Fuzzy Modelling and Consensus of Nonlinear Multiagent Systems With Variable Structure

Wenjun Xiong, Wenwu Yu, *Member, IEEE*, Jinhu Lü, *Fellow, IEEE*, and Xinghuo Yu, *Fellow, IEEE*

***Abstract*—The consensus problem of multiagent nonlinear systems (MANNs) with variable structure is discussed in this paper. T-S fuzzy models are first presented to describe MANNs with variable structure. The nodes of each T-S fuzzy model are rearranged so that the global fuzzy model is decomposed into independent and small-scale fuzzy models. It is shown that the consensus of the global fuzzy model is equivalent to that of its corresponding small-scale fuzzy models in which the continuous and sampled controllers are applied. Sufficient conditions are derived to ensure the consensus of the controlled fuzzy models. Finally, simulation results are given to illustrate the effectiveness of the proposed criteria.**

***Index Terms*—Fuzzy modeling, MANNs with variable structure, nodes rearrangement, graph Laplacian.**

I. Introduction

In recent years, the model of multiagent systems has been utilized more and more widely in the study of biological, social and engineering systems, such as group coordinated robots, sensor systems, fish school and so on. An important application area of multiagent systems is the distributed coordination problem, since the pioneering work stemming from management science and statistics in 1960s (see [1] and the references therein). One of the critical research problems is how to control all the agents in a system to reach a consensus. Consensus is a basic and fundamental research topic in the study of systems control of dynamic agents and has attracted great attention which is partly due to its broad applications in cooperative control of unmanned air vehicles, formation control of mobile robots, control of communication systems, design of sensor systems, flocking of social insects, swarm-based computing, etc. ([2]–[13]). It has been shown that consensus in a system with a dynamically changing topology can be reached if and only if the time-varying system topology contains a spanning tree frequently enough as the system evolves with time in [2]–[5]. In [9], a distributed linear consensus protocol with second-order dynamics has been designed, where both the current and some sampled past position data are utilized. It has also been shown in [12] that, for neutrally stable agents, there exists a protocol achieving consensus with a consensus region that is the entire open right-half plane if and only if each agent is stabilisable and detectable.

In the literature related to the consensus problem of multiagent systems, a connection between the nodes is assumed to either always exist or be always nonexistent. Obviously, this assumption is unrealistic. In many real cases, the structures of networks are always variable. For example, the existing connections between the nodes may not work properly, while the previous nonexistent connections may be joined from time to time due to all kinds of inherent and external influences. System structures may be changed due to some influences such as time, external environment, and temperature. Clearly, such influences are too complicated to be described clearly. Over the past few decades, the Takagi-Sugeno (T-S) fuzzy model has been proven to be an effective model to describe many nonlinear and complex systems with unstructured uncertainty ([14]–[25]). Motivated by the characteristics of the fuzzy model, we shall try to regard the obscure influences caused by the varying system connections as a fuzzy set. From the coordinated point of view, T-S fuzzy model can be used to describe the different systems communicating through the links which can cooperatively eliminate the uncertainties in a system. As a result, T-S fuzzy models will be applied to describe a system with variable structure.

As we know, MANNs may not reach a consensus when its connections are varying. Hence, effective control schemes have to be designed to force the complex system achieving a consensus. Generally, continuous controllers are usually designed due to their convenience and simplicity. However, some real-world applications can be modeled by continuous-time systems together with some discrete-time controllers such as impulsive responses, sampled data, quantization and so on. As a result, in this paper, the sampled controller will be also used, which is memoryless and easy to be designed since only information at some particular time intervals is needed. Hence, T-S fuzzy systems will be taken into account to describe multiagent models with variable structure in this paper. Moreover, the continuous

Manuscript received March 25, 2013; revised June 13, 2013; accepted July 09, 2013. Date of publication October 16, 2013; date of current version March 25, 2014. This work was supported in part by the educational commission of Sichuan provinceunder grant 12ZA198, in part by the National Natural Science Foundation of Chinaunder Grant 61203146 and 61104145, in part by the Natural Science Foundation of Jiangsu Province of Chinaunder Grant BK2011581, in part by the Research Fund for the Doctoral Program of Higher Education of Chinaunder Grant 20110092120024, and in part by the Fundamental Research Funds for the Central Universities of China. This paper was recommended by Associate Editor P. Shi.

W. Xiong is with the College of Science, Southwest Petroleum University, Chengdu 610500, China (e-mail: xwenjun2@gmail.com, xwenjun@swpu.edu.cn).

W. Yu is with the Department of Mathematics, Southeast University, Nanjing 210096, China and also with the School of Electrical and Computer Engineering, RMIT University, Melbourne VIC 3001, Australia (e-mail: wenwuyu@gmail.com; wwyu@seu.edu.cn).

J. Lü is with the Institute of Systems Science, Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing 100190, China and also with the School of Electrical and Computer Engineering, RMIT University, Melbourne VIC 3001, Australia (e-mail: jhlu@iss.ac.cn).

X. Yu is with the School of Electrical and Computer Engineering, RMIT University, Melbourne VIC 3001, Australia (e-mail: xinghuo.yu@rmit.edu.au).

Digital Object Identifier 10.1109/TCSI.2013.2283780

and sampled controllers will be designed to achieve the consensus of multiagent systems with variable structure respectively. The contribution of this paper is presented as follows:

1) T-S fuzzy systems will be first presented to address multiagent models with variable structure in this paper. For the proposed fuzzy models, a node-rearrangement method will be used to decompose the large-scale fuzzy models into independent and small-scale fuzzy models. Moreover, the consensus of every large-scale fuzzy model is equivalent to that of its corresponding small-scale fuzzy models.
2) Continuous pinning and sampled controllers will be designed for the small-scale fuzzy models. With sampled controllers, the controlled fuzzy models are hybrid systems. The sampled controllers will be then considered as continuous delayed controllers with the transformation in [26]. The hybrid fuzzy models are changed as continuous fuzzy systems with time-varying delays. A simple control method will be used to achieve a prescribed consensus.

The remainder of this paper is organized as follows. In Section 2, some definitions about directed graph are presented. The problem formulation and nodes rearrangement approach are addressed in Section 3. In Section 4, T-S fuzzy systems are first presented to address multiagent models with variable structure. Then, continuous and sampled controllers are designed to achieve the prescribed consensus. In Section 5, simulations are carried out to illustrate the effectiveness of the main results. Finally, conclusions are drawn in Section 6.

II. Preliminaries

Let $\mathcal{G}(\mathcal{V},\varepsilon,\mathcal{A})$ be a digraph of order n with the set of nodes $\mathcal{V}=\{v_1,v_2,\ldots,v_n\}$, the set of edges $\varepsilon\subseteq\mathcal{V}\times\mathcal{V}$, and a weighted adjacency matrix $\mathcal{A}=(a_{ij})_{n\times n}$. An edge of $\mathcal{G}$ is denoted by $e_{ij}=(v_i,v_j)$, where $e_{ij}\in\varepsilon$ means that there is a directed connection from node v_j to node v_i. That is, node v_j can send information to node v_i. The entry $a_{ij}>0$ if $e_{ij}\in\varepsilon$, and $a_{ij}=0$ otherwise. Moreover, it is assumed that $a_{ii}=0$ for all $i\in\{1,2,\ldots,n\}$. The Laplacian of the directed graph is defined as $L=(l_{ij})_{n\times n}=\Delta-\mathcal{A}$, and $\Delta=(\Delta_{ij})_{n\times n}$ is a diagonal matrix with $\Delta_{ii}=\sum_{j=1}^{n}a_{ij}$.

In a digraph, a directed path is an ordered sequence of vertices such that from each of its vertices there is an edge to the next vertex. If there is a directed path between any pair of distinct nodes, the digraph is said to be *strongly connected*. A digraph is *undirected* if $a_{ij}=a_{ji}$ for all $i,j\in\{1,2,\ldots,n\}$. Obviously, the Laplacian of an undirected graph is symmetric. A directed graph is called *weakly connected* if replacing all of its directed edges with undirected edges produces a connected undirected graph. A digraph is a *spanning tree* if it has m vertices and $m-1$ edges and there exists a root vertex with directed paths to all other vertices.

Assume that a system has n agents, and each agent is regarded as a node in a directed graph $\mathcal{G}$. Let $\bar{x}_i(t)\in R$ denote the state of agent v_i, then $\mathcal{G}_{\bar{x}}=(\mathcal{G},\bar{x}(t))$ with $\bar{x}(t)=(\bar{x}_1(t),\bar{x}_2(t),\ldots,\bar{x}_n(t))^T$ is a directed system. Agents v_i and v_j in the directed system are said to reach a (an) *consensus* (*agreement*) if and only if $\|\bar{x}_i(t)-\bar{x}_j(t)\|\to 0$ as $t\to+\infty$, for any $i,j\in\{1,2,\ldots,n\},i\neq j$. If the nodes are all in an agreement, the common value $\mathcal{X}(\bar{x})$ is called the *group decision value*.

The general multiagent nonlinear system without possible missing connections has the following dynamics

$$\frac{d\bar{x}_i(t)}{dt}=\sum_{v_j\in N_i}a_{ij}(f(\bar{x}_j(t))-f(\bar{x}_i(t)))\tag{1}$$

where $i=1,2,\ldots,n,N_i=\{v_j\in\mathcal{V}:(v_i,v_j)\in\varepsilon\}$ is the set of neighbors of node v_i, $i,j=1,2,\ldots,n,\bar{x}_i(t)$ is the state of agent v_i, and $f(\bar{x}_i(t))$ is a nonlinear function which has the same dimension with $\bar{x}_i(t)$. The dimension of $\bar{x}_i(t)$ could be arbitrary as long as it is the same for all agents. $A=(a_{ij})_{n\times n}\in R^{n\times n}$ is the weighted matrix. In this paper, for simplification, we only analyze the case when the dimension of $\bar{x}_i(t)$ is one. It is worth noticing that our analysis is valid for any dimension when the system models are rewritten with Kronecker products.

According to the definition of the Laplacian matrix L, (1) can be rearranged as

$$\frac{d\bar{x}(t)}{dt}=-L\bar{F}(\bar{x}(t))\tag{2}$$

where $\bar{x}(t)=(\bar{x}_1(t),\bar{x}_2(t),\ldots,\bar{x}_n(t))^T$ and $\bar{F}(\bar{x}(t))=(f(\bar{x}_1(t)),f(\bar{x}_2(t)),\ldots,f(\bar{x}_n(t)))^T$.

Notation: Throughout this paper, I stands for the identity matrix. The superscript "T" represents the transpose. For all $x=(x_1,x_2,\ldots,x_n)^T\in R^n$, $\|x\|=(\sum_{i=1}^{n}x_i^2)^{(1/2)}$. For a symmetric matrix A, $\lambda_m(A)$ and $\lambda_M(A)$ denote the minimal and maximal eigenvalues of matrix A respectively. $\|A\|$ denotes the spectral norm defined by $\|A\|=(\lambda_M(A^TA))^{(1/2)}$. For real symmetric matrixes X and Y, $X>Y$ (or $X\geq Y$) means that matrix $X-Y$ is positive definite (or positive semi-define).

III. Problem Formulation and Nodes Rearrangement Approach

A. Problem Formulation

In this paper, we consider that the connections between nodes may change in the process of information transmission due to some affections such as time, external environment, temperature and so on. Here, the mentioned affections are obscure and difficult to be elaborated clearly, which can be regarded as a fuzzy set. A fuzzy dynamic model has been proposed by Takagi and Sugeno [27] to represent different linear/nonlinear systems of different rules. Based on this, we shall construct T-S fuzzy models to describe multiagent systems with variable structure.

Similar to [18], [21], we consider a T-S fuzzy multiagent model, in which the ith rule is formulated in the following form:
Plant Rule i:
IF $\bar{x}_1(t)$ is η_{i1} and $\bar{x}_2(t)$ is η_{i2} and $\cdots$ and $\bar{x}_n(t)$ is η_{in},
THEN

$$\frac{d\bar{x}(t)}{dt}=-\bar{L}_i\bar{F}(\bar{x}(t)),\quad i=1,2,\ldots,r\tag{3}$$

where $\bar{x}(t)=(\bar{x}_1(t),\bar{x}_2(t),\ldots,\bar{x}_n(t))^T$, η_{ij} is the fuzzy set, and $\bar{L}_i$ is the ith Laplacian matrix corresponding to the ith rule.

Remark 1: Note that, the network structures may be changed due to some influences such as time, external environment and temperature. Many influences are too complicated to be de-

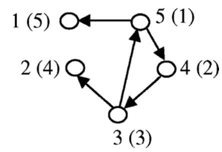

Fig. 1. A directed graph with 5 nodes.

scribed clearly. As we know, the T-S fuzzy model has been proven to be an effective model to describe many nonlinear and complex systems with unstructured uncertainty. Hence, the fuzzy model (3) is proposed to describe some MANNs with variable structure.

The defuzzified output of the T-S fuzzy system (3) is represented as shown in the following

$$\begin{aligned}\frac{d\bar{x}(t)}{dt} &= \frac{\sum_{i=1}^{r} v_i(\bar{x}(t))[-\bar{L}_i\bar{F}(\bar{x}(t))]}{\sum_{i=1}^{r} v_i(\bar{x}(t))}\\ &= -\sum_{i=1}^{r} h_i(\bar{x}(t))\bar{L}_i\bar{F}(\bar{x}(t)), \quad i=1,2,\ldots,r, \end{aligned} \tag{4}$$

where $v_i(\bar{x}(t)) = \Pi_{j=1}^{n}\eta_{ij}(\bar{x}_j(t))$, $h_i(\bar{x}(t)) = (v_i(\bar{x}(t)))/(\sum_{i=1}^{r} v_i(\bar{x}(t)))$, and $\eta_{ij}(\bar{x}_j(t))$ is the membership function of $\bar{x}_j(t)$ in η_{ij}.

A basic property of $v_i(\bar{x}(t))$ is that

$$v_i(\bar{x}(t)) \geq 0, \quad i=1,2,\ldots,r, \quad \sum_{j=1}^{r} v_j(\bar{x}(t)) > 0 \tag{5}$$

and therefore,

$$h_i(\bar{x}(t)) \geq 0, \quad i=1,2,\ldots,r, \quad \sum_{j=1}^{r} h_j(\bar{x}(t)) = 1 \tag{6}$$

for $\forall\ t \in R$.

B. Nodes Rearrangement and Control Law

In this paper, we do not assume that graph $\mathcal{G}$ contains a spanning tree. Clearly, fuzzy system (4) may not reach a consensus when its connections are varying. As a result, some control schemes have to be designed to achieve a consensus of fuzzy system (4). As mentioned in [28], it is difficult to know which nodes needed to be controlled for a large-scale system. Hence, we shall rearrange the node order of fuzzy system (4) in the following. In a graph, those root nodes are called the leaders, and the other nodes are called the followers.

For the original graph G_i, construct the rearranged graph $\hat{G}_i$ as follows:

Algorithm 1

1) Note that G_i may not be connected. For any $i \in \{1,2,\ldots,r\}$, find out all Strongly Connected Components G_i^j $(j \in \{1,2,\ldots,\Lambda\}, \iota = \sum_{j=1}^{\Lambda} i_j \leq n, i_j \geq 1$ is the number of nodes in G_i^j and $\Lambda \leq n$ is an integer) of graph G_i by using the algorithms in [29], [30]. Then, the nodes in G_i^j are all root nodes of graph G_i and the other nodes are all followers.
2) Rearrange the numerical orders of all nodes. Mark all root nodes as $1,2,\ldots$ and number the followers behind the root nodes.
3) Graph G_i is rearranged as graph $\hat{G}_i$. The Laplacian matrix L_i of graph $\hat{G}_i$ can be written as

$$\begin{pmatrix} L_i^1 & & & & 0 & \cdots & 0\\ & L_i^2 & & & 0 & \cdots & 0\\ & & \ddots & & \vdots & \cdots & \vdots\\ & & & L_i^{\Lambda} & 0 & \cdots & 0\\ L_i^{\iota+1,1} & L_i^{\iota+1,2} & \cdots & L_i^{\iota+1,\iota} & L_i^{\iota+1,\iota+1} & \cdots & 0\\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots\\ L_i^{n,1} & L_i^{n,2} & \cdots & L_i^{n,\iota} & L_i^{n,\iota+1} & \cdots & L_i^{n,n} \end{pmatrix}.$$

Here, L_i^j $(j \in \{1,2,\ldots,\Lambda\})$ are irreducible square matrices and in each line of the rest $n-\iota$ lines, there exists at least one entry satisfying $L_i^{\iota+i,j} \neq 0$ $(i = 1,2,\ldots,n-\iota, j = 1,2,\ldots,\iota+i)$.

Remark 2: A simple example is given to illustrate Algorithm 1. In Fig. 1, one knows that nodes 3, 4 and 5 are the root nodes, and nodes 1 and 2 are followers. Then, rearrange the numerical orders of all nodes. Number nodes 1, 2, 3, 4 and 5 to be nodes 5, 4, 3, 2 and 1, respectively. As a result, the followers are all numbered behind the root nodes. The Laplacian matrix of the rearranged graph can be written as

$$\begin{pmatrix} 1 & 0 & -1 & 0 & 0\\ -1 & 1 & 0 & 0 & 0\\ 0 & -1 & 1 & 0 & 0\\ 0 & 0 & -1 & 1 & 0\\ -1 & 0 & 0 & 0 & 1 \end{pmatrix}.$$

Clearly, the above matrix is similar to that in Algorithm 1. □

Based on Algorithm 1, the fuzzy system (3) can be rewritten as

Plant Rule i:
IF $\bar{x}_1(t)$ is η_{i1} and $\bar{x}_2(t)$ is η_{i2} and $\cdots$ and $\bar{x}_n(t)$ is η_{in},
THEN

$$\frac{dx(t)}{dt} = -L_iF(x(t)), \quad i=1,2,\ldots,r \tag{7}$$

where $x(t) = (x_1(t), x_2(t), \ldots, x_n(t))^T$ is a node-rearrangement of $\bar{x}(t)$, $F(x(t)) = (f(x_1(t)), f(x_2(t)), \ldots, f(x_n(t)))^T$. The defuzzified output of the T-S fuzzy system (7) is

$$\begin{aligned}\frac{dx(t)}{dt} &= \frac{\sum_{i=1}^{r} v_i(\bar{x}(t))[-L_iF(x(t))]}{\sum_{i=1}^{r} v_i(\bar{x}(t))}\\ &= -\sum_{i=1}^{r} h_iL_iF(x(t)), \end{aligned} \tag{8}$$

where $h_i(\bar{x}(t))$ is simplified as h_i.

Remark 3: According to Algorithm 1, one knows that the corresponding nodes in L_i^j $(j \in \{1,2,\ldots,\Lambda\})$ are root nodes, while the rest $n-\iota$ nodes are followers. According to the results in [28], the consensus of system (7) can be achieved if its root nodes can reach a consensus. As a result, one only needs to control the corresponding nodes of L_i^j ($j \in \{i_1, i_2, \ldots, i_\Lambda\}$) in system (7) to achieve a consensus.

We introduce a virtual leader such that system (3) wants to realize the prescribed consensus value $s_0 \in R$ (let $\bar{x}_0 = (s_0, s_0, \ldots, s_0)^T \in R^n$). Correspondingly, the prescribed consensus vector of system (8) is also $x_0 = (s_0, s_0, \ldots, s_0)^T \in R^n$). We consider two kinds of control laws for the fuzzy system (8) as follows:

i) The continuous control law is
Controller Rule i:
IF $\bar{x}_1(t)$ is η_{i1} and $\bar{x}_2(t)$ is η_{i2} and $\cdots$ and $\bar{x}_n(t)$ is η_{in}, THEN

$$u_i(t) = K_i(x(t) - x_0), \quad i = 1, 2, \ldots, r \tag{9}$$

where $K_i = diag(K_{i1}, K_{i2}, \ldots, K_{in})$ is a diagonal matrix and $K_{ij} \geq 0$ for any $j \in \{1, 2, \ldots, n\}$. The overall fuzzy controller can be given by $u(t) = \sum_{i=1}^r h_i K_i(x(t) - x_0)$. Note that $\sum_{i=1}^r h_i = 1$, the T-S fuzzy continuous control system of model (8) is governed by

$$\frac{dx(t)}{dt} = -\sum_{i=1}^r \sum_{p=1}^r h_i h_p [L_i F(x) + K_p(x(t) - x_0)]. \tag{10}$$

ii) The sampled control law is
Controller Rule i:
IF $\bar{x}_1(t)$ is η_{i1} and $\bar{x}_2(t)$ is η_{i2} and $\cdots$ and $\bar{x}_n(t)$ is η_{in}, THEN

$$u_i(t) = u_i(t_l) = S_i(x(t_l) - x_0), t_l \leq t < t_{l+1} \tag{11}$$

where $i = 1, 2, \ldots, r, S_i = diag(S_{i1}, S_{i2}, \ldots, S_{in})$ is a diagonal matrix. The discrete-time control signal is assumed to be generated by a zero-order hold function with a sequence of hold times $0 = t_0 < t_1 < \cdots < t_l < \cdots$. Here, $\forall l \geq 0, t_{l+1} - t_l \leq h$, $\lim_{t\to\infty} t_l = \infty$ and h is a positive number. Similar to the continuous control law, the overall sampled fuzzy controller can be given by $u(t_l) = \sum_{i=1}^r h_i S_i(x(t_l) - x_0)$. As a result, the T-S fuzzy discrete-time control system of model (8) is governed by

$$\frac{dx(t)}{dt} = -\sum_{i=1}^r \sum_{p=1}^r h_i h_p [L_i F(x) + S_p(x(t_l) - x_0)]. \tag{12}$$

In the following, our objective is to design continuous and sampled controllers to ensure the consensus of fuzzy systems (10) and (12).

IV. Main Results

The following assumption and lemma are needed for our main results.

Assumption 1: Nonlinear function $f(\cdot)$ satisfies that $f(\cdot) \in S$, where S denotes a set of nonlinear functions, and each $s \in S$ is continuous and strictly increasing. Moreover, for each $f(\zeta) \in S$, $f(\zeta) = 0 \Leftrightarrow \zeta = 0, \forall \zeta \in R$.

Lemma 1: For any vectors $x, y \in R^n$ and scalar $\varepsilon > 0$, the following inequality holds:

$$2x^T y \leq \varepsilon x^T x + \varepsilon^{-1} y^T y.$$

Let $y(t) = x(t) - x_0$, then Systems (10) and (12) can be respectively rewritten as

$$\frac{dy(t)}{dt} = -\sum_{i=1}^r \sum_{p=1}^r h_i h_p [L_i G(y(t)) + K_p y(t)], \tag{13}$$

and

$$\frac{dy(t)}{dt} = -\sum_{i=1}^r \sum_{p=1}^r h_i h_p [L_i G(y(t)) + S_p y(t_l)], \tag{14}$$

where $G(y(t)) = (g(y_1(t)), g(y_2(t)), \ldots, g(y_n(t)))^T = F(x(t)) - F(x_0)$ and $y(t_l) = x(t_l) - x_0$.

Following [26], we represent the digital control law in (14) as a delayed control as follows:

$$u_i(t_l) = u_i(t - \tau(t)), \tau(t) = t - t_l, t_l \leq t < t_{l+1} \tag{15}$$

where $i = 1, 2, \ldots, r$. For simplification, let $g(y_i(t)) = y_i(t)$ in (14), $i = 1, 2, \ldots, n$. As a result, (14) can be rewritten as

$$\frac{dy(t)}{dt} = -\sum_{i=1}^r \sum_{p=1}^r h_i h_p [L_i y(t) + S_p y(t - \tau(t))]. \tag{16}$$

In the following, our objective is to analyze the asymptotical stability of (13) and (16).

Theorem 1: Consider the fuzzy system (13). Under Assumption 1, for any initial values, system (13) can realize asymptotical stability (i.e., system (10) can achieve the prescribed consensus) if matrix $K_p^j = diag(K_{p,\iota_{j-1}+1}^j, K_{p,\iota_{j-1}+2}^j, \ldots, K_{p,\iota_{j-1}+i_j}^j)$ ($j \in \{1, 2, \ldots, \Lambda\}, \iota_j = \sum_{k=1}^j p_k, \iota_0 = 0$) satisfies that there exists at least one $K_{p\upsilon}^j > 0$, for any $p \in \{1, 2, \ldots, r\}, \upsilon \in \{\iota_{j-1}+1, \iota_{j-1}+2, \ldots, \iota_{j-1}+i_j\}$, where $K_p = diag(K_p^1, K_p^2, \ldots, K_p^\Lambda, 0 \ldots, 0)$ and $K_p^j \in R^{i_j \times i_j} (j \in \{1, 2, \ldots, \Lambda\}, i_j \geq 1$ is the number of nodes in G_i^j) are nonnegative diagonal matrices.

Proof: According to Remark 3, one only needs to control the corresponding nodes of $L_i^j (j \in \{1, 2, \ldots, \Lambda\})$ in system (13). That is, we can define $K_p = diag(K_p^1, K_p^2, \ldots, K_p^\Lambda, 0 \ldots, 0)$ and $K_p^j \in R^{i_j \times i_j}$ are nonnegative diagonal matrices. As a result, system (13) can realize asymptotical stability if and only if the following systems can reach asymptotical stability

$$\frac{dy^j(t)}{dt} = -\sum_{i=1}^r \sum_{p=1}^r h_i h_p \left[L_i^j G(y^j(t)) + K_p^j y^j(t)\right] \tag{17}$$

where $j \in \{1, 2, \ldots, \Lambda\}, y^j(t) = (y_{\iota_{j-1}+1}, y_{\iota_{j-1}+2}, \ldots, y_{\iota_{j-1}+i_j})^T, i_0 = 0$ and $G(y^j(t)) = (g(y_{\iota_{j-1}+1}), g(y_{\iota_{j-1}+2}), \cdots, g(y_{\iota_{j-1}+i_j}))^T$.

Clearly, $g(\cdot)$ satisfies the conditions in Assumption 1. Hence, one can construct the following Lyapunov function

$$V(y^j(t)) = \sum_{a=\iota_{j-1}+1}^{\iota_{j-1}+i_j} \xi_{i,a} \int_0^{y_a(t)} g(r)dr \tag{18}$$

where $j \in \{1,2,\dots,\Lambda\}, \xi_i = (\xi_{i,\iota_{j-1}+1}, \xi_{i,\iota_{j-1}+2}, \dots, \xi_{i,\iota_{j-1}+i_j})^T$ is the left eigenvalue of matrix L_i^j with zero eigenvalue. It is well know that each element of vector ξ_i is greater than 0 since L_i^j is an irreducible matrix [31].

Clearly, $V(y^k(t))$ is positive definite and radially unbounded. The time derivative of $V(y^k(t))$ along the solution of system (17) is

$$\frac{dV(y^k(t))}{dt} = -\sum_{i=1}^{r}\sum_{p=1}^{r} h_i h_p \Big[G^T(y^k(t))\tilde{B}_i^k G(y^k(t)) + G^T(y^k(t))\Xi K_p^k y^k(t)\Big], \tag{19}$$

where $k \in \{1,2,\dots,\Lambda\}, \Xi = diag(\xi_{i,\iota_{j-1}+1}, \xi_{i,\iota_{j-1}+2}, \dots, \xi_{i,\iota_{j-1}+i_j})$ and $\tilde{B}_i^k = (1/2)(\Xi L_i^k + (L_i^k)^T \Xi)$. It is easy to prove that $\tilde{B}_i^k$ is irreducible, symmetric and with zero-row-sum. That is, the eigenvalues of $\tilde{B}_i^k$ are greater than or equal to zero [31]. Note that there exists at least one $K_{p\upsilon}^k > 0$ in matrix K_p^k for $\upsilon \in \iota_{j-1}+1, \iota_{j-1}+2, \dots, \iota_{j-1}+i_j$. One has that $(dV(y^k(t)))/(dt) \le 0$ and $(dV(y^k(t)))/(dt) = 0$ if and only if $y_{\iota_{j-1}+1}(t) = y_{\iota_{j-1}+2}(t) = \cdots = y_{\iota_{j-1}+p_j}(t) = 0$.

From LaSalle's invariant principle, one can conclude that $x_{\iota_{j-1}+1}(t) = x_{\iota_{j-1}+2}(t) = \cdots = x_{\iota_{j-1}+i_j}(t) \to x_0, t \to \infty, j = 1,2,\dots,\Lambda$. As a result, $y_k(t) \to 0, t \to \infty$ and $x_k(t) \to x_0, t \to \infty, k = 1,2,\dots,\iota$. Note that, the rest $(n-\iota)$ nodes of system (13) are all followers. According to the results in [28], one can obtain that $y_k(t) \to 0, t \to \infty$ and $x_k(t) \to x_0, t \to \infty, k = 1,2,\dots,n$. That is, system (13) realizes asymptotical stability, i.e., system (10) achieves a consensus. The proof is completed.

Theorem 2: Consider the fuzzy system (16). If there exist matrices $U_{ip}^k > 0$, Q_{ip}^k and M_{ip}^k with appropriate dimension such that

$$\Phi_1 = \begin{pmatrix} \Upsilon_1 & Q_{ip}^k + M_{ip}^k L_i^k + M_{ip}^k S_p^k \\ * & hU_{ip}^k + M_{ip}^k + \left(M_{ip}^k\right)^T \end{pmatrix} < 0, \tag{20}$$

and

$$\Phi_2 < 0, \quad \forall\ i,p = 1,2,\dots,r, \tag{21}$$

where

$$\Phi_2 = \begin{pmatrix} \Upsilon_1 & Q_{ip}^k + M_{ip}^k L_i^k + M_{ip}^k S_p^k & -h(Q_{ip}^k)^T S_p^k \\ * & M_{ip}^k + (M_{ip}^k)^T & -hM_{ip}^k S_p^k \\ * & * & -hU_{ip}^k \end{pmatrix},$$

$\Upsilon_1 = (Q_{ip}^k)^T L_i^k + (L_i^k)^T Q_{ip}^k + (Q_{ip}^k)^T S_p^k + S_p^k Q_{ip}^k$, system (16) can realize asymptotical stability, i.e., system (12) can achieve the prescribed consensus. Here, $k \in \{1,2,\dots,\Lambda\}, S_p = diag(S_p^1, S_p^2, \dots, S_p^\Lambda, 0\dots,0)$ and $S_p^k \in R^{i_k \times i_k}$ ($i_k \ge 1$ is the number of nodes in G_i^j) are diagonal matrices.

Proof: Similar to Theorem 1, system (16) can realize asymptotical stability if and only if the following systems can reach asymptotical stability

$$\frac{dy^k(t)}{dt} = -\sum_{i=1}^{r}\sum_{p=1}^{r} h_i h_p \left[L_i^k y^k(t) + S_p^k y(t-\tau(t))\right] \tag{22}$$

where $k \in \{1,2,\dots,\Lambda\}$, $S_p = diag(S_p^1, S_p^2, \dots, S_p^\Lambda, 0, \dots, 0)$ and $S_p^k \in R^{i_k \times i_k}(k \in \{1,2,\dots,\Lambda\})$ are diagonal matrices. Construct the following Lyapunov function

$$V^k(t) = (h-\tau(t)) \int_{t-\tau(t)}^{t} (\dot{y}^k(s))^T U_{ip}^k \dot{y}^k(s)ds \tag{23}$$

where $k \in \{1,2,\dots,\Lambda\}$. Note that $V^k(t)$ does not increase along the jumps $t_0, t_1, t_2, \dots$ sine $V^k(t) \ge 0$ and $V^k(t) = 0$ at the jumps $t_1, t_2, \dots$. Thus, the condition $\lim_{t\to t_l^-} V^k(t) \ge V^k(t_l)$ holds.

Since $(dy^k(t-\tau(t)))/(dt) = (1-\dot{\tau}(t))\dot{y}^k(t-\tau(t)) = 0$, one has

$$\frac{dV^k(t)}{dt} = -\int_{t-\tau(t)}^{t} (\dot{y}^k(s))^T U_{ip}^k \dot{y}^k(s)ds + (h-\tau(t))(\dot{y}^k(t))^T U_{ip}^k \dot{y}^k(t). \tag{24}$$

Denoting $V_1 = (1)/(\tau(t)) \int_{t-\tau(t)}^{t} \dot{y}^k(s)ds$, one has that $\lim_{\tau(t)\to 0} V_1 = \dot{y}^k(t)$. From [26], one has that

$$\int_{t-\tau(t)}^{t} (\dot{y}^k(s))^T U_{ip}^k \dot{y}^k(s)ds \ge \tau(t)(V_1)^T U_{ip}^k V_1. \tag{25}$$

From (22) and $\sum_{i=1}^{r} h_i = 1$, one obtains that

$$\sum_{i=1}^{r}\sum_{p=1}^{r} h_i h_p \left[L_i^k y^k(t) + S_p^k y^k(t-\tau(t)) + \dot{y}^k(t)\right] = 0.$$

Then, one has

$$\sum_{i=1}^{r}\sum_{p=1}^{r} h_i h_p \left[(y^k(t))^T \left(Q_{ip}^k\right)^T + (\dot{y}^k(t))^T M_{ip}^k\right] \left[L_i^k y^k(t) + S_p^k y^k(t) - S_p^k \int_{t-\tau(t)}^{t} \dot{y}^k(s)ds + \dot{y}^k(t)\right] = 0, \tag{26}$$

where some $i_k \times i_k$ ($k \in \{1,2,\dots,\Lambda\}$) matrices Q_{ip}^k, M_{ip}^k are added into the left-hand side of (26).

Combining (24), (25) and (26), one has from $\sum_{i=1}^{r} h_i = 1$

$$\frac{dV^k(t)}{dt} = \sum_{i=1}^{r}\sum_{p=1}^{r} h_i h_p \Bigg\{-\int_{t-\tau(t)}^{t} (\dot{y}^k(s))^T U_{ip}^k \dot{y}^k(s)ds + (h-\tau(t))(\dot{y}^k(t))^T U_{ip}^k \dot{y}^k(t) + 2\left[(y^k(t))^T \left(Q_{ip}^k\right)^T + (\dot{y}^k(t))^T M_{ip}^k\right] \left[L_i^k y^k(t) + S_p^k y^k(t) - S_p^k \int_{t-\tau(t)}^{t} \dot{y}^k(s)ds + \dot{y}^k(t)\right]\Bigg\}$$

$$\begin{aligned} &\leq \sum_{i=1}^{r}\sum_{p=1}^{r} h_i h_p \left[-\tau(t)(V_1)^T U_{ip}^k V_1 + (h-\tau(t))(\dot{y}^k(t))^T\right. \\ &\times U_{ip}^k \dot{y}^k(t) + 2(y^k(t))^T \left(Q_{ip}^k\right)^T L_i^k y^k(t) + 2(y^k(t))^T \\ &\times \left(Q_{ip}^k\right)^T S_p^k y^k(t) - 2(y^k(t))^T \left(Q_{ip}^k\right)^T S_p^k \tau(t) V_1 \\ &+ 2(y^k(t))^T \left(Q_{ip}^k\right)^T \dot{y}^k(t) + 2((\dot{y}^k(t))^T M_{ip}^k L_i^k y^k(t) \\ &+ 2((\dot{y}^k(t))^T M_{ip}^k S_p^k y^k(t) - 2\tau(t)(\dot{y}^k(t))^T M_{ip}^k S_p^k V_1 \\ &\left. +2(\dot{y}^k(t))^T M_{ip}^k \dot{y}^k(t)\right]. \end{aligned} \tag{27}$$

Setting $\chi_1(t) = col\{y^k(t), \dot{y}^k(t), V_1\}$, one obtain that from (27)

$$\frac{dV^k(t)}{dt} \leq \sum_{i=1}^{r}\sum_{p=1}^{r} h_i h_p (\chi_1(t))^T \Phi \chi_1(t) \tag{28}$$

where

$$\Phi = \begin{pmatrix} \Upsilon_1 & \Upsilon_2 & -\tau(t)\left(Q_{ip}^k\right)^T S_p^k \\ * & \Upsilon_3 & -\tau(t) M_{ip}^k S_p^k \\ * & * & -\tau(t) U_{ip}^k \end{pmatrix} < 0 \tag{29}$$

and $\Upsilon_1 = (Q_{ip}^k)^T L_i^k + (L_i^k)^T Q_{ip}^k + (Q_{ip}^k)^T S_p^k + S_p^k Q_{ip}^k$, $\Upsilon_2 = Q_{ip}^k + M_{ip}^k L_i^k + M_{ip}^k S_p^k$, $\Upsilon_3 = (h-\tau(t))U_{ip}^k + M_{ip}^k + (M_{ip}^k)^T$. In (29), $\tau(t) \to 0$ and $\tau(t) \to h$ lead to the LMIs $\Phi_1 < 0, \Phi_2 < 0$, and Φ_1, Φ_2 are shown in (20) and (21). Let $\chi_0(t) = col\{y^k(t), \dot{y}^k(t)\}$, then (20) and (21) imply (29) since $(\chi_1(t))^T \Phi \chi_1(t) = (h-\tau(t))/(h)\chi_0^T(t)\Phi_1\chi_0(t) + (\tau(t))/(h)(\chi_1(t))^T \Phi_2 \chi_1(t) < 0, \forall\ \chi_1(t) \neq 0$.

One can conclude from (20) and (21) that the inequality (29) holds. That is, the fuzzy system (16) can realize asymptotical stability, i.e., system (12) can achieve the prescribed consensus. The proof is completed.

Remark 4: Combined with Theorems 1 and 2, the advantages of Algorithm 1 are as follows:

1) All nodes are rearranged in a system so that the root nodes and the followers are clearly separated. Moreover, one only needs to control one node in each L_i^j in Theorem 1 since L_i^j $(j \in \{i_1, i_2, \ldots, i_\Lambda\})$ are irreducible square matrices. This is consistent with the intuition that the root nodes must be controlled. As a result, the method in Algorithm 1 can reduce the number of nodes needed to be controlled effectively. Moreover, the rearranged systems do not change the consensus property of the system.
2) With Algorithm 1, the asymptotical stability of system (16) is equivalent to that of system (22). One can notice that the dimension of system (22) is smaller. As a result, conditions (20) and (21) can be easily obtained.

V. Illustrative Examples

In this section, a numerical example is presented to demonstrate the effectiveness of the developed results.

Consider a group of mobile agents with 8 agents where each agent has two sensors transmitting and receiving messages over the communication links. Here, the sensors monitor physical or environmental conditions, such as temperature, sound, pressure, etc, and cooperatively pass their data through the system. How to handle the different data to achieve the final asymptotic consensus state? As mentioned in Section 3, we will use the membership function in a fuzzy setting to describe the proportion of the data in different sensors and achieve the final data of the physical condition. For simplification, we only consider that one receives two different data for the same physical condition. Hence, a T-S fuzzy system with 8 nodes is proposed and its rules are as follows:

Plant Rule 1:
IF $\bar{x}_1(t)$ is $\eta_1(\bar{x}_1(t))$,
THEN

$$\frac{d\bar{x}(t)}{dt} = -\bar{L}_1 \bar{F}(\bar{x}(t)) \tag{30}$$

Plant Rule 2:
IF $\bar{x}_1(t)$ is $\eta_2(\bar{x}_1(t))$,
THEN

$$\frac{d\bar{x}(t)}{dt} = -\bar{L}_2 \bar{F}(\bar{x}(t)) \tag{31}$$

where $\bar{x}(t) = (\bar{x}_1(t), \bar{x}_2(t), \ldots, \bar{x}_8(t))^T$ and $\bar{x}_i(t)$ is the data state of the ith sensor. $f(\bar{x}_i(t))$ is the transmitted data of the ith sensor, which is assumed to be nonlinear and $f(\bar{x}_i(t)) = (1)/(4)\sin(\bar{x}_i(t)) + (1)/(4)\bar{x}_i(t), i = 1, 2, \ldots, 8$. The weight communication matrices $\bar{L}_1$ and $\bar{L}_2$ among the 8 sensors are assumed to

$$\bar{L}_1 = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -1 & 3 & 0 & 0 & 0 & -1 \\ 0 & 0 & -1 & 0 & 4 & -1 & -1 & -1 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & -2 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix},$$

$$\bar{L}_2 = \begin{pmatrix} 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & -2 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & -1 & -2 & -2 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 \end{pmatrix}.$$

The membership function is assumed to $h_1(\bar{x}_1(t)) = (1-\sin^2(\bar{x}_1(t)))/(2), h_2(\bar{x}_1(t)) = (1+\sin^2(\bar{x}_1(t)))/(2)$. Here, h_1 and h_2 can be seen the proportion of different data in deciding the final data of the physical condition. The defuzzified output of the T-S fuzzy systems (30) and (31) is

$$\frac{d\bar{x}(t)}{dt} = -\sum_{i=1}^{2} h_i \bar{L}_i \bar{F}(\bar{x}(t)). \tag{32}$$

For an arbitrary initial vector, let $x_0 = (2, 2, \ldots, 2)^T \in R^8$ be the prescribed consensus vector. Fig. 2 shows the state responses for the uncontrolled fuzzy system, which apparently

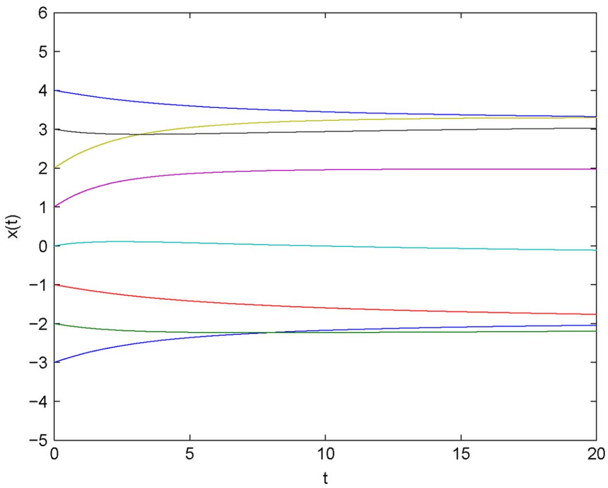

Fig. 2. The state trajectories of the uncontrolled fuzzy system (4) with 8 nodes.

cannot reach a consensus. According to Algorithm 1, $\bar{L}_1$ and $\bar{L}_2$ can be rearranged as

$$L_1 = \begin{pmatrix} 1 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & -2 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & -1 & 0 & 0 & -1 & 3 & 0 \\ 0 & 0 & -1 & -1 & -1 & -1 & 0 & 4 \end{pmatrix},$$

$$L_2 = \begin{pmatrix} 2 & 0 & -2 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & -2 & 0 & 0 & 0 & 3 & 0 \\ 0 & 0 & 0 & -1 & -2 & -2 & 0 & 5 \end{pmatrix},$$

where $L_1^1 = L_2^2 = \begin{pmatrix} 1 & 0 & -1 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$ and $L_1^2 = L_2^1 = \begin{pmatrix} 2 & 0 & -2 \\ -1 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix}$. The continuous control matrices in (17) are designed as follows: $K_1^1 = K_2^2 = diag(1, 0, 0)$ and $K_1^2 = K_2^1 = diag(0, 1, 0)$. According to Theorem 1, system (10) can achieve the prescribed consensus (i.e., system (13) can realize an asymptotical stability). Fig. 3 shows the state responses for the continuous control fuzzy system (10), which apparently reach a consensus.

The discrete-time control matrices in (17) are designed as follows: $S_1^1 = S_1^2 = S_2^1 = S_2^2 = 4I_{3\times 3}$, and define $h = 0.01$. By using the MATLAB LMI toolbox, LMIs (20) and (21) can be solved with feasible solutions. According to Theorem 2, system (16) can realize asymptotical stability. That is, system (12) (here, $F(x(t)) = x(t)$) can achieve the prescribed consensus. Fig. 4 shows the state responses for the discrete-time control fuzzy system (12), which apparently reach a consensus.

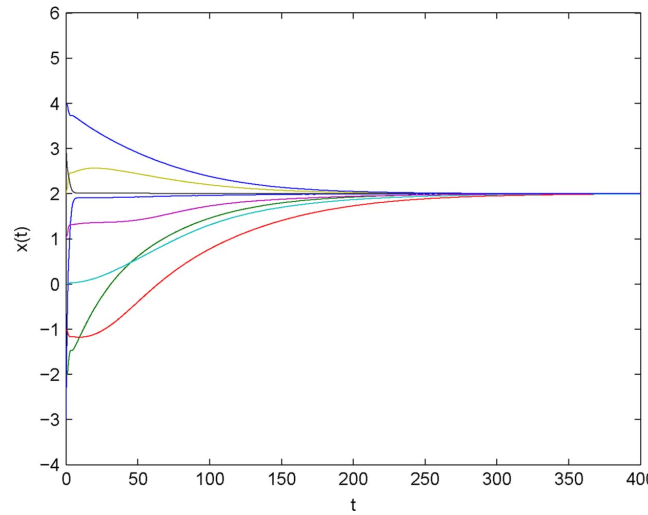

Fig. 3. The state trajectories of the continuous control fuzzy system (10) with 8 nodes.

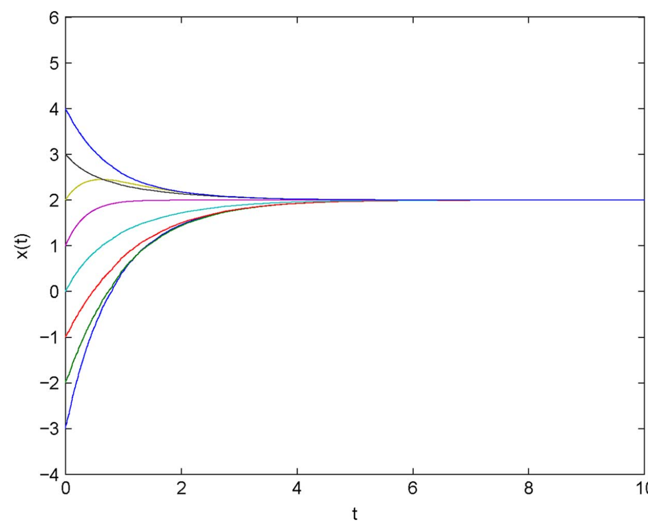

Fig. 4. The state trajectories of the sampled control fuzzy system (12) with 8 nodes.

In addition, consider a T-S fuzzy system (3) with 100 nodes and the rules (30), (31). For the weight matrix $\mathcal{A}_i$ $(i = 1, 2) = (a_{kj}^i)_{n\times n}$, let all of the connection weights be 1, that is, $a_{kj}^i = 1$ if $a_{kj}^i \neq 0$. L_i^k $(i = 1, 2)$ is the corresponding Laplacian matrix of $\mathcal{A}_i$. Define $f(x_i(t)) = (1)/(4)\tanh(x_i(t)) + (1)/(4)x_i(t), i = 1, 2, \ldots, 100$, $x(t) = (x_1(t), x_2(t), \ldots, x_{100}(t))^T$. We still let $x_0 = (2, 2, \ldots, 2)^T \in R^{100}$ be the prescribed consensus vector.

Fig. 5 shows the state responses for the uncontrolled fuzzy system, which apparently cannot reach a consensus. By designing the appropriate continuous controllers according to Theorem 1, Fig. 6 shows the state responses for the continuous control fuzzy system (10), which reach the prescribed consensus.

VI. Conclusion

In this paper, we have discussed on the consensus of a kind of multiagent nonlinear systems with variable structure. T-S fuzzy models have been first addressed to describe multiagent

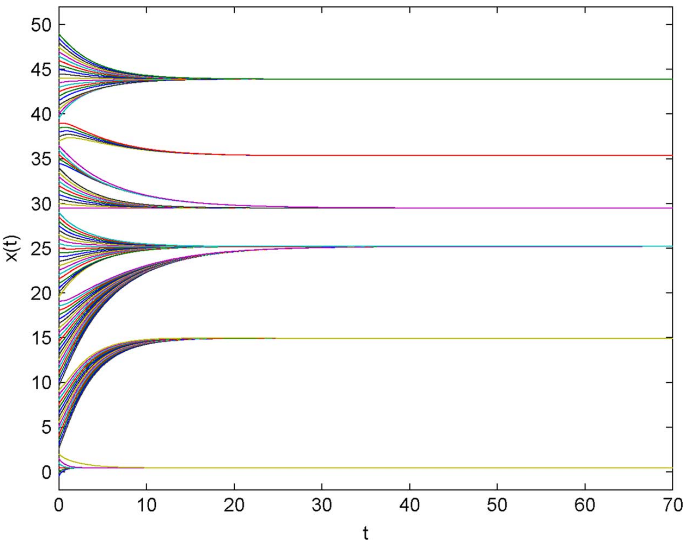

Fig. 5. The state trajectories of the uncontrolled fuzzy system (4) with 100 nodes.

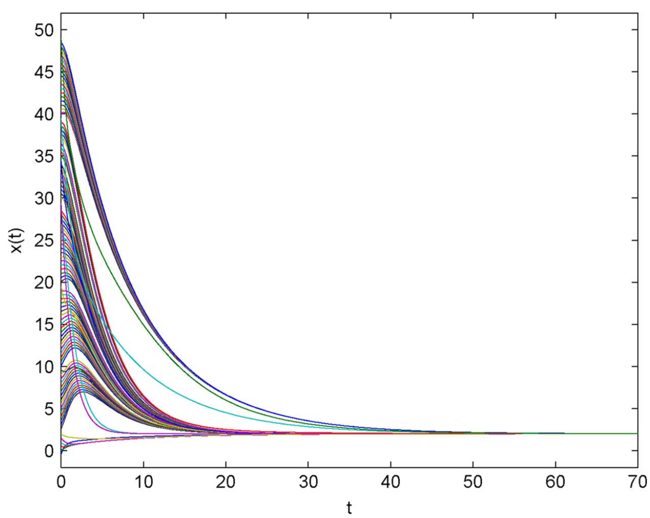

Fig. 6. The state trajectories of the appropriate continuous control fuzzy system (4) with 100 nodes.

nonlinear systems with variable structure. For the proposed models, a node-rearrangement algorithm has been applied to decompose every large-scale fuzzy model into independent and small-scale fuzzy models. Moreover, the consensus of every large-scale fuzzy model is equivalent to that of its corresponding small-scale fuzzy models. Then, continuous and sampled controllers have been applied in the small-scale fuzzy models. Sufficient conditions have been derived to ensure the consensus of the fuzzy models. Finally, numerical examples with the numerical simulations have been provided to illustrate the effectiveness of the obtained criteria.

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Wenjun Xiong was born in Hubei Province, China. She received the B.S. degree from Three Gorges University, Hubei, China, in 2002, the M.S. degree from Southeast University, Nanjing, Jiangsu Province, China, in 2005, and the Ph.D. degree from City University of Hong Kong, Hong Kong, in 2010.

She is currently a Associate Professor with the Department of Mathematics, College of Science, Southwest Petroleum University, Chengdu, China. Her current research interests include stability, synchronization and consensus of nonlinear systems and complex networks, cooperative control of autonomous agents and multiagent systems.

Wenwu Yu (S'07–M'12) received the B.Sc. degree in information and computing science and M.Sc. degree in applied mathematics from the Department of Mathematics, Southeast University, Nanjing, China, in 2004 and 2007, respectively, and the Ph.D. degree from the Department of Electronic Engineering, City University of Hong Kong, Hong Kong, China, in 2010.

Currently, he is an Associate Professor in the Research Center for Complex Systems and Network Sciences, Department of Mathematics, Southeast University, China. He held several visiting positions in Australia, China, Germany, Italy, The Netherlands, and the United States. He is the author or coauthor of about 60 referred international journal papers, and a reviewer of several journals. His research interests include multi-agent systems, nonlinear dynamics and control, complex networks and systems, neural networks, cryptography, and communications.

Dr. Yu is the recipient of the Best Master Degree Theses Award from Jiangsu Province, China in 2008, DAAD Scholarship from Germany in 2008, TOP 100 Most Cited Chinese Papers Published in International Journals in 2008, the Best Student Paper Award in 5th Chinese Conference on Complex Networks in 2009, the First Prize of Scientific and Technological Progress Award of Jiangsu Province in 2010, 2013 Best Theory Paper Award in Neural Systems and Applications Technical Committee of IEEE Circuits and Systems Society, The Best Paper Award in The 9th Asian Control Conference, and The Golden Award for Young Scientific Stars in Information Science in 2012.

Jinhu Lü (M'03–SM'06–F'13) received the Ph.D. degree in applied mathematics from the Academy of Mathematics and Systems Science, Chinese Academy of Sciences, Beijing, China, in 2002.

Currently, he is a Professor of the Academy of Mathematics and Systems Science, Chinese Academy of Sciences. He was a Visiting Fellow in Princeton University, USA from 2005 to 2006. He is the author of two research monographs and more than 100 international journal papers published in the fields of complex networks and complex systems, nonlinear circuits and systems, with more than 6000 SCI citations and h-index 39, including 20 papers with more than 100 SCI citations. He also has two authorized patents.

He is now serving as the Past Chair of TC on Neural Systems and Application and Chair Elect of TC on Nonlinear Circuits and Systems in the IEEE Circuits and Systems Society. Prof. Lü served and is serving as Editors in various ranks for 11 SCI journals including 5 IEEE Transactions: TCAS-I, TCAS-II, TNN, TIE, TII, PLOS Computational Biology, International Journal of Bifurcation and Chaos, and Asian Journal of Control. Prof. Lü received the prestigious National Natural Science Award twice from the Chinese government in 2008 and 2012, the 9th Guanghua Engineering Science and Technology Award from the Chinese Academy of Engineering in 2012, the 11th Science and Technology Award for Youth of China and the Australian Research Council Future Fellowships Award in 2009. Moreover, Prof. Lü attained the National Natural Science Fund for Distinguished Young Scholars and 100 Talents Program from the Chinese Academy of Sciences.

Xinghuo Yu (M'92–SM'98–F'08) received the B.Eng. and M.Eng. degrees from the University of Science and Technology of China, Hefei, China, in 1982 and 1984, and Ph.D. degree from Southeast University, Nanjing, China in 1988, respectively.

He is now with RMIT University (Royal Melbourne Institute of Technology), Melbourne, Australia, where he is the Foundation Director of RMIT's Platform Technologies Research Institute. Professor Yu's research interests include variable structure and nonlinear control, complex and intelligent systems, industrial electronics and smart grids. He has published over 450 refereed papers in technical journals, books and conference proceedings.

Prof. Yu has served as an Associate Editor of IEEE Transactions on Circuits and Systems —I: Regular Papers, IEEE Transactions on Industrial Informatics, IEEE Transactions on Industrial Electronics, and several other scholarly journals. He received a number of awards for his achievements, including the 2012 IEEE Industrial Electronics Magazine Best Paper Award, a Chang Jiang Scholar (Chair Professor) Award from the Ministry of Education of China in 2009, the 1995 Central Queensland University Vice Chancellor's Award for Research, and was made Emeritus Professor of Central Queensland University in 2002 for his long term contributions. He has been Vice-President for Publications (2012/2013) and an IEEE Distinguished Lecturer of IEEE Industrial Electronics Society. He is a Fellow of the Institution of Engineers, Australia, a Fellow of the Australian Computer Society, and a Graduate of the Australian Institute of Company Directors. He was Past Chair of Technical Committee on Smart Grids of IEEE Industrial Electronics Society.