Compressible Fluids I **AE 521 Fall 2022** **Instructor: P. Kreth**
Capstone Project **Due: 12/06/2022**

This project is worth 100 points and will count for a significant portion of your final average (approx. 20%). The goal of the project is to write a script that will help you solve Oblique Shock problems using Pressure-Deflection diagrams, especially ones where slip lines are involved. This will involve the use of some logic-based programming language (e.g., C, Python, MATLAB) or advanced use of Microsoft Excel.

To receive full credit, you will write **your own** algorithm and supporting script that will then be used to solve 3 oblique shock problems. You must provide (1) copies of your programs, (2) solutions to the given problems, and (3) detailed pressure-deflection diagrams that are output by your program and marked up for each problem. No written report is necessary for this project, though your scripts should include helpful comments that can be followed clearly.

For this Capstone Project, you will consider Oblique Shock problems that involve (1) shock reflections, (2) the confluence of two shocks at the trailing edge of an airfoil, and (3) the intersection of two shocks of opposite families. The latter two cases have a special discontinuity (the slip line) where no flow crosses it (i.e., flow is parallel to the slip line) and the pressure is continuous. Across the slip line, any other flow variable (e.g., velocity, Mach number, density, entropy, temperature) can be discontinuous.

One way to solve these types of problems is to employ a guess-and-check method, as you may have done / will do on the Homework assignments. Unfortunately, this process can be somewhat tedious. A more elegant solution methodology utilizes the Pressure-Deflection diagrams which are also called shock polars. Since the diagram graphically illustrates all viable O/S pressure ratios against corresponding deflection angles (for a given set of upstream conditions), we can use it to find the solutions to these problems graphically.

Note: useful discussions on this subject are available in Anderson's text in sections 4.7-4.9 (pages 157-161), though these materials are not entirely comprehensive. You may need to derive additional forms of the equations or seek out additional materials from other texts such as those listed in the syllabus as useful reference texts.

At a minimum, you must include the following content in your submission to receive full credit.

1) Copies of your own thoroughly commented programs, scripts, functions, and algorithms / equations that you used in solving these problems. I anticipate that a majority of you will use MATLAB, but other programs / languages can achieve the same solutions. If you plan to use Excel, however, please contact me in advance
2) If you construct or derive any new equations / expressions or simplify others, you must include hand-written derivations / algebraic manipulations that were used to do so
3) Correct solutions to the following problems including the pressure-deflection diagrams that your codes output. You must clearly indicate the solution point(s) and show the solution "path" as I had outlined in the lectures (the thicker lines I drew on top of the P-θ diagrams)

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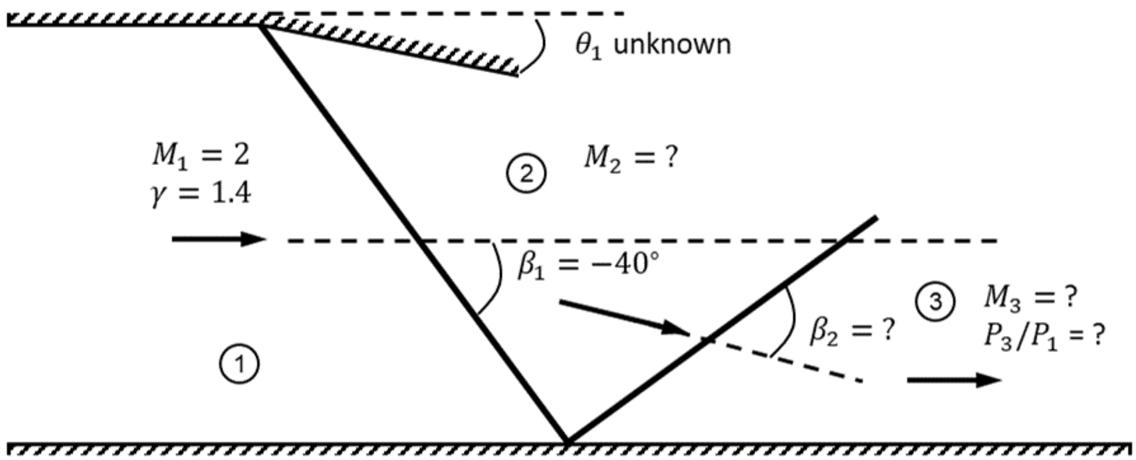
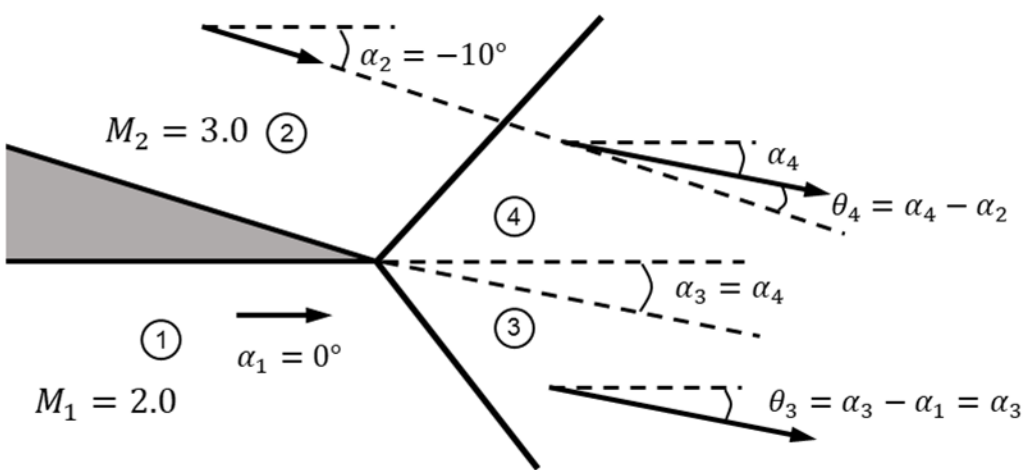
Problem 1: (reflected shock)

For a horizontal flow at $M_1 = 2.0$ and with $\gamma = 1.4$, an oblique shock occurs at a compression corner that has an unknown compression angle (i.e., flow deflection). The shockwave angle is measured to be $\beta = -40°$, however (note that I am using a negative sign to denote that the wave is right-running). This shockwave then reflects off the opposing wall, and you must determine (a) the Mach number between the shocks, M_2, (b) the angle of the reflected shock, β_2, (c) the Mach number downstream of the reflected shock, M_3, and (d) the pressure ratio across the shocks, P_3/P_1.



Problem 2: (confluence of two oblique shocks)

The figure below shows two supersonic flows that occur over the upper and lower surfaces of an airfoil and merge at the trailing edge. Because the flows must be turned in order to move in the same direction behind the airfoil, these streams pass through oblique shocks that sit on the trailing edge. Find the resulting flow direction (α_3 or α_4) and the downstream fluid properties in regions 3 and 4 (i.e., M_3, M_4, P_3 or P_4) if $M_1 = 2.0$ and $M_2 = 3.0$. Assume that both streams have the same pressure (i.e., $P_2 = P_1$ or $P_2/P_1 = 1$) and that $\gamma = 1.4$.



<u>Problem 3:</u> (intersection of two oblique shocks of opposite families)

A supersonic stream with $\boxed{\gamma = 5/3}$ and $M_1 = 3.5$ flows into an inlet as shown in the figure below. The lower lip of the inlet is a wedge with a $10°$ angle while the upper lip is a wedge with an angle of $-15°$ (negative to indicate flow turns down). The oblique shocks formed on these inlet lips intersect one another, and two shocks and a slip line emanate from the intersection point.

Five regions of uniform properties are created and numbered as shown in the figure. Determine the downstream flow directions (α_4 or α_5) and pressures (P_4 or P_5) that correspond to the shocks being the "weak shock" solutions.