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The effectiveness of a fuzzy mathematical programming approach for supply chain production planning with fuzzy demand

Josefa Mula*, David Peidro, Raul Poler

CIGIP (Research Centre on Production Management and Engineering), Universidad Politécnica de Valencia, Spain

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ABSTRACT

The main focus of this work is to prove the effectiveness of a fuzzy mathematical programming approach to model a supply chain production planning problem with uncertainty in demand. A fuzzy optimization model that takes into account the lack of knowledge in market demand is developed. This work uses an approach of possibilistic programming. Such an approach makes it possible to model the epistemic uncertainty in demand that could be present in the supply chain production planning problems as triangular fuzzy numbers. The emphasis is on obtaining more knowledge about the impact of fuzzy programming on supply chain planning problems with uncertain demand.

1. Introduction

The concept of supply chain management (SCM), since their appearance in 1982 (see Oliver and Weber, 1982), is associated with a variety of meanings. In the eighties, SCM was originally used in the logistical literature to describe a new integrated approach of logistics management through different business functions (Houlihan, 1984). Then, this integrated approach was extended outside of the company limits to suppliers and customers (Christopher, 1992). In accordance with the Global Supply Chain Forum (Lambert and Cooper, 2000), the SCM is the integration of key business processes, from final users to original suppliers providing products, services and information which add value to clients, shareholders, etc. This paper is related to one of these key business processes: the supply chain production planning.

Supply chain production planning consists of the coordination and the integration of key business activities carried out from the procurement of raw materials to the distribution of finished products to the customer (Gupta and Maranas, 2003). Here, tactical models concerning mainly about inventory management and resource limitations are the focus of our work. In this context, with the objective of obtaining optimal solutions related to the minimization of costs, several authors have studied the modelling of supply chain planning processes through mathematical programming models (see, for instance, Alemany et al. (2009) and Mula et al. (2010)). However, the complex nature and dynamics of the relationships among the different actors of supply chains imply an important grade of uncertainty in the planning decisions (Bhatnagar and Sohal, 2005). Therefore, uncertainty is a main factor that can influence the effectiveness of the configuration and coordination of supply chains (Davis, 1993). One of the key sources of uncertainty in any production–distribution system is the product demand. Thus, demand uncertainty is propagated up and down along the supply chain affecting sensibly to its performance (Mula et al., 2005).

Along the years many researches and applications aimed to model the uncertainty in production planning problems (Mula et al., 2006a). Different stochastic modelling techniques have been successfully applied in supply chain production planning problems with randomness (Escudero, 1994; Gupta and Maranas, 2003; Sodhi and Tang, 2009). However, probability distributions derived from evidences recorded in the past are not always available or reliable. In these situations, the fuzzy set theory (Bellman and Zadeh, 1970) represents an attractive tool to support the production planning research when the dynamics of the manufacturing environment limit the specification of the model objectives, constraints and parameters. Uncertainty can be present as randomness, fuzziness and/or lack of knowledge or epistemic uncertainty (Dubois et al., 2002). Randomness comes from the random nature of events and deals with uncertainty regarding membership or non-membership of an element in a set. Fuzziness is related to flexible or fuzzy constraints modelled by fuzzy sets. Epistemic uncertainty is concerned with ill-known parameters modelled by fuzzy numbers in the setting of possibility theory (Dubois and Prade, 1988).

In this paper, for the purpose of demonstrating the usefulness and significance of the fuzzy mathematical programming for production planning, a fuzzy approach is applied to a supply chain production planning problem with lack of knowledge in demand

* Correspondence to: Dpto. Organización de Empresas. Escuela Politécnica Superior de Alcoy, Edificio Ferrándiz y Carbonell, 2, 03801 Alcoy (Alicante), Spain. Tel.: +34 96 652 84 23; fax: +34 96 652 85 85.

E-mail addresses: fmula@cigip.upv.es (J. Mula), dapeipa@cigip.upv.es (D. Peidro), rpoler@cigip.upv.es (R. Poler).

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data. The main contribution of this paper is an application of known possibilistic programming in a supply chain planning case study. Other applications of possibilistic programming in production planning problems can be found in Inuiguchi et al. (1994), Hsu and Wang (2001), Wang and Fang (2001), Lodwick and Bachman (2005), Wang and Liang (2005), Mula et al. (2008) and Vasant et al. (2008). However, previous researches mentioned above did not consider supply chain production planning problems.

This paper is organized as follows. Firstly, in Section 2, the supply chain production planning model, which has been the basis of this work, is described. In Section 3, a fuzzy model is developed to incorporate the demand uncertainty in the supply chain production planning model. Then, Section 4 uses a supply chain case study to illustrate the potential savings and other benefits that can be attained by using fuzzy models in a fuzzy environment. In Section 5, conclusions are given.

2. Description of the problem formulation

The mixed integer linear programming (MILP) model for supply chain production planning originally proposed by McDonald and Karimi (1997) is adopted as the basis for this work. The aim of this tactical model is to determine the sources of the limited resources of a company and the optimal assignment to its production resources to satisfy market demands at a minimum cost. The considered supply chain consists of multiple production facilities, globally located and producing multiple products. The demand of those products exists for a set of customers. The midterm planning horizon embraces from 1 to 2 years. Each production facility is characterized by one or more resources of semi-continuous production with limited capacity. The diverse products that are grouped in product families, in order to reduce transition times and costs between products of a family, compete for the limited capacity of those resources. This decision making process can be divided into two different phases: the production phase and the distribution phase or logistics. The production phase is focused on the efficient allocation of the production capacity in each one of the production plants with the objective of determining the optimal operative politics. In the distribution phase, they have considered the post-production activities like the demand fulfilment and the inventory management to satisfy the demand. Safety stock is kept to provide a buffer against uncertainty in demand. Finally, the structure of the supply chain can be classified as a network (Huang et al., 2003). Two layers of the supply chain network are considered: (1) manufacturing facilities and (2) customers. The production facilities can manufacture both finished products and intermediate from the raw materials. The intermediate products can be shipped to other production facilities where they are transformed into finished products which are subsequently shipped to customers.

Let us consider the following fuzzy formulation of the McDonald and Karimi's (1997) model. Decision variables and parameters for the mathematical programming model are defined in Table 1.

$$\text{Minimize } Z=\sum_{i,j,s,t} v_{ijs}P_{ijst}+\sum_{i,s,t} p_{is}C_{ist}+\sum_{i,s,t} h_{ist}I_{ist}+\sum_{i,s,c,t} t_{sc}S_{isct} + \sum_{i,s,s',t} t_{ss'}\sigma_{iss't}+\sum_{i,s,t}\zeta_{is}I^{\wedge}_{ist}+\sum_{i,c,t}\mu_{ic}I^{-}_{ict}+\sum_{f,j,s,t} f_{fjs}Y_{fjst} \quad (1)$$

Subject to

$$P_{ijst}=R_{ijst}RL_{ijst} \quad \forall i\in N^{RM},\quad \forall j\in J,\quad \forall s\in S,\quad \forall t\in T \quad (2)$$

$$FRL_{fjst}\le H_{jst}Y_{fjst} \quad \forall f\in F,\quad \forall j\in J,\quad \forall s\in S,\quad \forall t\in T \quad (3)$$

Table 1
Decision variables and model parameters.

Sets of indices	
$T\equiv\{t\}$	Set of time periods
$I\equiv\{i\}$	Set of products. This set can be classified in raw materials, I^{RM}, intermediate products, I^{IP} and finished products, I^{FP}, so that $I=\{I^{RM}\cup I^{IP}\cup I^{FP}\}$. An intermediate product can also belong to the set of finished products
$F\equiv\{f\}$	Set of product families
$J\equiv\{j\}$	Set of resources
$S\equiv\{s\}$	Set of facilities
$C\equiv\{c\}$	Set of customers
Decision variables	
P_{ijst}	Quantity to produce product $i\in N^{RM}$ on resource j at site s in time period t
RL_{ijst}	Production time of product $i\in N^{RM}$ on resource j at site s in time period t
FRL_{fjst}	Production time of family f on resource j at site s in time period t
C_{ist}	Consumption of raw material or intermediate product $i\in N^{FP}$ at site s in time period t
I_{ist}	Inventory level of product $i\in N^{RM}$ at site s at the end of time period t
S_{isct}	Supply of finished product $i\in I^{FP}$ from site s to customer c in time period t
$\sigma_{iss't}$	Intermediate product flow $i\in I^{IP}$ from site s in time period t
I^{-}_{ict}	Shortage of finished product $i\in I^{FP}$ for customer c in time period t
$I^{\wedge}_{ist}$	Inventory deviation below safety stock target for product $i\in I$ at site s in time period t
Y_{ijst}	Binary variable which indicates if product I is produced on resource j at site s in time period t
Objective function cost coefficients	
μ_{ij}	Revenue per unit of product $i\in I^{FP}$ sold to customer c.
h_{ist}	Inventory cost of a unit of the product i at site s in time period t
p_{is}	Price of raw material $i\in I^{RM}$ at site s
ζ_{is}	Penalty for dipping below safety stock target of product i at site s
v_{ijs}	Variable cost of production of a unit of the product i on resource j at site s
$t_{ss'}\|t_{sc}$	Transportation cost to move a unit of product from site s to site s' or to customer c
f_{fjs}	Fixed cost of production for family f on resource j at site s
Technological coefficients	
R_{ijst}	Effective rate for product i using resource j at site s in time period t (it includes adjustment to the rate relating to efficiency, utility and/or yield)
$\beta_{i'is}$	Quantity of raw material or intermediate product $i\in N^{FP}$ that must be consumed to produce a unit of $i'\in N^{RM}$ at site s
General data	
k_{if}	0–1 parameter, which indicates if product i belong to family f
H_{jst}	Quantity of available time for production on resource j at site s in time period t
MRL_{fjs}	Minimum required time for family s on resource j at site s
$\tilde{d}_{ict}$	Fuzzy demand of finished product i for customer c in time period t
I^{L}_{ist}	Safety stock target for product i at site s in time period t
I_{is0}	Inventory of product i at site s at start of planning horizon

$$FRL_{fjst}\ge MRL_{fjs}Y_{fjst} \quad \forall f\in F,\quad \forall j\in J,\quad \forall s\in S,\quad \forall t\in T \quad (4)$$

$$FRL_{fjst}=\sum_{k_{if}=1} RL_{ijst},\quad \forall f\in F,\quad \forall j\in J,\quad \forall s\in S,\quad \forall t\in T \quad (5)$$

$$\sum_{f} FRL_{fjst}\le H_{jst} \quad \forall j\in J,\quad \forall s\in S,\quad \forall t\in T \quad (6)$$

$$C_{ist}=\sum_{i'\ni\beta i'is\neq 0}\beta_{i'is}\sum_{j}P_{i'jst} \quad \forall i\in N^{FP},\quad \forall j\in J,\quad \forall s\in S,\quad \forall t\in T \quad (7)$$

$$C_{ist}=\sum_{s'}\sigma_{is'st} \quad \forall i\in I^{IP},\quad \forall s\in S,\quad \forall t\in T \quad (8)$$

$$I_{ist}=I_{is(t-1)}+\sum_{j}P_{ijst}-\sum_{s'}\sigma_{iss't}-\sum_{c}S_{isct} \quad \forall i\in N^{RM},\quad \forall t\in T \quad (9)$$

$$I^-_{ict} \ge I^-_{ic(t-1)} + \tilde{d}_{ict} - \sum_c S_{isct} \quad \forall i \in I^{FP}, \quad \forall t \in T \tag{10}$$

$$\sum_{s,t' \le t} S_{isct'} \le \sum_{t' \le t} \tilde{d}_{ict'} \quad \forall i \in I, \quad \forall t \in T \tag{11}$$

$$I^-_{ict} \le \sum_{t' \le t} \tilde{d}_{ict'} \quad \forall i \in I, \quad \forall c \in C \tag{12}$$

$$I^{\wedge}_{ist} \ge I^L_{ist} - I_{ist} \quad \forall i \in I, \quad \forall s \in S, \quad \forall t \in T \tag{13}$$

$$I^{\wedge}_{ist} \le I^L_{ist} \quad \forall i \in I, \quad \forall s \in S, \quad \forall t \in T \tag{14}$$

$$P_{ijst}, RL_{ijst}, C_{ist}, I_{ist}, I^-_{ict}, \sigma_{iss't}, I^{\wedge}_{ist} \ge 0, \\ Y_{fjst} \in \{0,1\} \quad \forall i \in I, \quad \forall j \in J, \quad \forall c \in C, \quad \forall f \in F, \quad \forall s \in S, \quad \forall t \in T \tag{15}$$

where $\tilde{d}_{ict} = (d_{ict1}, d_{ict2}, d_{ict3})$ are positive triangular fuzzy numbers (TFNs). The parameters of a triangular possibility distribution represent the most pessimistic, the most possible and the most optimist values (see Section 3 for TFNs explanation).

The objective function (1) minimizes the production costs, the costs of raw materials or intermediate products consumption, the inventory and shortage penalties, the transportation costs and the fixed costs for product families. In Eq. (2), the production quantity is related to the production time through the correspondent production rate. Eqs. (3) and (4) provide the lower and upper limits of the production times for each product family, respectively. Eq. (5) establishes the relation of each product with its product family. Eq. (6) models the production capacity constraints. This model considers that the minimum run length of a product, MRL_{fjs}, is much less than the length of the time period. The cases in which this parameter is similar or much bigger than the length of the time period can be consulted in McDonald and Karimi (1997). Eq. (7) models the consumption of raw materials or intermediate products through the bill of materials. The raw materials come from an external supplier and it is assumed here that they are available when required, although this could easily be modified by including some lower and upper bound constraints. Intermediate products consumed at site s come from this site or from another site s'. Eq. (8) implies that all materials sent to site s will be consumed in the same time period. Thus, the inventory will be kept where the products are produced avoiding redundant material flows in the network. Eq. (9) represents the inventory balance constraint. The finished products are only sent between facilities if they are also intermediate products. Eq. (10) indicates that shortfalls in supply carry from one period to the next. Eq. (11) allows that demands of previous periods can be satisfied in the current time period. Eq. (12) ensures that the shortage of demand must be always inferior to the total cumulated demand in this time period. Demand is considered an uncertain data defined by a triangular fuzzy number in Eqs. (10)–(12). Eq. (13) determines the excesses and deviations of inventory with respect to the safety stock target established. Eqs. (14) and (15) establish superior and inferior limits to the different decision variables, respectively.

3. Fuzzy mathematical programming approach

In this section, a fuzzy decision model is developed for supply chain production planning where TFNs are used to model the lack of knowledge or epistemic uncertainty in demand.

In the context of possibility theory, there are different approaches to model the coefficients of the objective function and/or the constraints as fuzzy numbers (Tanaka and Asai, 1984; Inuiguchi et al., 1994; Vasant, 2005; Mula et al., 2008). Gen et al. (1992) propose a method to transform a fuzzy multiple objective linear programming (MOLP) problem model to crisp MOLP model. The authors consider fuzzy coefficients in the objective functions, fuzzy technical coefficients and fuzzy right-hand side in less than or equal, greater than or equal and equality type of constraints. All these fuzzy parameters are represented by TFNs. Here, the transformation method proposed by Gen et al. (1992) is adapted to transform a fuzzy LP model with a crisp objective function and fuzzy right-hand side numbers in less than or equal and greater than or equal type constraints to crisp LP model. The general model minimizing an objective function subject to m_1 less than or equal type constraints and m_2 greater than or equal type constraints with fuzzy right-hand side can be defined as follows:

$$\text{Minimize } z = \sum_{j=1}^{n} c_j x_j \tag{16}$$

Subject to

$$\sum_{j=1}^{n} a_{ij} x_j \ge \tilde{b}_i \quad i = 1,2,\ldots,m_1 \tag{17}$$

$$\sum_{j=1}^{n} a_{ij} x_j \le \tilde{b}_i \quad i = m_1+1,\ldots,m_2 \tag{18}$$

$$x_j \ge 0 \quad j = 1,\ldots,n \tag{19}$$

where x_j is the j-th decision variable; c_j is a crisp coefficient of the objective function and j the decision variable; a_{ij} is the crisp technical coefficient of the i-th constraint and the j-th decision variable; and b_i is the right-hand side term of the i-th constraint that represents the maximum ($\le$) or minimum ($\ge$) requirement and is represented as a TFN $\tilde{b}_i = (b_{i1}, b_{i2}, b_{i3})$, which membership function is defined in Gen et al. (1992) as

$$\mu_{\tilde{b}}(x) = \begin{cases} \dfrac{1}{b_2 - b_1}(x - b_2) + 1 & \text{if} \quad (b_1 \le x \le b_2) \\ \dfrac{1}{b_2 - b_3}(x - b_2) + 1 & \text{if} \quad (b_2 \le x \le b_3) \\ 0 & \text{if} \quad (x \le b_1, b_3 \le x) \end{cases} \tag{20}$$

This TFN $\tilde{b}$ can be represented as $\tilde{b} = (b_1, b_2, b_3)$ (Fig. 1).

In this transformation method, the joint conditional possibility distribution of the constraints with fuzzy right-hand side terms is defined using the min-operator (Bellman and Zadeh, 1970), as the minimum of the possibility of feasibility of the constraints.

The solution of the fuzzy problem may be achieved by solving the following linear programming problem:

$$\text{Minimize } z = \sum_{j=1}^{n} c_j x_j \tag{21}$$

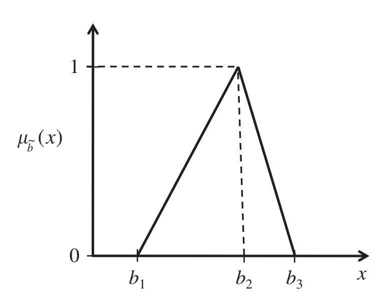

Fig. 1. A TFN $\tilde{b}$.

Subject to

$$\sum_{j=1}^{n} a_{ij}x_j \geq (1-\alpha)b_{i1} + \alpha b_{i2} \quad i=1,\ldots,m \tag{22}$$

$$\sum_{j=1}^{n} a_{ij}x_j \leq (1-\alpha)b_{i3} + \alpha b_{i2} \quad i=1,\ldots,m \tag{23}$$

$$x_j \geq 0 \quad j=1,\ldots,n \tag{24}$$

where $0 \leq \alpha \leq 1$ is a cutoff value that must be established parametrically.

Therefore, the transformation of the fuzzy model in Section 2 to an equivalent linear programming model is made by substituting Eqs. (10)–(12) for (25)–(27), respectively:

$$I^-_{ict} - I^-_{ic(t-1)} + \sum_{c} S_{isct} \geq (1-\alpha)d_{ict1} + \alpha d_{ict2} \quad \forall i \in I^{FP}, \quad \forall t \in T \tag{25}$$

$$\sum_{s,t' \leq t} S_{isct'} \leq \sum_{t' \leq t} ((1-\alpha)d_{ict'3} + \alpha d_{ict'2}) \quad \forall i \in I, \quad \forall t \in T \tag{26}$$

$$I^-_{ict} \leq \sum_{t' \leq t} ((1-\alpha)d_{ict'3} + \alpha d_{ict'2}) \quad \forall i \in I, \quad \forall c \in C \tag{27}$$

In order to solve the problem α is settled down parametrically to obtain the value of the objective function for each one of those $\alpha \in [0, 1]$. The result is, however, a fuzzy set and the planner has to decide which pair (α, z) considers optimal if he wants to obtain a crisp solution.

4. Supply chain planning case study

This section uses the Example 1 provided by McDonald and Karimi (1997) to illustrate the potential savings, which can be attained by using fuzzy models in a fuzzy environment. It is a representative supply chain of the chemical sector. There are two sites, s_1 and s_2, which produce 34 products. Each site contains only a resource or processor, so the J set is superfluous. The first site produces 23 products and there are 11 product families which rates minimum run lengths and fixed costs are provided. The second site, s_2, depends on s_1 and produces 11 products, which require a unit of the first product of each family of s_1. It is assumed that s_2 does not have capacity constraints. They have considered 12 monthly periods in the planning horizon with a required demand at the end of each time period. The demand for the 11 products at s_2 is derived as 50% of the demand for the products they consume from s_1. The target safety stock levels for s_2 are assumed equal to the average monthly demand. Also, they have provided the following information: bill of materials, transportation costs, fixed costs for family product, initial inventory, available capacity, raw material costs, safety stock penalties, inventory costs, production costs, production run times and demand.

4.1. Assumptions

In order to take into account the uncertainty in demand, we have assumed as the most possible value, d_{ict2}, a foreseen demand equal to the average real monthly demand. Therefore, the fuzzy model, dubbed *FMILP*, will generate results equivalent to the deterministic model, dubbed *MILP*, when α is established to 1. The most pessimistic value, d_{ict1}, is obtained by decreasing 20% the value of d_{ict2} and the most optimist value, d_{ict3}, is obtained by increasing 20% the value of d_{ict2}. Also, it is considered the demand is firm and known for period t at the beginning of this period t, i.e. d_{ict1}, d_{ict2} and d_{ict3} are equal for t at the beginning of t.

4.2. Implementation and resolution

The dynamic character of the supply chain planning problem and the integrity requirements on some variables are taken into consideration through the architecture used for the implementation and resolution of the model described before as illustrated in Fig. 2.

The model has been generated with the modelling language MPL. The problem is then solved by the CPLEX solver. CPLEX is capable to solve LP, integer programming and MILP problems. The input data and solutions of the model have been managed through the Microsoft Access database. The experiment has been carried out on a PC with Intel Pentium M processor at 1400 MHz and with 512 MB of RAM memory in the following way (Fig. 3).

It considers the technical and economic information of the products. Moreover, the demand information for a rolling horizon of 12 months. Models are executed for each one of the 12 months updating the demand values, the inventory and the delayed demand, which come from the planned launchings of the calculated periods.

The detailed data of this computational experiment can be found in McDonald and Karimi (1997).

4.3. Quantitative analysis of results

Next, we will validate if the fuzzy model for supply chain production planning, proposed in this paper, can be a useful tool for the decision making process of the production planners under demand uncertainty. Thus, we investigate the impact of demand uncertainty on the different supply chain decisions. To this end, the optimal decisions obtained by solving the fuzzy model, *FMILP*, are compared with those obtained by solving the deterministic model proposed by McDonald and Karimi (1997), *MILP*.

Table 2 reports the evaluation results according to a group of parameters defined originally in Mula et al. (2006b): (i) the service level; (ii) the levels of inventory; (iii) the planning nervousness respect to the planned period and the planned quantity; and (iv) the total costs.

In the case of the fuzzy model that provides a fuzzy solution, the fuzzy set of the decision has been obtained. Thus, *FMILP*, has been tested varying the α value from 0 to 1 in steps of 0.1. This fact allows the production planners to foresee the consequences of a sudden variation of the demands. Also, a defuzzification method must be used to get a compromise solution. The reader is referred to Lee (1990), Mendel (1995) and Jiménez et al. (2007). Table 2 presents the quantitative results of our experiment in terms of the following parameters:

(i) The average service level for finished products:

$$\text{Average service level (\%)} = \sum_{t=1}^{T} \frac{\left(1 - \frac{I^-_{ict}}{\sum_{t'=1}^{t} d_{ict'}}\right) \times 100}{T} \quad \forall i \in I, \quad \forall c \in C \tag{28}$$

(ii) The minimum and maximum inventory levels: for each model and item, if it presents the minimum inventory level, it is assigned the value of 1, to the rest we assign a null value. The model that obtains the highest number will have the minimum levels of inventory. The maximum inventory levels can be determined in a similar way but assigning the value of 1 to the maximum inventory level for item and model.

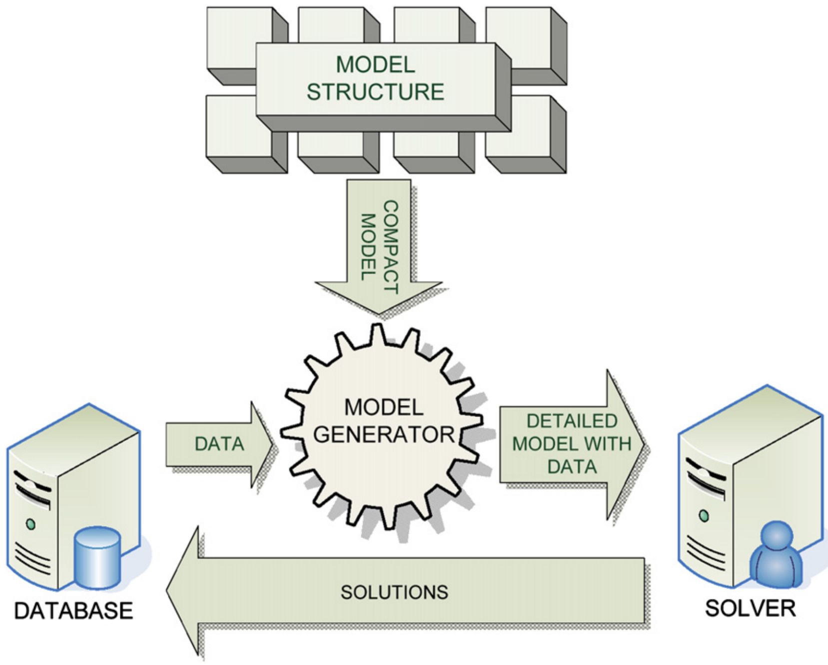

Fig. 2. Model architecture.

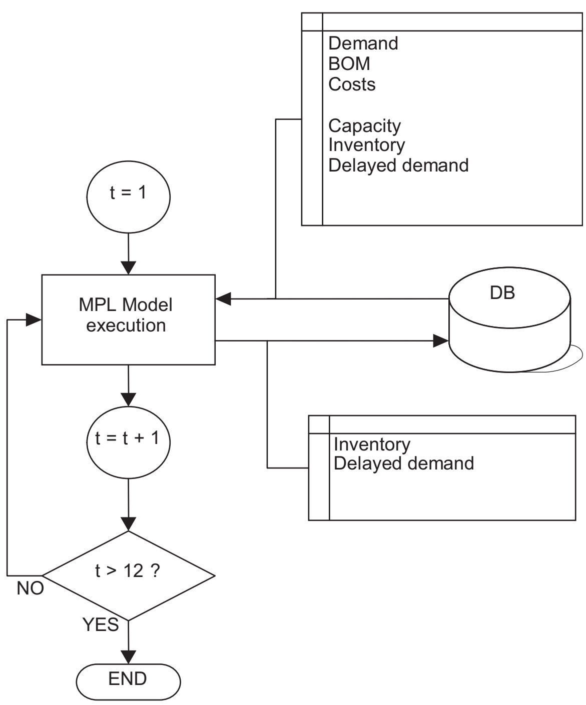

Fig. 3. Computational experiment.

(iii) The planning nervousness with respect to the planned period: a "nervous" or unstable planning is referred to a plan that suffers important variations when incorporating the demand changes between what is foreseen and observed in successive plans as defined by Sridharan et al. (1987). The planning nervousness can be measured according to the demand changes with respect to the planned period or with respect to the planned quantity. The demand changes in the planned period measure the number of times that a planned order is rescheduled independently of the planned quantity Heisig (1998). The next rule proposed by Donselaar et al. (2000) is summarized as follows:

At time t we check for each period $t+x$ ($x=0, 1, 2,\ldots,T-1$):

- If there is a planned order in $t+x$ and this order is not planned in the next planning run, we increase the number of reschedules by 1.
- If there was no planned order in $t+x$ and there is one in the next planning run, we increase the number of reschedules by 1.

(iv) The planning nervousness with respect to the planned quantity measures the demand changes in the planned quantity as the number of times that the quantity of a planned order is modified (De Kok and Inderfurth, 1997). The rule is described as follows:

In the period $t=1,\ldots,T$, where T is the number of periods that forms the planning horizon, it is checked for every period $t+x$ ($x=0, 1, 2,\ldots, T-1$):

- If a planned order exists in the period $t+x$, then if the quantity of the planned order is not the same as in the next planning run, we increase the number of reschedules by 1.

In the computation of planning nervousness, we are measuring the number of change. Another way to compute it would be taking into account the rate of the changes.

(v) Total costs are the sum of all the costs that are generated in every period of the considered planning horizon, derived from the production plans provided by the model.

Models present an average service level above 98%. *MILP* provides a 100% service level but a higher number of maximum inventory levels. *FMILP* model has presented less or equal nervousness with respect to the planned time period than the *MILP* model. On the other hand, the fuzzy model presents similar values of nervousness with respect to the planned quantity. Also, the fuzzy model generates lower total costs than the crisp model. These differences in the total costs are, mainly, due to the consideration of

Table 2
Evaluation of the results.

Model	Average service level (%)	Number of minimum inventory levels	Number of maximum inventory levels	Planning nervousness (period)	Planning nervousness (quantity)	Total costs (€)
FMILP (α=0)	99.71	23	1	2.64	2.45	2972.46
FMILP (α=0.1)	100.00	16	3	1.82	2.55	2542.38
FMILP (α=0.2)	99.34	9	5	1.82	2.27	2786.86
FMILP (α=0.3)	99.50	9	4	2.09	2.36	2595.64
FMILP (α=0.4)	99.51	9	4	2.00	2.55	2680.35
FMILP (α=0.5)	99.39	7	6	1.82	2.36	2848.75
FMILP (α=0.6)	99.95	6	4	1.64	2.09	2708.92
FMILP (α=0.7)	99.71	6	6	1.27	1.73	2856.40
FMILP (α=0.8)	98.79	7	7	2.00	2.45	2972.46
FMILP (α=0.9)	99.46	6	8	2.00	2.73	2751.67
FMILP (α=1) =*MILP*	100.00	5	18	2.00	2.64	3001.42

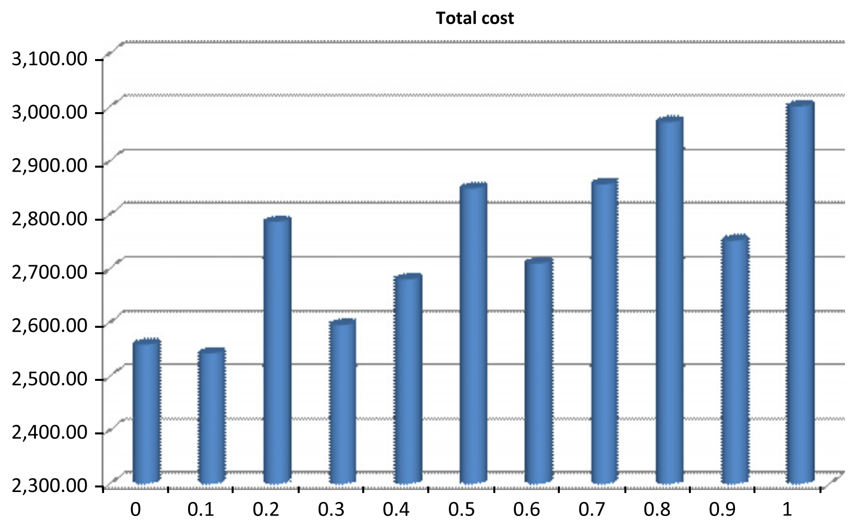

Fig. 4. Variation of total costs for each α.

possible future variations of the demand that originates larger production and inventories with the objective of avoiding the penalized demand backlogs, which suggests that cost savings can be obtained through the incorporation of demand uncertainty in supply chain production planning processes. In order to make the model better understandable, the variation of total costs for each step of α in the form of graph is depicted in Fig. 4.

Furthermore, we define a new parameter: the average capacity utilization. This parameter is calculated in two different ways depending on the available capacity in each site. If the site has a limited capacity, we use the following formula:

$$\text{Average capacity utilization (\%)} = \sum_{t=1}^{T} \frac{\left(\frac{RL_{ijst}}{H_{jst}}\right) \times 100}{T} \quad \forall i \in I, \quad \forall j \in J, \quad \forall s \in S \tag{29}$$

If the site has not a limited capacity, we use the following formula:

$$\text{Average capacity utilization (time)} = \sum_{t=1}^{T} \frac{RL_{ijst}}{T} \quad \forall i \in I, \quad \forall j \in J, \quad \forall s \in S \tag{30}$$

The average capacity utilization for each site provided by the two models, *MILP* and *FMILP*, is shown in Table 3; s_1 has a limited available capacity while s_2 does not have a specified limited available capacity. The foreseen capacity utilization is also different if the model considers the uncertainty in demand, which suggests different available capacity decisions if planners can take into account demand uncertainty in the planning process.

Table 3
Average capacity utilization.

Model	Average capacity utilization s_1 (%)	Average capacity utilization s_2 (time)
FMILP (α=0)	71.86	62.78
FMILP (α=0.1)	76.32	61.40
FMILP (α=0.2)	72.45	63.08
FMILP (α=0.3)	63.61	62.77
FMILP (α=0.4)	65.98	61.05
FMILP (α=0.5)	75.42	63.46
FMILP (α=0.6)	67.30	63.44
FMILP (α=0.7)	75.49	64.24
FMILP (α=0.8)	75.18	59.48
FMILP (α=0.9)	64.34	64.92
FMILP (α=1)=*MILP*	72.44	62.78

Table 4 shows the computational efficiency of the crisp model proposed by McDonald and Karimi (1997) and the fuzzy model proposed in this work. Both of the models can obtain the optimal solution of the *MILP* with a similar number of iterations in the first execution (planning period=1). Obviously, the number of

Table 4
Efficiency of computational experiments for the first MRP execution.

Model	Iterations	Decision variables	Integer	Constraints	Elements non zero	Array density (%)	CPU time (s)
MILP	27,6428	5118	528	9474	77,742	0.16	100
FMILP (α=0.8)	27,8017	5118	528	9474	77,742	0.16	100

iterations can change in the rest of executions depending on the input data. On the other hand, the fuzzy model has the same number of constrains, variables and integer variables, which is not implying greater requirements of information storage. With respect to the CPU time in both models, deterministic and fuzzy, a limit of 100 CPU seconds was set.

On the other hand, Gupta and Maranas (2003) propose a stochastic programming based approach to manage the demand uncertainty in supply chain planning using the same representative supply chain planning formulation given by McDonald and Karimi (1997) as the basis of their work. The demand is assumed to be normally distributed with a coefficient of variation of 20%. The stochastic approach generates a total of 500 scenarios by sampling the normal distributions and the resulting MILP has 136,000 constraints and 156,000 variables. When this stochastic approach is solved using the CPLEX solver, it fails to converge with a limit of 10,000 CPU seconds.

5. Conclusion

Supply chain environments imply the production planning decisions have to be made under conditions of uncertainty in parameters as important as demand. In this paper, a supply chain planning problem has been presented as a fuzzy MILP model with fuzzy demand. The proposed fuzzy mathematical programming approach extends the formulation originally presented by McDonald and Karimi (1997) considering uncertain demand. This approach is based on the method for solving multi-objective linear programming problems with fuzzy parameters represented by triangular fuzzy numbers proposed by Gen et al. (1992). We have adapted this approach for solving fuzzy linear programming problems with a crisp objective function and fuzzy right-hand side numbers in less than or equal and greater than or equal type constraints. This fuzzy mathematical approach has provided freedom of action with regard to supply chain production planning problems where epistemic uncertainty appears in demand with no increment of the requirements of information storage and the same specified resource limit of 100 CPU seconds as used by the deterministic formulation. Also, compared with the stochastic programming approach, a solution can be obtained in an easier manner.

Finally, as a result of the research carried out here, some circumstances have arisen that may open up possibilities for further research: (i) to adapt another fuzzy mathematical programming based approaches in order to prove their effectiveness to solve supply chain production planning problems; (ii) to use evolutionary computation with fuzzy optimization in order to solve more efficiently fuzzy supply chain production planning problems; and (iii) to integrate simulation models with fuzzy optimization and evolutionary computation models to better understand the behaviour and the results of the fuzzy supply chain production planning models.

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