Problem set 2: Digital Transmission Through AWGN Channels

Prob. 2-1 Problem 3-14 from the Textbook.

Prob. 2-2 Problem 3-19 from the Textbook.

Prob. 2-3 Problem 3-27 from the Textbook.

Prob. 2-4 Problem 4-5 from the Textbook.

Prob. 2-5 Problem 4-7 from the Textbook.

Prob. 2-6 Problem 4-10 from the Textbook.

Prob. 2-7 Problem 4-13 from the Textbook.

Prob. 2-8 Problem 4-19 from the Textbook.

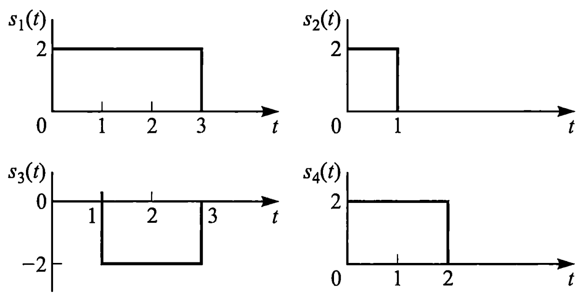
Prob. 2-9 Problem 4-22 from the Textbook.

Prob. 2-10 Problem 4-36 from the Textbook.

Prob. 2-11 Problem 4-43 from the Textbook.

Prob. 2-12 Problem 4-47 from the Textbook.

Prob. 2-13 Consider the four signals shown in figure below.



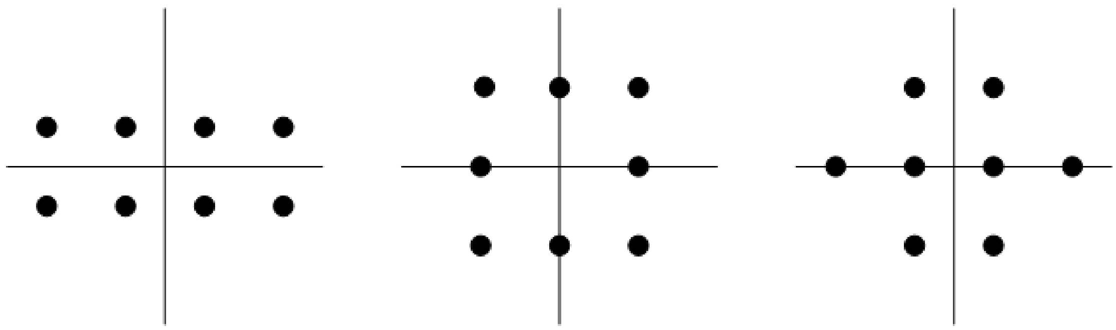
(a) Determine a set of orthonormal basis functions and dimensionality for this signal set.
(b) Use the basis functions to represent the four waveforms by vectors $\bar{s}_1, \bar{s}_2, \bar{s}_3, \bar{s}_4$. Plot the constellation, and using the constellation, find the energy in each signal. What is the average signal energy and what is E_{bavg}?
(c) Determine the minimum distance between any pair of vectors.

Prob. 2-14 Consider the three 8-point QAM signal constellations in the ϕ_1-ϕ_2 plane shown in figure below. Assume that the signal points are equally probable.

(a) The minimum distance between adjacent points is $2A$. Determine the average transmitted energy for each constellation. Which constellation is more efficient? Also, calculate the Peak to Average Power Ratio (PAPR) for each constellation. Further, calculate the Constellation Figure of Merit (CFM) defined as

$$CFM = \frac{d_{min}^2}{E_{av/2D}} = \frac{d_{min}^2}{E_{av}/(N/2}$$

where the denominator represents the average energy of the constellations per dimension pair. E_{av} is the average energy of the signal over $0 \le t \le T$.
(b) Now assume that the distances between signal points have been chosen such a way that all three constellations have the same average energy E_{av}. Determine the minimum distance for the three constellations as a function of E_{av}. Also, calculate the CFM.



(1) (2) (3)

(ϕ_1, ϕ_2): **(1):** (±A, ±A), (±3A, ±A); **(2):** (±2A, 0), (0, ±2A), (±2A, ±2A); **(3):** (±A, 0), (±3A, 0), (±A, ±2A);

Prob. 2-15 Problem 4-4 from the Textbook. Further assume that an OOK system is used to transmit two equiprobable messages over an AWGN channel with bit error probability $P_b = 10^{-6}$. As a result of fading, the channel loss is increased by 3 dB. Determine the bit error probability if (i) the receiver adaptively shifts the decision boundary to the optimum threshold, and (ii) the decision boundary remains unchanged.

Prob. 2-16 Consider signal transmission with bitrate R_b=1 Mbp*s* through an AWGN channel which is characterized by PSD -140 dBW/Hz. Channel loss is assumed to be 70 dB and the target bit error rate is 10^{-6}. For each of the following modulation schemes, calculate the required power and bandwidth.

	2-PAM	4-PSK	BDPSK Noncoherent	16-QAM	4-FSK Coherent	2-FSK Noncoherent
Power (W)						
Bandwidth (Hz)						

Prob. 2-17 Consider standard BPSK modulation with the two equally likely constellation points a distance 2*d* apart where *d* is a constant. However, the noise is uniformly distributed over [-*a* ,*a*], i.e.

$$p_N(n) = \frac{1}{2a} rect\left(\frac{n}{2a}\right)$$

where *a* is a constant.
(a) Specify the MAP and ML decision regions.
(b) What is the SNR?
(c) What is the probability of error of the ML detector?
(d) Identify, in terms of SNR, when uniform noise is better than Gaussian noise and vice versa. In other words, for a fixed transmit power, for what noise variance values is uniform noise preferable?
(e) Now, for the given SNR and $d=a/2$, determine the MAP regions and P_e when $P[x = +d] = 0.75$ and $P[x = -d] = 0.25$.
Hint: Consider two cases $a \geq d$ and $a < d$.

Prob. 2-18 (a) Consider a 4-ary modulation scheme with the constellation as: (d; $3d$); ($3d$; d); ($-d$; $-3d$), and ($-3d$; $-d$). For each dimension, the additive noise is Gaussian distributed with zero mean and power spectral density $N_0/2$. In the 2-D space, the noise is circular symmetric. What is the average symbol error probability for this modulation scheme? [Hint. An n-dimensional complex Gaussian random vector $\mathbf{x}$ is circular symmetric if for any θ; $e^{j\theta}\mathbf{x} \sim \mathbf{x}$.]
(b) Consider the rectangular 6-ary constellation: ($-d$; d), (0; d), (d; d), ($-d$; $-d$), (0; $-d$), and (d; $-d$), with equal a prior probability to be transmitted, as shown below. Assume the additive noise for each dimension follows the following distribution:

$$f_n(n) = \frac{1}{2\sigma} exp\left(\ \frac{|n|}{\sigma} \right)$$

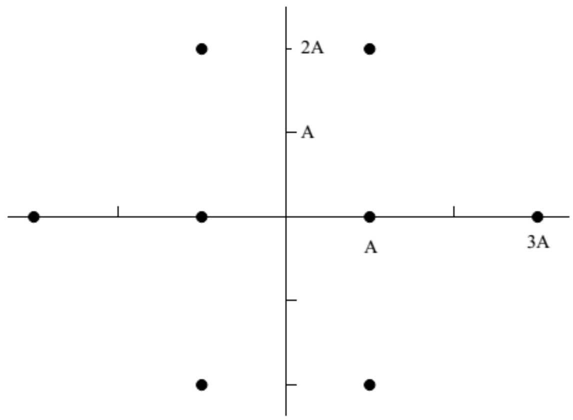
(i) What is the optimal decision region?
(ii) Find the mean and variance of the noise.
(iii) Find the pairwise error probability between Point 1 and Point 4.

Prob. 2-19 A QAM system with symbol rate $1/T$=10 MHz operates on an AWGN channel. The SNR is 24.5 dB and a symbol error probability $P_e<10^{\ \ 6}$ is desired.

(a) Find the largest constellation with integer k for which $P_e<10^{\ \ 6}$.
(b) What is the data rate for your design in part a?
(c) How much more transmit power is required (with fixed symbol rate at 10 MHz) in dB for the data rate to be increased to 60 Mbps? ($P_e<10^{\ \ 6}$)
(d) With SNR = 24 dB, an reduced-rate alternative mode is enabled to accommodate up to 9 dB margin or temporary increases in the white noise amplitude. What is the data rate in this alternative 9dB margin mode at the same $P_e<10^{\ \ 6}$?
(e) What is the largest QAM (with integer k) data rate that can be achieved with the same power, as in part d, but with $1/T$ possibly altered?

Prob. 2-20 Consider an 8 ary QAM system with the following signal coordinates relative to an orthonormal basis $\{\psi_1, \psi_2\}$: (±A, 0), (±3A, 0), (±A, ±2A). The signals are sent with equal probability, and are corrupted by AWGN with two-sided power spectral density $N_o/2$.

(a) Sketch the maximum likelihood decision regions on the following signal coordinate diagram.
(b) Express the average symbol energy and the average energy per bit in terms of A^2.
(c) Find a union bound on the probability of symbol error given that the signal with coordinates (A, 2A) is transmitted. Use the minimum number of terms required.
(d) Express the maximum symbol error probability exactly in terms of A, N_o, and the Q function, for the maximum likelihood receiver. (The maximum symbol error probability is the maximum, over all eight possible transmitted symbols, of the symbol error probability.)
(e) Compare the power efficiency of this system with an 8-level PAM system



Prob. 2-21 A data signal is multiplied by a sinusoidal carrier with a frequency f_c to be transmitted over a wireless communications channel. At the receiver this multiplication must be reversed to recover the baseband data signal. Ideally such recovery can be accomplished by multiplying the received data signal by a locally generated sinusoid whose frequency is also f_c Hz, and whose phase is identical to that of the carrier signal used at the transmitter. After passing the resultant signal through a lowpass filter, the original signal is recovered. In reality, the locally generated sinusoid will contain both a frequency and a phase offset. Let $f_{LO,Rx}=f_{LO,Tx} + \ f$ represent the Local Oscillator (LO) at the receiver where $\ f$ represents the frequency offset between $f_{LO,Rx}$ and $f_{LO,Tx}$. Ignoring the phase offset and using your basic knowledge of frequency mixing, find:

(a) f_{sum} , the frequency of the sum-frequency term (sometimes called the “sum image”) and f_{diff} , the frequency of the difference image.
(b) What are the results after lowpass filtering?
(c) Is the downconverted data signal still at baseband?
(d) Denoting the transmitted signal as $x(t)$ with an equivalent Fourier transform of $X(f)$, the “near baseband” signal can be denoted as $X(f \quad f)$. What is the time domain representation of the “near-baseband” signal?
(e) What effect will this have on the data signal’s constellation?
(f) Will this effect on the data signal constellation also affect the best symbol-by-symbol decision device (slicer)?

Prob. 2-22 Consider the 4-QAM signal set described by $s(t) = \text{Re}\{a\, g(t) \exp(j2\pi 9\times 10^{7} t)\}$, where the complex symbol is drawn from the 4-QAM alphabet $a \in \{-1-j, -1+j, 1-j, 1+j\}$, and where the pulse shape is g(t)=1 for t $\in[-0.5, 0.5]$ and zero elsewhere. Suppose a receiver observes a delayed and noiseless version of $s(t)$, $r(t) = s(t-\tau)$, for some delay $\tau \geq 0$. The receiver is unaware of the delay and it minimizes distance under the assumption that $\tau = 0$. Find the smallest delay τ (expressed in seconds) that causes the decision to be incorrect