, where customer preferences might have to be discarded. Related problems are studied by Agatz et al. (2011) and Hernandez et al. (2017), where the authors address the problem of geographical time window selection while maintaining a certain level of customer satisfaction. The problem of dynamically assigning delivery time windows is addressed by Mackert et al. (2019) and Lang et al. (2019), while Lang et al. (2020) study a multi-criteria version of the problem.

An application of time slot management in AHD problems, where service technicians have to be routed in different regions, is discussed by Bruck et al. (2018).

Köhler et al. (2020) address delivery time window management in a stochastic environment. They assume that customer requests are dynamically revealed, while a centralised system decides on time windows offered to incoming customers. These offers are based on tentative routing plans.

The main alternative to AHD is the usage of SDL, where companies deliver to, for example, digital smart lockers. From a logistical point of view, these systems are less complex, since parcels for different customers can be consolidated. This consolidation leads to a considerably reduced number of delivery locations and, consequently, decreased routing cost and travel distances. Thus, ecological objectives can also be satisfied. However, usage of SDL might lead to decreased perceived service quality (Lemke et al., 2016). A second disadvantage relates to the capacity limitations in SDL usage. An extensive discussion on public and private parcel locker systems is presented by Faugere and Montreuil (2016).

An analysis of the economic benefits of SDL-based distribution systems is provided by Morganti et al. (2014) and Iwan et al. (2016), while an analysis of the perceived value of this new trend to customers in parcel delivery is reported by Vakulenko et al. (2018). Despite the evident advantages of such systems, this concept has some drawbacks. In rural areas, for example, the diffusion of SDL is very limited. Customers are required to travel relatively long distances to pick up their parcels, which might decrease customer satisfaction considerably. Furthermore, elderly people or customers with disabilities may experience difficulties in relation to visiting an SDL, even if it is in close proximity. Consequently, logistics companies (e.g. Amazon) offer services where customers can choose between two delivery alternatives: (i) parcels can be received at home or at another place indicated by the customer or (ii) they are delivered to an SDL. For home deliveries, there is no indication about the time window in which the delivery will arrive. Thus, the SDL service gives customers more flexibility since no home attendance is required. However, literature on delivery systems integrating different delivery options is very limited. Zhang and Lee (2016) were the first to propose the inclusion of visits to SDL and to customers' homes within the same routing plan and show the potential benefit of such an approach. The authors consider that each customer chooses her preferred delivery location (a specific SDL or home location) and the company provides the minimum-cost routing plan. A similar problem is addressed by He et al. (2020), where the objective function also takes into account the waiting time at customers' locations due to customer absence. Both papers consider that each customer indicates only one preferred location, but in reality, many customers may have no strong preference among several locations. In this case, if they are flexible and accept to provide the company a set of compatible delivery locations, delivery operations can be performed at a globally lower cost for the company and within a shorter amount of time. It can be assumed that this will have a positive impact on customer satisfaction. Furthermore, a larger customer flexibility allows to provide more sustainable routing plans, with the benefit of reduced environmental impact. Therefore, considering a set of preferred delivery options for the customers shows potential benefits from several perspectives. This issue was first investigated by Orenstein et al. (2019), where the authors consider only deliveries to service points (SPs) and not to private locations (customers' home or offices). In the study of Dumez et al. (2021), customers provide an ordered list of preferred locations. A location positioned in a lower ranking in the list corresponds to a lower customer satisfaction. The goal is to minimise routing costs while ensuring a minimum global customer satisfaction level. The idea of asking customers to provide alternative delivery locations was introduced by Reyes et al. (2017), where each customer provides different private locations, each one characterised by a time window in which the delivery should be performed (e.g. house, office, gym). Grabenschweiger et al. (2020) study the problem with heterogeneous locker boxes, where total cost, consisting of routing and a fixed compensation cost, have to be minimised and packing of parcels into locker boxes is taken into account.

The integration of collection-and-delivery points in the strategic design of urban last-mile e-commerce distribution networks is analysed by Janjevic et al. (2019). These facilities can be seen as consolidation centres which are mainly used if home delivery is too expensive or does not meet economic goals. The authors offer recommendations on how to locate these centres and assume that customer demands are automatically delivered to them if they are within a given radius. They do not consider the case where AHD and SDL are alternative delivery options for each individual customer. Additionally, the usage of collection-and-delivery points from customers' perspective is discussed by Liu et al. (2019).

Since customers may receive packages of different sizes or shapes, terminal composition is a relevant issue in the field of operations management and logistics. Faugere and Montreuil (2020) address locker terminal design optimisation, integrating monolithic and modular configurations. Bailey et al. (2013) and Bailey et al. (2014) tackle a monolithic locker bank configuration for medical supply delivery within hospitals, while Pan and Lin (2017) propose an ergonomic optimisation approach for locker bank design on a university campus. From a mathematical modelling point of view, designing smart locker banks is related to two- and three-dimensional packing problems, where the goal is to efficiently pack differently sized items into heterogeneous bins, while minimising wasted space. The interested reader is referred to surveys presented by Lodi et al. (2002) and Martello et al. (2000). Smart locker bank design extends classical packing problems, since the location of lockers within the terminal has an impact on ergonomics and efficiency. If the lockers are placed too high or too low, customers might find it inconvenient or even impossible to access them. Moreover, amounts of future demands have to be taken into account, which builds a bridge to inventory problems.

Another alternative to AHD are roaming deliveries, where different delivery locations for each customer are considered, each one of which characterised by non-overlapping time windows. The newly proposed system includes the possibility of having parcels delivered to the trunk of the receiver's car. The authors show the cost advantage of such distribution systems compared to traditional home deliveries. It should be noted that detailed information on customers' habits is required and, furthermore, customers are limited in their ability to spontaneously change their plans.

Sitek and Wikarek (2019) address a routing problem with alternative delivery options in which customers can be served directly at home or at a compatible delivery point. Capacity constraints, expressed in the number of parcels that can be simultaneously assigned to a compatible delivery point, are taken into account. However, no delivery time windows, either at customer locations or at compatible delivery points, are considered. Zhou et al. (2018) investigate a multi-depot two-echelon VRP with alternative delivery options. The authors were the first to consider alternative delivery options at the second echelon, such as pickup facilities and home locations. However, no capacity restrictions or time windows are taken into account. In a study by Enthoven et al. (2020), a two-echelon distribution system is considered, in which customers may choose their goods to be delivered to a collection point, or to a satellite depot, where they are consolidated and delivered to the customers' home or alternative locations. In all these problems, no compensations are considered to encourage customers to choose alternative delivery locations. Another crucial issue to be considered is customers' willingness to pay for home delivery, or, more generally, for premium delivery services. Several sociological and psychological surveys cover this topic. In a study by Goethals et al. (2012), a survey on French consumers' willingness to pay for home delivery is reported. This analysis points out that 70% of customers are willing to pay for home delivery. An empirical study on e-customers from Bangladesh shows that customers are willing to pay more for a more efficient delivery system (Saha et al., 2020). Yuen et al. (2018) present results of a survey carried out in Singapore. The authors investigate factors influencing customers' intention to use self-collection delivery services. It is reported that the willingness of customers to use such services is strongly correlated with their perception of the potential benefit of this distribution system. This benefit is not only strictly related to purchasing experience but can also address environmental issues or global customers' satisfaction. This finding is very relevant in our context, since, if positive impact on the environment or on delivery speed is perceived, customers' willingness to provide more than one preferred delivery option might considerably increase.

Our study contributes to the literature by introducing an innovative last-mile distribution system, which combines the advantages of AHD and delivery to SDL, considering a compensation for customers served at SDLs. We show that letting the company decide on the delivery location while compensating customers for potential inconvenience can lead to considerable savings in total cost. The dominance of the newly proposed system is demonstrated in an extensive computational study. Moreover, we analyse the impact of customer preferences in terms of time windows and accepted walking distance to the SDL. To the best of our knowledge, none of these issues have been covered in the literature so far.

3. Problem definition and mathematical model

The mixed delivery system we propose can be modelled as an extension of the VRP with time windows (e.g. Bräysy and Gendreau, 2005). We consider the fulfilment of a set of parcel delivery requests I. Delivery routes start from a common depot (0). A set F of SDL is available, but each request i is compatible only with a subset of F, i.e. $V_i \subseteq F$. Each delivery must be performed either directly to a private location of customer i or to an SDL f, where $f \in V_i$.

Each SDL f is characterised by a limit B_f on the number of requests that can be handled within the planning horizon. This limit B_f corresponds to the number of available lockers at f. It depends not only on the size of SDL f but also on the number of lockers already occupied by parcels delivered on previous days which have not been picked up yet. A service time s_i is defined for each request being delivered to a private location or an SDL. We consider the service time at SDL fixed and independent from the number of parcels handled.

For each request i, where $i \in I$, the private location associated with it can be visited only within a fixed short time window $[E_i, L_i]$, indicated by the customer during the purchasing process. This type of delivery is an AHD since the customer guarantees his or her presence during the confirmed time window. This reduces the risk of unsuccessful deliveries, which typically occur due to customers being absent at the time of service. However, the SDL service is considered an unattended delivery, since the locker terminals can be accessed flexibly by customers.

Let us define the following sets of nodes in the distribution network: $N = I \cup F$ and $N_0 = N \cup 0$. Let us further denote the private locations indicated for AHD of requests i and j ($i, j \in I$) as nodes i and j, respectively. For each pair of nodes i, j in N_0, we assume travel time, t_{ij}, and travel cost, c_{ij}, to be constant and known in advance. Each vehicle can start at the depot in any period after the beginning of the time horizon and must return to it before the end of the time horizon (T_{max}). By δ we indicate the penalty charged to the company for each request delivered to an SDL. Thus, δ represents the compensation paid to a customer for having to perform the very last mile on his or her own. Moreover, each vehicle used for deliveries costs a fixed charge, which we denote by γ.

The objective is to minimise the total distribution costs, given by the sum of travel costs, vehicle usage costs and compensations paid to the customers. Without loss of generality, we assume that each SDL may be visited at most once. In fact, given the small size of packages that can be delivered to an SDL (compared to standard vehicle capacities), and the small number of lockers for each SDL, we assume that the demand of an SDL can always be fulfilled by a single vehicle. Therefore, in an optimal solution, multiple visits to the same SDL will never occur.

For the mathematical formulation, the following decision variables are needed.

X_{ij}: binary variable indicating whether j is visited directly after node i (1) or not (0)
Y_{if}: binary variable indicating whether i is delivered to SDL f (1) or not (0)
Z_f: binary variable indicating whether SDL f is visited (1) or not (0)
T_i: non-negative variable indicating the visit time at node i

The mathematical formulation is reported in the following:

$$\min \sum_{i \in N_0} \sum_{j \in N_0} c_{ij} X_{ij} + \delta \sum_{i \in I} \sum_{f \in F} Y_{if} + \gamma \sum_{j \in N} X_{0n} \tag{1}$$

$$\sum_{i \in N_0} X_{ij} + \sum_{f \in V(j)} Y_{jf} = 1 \quad \forall j \in I \tag{2}$$

$$\sum_{i \in N_0} X_{ij} = \sum_{i \in N_0} X_{ji} \quad \forall j \in N_0 \tag{3}$$

$$Z_f \geq \frac{1}{|I|} \sum_{i \in I} Y_{if} \quad \forall f \in F \tag{4}$$

$$\sum_{i \in N_0} X_{if} = Z_f \quad \forall f \in F \tag{5}$$

$$T_j \geq T_i + t_{ij} + s_j - 2T_{max}(1 - X_{ij}) \quad \forall j \in N \ \ \forall i \in N_0 \tag{6}$$

$$-T_{max} \sum_{f \in F} Y_{if} + E_i \leq T_i \leq +L_i + T_{max} \sum_{f \in F} Y_{if} \quad \forall i \in I \tag{7}$$

$$T_j + s_j + t_{j0} \leq T_{max} \quad \forall j \in N \tag{8}$$

$$\sum_{i \in I} Y_{if} \leq B_f \quad \forall f \in F \tag{9}$$

$$Y_{if} = 0 \quad \forall i \in I \quad \forall f \in F : f \notin V_i \tag{10}$$

The objective function (1) minimises the total distribution cost consisting of vehicle usage cost, travel cost and compensations due to requests delivered to an SDL.

Constraints (2) imply that each order must be delivered either to the customer's private location or to an SDL (V_i). Constraints (3) ensure route continuity. If at least one request has been assigned to an SDL, the SDL must be visited by a vehicle. This is specified by the combination of Constraints (4) and (5). Arrival time at the nodes is tracked by Constraints (6). Customers' time windows must be respected if and only if the orders are delivered directly to customers' private locations, while deliveries to an SDL may be performed at any time (7). Each vehicle must return to the depot before the end of the time horizon T_{max}. This is ensured by Constraints (8). Finally, the number of requests assigned to an SDL f must not exceed its capacity, which is expressed as the number of available lockers, B_f(9). Customer–SDL compatibility is handled by Constraints (10).

Note that the model covers the general case, where customers do not insist on one or the other service (AHD or SDL). However, this can of course be easily considered by adding Constraints (11) and Constraints (12), respectively.

$$Y_{if} = 0 \quad \forall f \in F \quad \forall i \in I : i \text{ explicitly selected AHD} \tag{11}$$

$$\sum_{f \in V_i} Y_{if} = 1 \quad \forall i \in I : i \text{ explicitly selected SDL} \tag{12}$$

4. Solution approaches

Since the newly introduced problem is an extension of the VRP with time windows – which is proven to be $\mathcal{NP}$-hard – it is $\mathcal{NP}$-hard too. Therefore, only small-sized instances may be efficiently handled by solving the problem to optimality. To solve larger instances, we propose a large neighbourhood search (LNS)-based matheuristic and an iterated local search (ILS) procedure. For ILS, we use LNS as a black box tool within the local-search phase. Whenever a local minimum is reached, a perturbation designed for the specific problem is applied to restart the procedure. In the last decade, matheuristics have become a very popular tool to address rich VRPs. The term *matheuristic* is generally used to address all hybrid approaches, where metaheuristics or heuristics are combined with mathematical programming (Boschetti et al., 2009). In order to apply matheuristics to rich VRPs, several frameworks have been proposed. For an extensive survey, we refer the interested reader to Doerner and Schmid (2010) and Archetti and Speranza (2014).

Recent approaches of matheuristics for rich VRPs can be grouped into two main categories. The first one includes *sequential* methods, in which a metaheuristic and an exact method are iteratively applied (Montoya et al., 2016) for the electric VRP with time windows and (Mancini, 2017a) for the VRP with time-dependent travel times. The second group covers approaches, where the mathematical model is used to efficiently explore neighbourhoods within a metaheuristic framework. By this, even very large neighbourhoods can be exhaustively searched within relatively short computational times. Such approaches have been successfully applied to several rich routing problems, e.g. the multi-depot multi-period VRP (Mancini, 2016), the hybrid VRP (Mancini, 2017b), the tourist-cruises itinerary planning (Mancini and Stecca, 2018) or the VRP with backhauls (Marques et al., 2020). In these approaches, neighbourhoods are implicitly defined by a destroy operator, which fixes the majority of variables to the values in the current best solution, while letting the model optimise the remaining variables. Destroy operators may work on different kinds of variables. For example, in studies by Mancini (2017b) and Marques et al. (2020), the destroy operators directly act on arc variables, fixing some of the routes and letting the model re-optimise the other routes. In other studies by Mancini (2016) and Mancini and Stecca (2018), however, the destroy operators act on a higher level, dealing with customer-to-vehicle and customer-to-period assignments or with node selection variables.

The matheuristic proposed in this paper, and the ILS based on it, belong to the second group and deal with the request-to-location assignment variables. These variables state which SDL is selected for a specific request, or whether the request has been scheduled for AHD service. Although the matheuristic framework we use is similar to the one used by Mancini (2016) and Mancini and Stecca (2018), the algorithm we propose in this paper is innovative and specifically suited for this problem. Particularly, in the study by Mancini and Stecca (2018), the algorithm fixes at each iteration a set of nodes that will not be in the solution, a set that will be necessarily included in the solution and a set that can be included or not, but they do not explicitly force any assignment of nodes to vehicles. Conversely, in the LNS proposed in this paper, at each iteration, we do not simply fix a set of customers to be served by an SDL but explicitly force their assignment to a particular SDL. This way, we reduce the size of the addressed neighbourhood (which remains quite large) in order to be able to efficiently explore it within a short time limit. If we applied the concept used by Mancini and Stecca (2018), the resulting neighbourhoods would become too large to be efficiently explored in a short time limit. This would consequently decrease solution quality. Compared to the study by Mancini (2016), where different destroy operators are provided, in this paper, we use a single random destroy operator. However, to avoid remaining trapped in local minima, we develop an ad-hoc perturbation mechanism, which exploits a particular feature of the problem, and perform a targeted perturbation which specifically guides the algorithm out of the current local minimum. The details of such a mechanism are described in Section 4.2.

4.1. The proposed matheuristic

To efficiently solve the newly introduced problem, an LNS matheuristic (MH) is developed. In each iteration, a destroy operator is applied to perturb the current solution and the mathematical model is exploited to reconstruct a feasible solution starting from the partially destroyed one.

An initial solution S^0 is computed by running the exact model for a short duration (T_{init}) and keeping the best feasible solution obtained so far. In case we are unable to find a feasible solution within this time limit, the problem is solved with a pure SDL delivery policy, i.e. all the orders are delivered to an SDL except those where AHD has been explicitly requested. This modified version of the problem can easily be solved to optimality in just a few seconds, which is due to the reduced number of nodes.

Customers are grouped in three sets, I^1, I^2, and I^3, containing requests for the delivery options, 1, 2, and 3, that have been chosen, respectively. In each iteration, a set P composed of p requests is randomly selected in $I^1 \cup I^2$. The remaining $|I| - |P|$ requests are assigned to the same SDL or a private location as defined in the current best solution. For each selected request, $i \in P$, we let the model free to assign them for AHD or SDL service, if they belong to I^3, or to assign them to any SDL, if they belong to I^2. This can be obtained by adding the following set of constraints to the mathematical model presented above.

$$Y_{if} = Y_{if}^{fix} \quad \forall i \in I \setminus P \tag{13}$$

where Y_{if}^{fix} is a constant, which equals 1 if request i has been assigned to SDL f in the current best solution, and 0 otherwise. The resulting new version of the model is run with a short time limit. The best solution is kept as long as there is an improvement in the current best solution.

The procedure is iterated until a maximum number of iterations, $ITER^{max}$, is reached, or after $NOIMPROVE^{max}$ iterations without improvement. The main advantage of this approach compared to classical metaheuristics is that very large neighbourhoods can be quickly explored by solving the mathematical model, which allows for leaving local minima.

The pseudocode of the MH is reported in Algorithm 1.

Algorithm 1 MH pseudocode

Require:
I^1 : set of customers choosing AHD
I^2 : set of customers choosing SDL service
I^3 : set of customers fine with both AHD and SDL
S^0 : initial solution (time limit T_{init})
$S^{best} \leftarrow S^0$
$NOIMPROVE \leftarrow 0$
$ITER \leftarrow 0$
while $ITER \leq ITER^{max}$ **do**
    while $NOIMPROVE \leq NOIMPROVE^{max}$ **do**
        randomly select a subset P of requests in $I^2 \cup I^3$
        $S^{iter} \leftarrow$ solve the model, with a timelimit TL ▷ including (13)
        if $S^{iter} < S^{best}$ **then**
            $S^{best} \leftarrow S^{iter}$
            $NOIMPROVE \leftarrow 0$
        else
            $NOIMPROVE \leftarrow NOIPROVE + 1$
        end if
        $ITER \leftarrow ITER + 1$
    end while
end while
return S^{best}

Alternative matheuristics approaches for VRP (e.g. Mancini, 2016) use proximity-based criteria to identify clustered sets of customers for the perturbation process. However, in this work we apply a pure random selection system, since requests can be fulfilled on different locations (private or SDL) and it is therefore not suitable to define a proper proximity measure. Note that even if the private locations associated to two requests i and j are very close, customer preferences are not necessarily identical, and thus the set of compatible SDL (V_i and V_j) could be different or even disjoint.

The main drawback of this approach is that it might be difficult to find a solution in which an SDL is not visited, starting from a solution in which it was visited. More precisely, if not all the customers assigned to the same SDL f are involved in the perturbation, this implies that at least one customer remains assigned to f. Thus, SDL f must be included in the routing plan of the new solution. Moreover, if f is visited, it could be convenient to assign more customers to it and, therefore, no new best solution with a considerable improvement can be found. This effect might yield to a premature convergence of the algorithm to a local minimum. In order to overcome this issue, we propose an ILS which provides a specific restart mechanism, allowing to move towards solutions where different sets of SDL are visited.

4.2. Iterated local search

For the ILS, the above described MH is used as a local search black-box tool. Each time the MH reaches a local minimum, a restart procedure is applied. This procedure consists of randomly closing one of the SDLs f, which is used in the current best solution, and letting the model reassign all the requests that have been assigned to f (this set of customers is denoted as Ω_f). All other assignments are kept. This perturbation contributes to solution diversification. Finally, MH is applied again using the best solution obtained by the perturbation mechanism as initial solution. The overall procedure ends following a maximum number of iterations, $ITER^{max}$, or after $PERT^{max}$ calls of the restart procedure.

This specific perturbation allows to reach promising areas of the solution space, especially solutions with new (and even disjoint) sets of visited SDL. Moreover, if visiting an SDL, which was closed during the perturbation phase, was beneficial, i.e. the restart procedure pushed the search into a non-promising area, the algorithm is able to immediately recover by moving back to more promising zones of the solution space. This ability is a strong advantage of the proposed approach, which turned out to be effective and very helpful in overcoming premature convergence of MH.

The pseudocode of ILS is reported in Algorithm 2.

Algorithm 2 ILS pseudocode

$S^{start} \leftarrow MH$
$S^{best} \leftarrow S^{start}$
$pert \leftarrow 0$
while $pert \leq PERT^{max}$ **do**
    S^{pert} ←run the exact model with a time limit TL, where Ω_f must be reassigned
    $S^{start} \leftarrow MH$ with starting solution S^{pert}
    if $S^{start} < S^{best}$ **then**
        $S^{best} \leftarrow S^{start}$
    end if
    $S^{start} \leftarrow S^{pert}$
    $pert \leftarrow pert + 1$
end while
return S^{best}

5. Experimental study

Our computational study consists of two parts: (i) the comparison of solution methods MH against ILS and an exact approach and (ii) a detailed managerial analysis of the newly introduced distribution strategy, where we compare its performance against standard distribution systems.

All computational experiments are conducted using instances which are publicly available in Mancini and Gansterer (2020).

We generate three sets of instances with 5 SDLs and 25, 50 and 75 requests, respectively. Customers are randomly distributed in a predefined area, represented as a 10*10 km square. The depot is located in the southern part of the customer area, while SDLs are located in the south-east, south-west, north-east, north-west and in the centre (see Fig. 2). Each set is composed of 10 instances. Sets are denoted as *rX_Y_Z*, where *X*, *Y* and *Z* are the number of customers, the number of SDLs and the number of the instances, respectively.

For all instances, we consider a time horizon of 12 h, i.e. 720 min. The time horizon consists of 12 slots with 60 min each. Customers can select their preferred time slot. The travel cost between two nodes, i and j, c_{ij}, is fixed equal to the travel time t_{ij}, which is computed as $3d_{ij}$, where d_{ij} is the euclidean distance between i and j. This corresponds to a travel speed of 20 km/h, which is standard in congested urban areas. The value of γ and δ are equal to 1 and 5, respectively. For home delivery, we consider a homogeneous service time (s_i) of 5 min, while service time at SDLs is 10 min.

The most challenging case, from a computational point of view, occurs if all customers choose Option 3, i.e. they let the system decide whether to serve them at home or use an SDL. For this reason, we assume, in all the instances, that all the customers choose Option 3. Clearly, it would be easy to add some customers who insist on Option 1 (AHD) or 2 (SDL).

The capacity of the SDL is homogeneous. It is always slightly larger than the total number of requests (30, 55 and 80 for instances with 25, 50 and 75, respectively), which allows for a pure SDL delivery strategy, even if only one SDL is provided. However, this requires a high level

Table 1
Comparison of solution approaches (MIP, MH, ILS) applied to small-sized instances. For MIP, we report the best known solution, the optimality gap, run times and times in which the best solution has been found (T.F.). For MH and ILS, we report percentage gap to the best solution found by MIP, average run times and average T.F. (all times in seconds).

Instance	MIP				MH			ILS		
	Sol	Opt.gap	Time	T.F.	Gap	Time	T.F.	Gap	Time	T.F.
r25_5_1	161.37	0.00%	1.6	1.2	0.00%	1.6	1.2	0.00%	1.6	1.2
r25_5_2	166.63	0.00%	127.7	33.6	0.00%	6.69	4.9	0.00%	6.69	4.9
r25_5_3	146.56	0.00%	11.2	6.3	0.00%	7.08	5.6	0.00%	7.08	5.6
r25_5_4	161.04	0.00%	36.16	16.8	0.44%	7.31	5.7	0.00%	62.27	16.6
r25_5_5	157.95	0.00%	2.4	1.9	0.00%	2.4	1.9	0.00%	2.4	1.9
r25_5_6	160.83	0.00%	4.9	3.7	0.00%	4.9	3.7	0.00%	4.9	3.7
r25_5_7	152.69	0.00%	9.12	9.0	1.76%	5.57	4.7	0.75%	30.44	8.4
r25_5_8	165.16	0.00%	59.36	45.3	1.07%	8.14	5.6	0.45%	60.43	19.6
r25_5_9	151.54	0.00%	4.34	3.9	0.00%	4.34	3.9	0.00%	4.34	3.9
r25_5_10	151.96	0.00%	5.16	3.6	0.00%	5.18	3.6	0.00%	5.18	3.6
Avg.	**157.58**	**0.00%**	**26.20**	**12.5**	**0.33%**	**5.32**	**4.1**	**0.12%**	**18.53**	**6.9**

Table 2
Comparison of solution approaches (MIP, MH, ILS) applied to medium-sized instances. For MIP, we report the best known solution, the optimality gap, run times and times in which the best solution has been found (T.F.). For MH and ILS, we report percentage gap to the best solution found by the MIP, average run times and average T.F. (all times in seconds).

Instance	MIP				MH			ILS		
	Sol	Opt.gap	Time	T.F.	Gap	Time	T.F.	Gap	Time	T.F.
r50_5_1	266.80	0.00%	1112	362.61	2.45%	74.90	57.4	0.94%	263.50	98.7
r50_5_2	267.97	0.00%	1465	170.91	3.57%	25.04	15.1	2.19%	224.99	61.0
r50_5_3	273.05	5.29%	3600	3365.71	5.28%	98.00	72.6	3.13%	352.20	131.5
r50_5_4	268.89	0.00%	526	386.31	5.51%	17.00	7.9	3.91%	171.50	42.5
r50_5_5	271.32	0.00%	3439	2686.61	3.43%	89.90	105.2	1.32%	303.90	170.0
r50_5_6	268.32	0.00%	1592	882.62	6.15%	90.40	73.3	3.62%	293.90	133.3
r50_5_7	253.18	0.00%	480	229.92	5.17%	53.40	36.9	3.41%	145.70	69.1
r50_5_8	266.68	0.00%	3114	1739.64	5.31%	47.50	32.7	3.42%	350.10	149.8
r50_5_9	267.56	6.81%	3600	82.65	2.17%	48.69	39.4	1.71%	328.60	59.9
r50_5_10	273.60	0.00%	3089	1412.54	6.08%	92.70	67.9	3.82%	332.60	204.6
Avg.	**267.74**	**1.21%**	**2202**	**1132**	**4.51%**	**63.75**	**50.8**	**2.75%**	**276.70**	**112**

of flexibility of customers in terms of compatibility with SDL. For each instance, we assume that each request is compatible with all the SDLs located within a travel time radius ρ from customers' private location. For the first part of the experiments, we keep the value of ρ fixed to an intermediate value of 15, while in the second part, we perform a deep analysis on the effect of the variation of ρ on solution quality.

Similarly, in the first part of the experiments, customers' preferred time windows are randomly generated, while in the second part, we define different user profiles, characterised by different time window preference distributions, and show how these affect solution quality. Parameters $ITER^{max}$ and $NOIMPROVE^{max}$ are set to 100 and 10, respectively. We set $PERT^{max}$ to 5 (small instances), 10 (medium instances) and 15 (large instances). For ILS, the maximum number of restarts is 5. For generating initial solutions, parameter T_{init} is set to 10 (seconds).

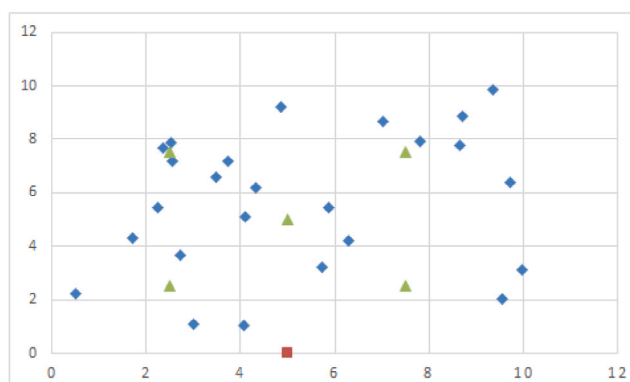

Fig. 2. Layout of instance *r25_5_1* with one depot (square), 5 SDL (triangles), and several customers (diamonds).

5.1. Comparison of solution approaches

In this part of our computational study, we compare the performance of the two proposed solution approaches (MH and ILS), as well as solutions found by applying a commercial solver (MIP) applied to the model presented in Section 3. Results are summarised in Table 1, Tables 2 and 3 for small (25 customers), medium (50 customers) and large (75 customers) instances, respectively.

We observe good solution quality for both MH and ILS. On the small instances (Table 1), both approaches yield a gap to the optimal solution of less than 0.5%, while runtimes are considerably shorter compared to MIP. On the medium-sized instances, both approaches yield a gap of less than 5% to the best known solution. While the gap of ILS is about 2 percentage points smaller, the runtime is about 4 times longer. On the large instances, the difference in solution quality is mitigated to a gap of about 1.73 percentage points, while the runtime of ILS is about 2.5 times longer compared to MH. For both MH and ILS, averaged values of T.F. are lower or equal to those required by the MIP for all the instances. Moreover, average T.F. values are strongly lower for the two heuristics. Resuming, MH is faster than ILS but less accurate. Therefore, none of them strictly dominates the other, while both of them strongly outperform the MIP either in effectiveness or in efficiency.

For the remaining computational study, we solve small- and medium-sized instances to optimality using MIP and using MH for larger instances. It should be noted that the underlying problem for AHD is a VRP with time windows, which is an $\mathcal{NP}$-hard optimisation problem (Bräysy and Gendreau, 2005). However, since we are considering tight time windows, AHD can be solved to optimality even for the large instances. The same applies to SDL. Thus, MH is needed for solving large PSDL instances.

Table 3
Comparison of solution approaches (MIP, MH, ILS) applied to large-sized instances. For MIP, we report the best known solution, the optimality gap, run times and times in which the best solution has been found (T.F.). For MH and ILS, we report percentage gap to the best solution found by the MIP, average run times and average T.F. (all times in seconds).

Instance	MIP				MH			ILS		
	Sol	Opt.gap	Time	T.F.	Gap	Time	T.F.	Gap	Time	T.F.
r75_5_1	362.33	9.70%	3600	2703	1.12%	176.00	124.5	0.60%	647.60	443.1
r75_5_2	383.73	11.10%	3600	504	0.43%	260.52	182.8	0.12%	660.72	393.2
r75_5_3	377.57	13.55%	3600	1602	4.62%	362.92	286.8	4.02%	804.05	562.0
r75_5_4	395.68	11.79%	3600	1149	1.34%	276.12	199.0	0.66%	840.28	434.5
r75_5_5	410.07	19.23%	3600	572	−1.99%	299.72	212.4	−2.11%	911.89	426.0
r75_5_6	386.60	14.52%	3600	3593	3.63%	363.68	266.0	2.18%	1001.57	779.9
r75_5_7	378.68	9.21%	3600	2968	3.19%	248.31	197.3	2.90%	670.05	333.2
r75_5_8	378.07	12.84%	3600	2525	3.86%	204.76	134.3	2.71%	752.47	377.8
r75_5_9	379.16	16.77%	3600	540	3.76%	343.97	244.9	2.93%	680.17	302.9
r75_5_10	360.79	3.25%	3600	3411	4.11%	298.69	244.5	3.33%	754.99	356.3
Avg.	**381.27**	**12.20%**	**3600**	**1957**	**2.41%**	**283.47**	**209.2**	**1.73%**	**772.38**	**440.9**

Table 4
Comparison of delivery strategies (AHD, PSDL, SDL) applied to small-sized instances with different radii (customer-accepted travel time needed to reach an SDL). We report objective values of AHD and the gap in solution quality of PSDL and SDL for each radius and instance set. *INF* indicates that the radius was too small to find a feasible solution.

Instance	AHD	Radius					
		5		15		25	
		PSDL	SDL	PSDL	SDL	PSDL	SDL
r25_5_1	264.9	−13.3%	INF	−39.1%	−30.7%	−44.0%	−30.7%
r25_5_2	270.4	−11.9%	INF	−38.4%	−32.2%	−43.9%	−32.2%
r25_5_3	245.5	−13.1%	INF	−40.3%	−25.3%	−40.3%	−25.3%
r25_5_4	256.4	−7.6%	INF	−37.2%	−28.5%	−42.3%	−28.5%
r25_5_5	240.0	−3.1%	INF	−34.2%	−19.9%	−40.5%	−23.6%
r25_5_6	245.9	−0.6%	INF	−34.6%	−19.3%	−42.7%	−25.4%
r25_5_7	241.7	−13.8%	INF	−36.8%	−24.1%	−38.5%	−24.1%
r25_5_8	293.2	−14.8%	INF	−43.7%	−34.4%	−49.4%	−37.4%
r25_5_9	245.4	−6.5%	INF	−38.2%	−25.2%	−40.4%	−25.2%
r25_5_10	243.4	−8.3%	INF	−37.6%	−24.6%	−37.9%	−24.6%
Avg.	**254.7**	**−9.5%**	INF	**−38.1%**	**−26.7%**	**−42.2%**	**−28.0%**

Table 5
Comparison of delivery strategies (AHD, PSDL, SDL) applied to medium-sized instances with different radii (customer-accepted travel time needed to reach an SDL). We report objective values of AHD and the gap in solution quality of PSDL and SDL for each radius and instance set. *INF* indicates that the radius was too small to find a feasible solution.

Instance	AHD	Radius					
		5		15		25	
		PSDL	SDL	PSDL	SDL	PSDL	SDL
r50_5_1	374.09	−3.6%	INF	−28.7%	−13.5%	−32.4%	−13.5%
r50_5_2	406.70	−6.0%	INF	−34.1%	−20.5%	−36.1%	−20.5%
r50_5_3	430.40	−10.0%	INF	−36.5%	−24.9%	−38.2%	−24.9%
r50_5_4	396.97	−8.5%	INF	−32.3%	−18.5%	−35.0%	−18.5%
r50_5_5	362.50	−4.7%	INF	−25.2%	−10.8%	−27.4%	−10.8%
r50_5_6	368.21	−3.7%	INF	−27.1%	−12.2%	−28.6%	−12.2%
r50_5_7	386.11	−5.2%	INF	−34.4%	−16.2%	−35.1%	−16.2%
r50_5_8	356.47	−4.5%	INF	−25.2%	−9.3%	−28.5%	−9.3%
r50_5_9	382.58	−6.0%	INF	−30.1%	−15.5%	−31.3%	−15.5%
r50_5_10	405.72	−9.0%	INF	−32.6%	−20.3%	−33.4%	−20.3%
Avg.	**386.98**	**−6.2%**	INF	**−30.8%**	**−16.4%**	**−32.8%**	**−16.4%**

Table 6
Comparison of delivery strategies (AHD, PSDL, SDL) applied to large-sized instances with different radii (customer-accepted travel time needed to reach an SDL). We report objective values of AHD and the gap in solution quality of PSDL and SDL for each radius and instance set. *INF* indicates that the radius was too small to find a feasible solution.

Instance	AHD	Radius					
		5		15		25	
		PSDL	SDL	PSDL	SDL	PSDL	SDL
r75_5_1	374.09	−5.9%	INF	−19.2%	−1.2%	−21.9%	−1.2%
r75_5_2	406.70	−8.0%	INF	−25.3%	−12.8%	−25.5%	−12.8%
r75_5_3	430.40	−0.5%	INF	−19.5%	−7.0%	−19.5%	−7.0%
r75_5_4	396.97	−4.0%	INF	−26.0%	−17.4%	−26.9%	−17.4%
r75_5_5	362.50	−2.3%	INF	−25.4%	−16.1%	−25.8%	−16.1%
r75_5_6	368.21	−3.2%	INF	−22.5%	−11.5%	−22.5%	−11.5%
r75_5_7	386.11	−4.6%	INF	−24.1%	−12.4%	−24.3%	−12.4%
r75_5_8	356.47	−3.6%	INF	−17.4%	−5.6%	−21.7%	−5.6%
r75_5_9	382.58	−0.9%	INF	−22.5%	−8.2%	−22.5%	−8.2%
r75_5_10	405.72	−4.9%	INF	−27.5%	−11.9%	−27.5%	−11.9%
Avg.	**386.98**	**−3.8%**	INF	**−23.1%**	**−10.7%**	**−23.9%**	**−10.7%**

5.2. Comparison with standard last-mile delivery systems

In this part of the computational study, we compare three distribution strategies: (i) AHD, (ii) SDL and (iii) the proposed mixed strategy, where private and shared delivery locations are used (PSDL). For SDL and PSDL, different accepted travel times needed to reach SDL (denoted as radii) are tested. Total costs are compared in Table 4–Table 6 for small, medium and large instances, respectively.

The results show that, if an acceptance level (i.e. radius) of 5 is assumed, no feasible SDL solution can be found. However, the PSDL strategy reduces total delivery cost by 9.5%, 6.2% and 3.8% on average for small (Table 4), medium (Table 5) and large (Table 6) instances, respectively. If the radius is increased to 15 or 25, SDL can also lead to feasible solutions.

For all instance sets, best solutions are found using the PSDL strategy. Depending on the radius, the AHD solution can be improved by about 40%, 30% and 20% on average for small (Table 4), medium (Table 5) and large (Table 6) instances, respectively. It should be noted that this holds even though customers receive a compensation payment δ for being served using an SDL, which increases total cost (see Eq. (1)). Thus, the experimental results reveal that the newly introduced PSDL strategy clearly outperforms both standard approaches (AHD and SDL). It should be emphasised that the newly proposed approach combines the advantages of the standard approaches, which is low delivery cost and high perceived service quality.

In Table 7, we report the cost, the average percentage usage of SDL and the average number of compatible SDLs per customer, for each instance set and for different values of radii. The aim of this analysis is to determine the level of flexibility required by the customers to achieve a reasonable gain in the objective function. We analyse five values of radii to make a more precise analysis (5, 10, 15, 20 and 25). What can be evinced from the table is that for a very small radius (i.e. 5), costs are very high and the usage of SDLs is very limited. Instead, by increasing the radius from 5 to 10, distribution costs can be strongly reduced and the usage of SDLs significantly increased. However, if the radius is increased from 10 to 25, solution quality as well as SDL usage do not significantly increase. This indicates that it is not necessary to use SDLs that are far away from customers' homes. Even if the accepted radius is reasonably small, transportation cost can be kept at a very low level. It is worth noticing that, even with the smallest radius, the mixed

Table 7
Cost, SDL percentage usage and average number of compatible SDLs per customer.

		Instances		
	Radius	Small	Medium	Large
Objective function	5	230.56	362.87	482.81
	10	162.63	278.26	390.29
	15	157.58	267.75	386.11
	20	147.93	261.39	382.39
	25	147.27	260.16	381.93
% Customers served at SDL	5	24.00	18.40	13.73
	10	64.80	64.60	39.90
	15	68.40	68.60	40.03
	20	71.20	71.00	40.97
	25	72.00	71.80	41.50
Avg. compatible SDL per customer	5	0.48	0.45	0.40
	10	1.59	1.53	1.53
	15	2.86	2.78	2.76
	20	3.98	3.95	3.90
	25	4.80	4.76	4.75

Table 8
Comparison of different profiles of customer-preferred time windows (P1–P4). We report the average gap of SDL and PSDL to AHD for large instances. The last 2 lines show aggregated results for small (s) and medium (m) instances.

Instance	P1		P2		P3		P4	
	SDL	PSDL	SDL	PSDL	SDL	PSDL	SDL	PSDL
r75_5_1	−10%	−19%	−20%	−31%	−10%	−21%	−12%	−24%
r75_5_2	−6%	−25%	−15%	−29%	−6%	−16%	−8%	−22%
r75_5_3	−10%	−19%	−5%	−10%	−1%	−10%	−8%	−17%
r75_5_4	13%	−26%	−3%	−15%	−5%	−19%	−12%	−22%
r75_5_5	8%	−25%	18%	5%	−16%	−27%	−11%	−20%
r75_5_6	13%	−22%	14%	0%	−4%	−17%	−11%	−22%
r75_5_7	−16%	−24%	−5%	−16%	−3%	−19%	−11%	−25%
r75_5_8	−5%	−17%	−1%	−12%	−6%	−16%	−14%	−22%
r75_5_9	13%	−23%	−13%	−23%	−5%	−19%	−12%	−21%
r75_5_10	11%	−28%	−10%	−24%	−3%	−23%	−9%	−22%
Avg.	0%	−23%	−5%	−17%	−6%	−19%	−11%	−22%
Avg. (s)	**−19%**	**−32%**	**−20%**	**−33%**	**−23%**	**−36%**	**−26%**	**−38%**
Avg. (m)	**−15%**	**−25%**	**−8%**	**−22%**	**−12%**	**−27%**	**−17%**	**−31%**

Table 9
Average percentage of SDL usage for different time window preference profiles.

Avg. % SDL usage	P1	P2	P3	P4
Small	64%	66%	68%	71%
Medium	49%	46%	55%	67%
Large	69%	74%	73%	72%

distribution strategy yields to lower costs compared to a pure AHD system. The number of average compatible SDLs needed to achieve a significant cost improvement is around 1.5.

We observe that the usage of SDL depends on the number of requests. While for small and medium instances, the average percentage usage is about 70%, it is considerably decreased in large instances (40%). This can be explained by the fact that carriers serving relatively dense customer areas can easily integrate additional requests into their tours. Hence, the need to use an SDL is reduced. This observation is in line with literature on collaborative logistics (e.g. Gansterer et al., 2019), where it is shown that multi-vehicle carriers serving larger areas of customer regions can easily adapt their tours to changes in customer locations. Carriers with only few vehicles and less customers, however, are considerably more vulnerable to changes in customer locations.

5.3. Comparison of different time window preferences

For the computational tests presented in Section 5.2, we assume randomly selected time windows. In order to gain more insights into the impact of time window preferences, we design 4 different time window profiles: (P1) customers available in the early morning and after 6pm, which reflects the typical availability of the working population, (P2) workers with an additional time window around noon, which reflects workers having a mid-day break at home, as is common in some European countries, (P3) customers available in the early morning and the whole afternoon and evening (e.g. teenagers) and (P4) customers who do not have specific preferences. This last set reflects availability of elderly, unemployed or quarantined persons.

In Table 8, we display detailed results for the large instances. Since Table 7 reveals that in larger instances, SDL usage is considerably lower, these instances seem to be most valuable for comparing time window preferences of attended home-delivery customers. However, aggregated results for small- and medium-sized instances are reported as well.

The results reveal that specific customer-preferred time window profiles do not affect the dominance of PSDL compared to AHD or SDL. We observe that PSDL improves solution quality by more than 30% for small- and medium-sized instances, and by more than 20% in large instances. This strongly emphasises the strength of the proposed innovative distribution system, since the results show that its dominance is not mitigated by specific time window preferences of AHD customers. The percentage of customers assigned to SDLs does not directly depend on the time window preference profile, as shown in Table 9 but it is more related to the combination of time window preference and customer location. In fact, it is intuitive that, if customers located very near to each other select completely different time windows, it would be better to assign some of them to SDLs in order to avoid visiting the same area several times. Similarly, if customers located very far from each other select the same time window, it would be rather impossible to serve all of them with AHD.

5.4. Comparison of different compensation schemes

In this section, we report an analysis on the impact of different compensation schemes on SDL usage and average travelled distances to reach them (cf. Coco, 2020). Experiments are carried out on small instances with a flexibility radius equal to 15. Each compensation scheme considers a different compensation threshold t. If the distance between a customer and an SDL is lower than the threshold, no compensation occurs. However, if it is equal or larger, compensation is paid. The value of this compensation is proportional to the value of the threshold, such that higher thresholds imply higher compensations. Thresholds are expressed in percentage of the total compatible distance. For example, a threshold of 0.5 means that the compensation occurs if the travelled distance is larger than 50% of the radius, i.e. if it is larger than 7.5. According to this scheme, no compensation is paid for distances lower than 7.5, while a compensation equal to 2 is paid for larger distances. In a compensation scheme with $t = 0$, a constant compensation equal to 1 is paid.

We test four different compensation schemes: f1, f2, f3 and f4. These are characterised by a threshold of $t = 0$, $t = 0.33$, $t = 0.5$ and $t = 0.75$, respectively. In Fig. 3, we report a graphical representation of the compensation schemes. In each sub-figure, the x-axes indicate the normalised distance to an SDL, Δ, where 1 represents the maximum allowed distance, i.e. the radius. On the y-axes, we report the value of the compensation offered. In order to have a fair comparison among different schemes, the compensation paid for distances over the threshold is set proportional to the value of the threshold, such that the area under the step function representing the compensation scheme is equal for all the schemes. In fact, if we consider the same compensation for different values of thresholds, schemes with higher thresholds would always be preferable or at least not be dominated. Hence, for every possible value of distance, they would imply a lower or equal compensation compared to schemes with a lower threshold and thereby distort the findings.

In Fig. 4, we report the average costs and average travel times to reach the SDL obtainable with each compensation scheme. To have a

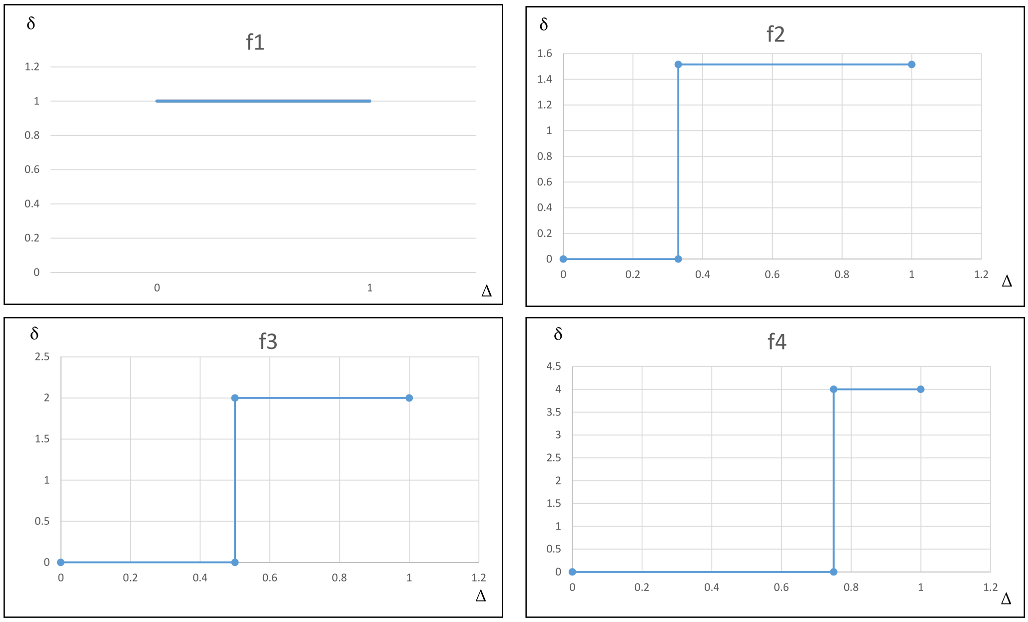

Fig. 3. Compensation schemes.

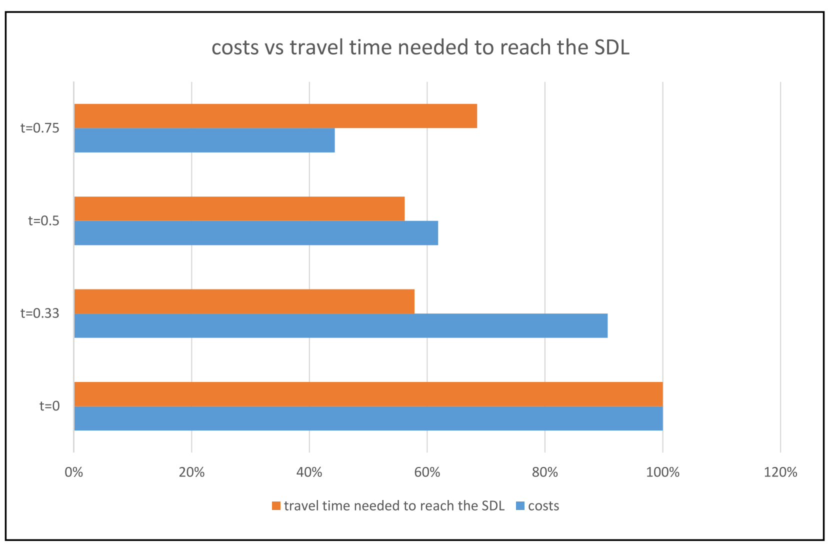

Fig. 4. Comparison between costs and average travel times to reach the SDL with different compensation schemes.

Table 10
Average percentage of SDL usage for different compensation schemes.

	f1	f2	f3	f4
Avg. % SDL usage	71%	74%	84%	84%

fair and homogeneous comparison among the two objectives, values are expressed as percentage of the costs and travel times related to the compensation scheme $t = 0$, which obtains the worse results on both objectives. What emerges is that, while costs decrease with increasing thresholds, the behaviour of the average distance covered to reach the SDL is different, since it is strongly higher for low and high thresholds, while it is considerably lower for intermediate thresholds. This leads to considering that, after a certain threshold (here 0.5), pursuing cost reduction is likely to cause loss of customer satisfaction. Consequently, the functions with thresholds set to 0.5 and 0.75 (namely f3 and f4) are non-dominated approaches, since f4 obtains lower costs but higher SDL distances, while f3 obtains higher costs but lower SDL distances. Schemes f1 and f2, whose thresholds are set at 0 and 0.33, respectively, are dominated by at least one of the previous approaches. It should be noted that f1 is dominated by all settings and f2 is dominated by f3. Concerning the percentage of customers assigned to SDLs (see Table 10), this is the highest for f3 and f4 (84%) compared to the 74% and 71% for f2 and f1, respectively. It is interesting to note that the percentage of customers assigned to SDLs is always lower than 100%. This further emphasises the fact that the mixed delivery system, where deliveries are performed through both SDL and AHS, performs better than models that allow only one of the two options to be applicable.

6. Conclusions

Changing customer habits and an increase in e-commerce forces logistics companies to develop cost- and time-efficient distribution systems. Standard approaches are attended home delivery (AHD) and usage of shared delivery locations (SDLs). While AHD systems might suffer from inefficiencies, SDL bears the risk of decreased perceived service quality. Thus, we proposed a new approach where AHD and SDL are combined. Customers can receive their deliveries at home, within a predefined time window, or pick them up at an SDL. However, we showed that this decision should be taken by the company and not the customers. In order to compensate for potential inconvenience, the company offers a payment to the customer. In our study, we focused on the scenario where the company has full flexibility in serving customers at home or using an SDL. However, customers insisting on one or the other type of service can easily be integrated.

The newly introduced problem was formulated mathematically. Furthermore, we proposed two matheuristic-based approaches in order to generate solutions in a short amount of time.

In an extensive computational study, these approaches were (i) bench-marked against exact solutions and (ii) used to evaluate the newly proposed distribution system against standard (AHD and SDL) approaches. Furthermore, the impact of different compensation schemes was investigated.

The results revealed that the newly proposed flexible delivery approach clearly outperforms the standard ones. We could show that distribution costs can be reduced by up to 40%, while perceived service quality remains unaffected. Several sensitivity tests showed the robustness of the obtained results in terms of customers' accepted travel time needed to reach an SDL as well as time window preferences for delivery. These experiments further emphasised the strength of the newly proposed delivery strategy.

Experimental data is made publicly available in order to enable and encourage future research on the topical problem of deliveries on the last mile.

CRediT authorship contribution statement

Simona Mancini: Problem definition, Modeling, Coding, Analyzing, Writing - original draft, Writing - review & editing. **Margaretha Gansterer:** Problem definition, Analyzing, Writing - original draft, Writing - review & editing.

Acknowledgement

This work is supported by FWF Austrian Science Fund: P 34502-N and P 34151-N.

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