Configuration Optimization of Underground Cables for Best Ampacity

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***Abstract*—This paper presents a method for configuring the locations of any number of cables, for the best total ampacity. The optimal configuration is determined through a proposed two-level optimization algorithm. At the outer level, a combinatorial optimization based on a genetic algorithm explores the different possible configurations. The performance of every configuration is evaluated according to its total ampacity, which is calculated by using a convex optimization algorithm. The convex optimization algorithm, which forms the inner level of the overall optimization procedure, is based on the barrier method. The proposed approach is tested for a duct bank installation containing 12 cables and 15 ducts, comprising two circuits and two cables per phase, and compared with a brute force method of considering all possible configurations. The proposed approach is also applied to an installation consisting of a single circuit inside a large magnetic steel casing.**

***Index Terms*—Barrier method, cable ampacity, combinatorial optimization, convex optimization, optimal configuration, vector immune system (VIS) algorithm.**

I. Introduction

THE USE OF electric power cables for power transmission and distribution is present in almost every geographical area. Power cables are installed overhead in the air or buried underground. Although the latter is more expensive to install and maintain than the former, it is the preferred method for urban areas. Due to the steep cost of underground cables installation and maintenance, it is critical that they are used to their full potential. However, to avoid overheating the cables, accurate knowledge of cable ampacity is required.

Significant work has been done in the field of ampacity computation for single and multiple cable installations. To determine the cable ampacities, analytical and empirical equations have been developed [1]–[5], commercial programs that implement these equations have been written [6], tables for specific cable designs and configurations have been published [7], and iterative and optimization methods for solving the ampacity equations for multiple cable installations have been proposed [8]–[10].

In large urban areas, cables are often laid in concrete duct banks to permit installation of several circuits in a fairly confined space. Fig. 1 shows an example of this installation.

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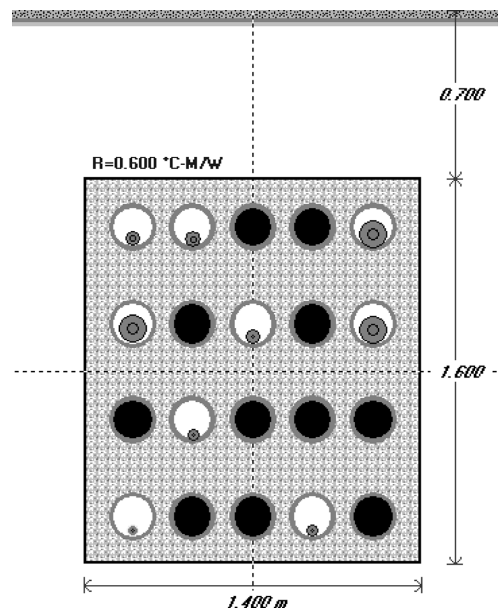

Fig. 1. Example of cables located in a duct bank buried underground.

In a duct bank installation with multiple available ducts, multiple cable configurations are possible. Each configuration may lead to a different circuit ampacity, because the mutual heating effect depends on cable locations as do the sheath and armor losses in each cable. The configuration that leads to the maximum total ampacity is desirable to maximize the usage of a limited duct bank space. On the other hand, the configuration with the smallest total ampacity is desirable when cables have already been installed and information regarding which ducts were used is lost, which happens quite often in practice when a large number of cables are located in one duct bank. In this case, a worst-case scenario is of interest.

Recently, another important installation configuration has been gaining some attention—namely, installing cables in steel or plastic casings. These are large pipes containing a number of plastic ducts as shown, for example, in Fig. 2.

In addition to the issue of mutual heating and sheath/armor losses discussed above, this installation may also result in very large hysteresis and eddy current losses in the magnetic pipe. A question arises regarding how the cables should be placed in the available ducts to minimize the losses in the magnetic pipe and to maximize overall circuit ratings.

A number of published works address the problem of cable ampacity calculations given cable locations or configurations. There have been no published papers on location optimization procedures that determine the best or worst cable configurations,

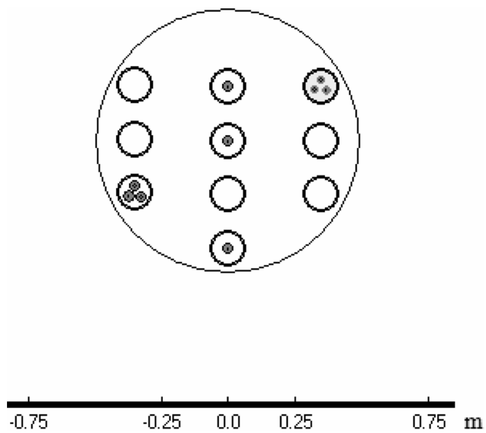

Fig. 2. Cables inside a casing.

from the total ampacity point of view. The aim of this paper is to present a procedure for finding the optimal cable configuration for cables located in a duct bank or a casing.

The issue of finding the best configuration for a number of circuits is by its nature a combinatorial optimization problem, due to the discrete solution space of possible configurations. Reference [11] solves a problem of optimizing cables configuration with the combined objectives of reducing the created magnetic fields and the current imbalance. This problem is similar to the problem at hand from the point of view of optimizing the cable configuration, but is different with regards to the objective of maximizing the total ampacity. Therefore, the approach in [11], utilizing the vector immune system (VIS) algorithm, will serve as the basis for the proposed algorithm that is presented in this paper.

Section II provides an overview of a method for calculating the ampacity of a system of cables placed in a fixed configuration. As a starting point, the method uses the Neher McGrath [1] and IEC [2] standard equations, which have been verified in [12], and solves them by using a convex optimization algorithm that provides always convergent results. The convex optimization algorithm finds the ampacity of a fixed cable configuration and, thus, serves as a single iteration within an outer-level combinatorial optimization algorithm that attempts to find the best cable configuration, from the point of view of the total system ampacity. The combinatorial optimization procedure is based on the VIS algorithm and is detailed in Section III. The proposed method is illustrated with a complex, real-life duct bank installation consisting of 12 cables and 15 available ducts, comprising two different circuits with two cables per phase. The results are presented in Section IV. Section V concludes this paper.

II. Ampacity Calculation

A. Ampacity Equation

This section presents a method for calculating the total ampacity of a group of cables inside a duct bank buried underground.

A typical power cable consists of a conductor, insulation, metallic sheath or screen, possible armor bedding, armor, and external serving layers, as shown in Fig. 3. The main sources of heat generated by a cable are the Joule losses in the conductor, sheath/screen, armor, and pipe. In addition, some cables may produce substantial dielectric losses [5]. This heat is dissipated through the various cable layers and the soil. The thermal resistances of the cable layers and its surroundings influence the rate at which the heat is dissipated and, hence, the rise of the conductor temperature above the ambient temperature. This thermal interaction can be represented for the steady-state conditions by a lump-parameter thermal circuit that incorporates the thermal resistances of the cable layers and the soil, the Joule and dielectric losses, and the ambient and conductor temperatures. The thermal circuit is then solved to obtain the maximum conductor current, given the allowable insulation temperature. The solution for the current $I(\mathrm{A})$ is formulated in (1), with a comprehensive derivation given in [5]

$$I=\left[\frac{\Delta\theta_{\max}-W_d(0.5T_1+N(T_2+T_3+T_4))-\Delta\theta_{\mathrm{int}}}{RT_1+NR(1+\lambda_1)T_2+NR(1+\lambda_1+\lambda_2)(T_3+T_4)}\right]^{0.5} \tag{1}$$

where

λ_1	sheath loss factor, which is the ratio of the total sheath losses to the total conductor losses;
λ_2	armor loss factor, which is the ratio of the total armor losses to the total conductor losses;
R	conductor ac resistance (Ω/m);
W_d	dielectric losses (in Watts per meter);
N	number of load-carrying conductors in the cable;
T_1	thermal resistance of the insulation (in Kelvin meter per watt);
T_2	thermal resistance of the armor bedding (in Kelvin meter per watt);
T_3	thermal resistance of the external serving (in Kelvin meter per watt);
T_4	thermal resistance of surrounding a medium (in Kelvin meter per watt);
$\Delta\theta_{\max}$	$\theta_{\max}-\theta_{\mathrm{amb}}$, $\theta_{\max}$ as the maximum allowable temperature of the cable conductor and θ_{amb} as the ambient temperature;
$\Delta\theta_{\mathrm{int}}$	conductor temperature reduction factor due to the heating from the neighboring cables.

The cable parameters depend on the material of each layer and its dimensions. The methods for calculating these parameters can be found in [5] and are out of the scope of this paper.

Calculating $\Delta\theta_{\mathrm{int}}$ for the ampacity equation of the cable of interest "i" is obtained by summing up the heat influences of all neighboring cables, as given by (2). The heat influence by each cable "j" on cable "i," $\Delta\theta_{ij}$, is calculated by using (3) by multiplying the heat produced by the cable "j," W_j, and the mutual thermal resistance T_{ij} between cables "j" and "i." W_j is the sum of the Joule and dielectric losses of cable "j," as expressed in (4). T_{ij} depends on the distance between the two

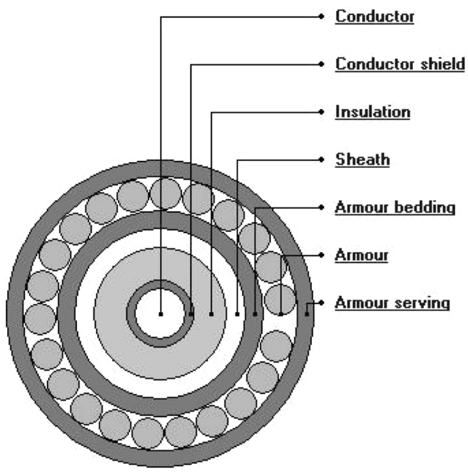

Fig. 3. Underground cable construction.

cables and their depth below the earth's surface and is calculated by using (5) [5]

$$\Delta\theta_{\text{int}} = \sum_{\substack{j=1 \\ j\neq i}}^{n} \Delta\theta_{ij} \tag{2}$$

$$\Delta\theta_{ij} = W_j \cdot T_{ij} \tag{3}$$

$$W_j = N\left[I_j^2 R_j(1+\lambda_{1j}+\lambda_{2j})\mu_j + W_{dj}\right] \tag{4}$$

$$T_{ij} = \frac{\rho_s}{2\pi}\ln\frac{d'_{ij}}{d_{ij}}. \tag{5}$$

In (2), n is the total number of cables in the system. For cable "j," I_j is its conductor current, R_j is its ac resistance (Ω/m), λ_{1j} and λ_{2j} are its sheath and armor loss factors, respectively, N is the number of conductors in the cable, μ_j is its loss factor, and W_{dj} is its dielectric loss. The calculation of the mutual thermal resistance becomes somewhat more involved when the cables are located in a duct bank, backfill, or a large casing [5].

It is evident from (1)–(5) that in order to calculate the ampacity of a cable "i," the currents of all the other cables must be known. However, these currents are not known *a priori* because the objective is to compute the ampacities of all the cables in the system. The application of (1) to every cable will result in a system of interrelated equations. In practice, these equations are solved iteratively [5], [6], [8]. However, the iterative method is not always convergent. A recently postulated alternative is to express these equations as an optimization problem and then solve them to obtain the ampacities [10]. This method is always convergent and is summarized next.

B. Convex Optimization Problem

Because the ampacity of a cable is the largest current that it can carry while not causing its conductor temperature to rise above a specified maximum, the total ampacity of a group of cables is the largest sum of the currents that do not cause *any* cable to overheat. Using (1) for every cable, the resulting system of interrelated equations can be expressed, through algebraic manipulations, as an optimization problem with an objective function of maximizing the sum of all cable currents and with the constraints of having every cable conductor temperature below a specified maximum. The resulting formulation is given in (6), with a complete mathematical derivation detailed in [9]

$$\begin{aligned}
\text{Minimize}\quad & -I_1 - I_2 - \cdots - I_n \\
\text{subject to}\quad & \frac{1}{d_1}I_1^2 + \frac{c_{12}}{d_1}I_2^2 + \ldots + \frac{c_{1n}}{d_1}I_n^2 \leq 1 \\
& \frac{c_{21}}{d_2}I_1^2 + \frac{1}{d_2}I_2^2 + \ldots + \frac{c_{2n}}{d_2}I_n^2 \leq 1 \\
& \vdots \\
& \frac{c_{i1}}{d_i}I_1^2 + \frac{c_{i2}}{d_i}I_2^2 + \ldots + \frac{1}{d_i}I_i^2 + \ldots + \frac{c_{\text{in}}}{d_i}I_n^2 \leq 1 \\
& \vdots \\
& \frac{c_{n1}}{d_n}I_1^2 + \frac{c_{n2}}{d_n}I_2^2 + \ldots + \frac{1}{d_n}I_n^2 \leq 1
\end{aligned} \tag{6}$$

where c_{ij} and d_i are defined in (7) and (8) at the bottom of the page.

This problem belongs to the class of continuous convex optimization problems. Such type of problems can be solved effectively using the barrier method algorithm, which is briefly described in the Appendix and is detailed in [9].

So far, ampacities have been calculated for multiple cables placed at the specific known positions, by solving a corresponding convex optimization problem. However, in some cases, there are multiple available laying locations for the cables, and the engineer might be interested in the optimal cable configuration, which would lead to the largest or the smallest possible total ampacity. The need to find this optimal configuration leads to another optimization problem at a different, outer level than the aforementioned convex optimization problem. The convex optimization problem simply provides

$$c_{ij} = \frac{N_j R_j(1+\lambda_{1j}+\lambda_{2j})\mu_j \dfrac{\rho_s}{2\pi}\ln\dfrac{d'_{ij}}{d_{ij}}}{R_i T_{1i} + N_i R_i(1+\lambda_{1i})T_{2i} + N_i R_i(1+\lambda_{1i}+\lambda_{2i})(T_{3i}+T_{4i})},\quad \begin{matrix} i\neq j \\ i=1,\ldots,n \\ j=1,\ldots,n \end{matrix} \tag{7}$$

$$d_i = \frac{\Delta\theta_{i\max} - W_{di}\left[0.5T_{1i} + N_i(T_{2i}+T_{3i}+T_{4i})\right] - \dfrac{\rho_s}{2\pi}\left[\displaystyle\sum_{\substack{j=1\\ j\neq i}}^{n}\left(N_j W_{dj}\ln\frac{d'_{ij}}{d_{ij}}\right)\right]}{R_i T_{1i} + N_i R_i(1+\lambda_{1i})T_{2i} + N_i R_i(1+\lambda_{1i}+\lambda_{2i})(T_{3i}+T_{4i})}. \quad i=1,\ldots,n \tag{8}$$

the total ampacities for a specific configuration and, therefore, constitutes a single iteration in the outer-level combinatorial optimization problem that seeks the best or worst configuration resulting in the optimal attainable total ampacity. Considering all the possible cable configurations and calculating the total ampacity for each can be a very time-consuming task. Instead, combinatorial optimization algorithms can be invoked to solve this problem. The following section presents this outer-level combinatorial optimization problem and provides an algorithm for solving it.

III. Cable Configuration Optimization

This section presents a method for configuring cables for the total ampacity. The method is based on a Vector Immune System (VIS) combinatorial optimization algorithm. Customization of this algorithm to suit the problem being solved is also shown.

A. Vector Immune System

In most duct bank installations, the cables can be placed in various fixed empty ducts. The task of finding the best duct allocation for each cable that would result in the maximum possible total ampacity is by its nature a combinatorial optimization problem. This is because the solution space, which is the set of duct allocations of cables, is discrete. A combinatorial optimization algorithm that has been applied to a similar problem is described in [11]. Thus, the VIS algorithm will serve as basis for the algorithm presented here and will be customized to suit our problem. The customization of the VIS algorithm is presented in Section III.B.

Evolutionary algorithms such as the VIS are based on the survival of the fittest, where the traits of only the fittest population are passed on from one generation to the next. This is achieved in the VIS algorithm by structuring the operations into two nested loops, as illustrated by the flowchart in Fig. 4. In the inner loop, two operations are applied to the population, namely, cloning and mutation and clonal selection. The parent population from the previous iteration is copied into N_{clones} clones, and each clone is mutated by random modifications. The mutation operations are discussed in Section III.B. The parents and their mutated clones, also called children, are each given a measure of fitness, according to their respective values of the objective function. Among each parent and its children, the one with the best fitness is selected to become the parent of the next generation, or inner iteration. This process ensures the fitness of the population improves with every new generation. In the outer loop, the population undergoes affinity, suppression, and random replacement. This is done by measuring the Euclidean distance between the memory cells in the objective space, which in our case is the change in the total system ampacity. All the memory cells, except the ones with distances below a preset threshold, are suppressed, or deleted, and are replaced by randomly generated new ones. The optimization algorithm ends when the numbers of outer and inner iterations reach their preset limits, N_{out} and N_{in}. At this point, the parent solution with the best fitness is output as the final solution. The greater the number of inner and outer iterations, the larger the probability that the output solution is the sought optimal for the problem.

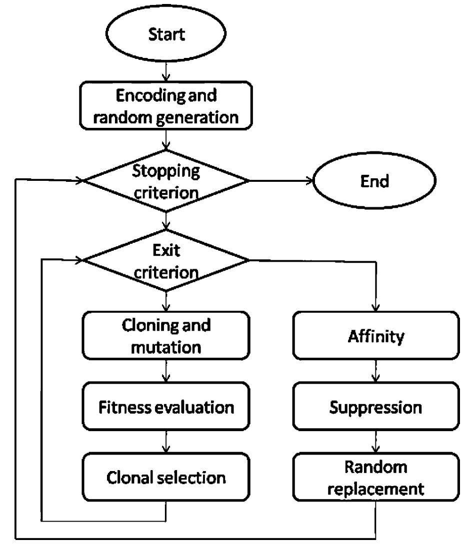

Fig. 4. Flowchart illustrating the VIS algorithm [11].

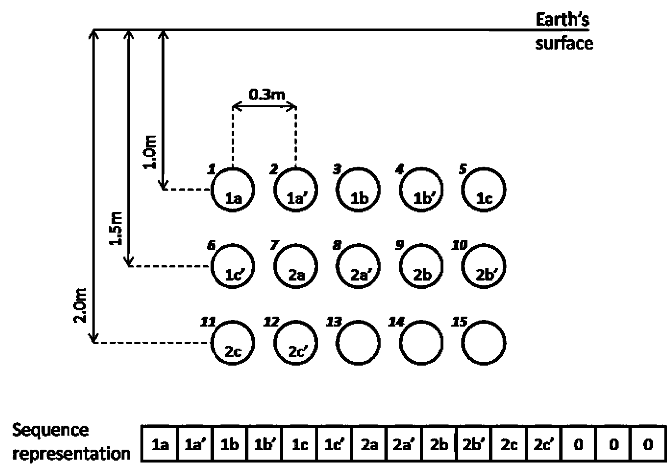

Fig. 5. Duct bank installation showing twelve cables in fifteen ducts and the corresponding sequence representation.

The following subsections detail some of the steps in the aforementioned algorithm and its customization to suit the problem being solved here.

B. Algorithm Customization

1) Encoding and Random Generation: Each different cable configuration is a viable solution to the combinatorial optimization problem, and should be unique in its representation as a solution candidate in the VIS algorithm. Because the cable configurations are simply the positions of the cables in fixed ducts, a natural way of representing each configuration is through a sequence of cable identifiers. Each identifier is assigned to a specific cable, and each position within the sequence is preset to a specific duct. An illustration of the sequence corresponding to a cable configuration is shown in Fig. 5.

The identifiers specify the cable's circuit and phase. If there are multiple cables per phase, then they are distinguished through the addition of an extra apostrophe. For example, the identifier " 1a' " represents one of the two cables belonging to phase "a" of circuit 1, the other cable having the identifier

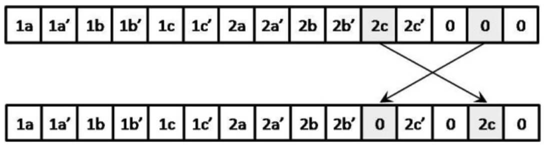

Fig. 6. *Exchange* mutation operation.

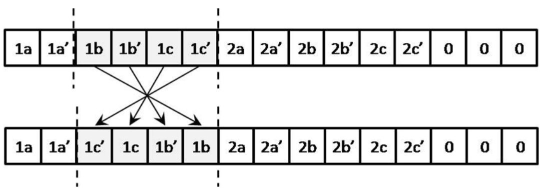

Fig. 7. *Inversion* mutation operation.

" 1a ". If there is no cable filling a particular duct, then this non-existent cable is represented by a zero, "0". As can be seen in Fig. 5 each duct contains a cable identifier. Each duct is also preset to a position in the sequence. This preset position is given by a number at the top left corner of the duct, corresponding to a position index within the sequence going from left to right.

In the first step of the algorithm, a number of allowable sequences, N_{pop}, are generated randomly to represent the initial population. Next, the population undergoes cloning and mutation operations in the inner loop of the algorithm, as discussed next.

2) Mutation: Evolutionary algorithms explore the solution space generally through crossover and mutation of the population. However, algorithms that are based on the immune system do not have a crossover analogy, and thus only mutation is implemented in the VIS solution. Mutation is achieved basically through random local modifications of each sequence. The mutation operation should be closed—that is, the resulting sequence should also be legal. Two mutation operations are implemented in the algorithm—namely, *Exchange* and *Inversion* mutation. These two operations are detailed below.

The *Exchange* mutation operation exchanges the contents of two random elements in the sequence. This operation is illustrated in Fig. 6.

As can be seen in Fig. 6, two elements are exchanged such that the cable in duct "11" is removed and placed in the previously empty duct "14".

Inversion mutation is implemented by choosing two random edges within the sequence and reversing the order of the elements in between. This operation is illustrated in Fig. 7.

As can be seen from Fig. 7, the cables inside ducts 3–6 are reversed in order.

3) Fitness: The measure of fitness in our problem is the total ampacity, which is computed using the method discussed in Section II.B for each cable configuration. Depending on whether the objective is to minimize or maximize the total ampacity, a solution with a better fitness will have a lower or higher total ampacity, respectively.

4) Affinity, Suppression, and Random Replacement: The affinity and suppression operations implemented in [11] are based on comparing sequences together, element by element, and determining how many elements differ. If the two sequences differ by less than a preset threshold number of elements, then one of the sequences is deleted and replaced by a randomly generated one. These operations filter out any similar sequences in the population.

However, this approach does not result in optimal performance for the problem being solved here. This is because two sequences differing by just two elements can correspond to two different cable configurations with a large difference in the total ampacity. Thus, the suppression operation is implemented by making an exact comparison between every two sequences. If the two sequences are exactly the same, then one of them is deleted and replaced by a randomly generated new one.

IV. Numerical Examples

Two real-life installations for which the cable location plays an important role are examined here. The first is a duct bank with several empty ducts and the second is a steel casing where the losses in the steel pipe are heavily dependent on the location of the cables.

The system of cables and ducts is shown in Fig. 5. The size of the population and the number of clones are chosen to be equal to: $N_{\text{pop}} = 50$ and $N_{\text{clones}} = 5$, respectively. There is no proof for the optimality of these choices, but rather they are picked based on the experience of the authors with solving this combinatorial optimization problem. The number of the inner and outer loop iterations are: $N_{\text{in}} = 12$ and $N_{\text{out}} = 12$. Because the total number of iterations directly corresponds to the probability that the final solution is identical to the sought optimal, these choices for the number of iterations are important in determining the accuracy and simulation time of the proposed method. Generally, the greater the number of iterations, the better the accuracy but the longer the simulation time will be. The authors have chosen the number of iterations such that at least for every ten consecutive simulations, the same final solution is obtained.

A sensitivity analysis was carried out to examine the effect of the number of iterations on the accuracy and simulation time, with the results for this case presented in Fig. 8. The total number of iterations represent the product of N_{in} and N_{out}. While performing the analysis, the authors found that an equal change in the total number of iterations due to an increment of N_{in} or N_{out} had almost the same effect on the accuracy and simulation time. Thus, only the total number of iterations is shown. Due to the probabilistic nature of the applied algorithm, the data points represent the total ampacity averaged over five simulations for a given total number of iterations. The dashed horizontal line shows the optimal solution (with very high probability), which is obtained after simulating for a very long time (i.e., $N_{\text{in}} = 30$ and $N_{\text{out}} = 30$). The results in Fig. 8 justify the selection of the number of iterations used by the authors.

The simulation time on a Intel Pentium Dual Core 2 Ghz computer was about 20 minutes, and the final solutions computed for the largest and the smallest ampacity cases are shown in Figs. 9 and 10, respectively. The largest total ampacity is 5 703 A, whereas the smallest total current is 3 437 A. This large

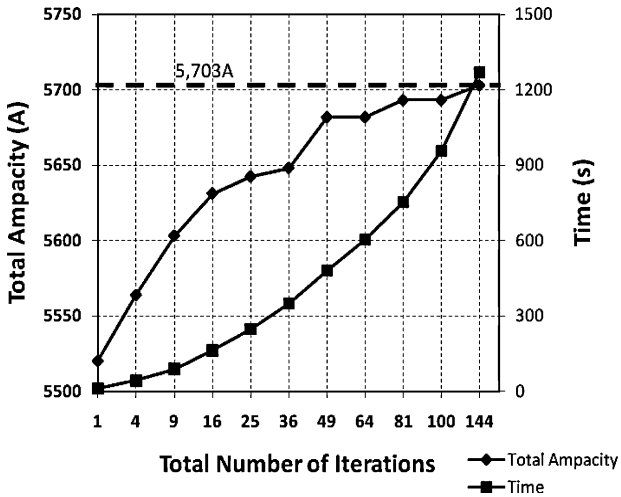

Fig. 8. Total ampacity and simulation time for different number of iterations.

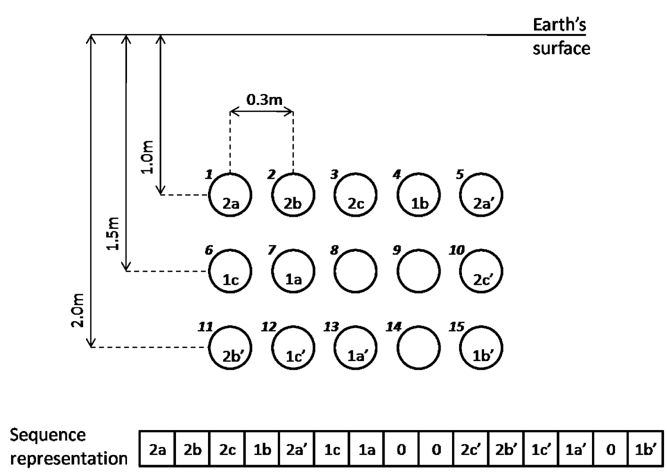

Fig. 9. Configuration of cables for largest total ampacity of 5 703A.

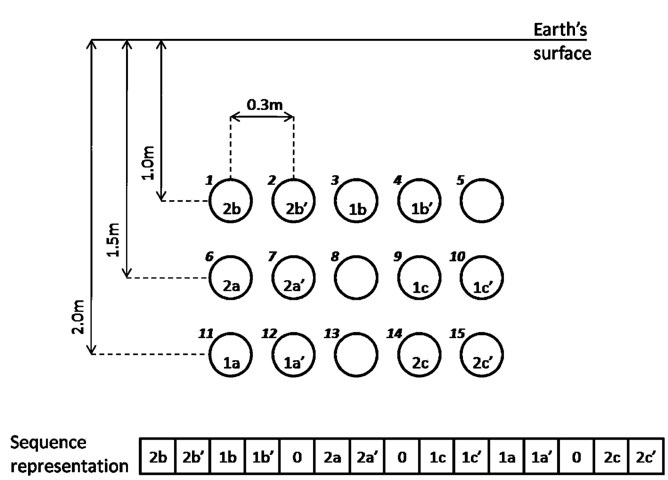

Fig. 10. Configuration of cables for smallest total ampacity of 3 437A.

TABLE I
CABLE DIMENSIONS AND CURRENT-INDEPENDENT PARAMETERS

Cable and Duct Bank Dimensions		Cable and Ambient Parameters (current-independent)	
Cu conductor external diameter /mm	27.0	λ_2	0
XLPE insulation external diameter /mm	49.8	N	1
Lead sheath external diameter /mm	53.8	W_d (W/m) (44 kV cable)	0
PE jacket external diameter /mm	59.4	T_1 (Km/W)	0.341
PVC duct internal diameter /mm	170.1	T_2 (Km/W)	0
Duct external diameter /mm	199.0	T_3 (Km/W)	0.095
Concrete duct bank horizontal width /m	1.6	θ_{max} $(^{\circ}C)$	90
		θ_{amb} $(^{\circ}C)$	20
Duct bank vertical height /m	1.4	ρ_{soil} (Km/W)	1
		$\rho_{concrete}$ (Km/W)	0.6

TABLE II
CABLE AMPACITIES, TEMPERATURES, SHEATH LOSS FACTORS, AC RESISTANCES, AND THERMAL RESISTANCES FOR THE CONFIGURATION IN FIG. 9

Cable	Ampacity (A)	Temperature (°C)	λ_1	R $(\mu\,\Omega/m)$	T_4 (Km/W)
1a	$458\angle{-118^o}$	90.0	0.93	41.5	1.06
1a'	$450\angle{-122^o}$	89.0	1.00	41.3	1.10
1b	$496\angle{0.45^o}$	88.3	1.05	41.3	1.04
1b'	$412\angle{-0.54^o}$	87.1	1.74	41.1	1.15
1c	$443\angle{117^o}$	87.2	1.06	41.1	1.08
1c'	$466\angle{123^o}$	89.7	0.88	41.4	1.09
2a	$486\angle{-119^o}$	88.2	1.31	41.3	1.09
2a'	$508\angle{-121^o}$	88.2	1.18	41.3	1.08
2b	$518\angle{5.04^o}$	87.8	0.72	41.2	1.05
2b'	$480\angle{-5.44^o}$	89.3	0.95	41.4	1.12
2c	$530\angle{123^o}$	90.0	0.78	41.5	1.04
2c'	$465\angle{116^o}$	88.5	1.16	41.3	1.11

difference in ampacity illustrates the importance of selecting proper cable configuration.

Cable dimensions and parameters that are common to all cables used in the aforementioned two cases are given in Table I. The cable ampacities, conductor temperatures, sheath loss factors, ac resistances, and thermal resistances of the surroundings are given in Tables II and III, corresponding to the configurations in Figs. 9 and 10, respectively.

The simulation time for calculating the total ampacity of a single configuration is about 0.06 s. Solving the combinatorial

TABLE III
CABLE AMPACITIES, TEMPERATURES, SHEATH LOSS FACTORS, AC RESISTANCES AND THERMAL RESISTANCES FOR THE CONFIGURATION IN FIG. 10

Cable	Ampacity (A)	Temperature (˚C)	λ_1	R ($\mu\ \Omega / m$)	T_4 (Km / W)
1a	$268\angle\text{-}123^o$	88.3	6.23	41.26	1.14
1a'	$340\angle\text{-}117^o$	90.0	3.53	41.46	1.07
1b	$297\angle 10.5^o$	81.4	4.21	40.50	1.05
1b'	$320\angle\text{-}9.72^o$	80.26	3.62	40.38	1.04
1c	$337\angle 116^o$	84.5	2.48	40.84	1.04
1c'	$272\angle 125^o$	81.2	4.25	40.48	1.09
2a	$233\angle\text{-}121^o$	79.3	5.19	40.27	1.08
2a'	$306\angle\text{-}119^o$	82.3	2.63	40.60	1.03
2b	$306\angle 2.63^o$	74.0	2.96	39.69	1.03
2b'	$233\angle\text{-}3.46^o$	76.6	5.58	39.98	1.06
2c	$289\angle 113^o$	90.0	5.35	41.46	1.08
2c'	$254\angle 128^o$	87.9	7.22	41.23	1.12

TABLE IV
CASING PARAMETERS AND DUCT LOCATIONS

Casing Parameters		Duct Locations (polar coordinates with respect to casing center)	
Steel casing internal diameter /mm	750	Duct 1	(152.4 mm, 139.0°)
Casing external diameter /mm	770	Duct 2	(152.4 mm, 41.0°)
Steel relative magnetic permeability [13], [14]	1350	Duct 3	(237.7 mm, -157.8°)
Steel electric conductivity /Sm^{-1}	9.0×10^6	Duct 4	(237.7 mm, -22.2°)
Casing center depth below earth surface /m	16	Duct 5	(250.0 mm, -90.0°)

optimization problem through a brute force method by considering all possible cable configurations and comparing their total ampacities would require 22×10^7 minutes (or 415 years) for 2×10^{11} different permutations. Even if vertical symmetry is considered, the simulation time would be halved and still be on the order of an unreasonable 207 years.

The proposed method is also applied for an installation consisting of a single circuit inside a large steel casing, with a single cable per phase and five available ducts. This installation is illustrated in Fig. 12, and its relevant parameters are given in Table IV. The same cables, ducts and ambient conditions are used as in the duct bank installation, but the cable sheaths are single-point bonded and, thus, no sheath circulating current losses are incurred (i.e., $\lambda_1 \approx 0$). The cable currents

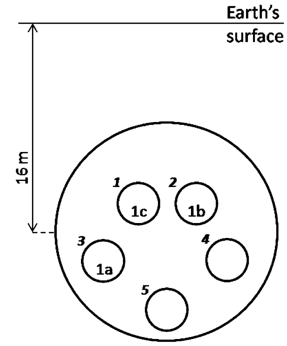

Fig. 11. Cables inside a steel casing, with a largest ampacity of 2 851A.

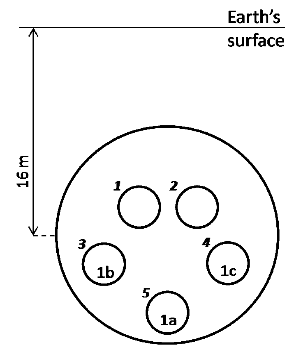

Fig. 12. Cables inside a steel casing, with a smallest ampacity of 2 766A.

TABLE V
CABLE AMPACITIES, TEMPERATURES, AC RESISTANCES, THERMAL RESISTANCES, AND TOTAL PIPE LOSS RATIO FOR THE CONFIGURATION IN FIG. 11

Cable	Ampacity (A)	Temperature (˚C)	R ($\mu\Omega / m$)	T_{41}; T_{42} (Km / W)	λ_p
1a	950.4	88.5	22.01	1.81; 0.70	0.69
1b	950.4	88.3	21.91	1.82; 0.70	
1c	950.4	90.0	22.60	1.81; 0.70	

TABLE VI
CABLE AMPACITIES, TEMPERATURES, AC RESISTANCES, THERMAL RESISTANCES, AND TOTAL PIPE LOSS RATIO FOR THE CONFIGURATION IN FIG. 12

Cable	Ampacity (A)	Temperature (˚C)	R ($\mu\Omega / m$)	T_{41}; T_{42} (Km / W)	λ_p
1a	921.9	88.9	21.96	1.79; 0.70	1.07
1b	921.9	88.9	21.96	1.79; 0.70	
1c	921.9	90.0	22.44	1.79; 0.70	

cause eddy-current and hysteresis losses in the magnetic steel pipe. These losses are computed applying an analytical method outlined in [14]. Different cable configurations give rise to different pipe losses and hence different cable ampacity. The configuration resulting in the largest ampacity is shown in Fig. 11 whereas the one giving rise to the smallest ampacity is shown in Fig. 12. Cable ampacities, temperatures and current dependent parameters, pertaining to the configurations in Figs. 11 and 12 are presented in Tables V and VI, respectively. λ_p is the ratio of the total pipe losses to the total cable conductor losses in the circuit. T_{41} is the thermal resistance outside of a cable but inside the casing, whereas T_{42} is the thermal resistance outside the casing. The combinatorial optimization algorithm parameters are as follows: $N_{\text{pop}} = 50, N_{\text{clones}} = 5, N_{\text{in}} = 1$ and $N_{\text{out}} = 1$.

V. Conclusion

This paper presents a method for configuring the locations of any number of cables, with the objective of obtaining either the largest or smallest total ampacity. The largest ampacity configuration is useful when designing for a limited space, whereas the lowest ampacity configuration is important when information regarding the actual installed configuration is lost, and thus a worst-case scenario is considered. The optimal configuration is obtained through a two-level optimization algorithm. At the outer level, a combinatorial optimization algorithm that is based on the VIS algorithm explores the different possible configurations. The performance of every configuration is evaluated according to its total ampacity that is calculated using a convex optimization algorithm. The convex optimization algorithm, which forms the inner level of the overall optimization algorithm, is based on the barrier method. The proposed method is tested for a duct bank installation containing twelve cables and fifteen ducts, comprising two circuits and two cables per phase, and for an installation consisting of a single circuit in a large steel casing. The results obtained show that the proposed method for the duct bank case is on the order of 10^6 times faster than using a brute force method of trying all possible configurations. The steel casing example shows the effect of cable configuration on casing losses and thus the ampacity of the circuit.

Appendix
Barrier Method Algorithm

The convex optimization problem given in (6) is of the following form:

$$\begin{aligned} &\text{Minimize} \quad f_0(x) \\ &\text{subject to} \quad f_i(x) \le 0 \quad i = 1, \ldots, n \end{aligned}$$

where $x = [I_1 \quad I_2 \quad \cdots \quad I_n]^T$.

This problem can be solved by using the barrier method algorithm as follows:

Given: strictly feasible $x, t = t^{(0)}, \mu > 1$, tolerance $\varepsilon_{\text{out}} > 0$

Repeat:

1) Compute the solution of $\text{Minimize } f_0(x) + (1/t)\varphi, x^*(t)$, starting at x, using Newton's method as follows:

   Given: starting point feasible $x^*, (x^* = x)$, tolerance $\varepsilon_{\text{in}} > 0, \alpha \in (0, 1/2), \beta \in (0, 1)$.

   Repeat:

   1) Compute Δx^* by solving the matrix equation:

$$\left[\nabla^2 f_0(x^*) + \frac{1}{t}\nabla^2 \varphi(x^*)\right] [\Delta x^*] = \underbrace{\left[-\nabla f_0(x^*) - \frac{1}{t}\nabla \varphi(x^*)\right]}_{r(x^*)}$$

   2) Compute damping factor t_{in} for updating x^*:
      $t_{\text{in}} = 1$
      while norm $(r(x^* + t_{\text{in}}\Delta x^*)) \ge (1 - \alpha \cdot t_{\text{in}}) \cdot norm(r(x^*))$
      $t_{\text{in}} = \beta \cdot t_{\text{in}}$
   3) Update $x^* = x^* + t_{\text{in}}\Delta x^*$.
   4) Stopping criterion: stop if $norm(r(x^*)) \le \varepsilon_{\text{in}}$.
2) Update $x = x^*(t)$
3) Stopping criterion: stop if $n/t < \varepsilon_{\text{out}}$
4) Increase $t = \mu \cdot t$

where $\varphi = -\sum_{i=1}^{n} \log(-f_i(x))$. ∇f_0 and $\nabla^2 f_0$ are the gradient and hessian of the objective function f_0, respectively, and $\nabla \phi$ and $\nabla^2 \phi$ are the gradient and hessian of the function ϕ, respectively.

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