project 2

Tuttle Creek Reservoir was constructed in 1962 and has lost approximately 50% of its conservation pool capacity. The State of Kansas is interested in assessing the trapping efficiency (T_E) of the reservoir. To achieve this, they installed turbidity sensors and stream gages at the upstream river inlet and directly downstream of the dam outlet. The discharge and sensor data are attached as an Excel file to this assignment. Engineers also collected some data regarding the properties of the river channels and sediment at the two locations, which can be found in the Excel sheet.

Answer the following …

1. Plot the upstream flow depth and concentration time-series on a single figure. Do the same for the downstream data. Use log-scale axes where helpful.
2. Assuming fully-rough flow, plot the velocity and concentration profiles for a time-step of your choosing at the upstream location. Do the same for the downstream location.
3. Using Einstein's Q_{SS} method, estimate sediment loading into and out of the reservoir. Report your results in tons/yr as well as m^3/yr.
4. Report the T_E of the reservoir over the study period duration.
5. How does the value of T_E at Tuttle Creek Lake compare to a couple other reservoirs? What could be the reasons for observed differences in T_E?

project 2

The Kansas Water Office and the Army Corps of Engineers want to implement Water Injection Dredging (WID) at Tuttle Creek Reservoir, which has lost more than 50% of its conservation pool capacity since construction in 1962. A cartoon of how WID works is shown on the right in Figure 1. Because this technology is so new, the Water Office and Army Corps are not sure how effective it will be. From historical surveys of sediment accumulation in Tuttle Creek Lake, engineers estimate that the reservoir is filling at a rate of 3.9×10^6 m^3 per year. Successful dredging would mean that a significant fraction of this infilling can be passed through to the downstream channel using WID, thus prolonging reservoir life.

Figure 1 Water Injection Dredging Concept

Engineers have done some preliminary measurements of the reservoir, the WID apparatus, and the resulting turbidity current. They report the following findings:

- The dredging of the reservoir bed occurs 1 km upstream of the dam outlet.
- The mean bed slope over this stretch of reservoir is 0.0167 m/m.
- The WID apparatus generates a turbidity current with an initial height (h), Froude number (Fr), and concentration ($C_{g/L}$) of 0.4 m, 1.1, and 225 g/L, respectively.
- The turbidity current starts at a 10-m width and remains that width during transport.
- To simplify the model considerably, assume that the WID remains stationary and that there is an infinite amount of sediment at the point-of-injection.
- Your model will only work with a supercritical turbidity current (Fr > 1).

You are tasked with developing a numerical model of the WID-induced turbidity current and assessing its performance. The Kansas Water Office wants you to do the following:

1. Plot the thickness of the turbidity current from the WID down to the dam. On this same plot, on a secondary axis, plot the longitudinal development of the current velocity.
2. Plot the concentration of the turbidity current from the WID down to the dam. On this same plot, on a secondary axis, plot the sediment load (kg/s) of the current.
3. What is the production rate of sediment (m^3/hr) at 1 km downstream of the WID? How does this production rate value compare to other dredging approaches?
4. To cancel out the annual infilling rate of the reservoir, how many days of the year would the WID need to operate given the production rate you calculated? Is it reasonable to think that WID could be successful for Tuttle Creek based on this result?
5. Assuming the WID remains at 1-km upstream of the dam, how could the WID operator change the initial height or concentration of the current to get greater transport?

A description of the modeling equations can be found on the next few pages. Gary Parker and Leo van Rijn have good writings on turbidity currents. These links can help you understand the problem and provide context, but you should **use the equations I provide** for modeling purposes. Much of the following text is borrowed from their formulations.

MATHEMATICAL MODEL FOR TURBIDITY CURRENTS

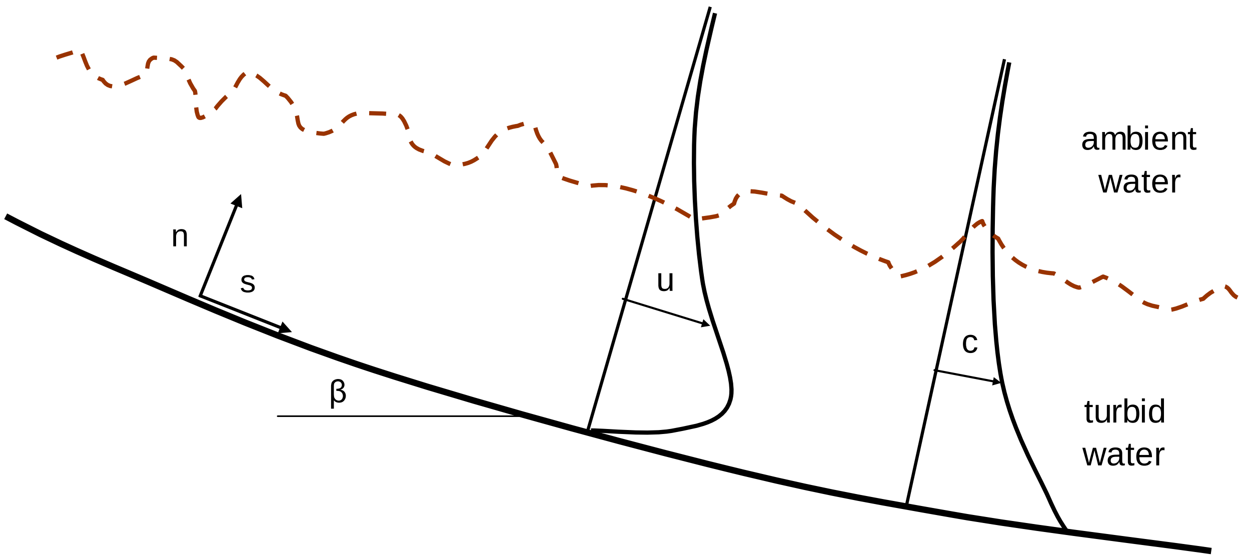

Figure 2 Turbidity current velocity and concentration profile showing ambient water above the turbid current.

Our goal is to estimate the velocity, concentration, and sediment load of the turbidity current as it travels downstream in a reservoir. To make this problem easier, we can consider two layers: (1) low-concentration overlying water layer and (2) high-concentration current layer. In the case of a 1D channel with a constant slope angle (β) it is most easy to formulate the equations with respect to a tilting coordinate system.

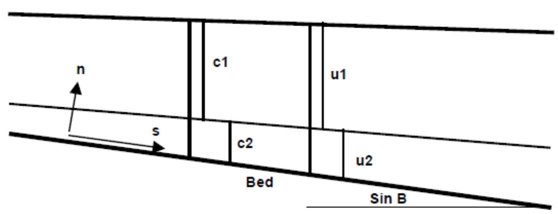

Figure 3 Two-layer depth-averaged formulation.

Assuming (1) that the flow is steady, (2) that the velocities (u_1) and sediment concentrations (c_1) in the upper layer are negligibly small ($c_1 \cong 0$, density is equal to the fluid density, and $u_1 \cong 0$, water is still), (3) that the flow in the lower layer is fully turbulent, and (4) that the pressure is hydrostatic, the equations for 1D conditions can be expressed by Eqs. (1) to (3). Because we have eliminated all c_1 and u_1 terms, we now refer to h_2, c_2, and u_2 as simply h, c, and u.

Equation (1): mass balance for fluid in lower layer 2 in s-direction

$$\frac{\partial\big(uh(1-c)\big)}{\partial s} - W_i - W_b = 0$$

with:
s = coordinate along bed slope,
h = thickness of lower layer (m),
c = dimensionless volumetric sediment concentration in lower layer ($C_{g/L}/\rho_s$) (-),
u= q/h = velocity in lower layer (m/s),
q = unit-discharge in lower layer (m^2/s),
W_i = exchange of fluid at the interface (m/s),
W_b = exchange of fluid at the bed (m/s).

Equation (2): mass balance for sediment in lower layer 2 in s-direction

$$\frac{\partial(uhc)}{\partial s} - S_i - S_b = 0$$

with:
S_i = exchange of sediment at the interface (m/s),
S_b = exchange of sediment at the bed (m/s).

Equation (3): momentum balance of mixture in lower layer 2 in s-direction

$$\rho\frac{\partial\left(u^2h\right)}{\partial s} + \left(\rho_s - \rho_w\right)hc\left[(g\cos\beta)\frac{\partial h}{\partial s} - g\sin\beta\right] + \left[\left(\rho_s - \rho_w\right)\left(0.5gh^2\cos\beta + u^2h\right)\right]\frac{\partial c}{\partial s} + (\tau_i + \tau_b) = 0$$

with:
ρ = mixture density of lower layer (water and sediment) = $\rho_s c + (1 - c)\rho_w$ (kg/m^3),
ρ_w = fluid density (clear water) (kg/m^3),
ρ_s = sediment density (kg/m^3),
β = angle of bed slope in s-direction (-),
τ_i = shear stress at interface (= $\rho C_{di}u^2$) (Pa),
τ_b = bed shear stress (= $\rho C_d u^2$) (Pa),
C_d = bottom friction coefficient (= g/C_z^2), C_z = Chézy coefficient,
C_{di} = interface friction coefficient (= $C_d/3$).

The equations (1), (2) and (3) define a set of three equations with three unknown parameters u, h and c, which can be solved for given boundary conditions and closure expressions. We can further re-work the equations to make them simpler…

The mass balances of fluid and sediment (Eqs. 1 and 2) and momentum balance (Eq. 3) can be reformulated as below. Discretize each left-hand term as backward in space. Assume that at the very first spatial step (i = 1) that some derivatives (e.g., $\partial c/\partial s$) are equal to 0, this will help you initiate a solution of $\partial h/\partial s$. **These are the forms of the equations you will solve at each spatial location:**

$$\frac{\partial q}{\partial s} = \frac{\partial q_s}{\partial s} + (W_i + W_b)$$

$$\frac{\partial q_s}{\partial s} = S_b + S_i$$

$$\frac{\partial h}{\partial s} = \frac{\left[1 - \left(h_{eq}/h\right)^3 - \alpha_1\frac{(W_i + W_b)}{u} - \alpha_2 h\frac{\partial c}{\partial s}\right]\beta}{1 - (h_{cr}/h)^3}$$

where:

$$\alpha_1 = \frac{2\rho u^2}{(\rho_s - \rho_w)(1 - c)hcg\beta}$$

$$\alpha_2 = \frac{2\rho u^2 + (\rho_s - \rho_w)(1 - c)u^2 + 0.5(\rho_s - \rho_w)(1 - c)hg}{(\rho_s - \rho_w)(1 - c)hcg\beta}$$

with
$u = q/h$, current velocity (m/s)
$c = q_s/q$, sediment concentration (-)
$h_{cr} = [(q^2\rho)/\{(\rho_s-\rho_w)cg\}]^{1/3}$, critical depth (defined by Froude number Fr = 1) (m),
$h_{eq} = [(C_d+C_{di})q^2/\{((\rho_s-\rho_w)/\rho)cg\beta\}]^{1/3}$, equilibrium depth (water and sediment flow) (m).

We must define **upstream boundary conditions**:

$$(u,h,c)|_{x=0} = (u_o,h_o,c_o)$$

These are the upstream boundary conditions which you will input into your turbidity current model to estimate the distance the plume will travel.

Assuming supercritical flow (Fr > 1), $\partial h/\partial s$ increases (h grows more rapidly) with

- increasing entrainment ($W_i + W_b$) of fluid into the lower layer;
- increasing shear stresses ($\tau_i + \tau_b$) at the bed and at the interface;
- increasing concentration in downstream direction (increasing $\partial c/\partial s$).

We also need **dimensionless numbers** to see if our solution will be stable and **closure relations**, i.e., additional equations to solve for some other unknowns.

The Froude number can be solved with the following expression

$$Fr = \frac{u}{\sqrt{(\Delta\rho/\rho)gh}}$$

The Richardson number can be solved with the following expression

$$Ri = \frac{1}{Fr^2}$$

The Reynolds number can be solved with the following expression

$$Re = \frac{uh}{v}$$

The density difference between the sediment suspension and the clear water is given by

$$\Delta\rho = \rho - \rho_w = [\rho_s c + (1 - c)\rho_w] - \rho_w = c(\rho_s - \rho_w)$$

The kinematic viscosity varies as a function of sediment concentration as

$$v = v_w \left(1 - \frac{c}{c_{max}}\right)^{-3}$$

The exchange of water between the interface and the turbidity current can be expressed as

$$W_i = R_{ewi}(Ri/500)u$$

The exchange of water between the bed and the turbidity current can be calculated as

$$W_b = (1 - \eta)/10^5$$

The exchange of sediment between the interface and the turbidity current can be calculated as

$$S_i = -\frac{R_{esi}}{40}\frac{\sqrt{g}}{C_z}uc$$

The exchange of sediment between the bed and the turbidity current can be expressed as

$$S_b = v_{be} + v_{bs}$$

where the erosion velocity (v_{be}) and the depositional velocity (v_{bs}) can be calculated as

$$v_{be} = \max\left[0, e\left(\frac{\tau_b - \tau_{cr}}{\tau_{cr}}\right)\right]$$

$$v_{bs} = -w_s c(1 - c)^4$$

Below is a table of important parameters and their units.

Description	**Variable**	**Value**	**Units**
Length of study domain	L	1000	m
Width of turbidity current	B	10	m
Bed slope	β	0.0167	m/m
Maximum possible concentration	c_{max}	0.2	-
Settling velocity	w_s	0.0005	m/s
Sediment density	ρ_s	2650	kg/m^3
Clear-water density	ρ_w	1000	kg/m^3
Clear-water kinematic viscosity	v_w	1×10^{-6}	m^2/s
Gravitation acceleration constant	g	9.81	m/s^2
Chézy coefficient	C_z	50	m$^{0.5}$/s
Critical erosion shear stress	τ_{cr}	0.7	Pa
Erosion coefficient	e	5×10^{-6}	-
Bed porosity	η	0.6	-
Coefficient to correct fluid exchange at interface	R_{ewi}	0.1	-
Coefficient to correct sediment exchange at interface	R_{esi}	0.2	-