University of Massachusetts Dartmouth

Department of Electrical and Computer Engineering

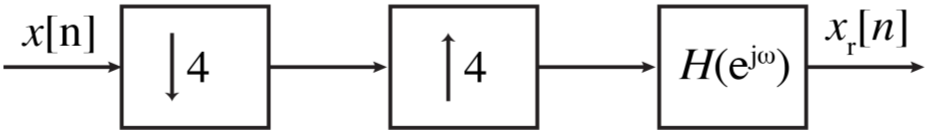
ECE 475/574 DISCRETE-TIME SIGNAL PROCESSING

Problem Set No. 4

Issued: Wednesday, September 25, 2024 **Due:** Friday, October 11, 2024

ECE 475: Problems 4.1, 4.2, 4.3(a) and 4.3(b), and 4.4(a)
ECE 574: All Problems

Problem 4.1 Consider the system shown in the figure below



where

$$H(e^{j\,w}) = \begin{cases} 2, & |w| \le \pi/4 \\ 0, & \dfrac{\pi}{4} < |w| \le \pi \end{cases}$$

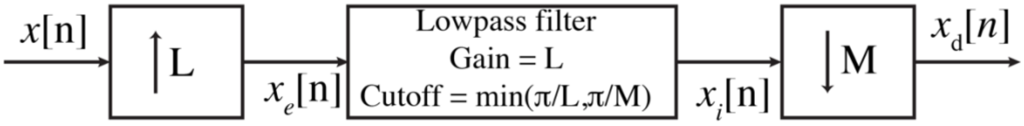
For each of the following input signals, x[n], indicate whether the output $x_r[n] = x[n]$:

(a) $x[n] = \cos\left(\pi n/_5\right)$
(b) $x[n] = \cos\left(\pi n/_2\right)$
(c) $x[n] = \left[\frac{\sin(\pi n/_8)}{\pi n}\right]^2$
Tip for (c) - use the Fourier Transform properties

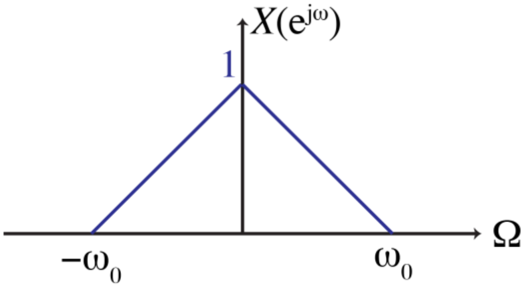
Rubric for Problem 4.1

Criterion/section	Points ECE 475	Points ECE 574
Use correct procedure to determine if the output is equal to the input	4 pt/section	3 pt/section
Provide correct solution whether the output is equal to the input or not	2 pt/section	1.5 pt/section
	18 pt	13.5 pt

Problem 4.2 Consider the below system



and the below Fourier Transform of the input signal x[n],



Specify the maximum value of the cutoff frequency of the input signal so that $X_d(e^{j\,w}) = a\,X(e^{j\,M\,w/L})$ where a is a constant value, for each of the following cases:

(a) $M = 5$ and $L = 3$
(b) $M = 3$ and $L = 5$

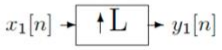
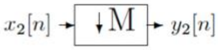
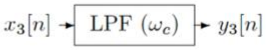
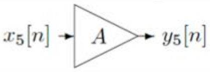
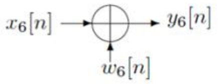
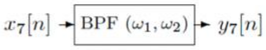
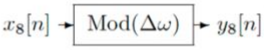
Rubric for Problem 4.2

Criterion/section	Points ECE 475	Points ECE 574
Use correct procedure to determine if the maximum possible value of the cutoff frequency of the signal	6 pt/section	6 pt/section
Provide correct solution	3 pt/section	3 pt/section
	18 pt	18 pt

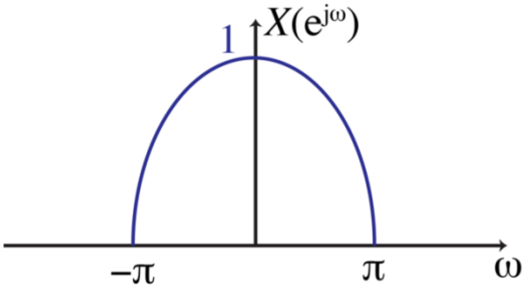
Problem 4.3 Let $x[n]$ be samples of a continuous-time signal $x_c(t)$ sampled at $T = 1/28{,}000$ sec (i.e., 28 kHz). Assume that $x_c(t)$ is bandlimited so that $X_c(\mathrm{j}\,\Omega) = 0$ for $|\Omega| \geq 2\pi(14{,}000)$ rad/sec.

(a) Design a system that will produce $y[n] = x_c(n\,T_2)$ where $T_2 = 1/30{,}000$. Your system **must only use** the modules in Table 1. Specify all parameters for all the modules implemented in your system. Calculate the total cost of your system based on Table 4.1.

Table 1: Modules

Module	Specification	Symbol	Param.	Cost
Expander	$y_1[n] = \begin{cases} x_1[\frac{n}{L}] & n = 0, \pm L, \pm 2L, \dots \\ 0 & \text{otherwise} \end{cases}$		L	50
Compressor	$y_2[n] = x_2[nM]$		M	50
Lowpass Filter	$Y_3(e^{j\omega}) = \begin{cases} X_3(e^{j\omega}), & \|\omega\| \le \omega_c \\ 0, & \text{otherwise} \end{cases}$		ω_c	$\frac{50\pi}{\omega_c}$
Highpass Filter	$Y_4(e^{j\omega}) = \begin{cases} X_4(e^{j\omega}), & \omega_c \le \|\omega\| \le \pi \\ 0, & \text{otherwise} \end{cases}$		ω_c	$\frac{50\pi}{\omega_c}$
Gain	$y_5[n] = Ax_5[n]$		A	10
Adder	$y_6[n] = x_6[n] + w_6[n]$		None	10
Bandpass Filter	$Y_7(e^{j\omega}) = \begin{cases} X_7(e^{j\omega}), & \omega_1 \le \|\omega\| \le \omega_2 \\ 0, & \text{otherwise} \end{cases}$		ω_1, ω_2	$\frac{100\pi}{\omega_2 - \omega_1}$
Modulate	$y_8[n] = \cos(\Delta\omega n)x_8[n]$		$\Delta\omega$	50

(b) Consider that the below DTFT of the original input, $x[n]$. Sketch output spectra of each module to demonstrate that your design/system produces the desired signal. Label all important frequencies and amplitude for your sketches to receive full credit.



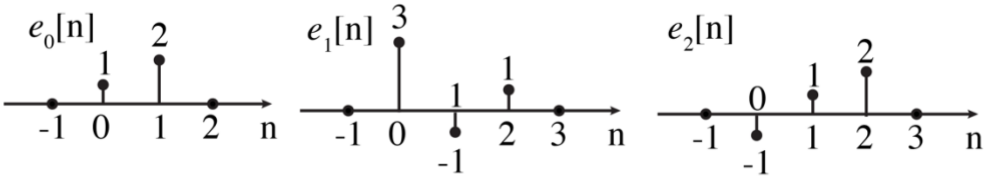
(c) **(Bonus for ECE475)**. To receive additional 10 points, the system in (a) should be cost-effective. Prove that the design in (a) is the cheapest one. If it is not, re-design the system to be the cheapest one.

Rubric for Problem 4.3

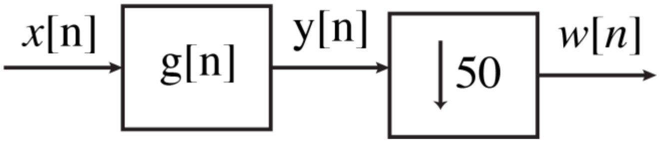
Criterion/section	Points ECE 475	Points ECE 574
Use correct procedure to design a system that provides y[n] = $x_c(nT_2)$	25	20
Provide correct design	10	5
Sketch spectra of your above design, labeling all important magnitudes and frequencies	15 pt	15 pt
Investigate/Prove/Discuss multiple designs so the cost is the cheapest one whereas still satisfying y[n] = $x_c(nT_2)$	+10 pt (Bonus)	10
	50 pt (+10 bonus)	50 pt

Problem 4.4 – ECE475 students only part (a)

(a) The below figure shows the three L = 3 polyphase components for a polyphase implementation of an FIR LTI system with impulse response $h[n]$. Sketch the impulse response $h[n]$



(b) You are a DSP engineer at NUWC. Your job is to implement the system shown below on a DSP chip, so it runs in real-time and provides an output sample $w[n]$ at a rate of 40 kHz. The input signal is real-valued and sampled at 2 MHz. The DSP chip that you must implement the system on can multiply two real numbers in T_{mult} = 10 ns. You may assume that the time required for any other operations is negligible compared to T_{mult}. You may also assume that $g[n]$ is a real-valued, FIR impulse response of length 500 implementing a lowpass filter with cutoff frequency $w_c = \pi/50$. What is the minimum polyphase factor you can use in implementing the system to satisfy these requirements? Sketch a system that will satisfy the real-time requirements given and be equivalent to the system shown in the below figure.



Rubric for Problem 4.4

Criterion/section	Points ECE 475 (only a)	Points ECE 574
Use correct procedure to determine the impulse response	10	8
Provide correct impulse response	4	3
Provide correct procedure to determine the minimum polyphase factor		4
Sketch a system that satisfies the real-time requirements and is equivalent to Fig. P4.5.2		3.5
	14 pt	18.5 pt