Exploiting the Potential of Energy Hubs in Power Systems Regulation Services

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***Abstract*—Smart grid infrastructures enable consumers with technologies such as electric vehicle and energy storage to participate in electric regulation services. Usually with such technologies, the implementation of large-scale regulation services confronts high interruption cost, uncertainties in availability, and batteries' degradation cost. This motivates us to explore an alternative solution by participating energy hubs with energy conversion technologies to adjust the conversion of natural gas into electricity if the electric grid calls for demand shaping and regulation services. To exploit the potential of energy hubs, we propose an auction for their participation in regulation services. The energy hubs' interaction in the auction is modeled as a non-cooperative game with coupling constraints. To study the existence and uniqueness of the generalized Nash equilibrium (GNE) for such a game, we show that it admits a best response potential function, whose global minimum corresponds to the GNE. We also design a distributed algorithm to achieve that equilibrium. Simulations are performed to illustrate the convergence properties and scalability of the proposed algorithm. Results show that if a participant becomes an energy hub, its profit increases by 60% on average. The electric system operator also benefits from 31% payment reduction to the participants.**

***Index Terms*—Energy hub, regulation services, generalized Nash equilibrium, potential game, distributed algorithm.**

I. Introduction

Addressing unforeseen supply-demand imbalance at short notice is of prominent importance to guarantee the stable operation of power grids. This goal can be achieved through procuring regulation services comprising a variety of control actions to maintain the grid's secure operation at the nominal frequency. Regulation services have conventionally been provided by generation facilities with automatic generation control (AGC) capability to adjust their output power. The recent advancement in smart grid facilities has promoted effective regulation services by electricity consumers adjusting their demand in response to an unexpected supply variation [1].

The evolving regulatory frameworks such as the recent orders issued by the United States (U.S.) Federal Energy Regulatory Commission (FERC) have opened ancillary services markets for new technologies [2]. This has motivated a recent rich body of literature on the participation of electricity consumers in regulation services programs. In [3]–[5], responsive thermal loads such as heating, ventilation, and air-conditioners (HVACs) are suggested to be switched off when the frequency drops. Nevertheless, the consumers often experience a high interruption cost for curtailing their demand. To mitigate the high interruption cost, the application of electric vehicles (EVs) for regulation services has been studied by using different techniques such as stochastic optimization [6], robust control [7], game theory [8], and Markov decision process [9]. Using EVs, however, involves uncertainty in the arrival and departure times. To avoid these uncertainties, energy storage systems (ESSs) has been suggested for regulation services. To do so, different techniques such as robust optimization [10], convex optimization [11], and dynamic programming [12] have been used. Nonetheless, ESSs have strict constraints for their battery's cycle life cost and capacity degradation.

The low energy efficiency of the fossil fueled power plants have accelerated the proliferation of new high-efficiency energy conversion technologies such as combined heat and power (CHP) units in energy hub [13]. CHPs in energy hubs are equipped with micro turbine and gas furnace to convert natural gas into electricity locally, enabling them to adjust the conversion process of natural gas into electricity if the electric grid is in need of demand shaping. In other words, CHPs in energy hubs can decouple the electricity consumption at the customer-side from the electrical power provided by the power grid. Hence, an energy hub is able to change the amount of purchased electric power without a major effect on the customer side's power consumption. This unique flexibility can make energy hubs applicable source of regulation services with high availability and low customers' interruption cost.

A rich body of literature including [14]–[16] studied the load management for an energy hub with CHPs to address the operation scheduling problem in a demand response program for power systems. These studies, however, cannot be directly applied for regulation services as they did not mention how the proposed approaches can be extended to a system with multiple energy hubs. A main challenge in regulation services is to cope with the demand shaping of multiple participants. Several studies including [17]–[19] addressed the interaction of multiple energy hubs for energy management in an energy system to tackle the problem of steady state balancing the supply and demand. Nevertheless, addressing *unforeseen* supply-demand imbalance needs a proper mechanism to incentivize the participants for a *near real-time* demand shaping. The recent work in [20] has studied the participation of energy hubs in tertiary regulation services, where the reserves are traded for capacity and energy. This is a long-run optimization problem, which is formulated as a mixed-integer linear program.

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Despite a high potential of energy hubs for regulation services, their participation would not be without challenges. Specifically, system operators should motivate a fair competition through a well-designed regulation services mechanism. This paper focuses on proposing a viable auction-based mechanism, where the energy hubs submit their bids to indicate their amount of electric power variations in response to pricing offers. The bidding decision of an energy hub affects the clearing price, and thereby the profit of other participating energy hubs. Reaching a stable equilibrium in the auction is a challenge due to the coupling among the competing energy hubs. The main contributions of this paper are as follows:

- *Regulation Services Mechanism Design*: We design a framework specifically for the participation of energy hubs in regulation services. Motivated by the application of bidding mechanisms in wholesale markets [21], we propose an auction-based regulation services mechanism for energy hubs, in which each energy hub expresses its willingness to provide regulation services by submitting a bid. Based on all submitted bids as well as the power network's operating constraints, the auction clearing price and the contribution of each energy hub are determined.
- *Solution Concept*: The energy hubs' interaction in the proposed auction-based regulation services is captured by a non-cooperative game with coupling constraints. We study the *generalized Nash equilibrium (GNE)* [22] of such a game. We prove that the game is a *best response potential game* [23] with a strictly convex potential function, whose global minimum coincides with the GNE.
- *Distributed Algorithm Design*: Characterizing the potential function enables us to develop an efficient algorithm executed by the energy hubs in a distributed fashion. We show that the proposed algorithm globally converges to the GNE from any initial condition in the auction. The convergence is guaranteed even with delay/interruption in the communication network.

The rest of this paper is organized as follows. Section II introduces the system model. In Section III, an auction-based regulation services program is proposed. In Section IV, a distributed algorithm is developed. Simulations are performed in Section V, and the paper is concluded in Section VI.

II. System Model

Consider an energy network shown in Fig. 1(a) comprises a set $\mathcal{N}$ of $N = |\mathcal{N}|$ energy hubs, one electricity utility company (EUC), and one natural gas utility company (GUC). The energy hubs are scattered in a power network with a set $\mathcal{B}$ of $B = |\mathcal{B}|$ buses and a set $\mathcal{L} \in \mathcal{B} \times \mathcal{B}$ of power transmission lines. An energy hub has multiple inputs and outputs. The inputs correspond to the purchased energies from the utility companies, and the outputs correspond to the customer side. Fig. 1(b) depicts the schematic of an energy hub that couples electricity and natural gas infrastructures to provide the customer side with electricity and heating powers.

A. Energy Hub's Operation Model

Consider energy hub $n \in \mathcal{N}$. E_n^{in} and G_n^{in} denote the input electricity and natural gas powers, respectively. Also, E_n^{out}

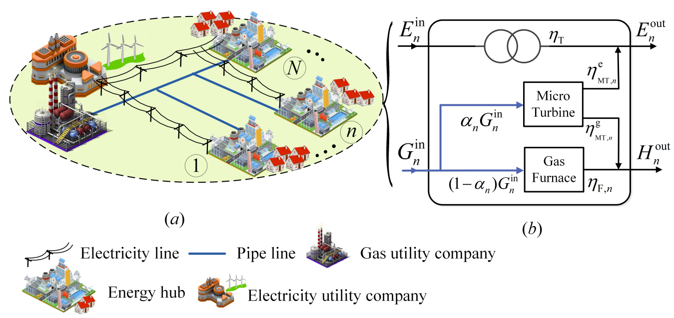

Fig. 1. (a) Energy market with N energy hubs, one electricity utility company, and one gas utility company; (b) Schematic of an energy hub consisting of transformer, gas furnace, and micro turbine as the energy conversion devices.

and H_n^{out} denote the output electricity and heating powers, respectively. Denote the dispatch factor in energy hub n by $\alpha_n \in [0, 1]$. It defines the dispatch of the natural gas input to the micro turbine and the gas furnace [17]. The energy conversion devices are transformer, gas furnace, and micro turbine with efficiency parameters $\eta_{\text{T},n}$, $\eta_{\text{F},n}$, $\eta_{\text{MT},n}^{\text{e}}$ (electrical efficiency of the micro turbine), and $\eta_{\text{MT},n}^{\text{g}}$ (thermal efficiency of the micro turbine), respectively. The power conversion can be expressed by the following matrix equation:

$$\begin{bmatrix} E_n^{\text{out}} \\ H_n^{\text{out}} \end{bmatrix} = \begin{bmatrix} \eta_{\text{T},n} & \alpha_n\, \eta_{\text{MT},n}^{\text{e}} \\ 0 & \eta_{\text{F},n}(1-\alpha_n) + \alpha_n \eta_{\text{MT},n}^{\text{g}} \end{bmatrix} \begin{bmatrix} E_n^{\text{in}} \\ G_n^{\text{in}} \end{bmatrix}. \quad (1)$$

Next we study the participation of energy hub $n \in \mathcal{N}$ in the electrical regulation services program. Let $\widehat{z}_n$ denote the of value an arbitrary variable z_n for energy hub n after participating in the ancillary services program. Let $\Delta e_n^{\text{in}} = E_n - \widehat{E}_n$ and $\Delta e_n^{\text{out}} = E_n^{\text{out}} - \widehat{E}_n^{\text{out}}$ denote the *reduction* in the input electric power and the customer side's electric demand in energy hub $n \in \mathcal{N}$, respectively. Energy hub n may change its input natural gas by $\Delta g_n^{\text{in}} = G_n^{\text{in}} - \widehat{G}_n^{\text{in}}$ and the dispatch factor by $\Delta\alpha_n = \alpha_n - \widehat{\alpha}_n$. We make the following assumption:

Assumption 1: In the electricity regulation services program, the customer side's heating power consumption will not be interrupted, i.e., $\Delta H_n^{\text{out}} = 0,\ n \in \mathcal{N}$.

Considering Assumption 1 and using (1), we can express Δe_n^{out} and Δg_n^{in} as follows:

$$\Delta e_n^{\text{out}} = \eta_{\text{T},n} \Delta e_n^{\text{in}} + \eta_{\text{MT},n}^{\text{e}} \left(\alpha_n G_n^{\text{in}} - \widehat{\alpha}_n \widehat{G}_n^{\text{in}} \right), \quad (2a)$$

$$\Delta g_n^{\text{in}} = \frac{(\eta_{\text{F},n} - \eta_{\text{MT},n}^{\text{g}})}{\eta_{\text{F},n}} \left(\alpha_n G_n^{\text{in}} - \widehat{\alpha}_n \widehat{G}_n^{\text{in}} \right). \quad (2b)$$

From (2a) and (2b), we obtain

$$\Delta e_n^{\text{out}} = \eta_{\text{T},n} \Delta e_n^{\text{in}} + \frac{\eta_{\text{F},n}\, \eta_{\text{MT},n}^{\text{e}}}{(\eta_{\text{F},n} - \eta_{\text{MT},n}^{\text{g}})} \Delta g_n^{\text{in}}, \quad n \in \mathcal{N}. \quad (3)$$

Remark 1 (Energy Hubs' Flexibility): Equation (3) implies that in the scenario where energy hub n increases its input natural gas (i.e., $\Delta g_n^{\text{in}} < 0$), then we have $\Delta e_n^{\text{out}} < \Delta e_n^{\text{in}}$. This demonstrates the flexibility of an energy hub to avoid a large interruption cost in shedding electric demands.

In energy hub n, the value of Δe_n^{out} is always nonnegative.

Thus, we obtain the following lower bound for $\Delta g_n^{\rm in}$:

$$\Delta g_n^{\rm in} \geq -\eta_n \, \Delta e_n^{\rm in}, \qquad n \in \mathcal{N}, \tag{4}$$

where $\eta_n = \eta_{{\rm T},n}\big(\eta_{{\rm F},n} - \eta^{\rm g}_{{\rm MT},n}\big)/(\eta_{{\rm F},n}\,\eta^{\rm e}_{{\rm MT},n})$. Also, (2b) implies that $|\Delta g_n^{\rm in}|$ is maximized if $\widehat{\alpha}_n = 1$. From equality $\Delta H_n^{\rm out} = 0$ obtained from Assumption 1 and using (1), we get $\widehat{G}_n^{\rm in} = (\eta_{{\rm F},n}(1-\alpha_n) + \alpha_n \eta^{\rm g}_{{\rm MT},n})\, G_n^{\rm in}/\eta^{\rm g}_{{\rm MT},n}$. Substituting $\widehat{G}_n^{\rm in}$ and $\widehat{\alpha}_n = 1$ into (2b), we obtain the following inequality:

$$\Delta g_n^{\rm in} \geq -\Delta g_n^{\rm min}. \tag{5}$$

where $\Delta g_n^{\rm min} = (\eta_{{\rm F},n} - \eta^{\rm g}_{{\rm MT},n})(1-\alpha_n)\, G_n^{\rm in}/\eta^{\rm g}_{{\rm MT},n}$. Depending on $\Delta e_n^{\rm in}$, α_n, and $G_n^{\rm in}$, one of (4) and (5) becomes tighter.

The cost of energy hub $n \in \mathcal{N}$ involves three terms: i) the customer side's interruption cost $\mathcal{I}_n(\Delta e_n^{\rm out})$ for shedding $\Delta e_n^{\rm out}$ of electrical load demand, ii) the payment to the GUC for purchasing $|\Delta g_i^{\rm in}|$ extra natural gas, and iii) the reward from the EUC for reducing $\Delta e_i^{\rm in}$ of electrical input power.

Determining the interruption cost $\mathcal{I}_n(\Delta e_n^{\rm out})$ requires well-designed surveys. We make the following assumption:

Assumption 2: The customers' interruption cost $\mathcal{I}_n(\Delta e_n^{\rm out})$ in energy hub $n \in \mathcal{N}$ is an affine function of $\Delta e_i^{\rm out}$. That is

$$\mathcal{I}_n\left(\Delta e_n^{\rm out}\right) = c_n\, \Delta e_n^{\rm out}, \tag{6}$$

where $c_n \geq 0$, $n \in \mathcal{N}$ is the marginal interruption cost.

In nowadays natural gas markets, the GUCs typically implement a fixed price scheme. Let $p_{\rm g}$ denote the natural gas market price in \$/kWh. Energy hub n pays the GUC for purchasing $|\Delta g_n^{\rm in}|$ of extra natural gas. The EUC rewards energy hubs with price $p_{\rm e}$. The cost function of energy hub $n \in \mathcal{N}$ in the ancillary services program can be obtained as

$$\mathcal{C}_n = \mathcal{I}_n\left(\Delta e_n^{\rm out}\right) - p_g\, \Delta g_i^{\rm in} - p_{\rm e}\, \Delta e_n^{\rm in}. \tag{7}$$

B. Power Network's Model

Participation of energy hubs in the electricity regulation services changes the voltage of the buses and the power flow in transmission lines of the power network. Let $\boldsymbol{v} = (v_b,\, b \in \mathcal{B})$ denote the column vector of voltage magnitude of all buses. Let $\boldsymbol{E}^{\rm flow}$ denote the column vector of active power flow into the all transmission line $(r,s) \in \mathcal{L}$. We define $\boldsymbol{E}^{\rm inj}$ as the column vector of injected active powers into the buses $b \in \mathcal{B}$. The elements corresponding to bus n with an energy hub in vector $\Delta\boldsymbol{E}^{\rm inj}$ is equal to $\Delta e_n^{\rm in}$. Other elements of $\Delta\boldsymbol{E}^{\rm inj}$ are zero. Similar to [24], we can use linearized ac power flow model and show that there exist a predetermined matrices $\boldsymbol{R}$ and $\boldsymbol{L}$, such that $\Delta\boldsymbol{v} = \boldsymbol{R}\,\Delta\boldsymbol{E}^{\rm inj}$ and $\Delta\boldsymbol{E}^{\rm flow} = \boldsymbol{L}\,\Delta\boldsymbol{E}^{\rm inj}$. The voltage magnitude of buses and power flow of lines should maintain between their lower and upper limits. We have

$$\Delta\boldsymbol{v}^{\rm min} \leq \boldsymbol{R}\,\Delta\boldsymbol{E}^{\rm inj} \leq \Delta\boldsymbol{v}^{\rm max}, \tag{8a}$$

$$\Delta\boldsymbol{E}^{\rm flow,min} \leq \boldsymbol{L}\,\Delta\boldsymbol{E}^{\rm inj} \leq \Delta\boldsymbol{E}^{\rm flow,max}. \tag{8b}$$

III. Regulation Services Program

Suppose that the EUC broadcasts the request signal to decrease the aggregate electrical load demand with amount of $D > 0$. The EUC establishes a proper mechanism to determine the price $p_{\rm e}$ and the contribution of each participating energy hub n, (i.e., $\Delta e_n^{\rm in}$) in the load shedding. Inspired by the work in [25], we propose an auction-based electrical load control using the supply function bidding mechanism. In the proposed auction mechanism, $\Delta e_n^{\rm in}$ for energy hub n depends on the offered unit price $p_{\rm e}$ by the EUC and the submitted bid $b_n \geq 0$ (in kW/\$) as $\Delta e_n^{\rm in}(p_{\rm e}, b_n) = b_n p_{\rm e}$, $n \in \mathcal{N}$. The bid b_n indicates the elasticity of energy hub n to the auction price $p_{\rm e}$. Let $\boldsymbol{b} = (b_n,\, n \in \mathcal{N})$ denote the vector of bids for all energy hubs. To clear the auction, the total amount of change in the electrical loads should be equal to D. That is $\sum_{n\in\mathcal{N}} \Delta e_n^{\rm in}(p_{\rm e}, b_n) = D$. We have

$$p_{\rm e}(\boldsymbol{b}) = \frac{D}{\sum_{i\in\mathcal{N}} b_i}. \tag{9}$$

From (9), the contribution of energy hub n is

$$\Delta e_n^{\rm in}(\boldsymbol{b}) = \frac{D\, b_n}{\sum_{i\in\mathcal{N}} b_i}, \qquad n \in \mathcal{N}, \tag{10}$$

which depends on the bid of energy hub n as well as the bids of other energy hubs. We capture the interactions among energy hubs by the following non-cooperative game.

Game 1 *(Energy Hubs' Strategic Bidding Game)*:

- ***Players***: The set $\mathcal{N}$ of all energy hubs.
- ***Strategies***: The strategy of energy hub $n \in \mathcal{N}$ is the tuple $\boldsymbol{x}_n = (b_n,\, \Delta g_n^{\rm in})$. Let $\boldsymbol{x} = (\boldsymbol{x}_n,\, n \in \mathcal{N})$ denote the joint strategy profile of all energy hubs. Also, let $\boldsymbol{x}_{-n}$ denote the strategy profile of energy hubs except energy hub n. To determine the strategy space of energy hub n, we substitute (10) into (4) to obtain

$$\Delta g_n^{\rm in} \geq -\eta_n \frac{D\, b_n}{\sum_{i\in\mathcal{N}} b_i}, \;\; n \in \mathcal{N}. \tag{11}$$

  Let $\mathcal{X}_n$ denote the strategy space for energy hub n defined by constraints (5), (8a), (8b), (11), and $b_n \geq 0$.
- ***Costs***: Energy hub $n \in \mathcal{N}$ aims to minimize (7). We substitute (10) into (3), and the result into (6) and (7). Setting $p^{\rm thr}_{{\rm g},n} = c_n\, \eta^{\rm e}_{{\rm MT},n}\eta_{{\rm F},n}/(\eta_{{\rm F},n} - \eta^{\rm g}_{{\rm MT},n})$, we can express the cost of energy hub n as follows:

$$\mathcal{C}_n(\boldsymbol{x}_n, \boldsymbol{x}_{-n}) = \frac{c_n\,\eta_{{\rm T},n}\, D\, b_n}{\sum_{i\in\mathcal{N}} b_i} + p^{\rm thr}_{{\rm g},n}\Delta g_n^{\rm in} - p_{\rm g}\Delta g_n^{\rm in} - \frac{D^2 b_n}{\left(\sum_{i\in\mathcal{N}} b_i\right)^2}. \tag{12}$$

Analyzing Game 1 is challenging, as the inequality constraints (8a), (8b), and (11) depend on the energy hubs' joint bids. Thus, the strategy space $\mathcal{X}_n(\boldsymbol{x}_n, \boldsymbol{x}_{-n})$ of energy hub n depends on the decision of other energy hubs in set $\mathcal{N}\setminus\{n\}$. Generalized Nash equilibrium (GNE) is a commonly used solution concept for such a game [22].

Definition 1 (Generalized Nash Equilibrium): A joint *pure strategy* profile $\boldsymbol{x}^\star = (\boldsymbol{x}_n^\star,\, n \in \mathcal{N})$ is a GNE for Game 1 if and only if for each energy hub $n \in \mathcal{N}$, we have

$$\boldsymbol{x}_n^\star \in \underset{\boldsymbol{x}_n \in \mathcal{X}_n(\boldsymbol{x}_n, \boldsymbol{x}^\star_{-n})}{\rm argmin}\; \mathcal{C}_n(\boldsymbol{x}_n, \boldsymbol{x}^\star_{-n}), \quad n \in \mathcal{N}. \tag{13}$$

That is, $\boldsymbol{x}^*$ is a GNE if and only if it satisfies the individual and coupling constraints and no energy hub can benefit by *unilaterally* deviating from its own strategy $\boldsymbol{x}_n^\star$.

A. Preliminary Analysis of the GNE

Problem (13) implies that the GNE is a fixed point solution of $|\mathcal{N}|$ optimization problems with coupling constraints. To analyze the GNE of Game 1, we consider problem (13) for energy hub n under the given profile $\boldsymbol{x}^\star_{-n}$. We define $B^\star_{-n} = \sum_{i\in\mathcal{N}\setminus\{n\}} b^\star_i$ as the sum of bids for all energy hubs other than energy hub n. Considering the bounds in (5) and (11) and the linearity of cost function (12) in Δg_n^{in}, we consider the following three cases for energy hub n in the GNE:

Case 1: In this case, we suppose that

$$\mathbf{C}1: \quad p_{\text{g}} \geq p_{\text{g},n}^{\text{thr}}.$$

Minimizing (12) for energy hub n results in

$$\Delta g_n^{\star,\text{in}} = 0. \tag{14}$$

That is, energy hub n prefers not to convert natural gas into electricity due to a high natural gas price p_{g}. Substituting $\Delta g_n^{\star,\text{in}} = 0$ into (12), we have

$$\mathcal{C}_n(b_n, \boldsymbol{x}^\star_{-n}) = \frac{c_n\,\eta_{\text{T},n}\,D\,b_n}{b_n + B^\star_{-n}} - \frac{D^2 b_n}{(b_n + B^\star_{-n})^2}. \tag{15}$$

Under the given strategy profile $\boldsymbol{x}^\star_{-n}$, the optimal bid b_n^* should satisfy the optimality condition for problem (13), i.e., $\frac{\partial \mathcal{C}_n(b_n^*, \boldsymbol{x}^*_{-n})}{\partial b_n^*}(b_n - b_n^*) \geq 0$ for all feasible b_n. By performing some algebraic manipulations, for all feasible b_n, we have

$$\left(\frac{c_n\,\eta_{\text{T},n} B^*_{-n}}{D} - \frac{B^*_{-n} - b_n^*}{B^*_{-n} + b_n^*}\right)(b_n - b_n^*) \geq 0. \tag{16}$$

Case 2: By setting $\Delta e_n^{\text{thr}} = (1-\alpha_n)\,\frac{\eta_{\text{F},n}\eta^{\text{e}}_{\text{MT},n}}{\eta_{\text{T},n}\eta^{\text{g}}_{\text{MT},n}}\,G_n^{\text{in}}$, we suppose that

$$\mathbf{C}2: \begin{cases} p_{\text{g}} < p_{\text{g},n}^{\text{thr}}, \\ \dfrac{D\,b_n}{b_n + B^\star_{-n}} < \Delta e_n^{\text{thr}}. \end{cases}$$

Based on $\mathbf{C}2$, minimizing $\mathcal{C}_n(\boldsymbol{x}_n, \boldsymbol{x}_{-n})$ in (12) is equivalent to set $\Delta g_n^{\star,\text{in}}$ to the lower bound in (11). Hence, we have

$$\Delta g_n^{\star,\text{in}} = -\eta_n \frac{D\,b_n^\star}{b_n^\star + B^\star_{-n}}. \tag{17}$$

Substituting (17) into (12), cost of energy hub n becomes

$$\mathcal{C}_n(b_n, \boldsymbol{x}^\star_{-n}) = \frac{p_{\text{g}}\eta_n D\,b_n}{b_n + B^\star_{-n}} - \frac{D^2 b_n}{\left(b_n + B^\star_{-n}\right)^2}. \tag{18}$$

The objective function (18) has the same structure as of the objective function (15). The following optimality condition can be derived for all feasible b_n:

$$\left(\frac{p_{\text{g}}\eta_n B^\star_{-n}}{D} - \frac{B^\star_{-n} - b_n^\star}{B^\star_{-n} + b_n^\star}\right)(b_n - b_n^\star) \geq 0. \tag{19}$$

Case 3: In this case, we have

$$\mathbf{C}3: \begin{cases} p_{\text{g}} < p_{\text{g},n}^{\text{thr}}, \\ \dfrac{D\,b_n}{b_n + B^\star_{-n}} \geq \Delta e_n^{\text{thr}}. \end{cases}$$

Minimizing (12) for energy hub n results in setting $\Delta g_n^{\star,\text{in}}$ to the lower bound in (5). That is, we have

$$\Delta g_n^{\star,\text{in}} = -\Delta g_n^{\min}. \tag{20}$$

In this case, $\Delta g_n^{*,\text{in}}$ is constant. We remove the terms with $\Delta g_n^{*,\text{in}}$ from (12). The cost function of energy hub n becomes

$$\mathcal{C}_n(b_n, \boldsymbol{x}^\star_{-n}) = \frac{c_n\,\eta_{\text{T},n}\,D\,b_n}{b_n + B^\star_{-n}} - \frac{D^2 b_n}{(b_n + B^\star_{-n})^2}, \tag{21}$$

which is the same as (15) for Case 1. Thus, we have the same optimality condition as (16) for all feasible b_n:

$$\left(\frac{c_n\,\eta_{\text{T},n} B^\star_{-n}}{D} - \frac{B^\star_{-n} - b_n^\star}{B^\star_{-n} + b_n^\star}\right)(b_n - b_n^\star) \geq 0. \tag{22}$$

B. Existence and Uniqueness of the GNE

In this subsection, we study the existence and uniqueness of the GNE. In the following theorem, we show that there is no GNE for a system with two energy hubs.

Theorem 1: For an ancillary services market with $N = 2$ energy hubs, the strategic bidding game among energy hubs (Game 1) does not have a GNE in pure strategies.

The proof can be found in Appendix A. From the proof of Theorem 1, if a GNE exists for Game 1, then we have $0 \leq \Delta e_n^{\star,\text{in}} < \frac{D}{2}$ for all energy hubs $n \in \mathcal{N}$ in the GNE.

Next we show that a GNE exists for an ancillary services market for $N \geq 3$. To do so, we show that Game 1 can be categorized in a class of best response potential games [23].

Definition 2 (Best Response Potential Game): Game $\mathcal{G}\big(\mathcal{N}, \boldsymbol{x}, (\mathcal{C}_n, n\in\mathcal{N})\big)$ is a best response potential, if it is equivalent to game $\widetilde{\mathcal{G}}\big(\mathcal{N}, \boldsymbol{y}, (\Phi, n\in\mathcal{N})\big)$, and there exists a one-to-one mapping $\mathcal{F}$ between the joint strategy profiles $\boldsymbol{x}$ and $\boldsymbol{y}$. Also, cost functions $\mathcal{C}_n$ for all agents $n \in \mathcal{N}$ are replaced with a function Φ called a potential function. Agent n has the same best response to arbitrary strategy profiles $\boldsymbol{x}_{-n}$ in $\mathcal{G}$ and $\boldsymbol{y}_{-n} = \mathcal{F}(\boldsymbol{x}_{-n})$ in $\widetilde{\mathcal{G}}$, i.e., $\widetilde{\mathcal{B}}_n(\boldsymbol{y}_{-n}) = \mathcal{B}_n(\boldsymbol{x}_{-n})$.

To show that Game 1 is a best response potential game, we assign the new strategy profile $\boldsymbol{y}_n = (\Delta e_n^{\text{in}}, \Delta g_n^{\text{in}})$ to energy hub $n \in \mathcal{N}$. The strategy space $\mathcal{Y}_n$ is defined by $0 \leq \Delta e_n^{\text{in}} < \frac{D}{2}$, $\sum_{n\in\mathcal{N}} \Delta e_n^{\text{in}} = D$, $\Delta g_n^{\text{in}} \leq 0$, and the power flow constraints (8a) and (8b). Consider the following theorem:

Theorem 2: Game 1 is a best response potential game. It is equivalent to Game 2 with the new strategy profile $\boldsymbol{y} = (\boldsymbol{y}_n, n\in\mathcal{N})$ and the following potential function:

$$\Phi(\boldsymbol{y}) = \sum_{n\in\mathcal{N}} \varphi_n(\boldsymbol{y}_n), \tag{23}$$

where for $p_{\text{g}} \geq p_{\text{g},n}^{\text{thr}}$, we have

$$\varphi_n(\boldsymbol{y}_n) = c_n\eta_{\text{T},n}\left(\frac{\Delta e_n^{\text{in}}}{2} - \frac{D}{4}\ln\left(D - 2\Delta e_n^{\text{in}}\right)\right) + \left(\Delta g_n^{\text{in}}\right)^2, \tag{26a}$$

and for $p_{\text{g}} < p_{\text{g},n}^{\text{thr}}$, if $\Delta e_n^{\text{in}} < \Delta e_n^{\text{thr}}$, then

$$\varphi_n(\boldsymbol{y}_n) = p_{\text{g}}\eta_n\left(\frac{\Delta e_n^{\text{in}}}{2} - \frac{D}{4}\ln\left(D - 2\Delta e_n^{\text{in}}\right)\right) + \left(\Delta g_n^{\text{in}} + \eta_n\Delta e_n^{\text{in}}\right)^2, \tag{26b}$$

and if $\Delta e_n^{\text{in}} \geq \Delta e_n^{\text{thr}}$, then we have

$$\varphi_n(\boldsymbol{y}_n) = c_n\, \eta_{\text{T},n} \left(\frac{\Delta e_n^{\text{in}}}{2} - \frac{D}{4} \ln \left(D - 2\Delta e_n^{\text{in}}\right) \right) + \left(\Delta g_n^{\text{in}} + \Delta g_n^{\min}\right)^2 . \tag{26c}$$

For the proof see Appendix B.

Theorem 3: For nonempty strategy spaces $\mathcal{Y}_n$, $n \in \mathcal{N}$ and $N \geq 3$, Game 1 has a unique GNE.

The proof can be found in Appendix C. The proof of Theorem 3 is based on the the strictly convexity of the potential function (23) and the coincidence of the GNE and the unique global minimum of the potential function.

IV. Distributed Algorithm Design

In this section, we develop a distributed algorithm executed by the energy hubs in a parallel fashion to converge to the unique GNE of Game 1. Based on Theorems 2 and 3, we develop a distributed algorithm that converges to the global optimum of $\Phi(\boldsymbol{y})$. Note that $\Phi(\boldsymbol{y})$ is a sum of N distinct functions $\varphi_n(\boldsymbol{y}_n)$. The coupling constraint $\sum_{n\in\mathcal{N}} \Delta e_n^{\text{in}} = D$ is also linear and decomposable. Hence, we can develop a distributed algorithm based on *dual decomposition* method [26, Sec. 3.5] to determine the global optimal point of $\Phi(\boldsymbol{y})$.

Denote the iteration index of Algorithm 1 by k. The loop involving Lines 3 to 8 describes the interactions among energy hubs and the EUC in the auction. The price in the GNE corresponds to the dual variable for $\sum_{n\in\mathcal{N}} \Delta e_n^{\text{in}} = D$. By receiving the bids from all energy hubs, the EUC determines the updated price p_{e}^{k+1} in Line 5 as follows:

$$p_{\text{e}}^{k+1} = \left[p_{\text{e}}^{k} - \rho \left(\sum_{n\in\mathcal{N}} b_n^k p_{\text{e}}^k - D \right) \right]_{\wp}, \tag{27}$$

where $\rho > 0$ is the step-size and $[\cdot]_{\wp}$ is the projection on to nonnegative orthand. In Line 6, when energy hub n receives the updated price and uses the optimality conditions (26a)−(26c) to compute $\Delta e_n^{\text{in},k+1}$ and $\Delta g_n^{\text{in},k+1}$as follows:
If $p_{\text{g}} \geq p_{\text{g},n}^{\text{thr}}$, then energy hub n sets

$$\Delta g_n^{\text{in},k+1} = 0. \tag{28}$$

According to (38) in Appendix B, energy hub n sets

$$\Delta e_n^{\text{in},k+1} = \left[\frac{D(p_{\text{e}}^{k+1} - c_n\, \eta_{\text{T},n})}{2p_{\text{e}}^{k+1} - c_n\, \eta_{\text{T},n}} \right]_{\wp}, \tag{29}$$

where $[\cdot]_{\wp}$ is the projection on to the feasible space defined by $0 \leq \Delta e_n^{\text{in}} < \frac{D}{2}$ and the power flow constraints (8a) and (8b) under the given $\Delta e_{n'}^{\star,\text{in}}$, $n' \in \mathcal{N}\setminus\{n\}$. The EUC provides energy hub n with the limits for Δe_n^{in} using (8a) and (8b). For $p_{\text{g}} < p_{\text{g},n}^{\text{thr}}$, then if $\Delta e_n^{\text{in},k} < \Delta e_n^{\text{thr}}$, energy hub n sets

$$\Delta g_n^{\text{in},k+1} = -\frac{\eta_{\text{T},n}(\eta_{\text{F},n} - \eta_{\text{MT},n}^{\text{g}})}{\eta_{\text{F},n}\eta_{\text{MT},n}^{\text{e}}} \Delta e_n^{\text{in},k}. \tag{30}$$

According to (39) in Appendix B, energy hub n sets

$$\Delta e_n^{\text{in},k+1} = \left[\frac{D\left(p_{\text{e}}^{k+1} - p_{\text{g}}\eta_n\right)}{2p_{\text{e}}^{k+1} - p_{\text{g}}\eta_n} \right]_{\wp}. \tag{31}$$

Algorithm 1 Decentralized Load Control Algorithm.

1: Set $k := 0$ and $\xi := 10^{-3}$.
2: Each energy hub n randomly initializes its bid b_n^0 and sets $\Delta e_n^{\text{in},0} = \Delta g_n^{\text{in},0} = 0$.
3: **Repeat**
4: Each energy hub n submits its bid b_n^k to the EUC.
5: EUC determines the updated price p_{e}^{k+1} according to (27) and broadcasts to the energy hubs.
6: Each energy hub n, depending on the value of p_{g} and $\Delta e_n^{\text{in},k}$, determines $\Delta g_n^{\text{in},k+1}$ and $\Delta e_n^{\text{in},k+1}$ according to (28)−(32). Energy hub n computes b_n^{k+1} as (33).
7: $k := k + 1$.
8: **Until** $|p_{\text{e}}^k - p_{\text{e}}^{k-1}| \leq \xi$.

If $\Delta e_n^{\text{in},k} \geq \Delta e_n^{\text{thr}}$, then energy hub n sets

$$\Delta g_n^{\text{in},k+1} = -\Delta g_n^{\min}, \tag{32}$$

Energy hub n sets $\Delta e_n^{\text{in},k+1}$ as (29). In all above-mentioned scenarios, energy hub n sets its updated bid as

$$b_n^{k+1} = \frac{\Delta e_n^{\text{in},k+1}}{p_{\text{e}}^{k+1}}. \tag{33}$$

The potential function is strictly convex. Hence, with any initial condition, the Algorithm 1 converges to the global minimum of the potential function [26, Sec. 3.5]. That is

Theorem 4: Algorithm 1 converges to the GNE from any initial point, and the convergence rate is $O(1/\sqrt{k})$.

This is an important result, since if the communication interrupted or delayed, the GNE would not be affected. Specifically, if the delays/interruptions happen in a finite number of iterations, then there exists an iteration number, after which all hubs update their strategy profile without delay. That iteration can be interpreted as a new initial point for Algorithm 1.

The number of iterations for convergence depends on the initial conditions. In Section V, we show that the number of iterations is acceptable for practical implementations. One can also apply the approach in [27, Algorithm 4] and [28, Sec. IV.B] to enhance the convergence rate of Algorithm 1 to $O(1/k)$ using proximal Jacobian alternating method of multipliers (PJ-ADMM). We omit the modification and only present the comparison in Section V.

V. Performance Evaluation

In this section, we evaluate the performance of the proposed distributed algorithm on an ancillary market with $N = 5$ energy hubs. We consider the request from the EUC for $D = 2$ MW load reduction due to the supply shortage. The natural gas wholesale price is set to $p_{\text{g}} = 17$ \$/MWh. The parameters $\eta_{\text{T},n}$, $\eta_{\text{F},n}$, $\eta_{\text{MT},n}^{\text{e}}$, and $\eta_{\text{MT},n}^{\text{g}}$ for energy hub $n \in \mathcal{N}$ are selected randomly from intervals $[0.92, 0.98]$, $[0.6, 0.8]$, $[0.18, 0.27]$, and $[0.2, 0.3]$ [29], respectively. The initial dispatch factors α_n, for $n = 1, \ldots, 5$ are set to 0.2, 0.7, 0.4, 0.65, and 0.8, respectively. The gas demand G_n^{in} for $n = 1, \ldots, 5$ are set to 2, 2.5, 5.2, 7.8, and 6 MWh, respectively. The coefficients c_n are set to 100, 50, 105, 100, and 110 \$/MWh, respectively. The stepsize ρ is set to 0.2 \$/MWh.

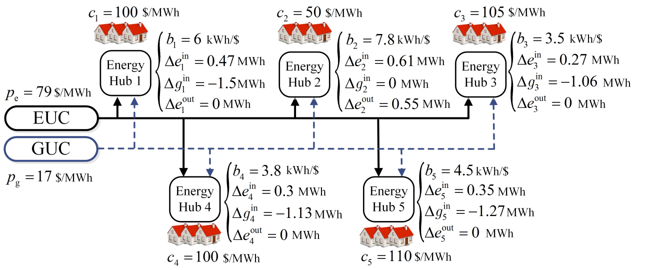

Fig. 2. The equilibrium of the energy market in the underlying auction-based regulation services program.

First we study the participation of the energy hubs in the proposed auction-based regulation services program. Fig. 2 shows the underlying energy network with five energy hubs in the GNE. The auction clearing price in the GNE is $p_{\mathrm{e}} = 79$ \$/MWh. Different factors such as the marginal interruption cost c_n, the dispatch factor α_n, and the natural gas demand G_n^{in} affect the contribution of each energy hub in the regulation services program. For example, energy hub 2 has the largest Δe_n^{in} due to its low marginal interruption cost. Energy hub 1 has also a large contribution due its small initial dispatch factor. As expected, the energy hub with a larger contribution would submit a larger bid in the auction. We can observe that $\Delta g_n^{\mathrm{in}} \leq 0$ for all energy hubs. Energy hub 1 has the highest $|\Delta g_n^{\mathrm{in}}|$ due to its low dispatch factor. Furthermore for energy hub 2, we have $\Delta g_n^{\mathrm{in}} = 0$, since $p_{\mathrm{g}} \geq< p_{\mathrm{g},2}^{\mathrm{thr}}$ (condition **C**1). Energy hub 2 has the lowest interruption cost, and hence it does not prefer to pay to the GUC for converting natural gas into electricity. Instead, it prefers to interrupt its customer side's electricity consumption, i.e., $\Delta e_n^{\mathrm{out}} = 0.55$ MWh. Other energy hubs have relatively high interruption costs, and thereby preferring to use their CHP technology for energy conversion. We can observe that $\Delta e_n^{\mathrm{out}} = 0$ for energy hubs 1, 3, 4, and 5. This perfectly demonstrates the potential of energy hubs in a regulation services program, as they can actively participate in the program without affecting their customer side's electricity consumption habits.

We now study the convergence of Algorithm 1 to the GNE. Fig. 3(a) shows the convergence of the auction clearing price p_{e} from the initial value 120 \$/MWh to 79 \$/MWh in the GNE within 27 iterations. Fig. 3(b) shows the convergence of electrical load change Δe_n for the participating energy hubs. The initial value of Δe_n is zero for all energy hubs. The value of Δe_n is higher in the initial iterations, as the auction price is still large compared with the clearing price in the GNE. Fig. 3(c) shows the convergence of Δg_n^{in}. For hub 2, we have we have $\Delta g_n^{\mathrm{in}} = 0$ in all iterations, as condition **C**1 is always satisfied for this energy hub. Fig. 3(d) shows the convergence of the submitted bids. A larger bid implies a larger market share. We can observe that energy hub 2 gains a larger market share gradually due its lowest interruption cost.

We study the impact of communication delay/interruption on the convergence of Algorithm 1 to the GNE. We consider four scenarios: In Scenario 1, no delay or interruption is occurred. In Scenario 2, energy hubs 1, 2, and 3 experience an interruption in their communication link from iteration 4 to 20.

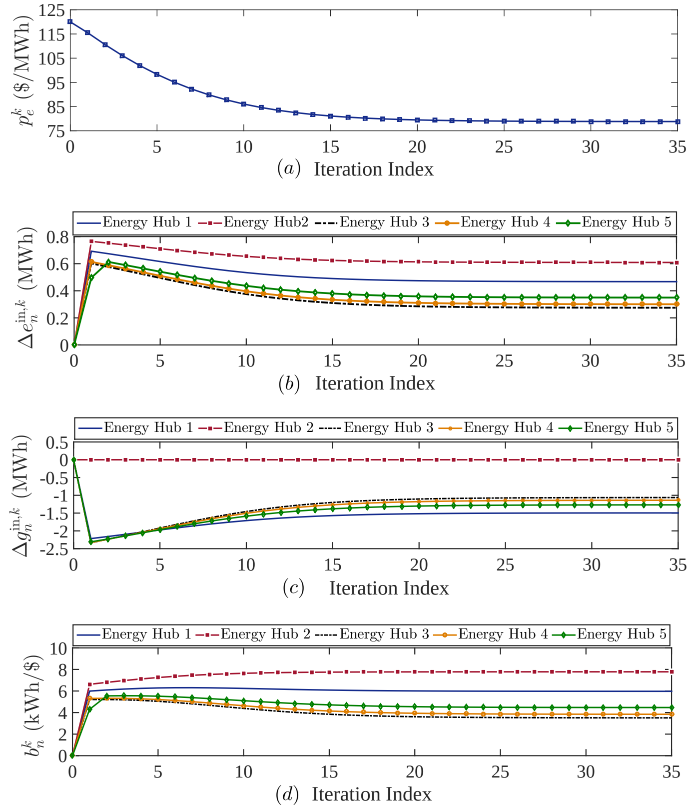

Fig. 3. Convergence of (a) market price; (b) electrical load change; (c) natural gas load change; and (d) submitted bids to the GNE of Game 1.

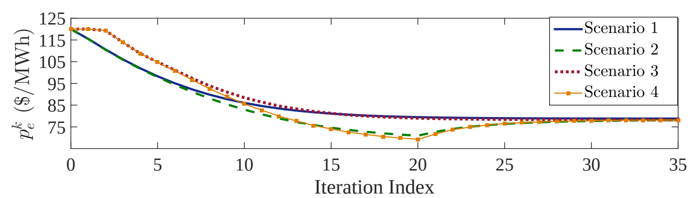

Fig. 4. The impact of communication delay and interruption on the convergence of Algorithm 1 to the GNE.

In Scenario 3, energy hubs 4 and 52 experience a delay in their communication link and update their bids in every 2 iterations; and Scenario 4 is the combination of the second and third scenarios. Fig. 4 depicts that in all aformentioned scenarios, the clearing market price converges to a same value in the unique GNE though the convergence rate changes. This shows that the GNE is asymptotically stable and the convergence of Algorithm 1 is independent to the initial conditions.

Next we study the profit of the energy hubs and the total payment of the EUC to the energy hubs in the GNE. We consider three scenarios. In Scenario 1, the energy hubs do not convert gas to electricity. In Scenario 2, energy hub 1 uses its ability to convert gas to electricity, but other energy hubs do not convert natural gas to electricity. In Scenario 3, all energy hubs use their ability to convert gas to electricity. Fig 5(a) shows that in Scenario 1, energy hub 2 gains the highest profit due to its lowest marginal interruption cost. When energy hub 1 uses CHP technology in the second scenario, it can get a larger market share to increase its profit. When energy hubs

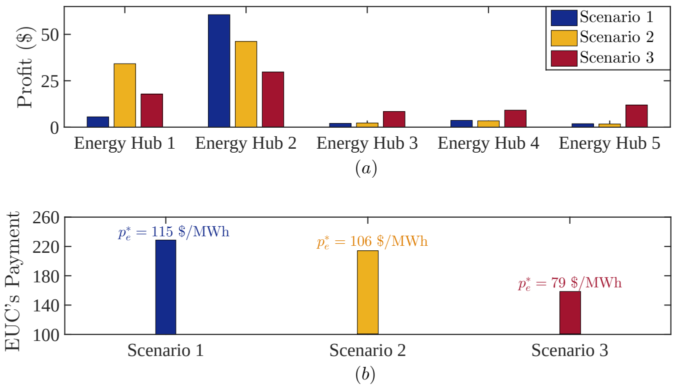

Fig. 5. (a) Profit of the energy hubs; and (b) payment of the EUC.

3, 4, and 5 also use CHP in the third scenario, they can increase their market share, and thus their profit (by about 60%). Results for all five energy hubs show that using the CHP enables them to avoid a large interruption cost and increase their profit by about 7.6% compared to Scenario 1.

Fig 5(b) shows the EUC's payment in the mentioned three scenarios. The market clearing price is 115 \$/MWh and 79 \$/MWh in Scenarios 1 and 3, respectively. That is, the EUC's payment to the participating consumers is about 31% lower when energy hubs convert natural gas to electricity in the third scenario. This shows the advantage of using energy conversion technologies in the regulation services programs. The CHP technology enables the energy hubs' active participation with lower bids through converting natural gas to electricity. Scenario 2 demonstrates that if energy hub 1 deploys the CHP technology, then it can damp the market by reducing the market clearing price from 115 to 106 \$/MWh. This scenario shows the fact that even if the EUC motivates one energy hub to participate in the ancillary services market, then its payment will be reduced. Other participants are also motivated to deploy the CHP; they will otherwise lose the market share to their competitors with CHP technology.

Now we study the impact of power network constraints on the auction results. Suppose that the energy hubs are located in an IEEE 37-bus test feeder, as shown in Fig. 6(a). The limits for the bus voltage are set to 0.95 pu and 1.05 pu. The line flow limit is set to 1.05 pu. The initial operating conditions are shown in Fig. 6(a). Figs. 6(b) and (c) show the auction clearing price and the energy hubs contributions to curtail 2 MW of electric demand. Energy hubs 2 and 3 are located in buses with low voltage magnitudes. Thus, they cannot reduce their demand too much due to constraint (8a). In particular, when the network constraints are taken into account, the auction price increases from 79 \$/MWh to 90 \$/MWh (i.e., the DNO pays more to the participants). Also, the contributions of energy hubs 2 and 3 decrease in the regulation services. That is, the network constraints force a new operating condition in the regulation services that might limit high-potential participants (e.g., energy hub 2) not to schedule their demand as much as they can.

We study the scalability of Algorithm 1 by considering test systems with 37, 123, and 8500 buses [30] with different

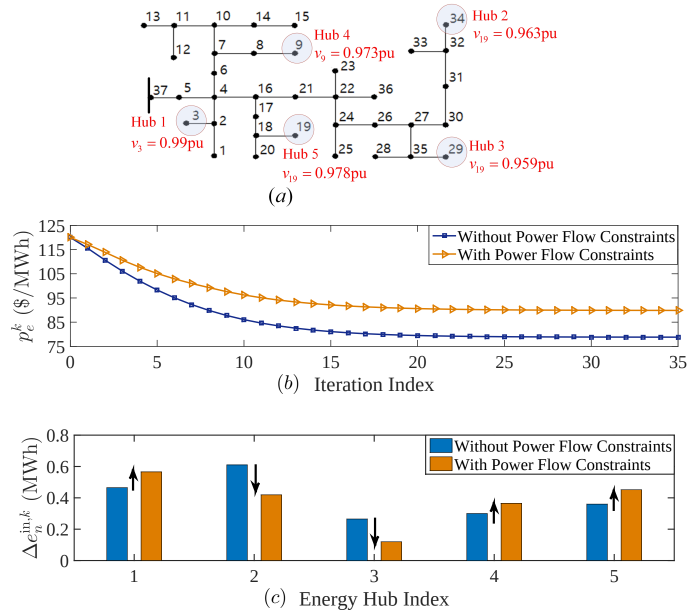

Fig. 6. (a) Energy hubs' locations in the distribution feeder; (b) auction clearing price; and (c) contributions of each energy hub, with and without considering power flow constraints.

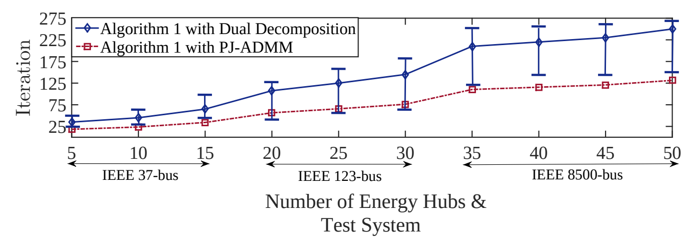

Fig. 7. Required number of iterations for convergence to the GNE.

number of energy hubs. The characteristics of the energy hubs are randomly chosen to be similar to our simulation setup. Fig. 7 depicts the required number of iterations for convergence to the GNE. For 100 random initial conditions, the minimum and maximum number of iterations are shown, which are even in large test systems. We also apply the approach in [27, Algorithm 4] and [28, Sec. IV.B] to enhance the convergence rate of Algorithm 1. We can observe that PJ-ADMM can reduce the average number of iterations by about 40%. Usually, in practical energy markets, the number of energy hubs is not too large. Thus, Algorithm 1 has a potential to be used in practice. In all scenarios, the running time per iteration is less than 0.5 seconds thanks to the closed form update rules in (28)$-$(33), which do not depend on the the network scale and number of energy hubs. In other words, the *number of iterations* for convergence is the bottleneck for the algorithm's aggregate running time.

Finally, we compare performance of the proposed auction mechanism with existing demand response algorithms (e.g., [17]–[20]). We consider an optimization problem with the objective function $\sum_{n\in\mathcal{N}} \mathcal{I}_n(\Delta e_n^{\text{out}}) - p_g\, \Delta g_i^{\text{in}}$ and constraints (4), (5), (8a), and (8b), as well as the constraint $\sum_{n\in\mathcal{N}} \Delta e_n^{\text{in}} = D$. We focus on the profit of energy hubs in the GNE of Game 1 and the underlying demand response framework in Fig. 8. Two facts can be observed. $i)$ Energy hubs 3, 4, and

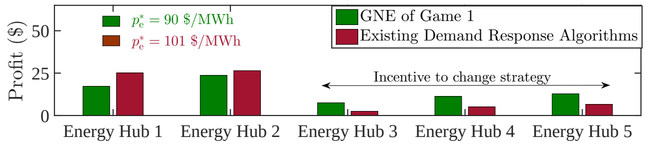

Fig. 8. Energy hubs' profit in the GNE of Game 1 and minimum social cost.

5 have lower profit in the underlying demand response framework. They have incentive to change their strategy unilaterally to increase their own profit. In the GNE, however, the energy hubs are satisfied with their profit and do not have incentive to change their strategy unilaterally, i.e., the GNE is *stable*. *ii)* As shown in Fig. 8, the market clearing price $p_e^\star$ (i.e., the dual variable associated with constraint $\sum_{n\in\mathcal{N}} \Delta e_n^{\rm in} = D$) is lower in the GNE. The reason is that in our framework, the energy hubs are allowed to submit bids considering their interruption costs. But when the EUC minimizes the social cost, all energy hubs should be satisfied; thereby the clearing price is largely biased by the participants with higher interruption costs.

VI. Conclusion

In this paper, we studied the potential of energy hubs for power systems regulation services. We considered the scenario of electric supply shortage and showed that energy hubs can effectively adjust energy conversion process to reduce their input electric power without a major effect on the customer side's electric power consumption. We proposed a bidding mechanism and modeled the interaction among energy hubs as a non-cooperative game with coupling constraints. We prove the existence of the GNE in such a game by constructing a best response potential function, whose global minimum corresponds to the GNE. We also developed a distributed algorithm to reach the GNE. Simulation results showed that the proposed algorithm converges to the GNE in a reasonable number of iterations even with interruption in the communication network. If all participants use CHP technology, they can take advantage of 7.6% increase in their profit on average. Meanwhile, the EUC can reduce its payment by 31% by motivating energy hubs to participate in the regulation program. For future work, we plan to extend our proposed algorithm by considering the energy hubs' interaction in capacity markets.

Appendix A
Proof of Theorem 1

We prove the result by contradiction. Suppose that for a system with two energy hubs, there exists a GNE for Game 1. From the optimality conditions (16), (19), and (22), we conclude that $b_n^\star < B_{-n}^\star$, $n \in \mathcal{N}$. Specifically, if $b_n^\star \geq B_{-n}^\star$ for energy hub n, then we have $\frac{\partial \mathcal{C}_n(b_n^\star, \boldsymbol{x}_{-n}^\star)}{\partial b_n^\star} > 0$. That is, energy hub n has an incentive to decrease its bid $b_n^\star$, which contradicts with the definition of the GNE. Consequently, from (10), we obtain $\Delta e_n^{\rm in}(\boldsymbol{b}^\star) = \frac{D\, b_n^\star}{b_n^\star + B_{-n}^\star} < \frac{D}{2}$. Thus, for a system with $N = 2$ energy hubs, we have $\sum_{n\in\mathcal{N}} \Delta e_n^{\rm in}(\boldsymbol{b}^\star) < D$, which contradicts with the power balance. We conclude that the GNE does not exits. The proof is completed. ∎

Appendix B
Proof of Theorem 2

The proof involves the following two steps:

Step a) Considering $\Delta e_n^{\rm in}(\boldsymbol{b}) = \frac{D\, b_n}{b_n + B_{-n}}$, the conditions for (26a)−(26c) correspond to the conditions **C**1, **C**2, and **C**3 for Cases 1, 2, and 3 in Section III-A, respectively. $\Delta g_n^{\star,\rm in} = 0$ is the optimal point for (26a) which also corresponds to (14). $\Delta g_n^{\star,\rm in} = -\eta_n \Delta e_n^{\star,\rm in}$ is the optimal point for (26b) which also corresponds to (17). $\Delta g_n^{\star,\rm in} = -\Delta g_n^{\rm min}$ is the optimal point for (26c) which also corresponds to (20).

Step b) Next we show that the optimality condition for $\Delta e_n^{\rm in}$, $n \in \mathcal{N}$ in Game 2 is equivalent to the optimality conditions (16), (19), and (22) for b_n, $n \in \mathcal{N}$ in Game 1.

Consider (16) and (22). By exchanging the denominator of the first term and the nominator of the second term, we can rewrite the optimality conditions (16) and (22) as follows:

$$\left(\frac{c_n\, \eta_{{\rm T},n} B_{-n}^*}{B_{-n}^* - b_n^*} - \frac{D}{B_{-n}^* + b_n^*}\right)(b_n - b_n^*) \geq 0, \qquad (34)$$

and for all feasible b_n:

$$\left(\frac{p_{\rm g}\eta_n B_{-n}^*}{B_{-n}^* - b_n^*} - \frac{D}{B_{-n}^* + b_n^*}\right)(b_n - b_n^*) \geq 0. \qquad (35)$$

It enables us to express the optimality condition (16) and (22) in terms of $\Delta e_n^{\star,\rm in}$. By substituting $\Delta e_n^{\star,\rm in} = \frac{D\, b_n^\star}{B_{-n}^\star + b_n^\star}$ and $p_{\rm e}^\star = \frac{D}{B_{-n}^\star + b_n^\star}$ into (34), for all feasible b_n, we obtain

$$\left(c_n\, \eta_{{\rm T},n} \frac{D - \Delta e_n^{*,\rm in}}{D - 2\Delta e_n^{*,\rm in}} - p_{\rm e}^*\right)(p_{\rm e}^* b_n - \Delta e_n^{*,\rm in}) \geq 0. \qquad (36)$$

Similarly, the optimality condition (35) is equivalent to

$$\left(p_{\rm g}\eta_n \frac{D - \Delta e_n^{*,\rm in}}{D - 2\Delta e_n^{*,\rm in}} - p_{\rm e}^*\right)(p_{\rm e}^* b_n - \Delta e_n^{*,\rm in}) \geq 0. \qquad (37)$$

Now we study the optimality conditions for $\Delta e_n^{\rm in}$ in Game 2. Let $\boldsymbol{y}^\star$ denote the optimal point of potential function $\Phi(\boldsymbol{y})$. Let λ^* denote the dual variable for constraint $\sum_{n\in\mathcal{N}} \Delta e_n^{\rm in} = D$ in the optimal point of $\Phi(\boldsymbol{y})$. The derivative of $\frac{\Delta e_n^{\rm in}}{2} - \frac{D}{4}\ln(D - 2\Delta e_n^{\rm in})$ w.r.t. $\Delta e_n^{\rm in}$ is $\frac{D - \Delta e_n^{\rm in}}{D - 2\Delta e_n^{\rm in}}$. Thus, the optimality condition for $\Delta e_n^{\rm in}$ in (26a) (and (26c)) is obtained as

$$\left(c_n\, \eta_{{\rm T},n} \frac{D - \Delta e_n^{\star,\rm in}}{D - 2\Delta e_n^{\star,\rm in}} - \lambda^\star\right)(\Delta e_n^{\rm in} - \Delta e_n^{\star,\rm in}) \geq 0, \qquad (38)$$

which is the same as (36) by setting $\lambda^* = p_{\rm e}^*$. In a similar manner, the optimality condition for $\varphi_n(\boldsymbol{y}_n)$ in (26b) after substituting $\Delta g_n^{\star,\rm in} = -\eta_n \Delta e_n^{\star,\rm in}$ is as

$$\left(p_{\rm g}\eta_n \frac{D - \Delta e_n^{*,\rm in}}{D - 2\Delta e_n^{*,\rm in}} - \lambda^*\right)(\Delta e_n^{\rm in} - \Delta e_n^{*,\rm in}) \geq 0, \qquad (39)$$

which is the same as (37) by setting $\lambda^* = p_{\rm e}^*$. That is, we have a one-to-one mapping between $\Delta e_n^{\star,\rm in}$ and $b_n^\star$.

We can conclude that Game 1 is a best response potential game, and the potential function is given in (23). ∎

Appendix C
Proof of Theorem 3

The GNE of Game 1 is the fixed point of the best response strategies of energy hubs. According to Theorem 2, Game 1 is a best response potential game. Hence, the fixed point of the best response of the energy hubs in Game 1 corresponds to global optimal point of the potential function $\Phi(\boldsymbol{y})$ in (23) over the feasible space $\mathcal{Y}$ defined by constraints $0 \leq \Delta e_n^{\text{in}} < \frac{D}{2}$, $\Delta g_n^{\text{in}} \leq 0$, $n \in \mathcal{N}$, $\sum_{n \in \mathcal{N}} \Delta e_n^{\text{in}} = D$, and the power flow constraints (8a) and (8b). If this feasible space is non-empty when $N \geq 3$, then, the global optimal point of $\Phi(\boldsymbol{y})$ exists, and thereby Game 1 has at least one GNE.

Next we show that the potential function $\Phi(\boldsymbol{y})$ in (23) is strictly convex. It is sufficient to show that function $\varphi_n(\boldsymbol{y}_n)$ defined in (28a)$-$(28c) is strictly convex in $\boldsymbol{y}_n = (\Delta e_n^{\text{in}}, \Delta g_n^{\text{in}})$. The second derivative of $\frac{\Delta e_n^{\text{in}}}{2} - \frac{D}{4}\ln(D - 2\Delta e_n^{\text{in}})$ with respect to Δe_n^{in} is $\frac{D}{(D-2\Delta e_n^{\text{in}})^2}$, which is positive and continuous for $\Delta e_n^{\text{in}} < D/2$. That is, function $\frac{\Delta e_n^{\text{in}}}{2} - \frac{D}{4}\ln(D - 2\Delta e_n^{\text{in}})$ is strictly convex. Other terms in (28a)$-$(28c) are quadratic functions of $\boldsymbol{y}_n$, $n \in \mathcal{N}$. Hence, function $\varphi_n(\boldsymbol{y}_n)$ is strictly convex in $\boldsymbol{y}_n = (\Delta e_n^{\text{in}}, \Delta g_n^{\text{in}})$.

The feasible space is also a convex polyhedron, and thus, the potential function has a unique global minimum corresponding to the unique GNE of Game 1. ■

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