

[REDACTED]

- (2) Consider energy fluctuations in a canonical ensemble. The relative magnitude of these energy fluctuations is given by the expression:

$$\frac{\sigma_E}{\bar{E}} = \frac{(kT^2 C_v)^{1/2}}{\bar{E}}$$

where \bar{E} is the average energy, k is the Boltzmann constant, T is the temperature and C_v is the constant volume molar heat capacity.

- (a) Derive an expression to estimate the relative magnitude of the energy fluctuations in an ideal gas system.

⇒ In ideal gas system $\bar{E} \approx N \cdot k \cdot T$

$$\left. \frac{\partial \bar{E}}{\partial T} \right|_{N,P} = C_v \approx N \cdot k$$

$$\frac{\sigma_E}{\bar{E}} \approx \frac{[k \cdot T^2 \cdot (N \cdot k)]^{1/2}}{N \cdot k \cdot T}$$

$$\frac{\sigma_E}{\bar{E}} \approx \frac{k \cdot T \cdot N^{1/2}}{N \cdot k \cdot T} \approx \frac{1}{\sqrt{N}} \quad *$$

- (b) Use the expression derived in part (a) to estimate the relative magnitude of the energy fluctuations in a macroscopic sample of an ideal gas.

	Number of Molecules	$\frac{\sigma_E}{\bar{E}}$
Microscopic →	25	$\frac{1}{\sqrt{25}} = 0.2$
	2,500	$\frac{1}{\sqrt{2500}} = 0.02$
	250,000	$\frac{1}{\sqrt{250000}} = 0.002$
Macroscopic →	25,000,000	$\frac{1}{\sqrt{25,000,000}} = 0.0002$