

# Four degrees of freedom SCARA robot kinematics modeling and simulation analysis

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## Abstract

For SCARA robot multi-variable, nonlinear, difficult to verify the correctness of model problem, in this paper, for four degrees of freedom SCARA robot kinematics modeling, and then in the MATLAB environment, using Robotics Toolbox forward kinematics of the robot inverse kinematics simulation. Through simulation, the observed motion of each joint SCARA robot state verification the proposed model is correct, to achieve the desired goal.

## 1. Introduction

SCARA industrial robot with a high degree of flexibility and versatility, has been in the electronics, semiconductor, pharmaceutical pharmaceuticals, automotive, consumer goods and other industries so as to obtain a wide range of applications. Multi-DOF SCARA industrial robot is a highly non-linear, strong coupling, time-varying systems, so to design a high-precision SCARA robot control system, for the corresponding robot body kinematics modeling is one of the key technologies used to verify the correctness of the model. More and more Control Session scholars began to be interested in studying SCARA robots. Literature [1] designed an open-robot system

development tools VB6.0, through the ActiveX technology, and Matlab combined with graphic to describe the planar articulated SCARA robot kinematics model. Literature [2] described a 6-DOF articulated robot kinematics model. These documents do not verify the correctness and validity of the model. Literature [3] described how to use secondary function of Solid Edge to realize the robot graphic simulation analysis method. But there is no specific robot modeling and simulation analysis. Literature [4] established six DOF robot dynamics model, and used the combination of OpenGL and MATLAB modeling, and dynamic simulation consecutive points to get solutions , but not for the kinematic analysis. Literature [5] , with one kind of a cylindrical coordinate robot design parameters, discussed the problem of robot kinematics and the kinematics of the simulation, but there is no dynamic process simulation.

This paper describes the laboratory four DOF SCARA robot structure and the technical specifications, and the application of DH method to establish a robot kinematics model in MATLAB environment with the Robotics Toolbox kinematics simulation model. By MATLAB simulation, the observed motion of each joint SCARA robot status verified the proposed model is correct.

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## 2. SCARA Robot Body Structure

This robot studied in this paper is to adopt Adept SCARA robot similar joint structure design. Joints 1,2,4 are rotary joints driven by using AC servo motor and harmonic reducer, and joint 3 is a straight joint driven by AC servo motor and ball screw. Joints control shaft 1,2,3 also has photoelectric switch limit on the motor, and are combined with incremental optical encoder as the relative position of the robot to control shaft positioning; the joint shaft on the hardware travels within its range to ensure the robot body and operator safety. Joint 1 rod length is 200mm, and moves range of  $\pm 120^\circ$ ; joint 2 rod length is 200mm, and moves range of  $\pm 150^\circ$ ; straight joint 3 strokes about  $\pm 48\text{mm}$ ; motion range of joint 4 is about  $\pm 360^\circ$ . According to the actual needs of the job the end tool of the robot joint 4 can be installed with an electromagnetic gripper. Four joints from robot control the motor encoder feedback signal and the joint axis angle limit signal connected via the connecting cable and control. [6] SCARA robot real photos image is shown in figure 1.



Figure 1: SCARA robot body kind photos

## 3. SCARA Robot Kinematics Modeling

### 3.1 SCARA Robot Arm Coordination System and D-H Parameters

SCARA robot has three rotational joints,  $\theta_{1,2,4}$  are joint variables. A moving joint,  $d_3$ , is articular variable. Specific linkage coordination system is shown in Figure 2-2. Its corresponding link parameters are shown in Table 1.

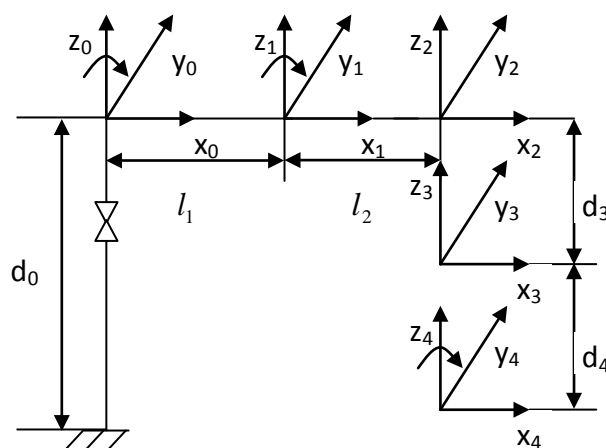


Figure 2: SCARA link coordinate system

**Table 1: SCARA robot parameters lever**

Link	Joint variables	$\alpha_i$	$a_i$	$d_i$	$\cos \alpha_i$	$\sin \alpha_i$
1	$\theta_1$	0	$L_1$	0	1	0
2	$\theta_2$	0	$L_2$	0	1	0
3	$d_3$	0	0	$-d_3$	1	0
4	$\theta_4$	0	0	$-d_4$	1	0

Fig 2 is as the basis for modeling, parameters of the corresponding lever listed in Table 1. We can find the respective variables. Each joint uses the DH method to coordinate system to establish a link at the odd transform matrix, which means that the linkage is between the coordinate and previous system.

Let a base be connected to the robot reference coordinate  $x_0, y_0, z_0$  to the coordinate system, According to Table 1, the relationship between each rod can get the corresponding transformation matrix  ${}^{n-1}A_n$  Pose ( ${}^{n-1}A_n$  rod n represents a linkage

relative to the previous n-1 position and posture, where  $n = 1 \sim 4$ .)

### 3.2 Kinematic Equations

According to the characteristics of the SCARA robot and requirements to achieve the goal, we consider their claws and gripper rotate cause at the end of move up and down in the space within the scope of the change of the posture, and according to the robot rod parameters and D-H transformation matrices derived, gripper kinematics equations are:

$$\begin{aligned}
 T &= {}^0A_1 = {}^0A_1 {}^1A_2 {}^2A_3 {}^3A_4 \\
 &= \begin{bmatrix} c_1 & -s_1 & 0 & L_1 c_1 \\ s_1 & c_1 & 0 & L_1 s_1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_2 & -s_2 & 0 & L_2 c_2 \\ s_2 & c_2 & 0 & L_2 s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & -d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1) \\
 &= \begin{bmatrix} c_{124} & -s_{124} & 0 & L_2 c_{12} + L_1 c_1 \\ s_{124} & c_{124} & 0 & L_2 s_{12} + L_1 s_1 \\ 0 & 0 & 1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}
 \end{aligned}$$

The formula :  $c_{124} = \cos(\theta_1 + \theta_2 + \theta_4)$  ,  $s_{124} = \sin(\theta_1 + \theta_2 + \theta_4)$  ,  $s_{12} = \sin(\theta_1 + \theta_2)$  ,  $c_{12} = \cos(\theta_1 + \theta_2)$  ,  $L_1$  = Boom length ,  $L_2$  = Arm length,  $\theta_1$  = The length of the arm and forearm

coordinate system X-axis origin,  $\theta_2$  = The angle between the boom and arm,  $d_4$  = Gripper initial ordinate,  $d_3$  = Longitudinal position of the gripper,  $\theta_4$  = Angle of rotation of the gripper.

### 3.3 SCARA Robot Inverse Kinematics Equation

In the preparation of the robot control program, it is always in general to specify the coordinate system of the robot position and posture of the tip end tool. To enable the robot to reach the designated end tool with the specified location and attitude, it must be driven by the current position of each joint robot reaches the end tool position and orientation corresponding position. Let the position and posture of the robot type (2) determine the joint variables[7].

$$T = \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (2)$$

According to equation (1) with  ${}^0A_1^{-1}$  Left Riding (2) of the robot inverse kinematics is derived as follows:

$${}^1T_4 = \begin{bmatrix} c_2 & -s_2 & 0 & L_2c_2 \\ s_2 & c_2 & 0 & L_2s_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & -d_3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c_4 & -s_4 & 0 & 0 \\ s_4 & c_4 & 0 & 0 \\ 0 & 0 & 1 & -d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (7)$$

$$= \begin{bmatrix} c_{24} & -s_{24} & 0 & L_2c_2 \\ s_{24} & c_{24} & 0 & L_2s_2 \\ 0 & 0 & 1 & -d_3 - d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Using (4) and (7) obtained in correspondence of the elements:

$${}^0A_1^{-1}T = \begin{bmatrix} c_1 & s_1 & 0 & -L_1 \\ -s_1 & c_1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (3)$$

It is expressed as

$${}^0A_1^{-1}T = \begin{bmatrix} f_{11}(n) & f_{11}(o) & f_{11}(a) & f_{11}(p) \\ f_{12}(n) & f_{12}(o) & f_{12}(a) & f_{12}(p) \\ f_{13}(n) & f_{13}(o) & f_{13}(a) & f_{13}(p) \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (4)$$

Of which:

$$f_{11}(j) = c_1j_x + s_1j_y \quad (5)$$

$$j = n, o, a$$

$$f_{11}(p) = c_1p_x + s_1p_y - L_1$$

$$f_{12}(i) = -s_1i_x + c_1i_y \quad (6)$$

$$f_{13}(i) = i_z$$

$$i = n, o, a, p$$

We know  ${}^0A_1^{-1}T = {}^1A_2 {}^2A_3 {}^3A_4 = {}^1T_4$ , However

$$f_{11}(p) = c_1p_x + s_1p_y - L_1 = L_2c_2 \quad (8)$$

$$f_{12}(p) = -s_1p_x + c_1p_y = L_2s_2$$

$$\begin{aligned}
 f_{11}(p)^2 + f_{12}(p)^2 &= L_2^2 = c_1^2 p_y^2 + s_1^2 p_x^2 - 2c_1s_1p_xp_y + c_1^2 p_x^2 + s_1^2 p_y^2 + \\
 &2c_1s_1p_xp_y + L_1^2 - 2L_1(c_1p_x + s_1p_y) \\
 &= p_y^2 + p_x^2 + L_1^2 - 2L_1(c_1p_x + s_1p_y)
 \end{aligned}
 \tag{9}$$

Order was:

$$\begin{cases}
 \theta_1 = \arccos \frac{r^2 + L_1^2 - L_2^2}{2L_1r} + \phi = \arccos \frac{r^2 + L_1^2 - L_2^2}{2L_1r} + \arccos \frac{p_x}{r} \\
 \theta_2 = \arccos \frac{c_1p_x + s_1p_y - L_1}{L_2} \\
 d_3 = p_z - d_4 \\
 \theta_4 = \arccos(c_1n_x + s_1n_y) - \theta_2
 \end{cases}
 \tag{10}$$

It can be seen from the above that formula is derived, point-to-point in a plane within the movement, from a known point (e.g., zero) movement to another point, in the control interface inputting any point (within the scope of the robot's movement) coordinates, so we can coordinate values according to this point (constant) calculated the value of the joint angle of the point; then this is the starting point of the joint angle value with the value of joint angle drawn differencing needed to rotate each joint of the robot angle increment.

coordinates, and demonstrates this state robot pose state; The latter is input to reach the desired end point of space coordinate system point in the output end of the robot joint angle change point. MATLAB simulation flow chart is shown in Figure 3.

## 4. SCARA Robot Kinematics Simulation Analysis

### 4.1 Simulation Process

SCARA simulation model is a reference to the robot robotic toolbox in Matlab toolbox to establish. Choose operating systems Windows XP, operating system platform for Matlab7.0. For SCARA robot kinematics simulation it includes forward kinematics and inverse kinematics simulation of two parts. The former is to enter the desired joint angles, and the system output end of the robot is to reach the space

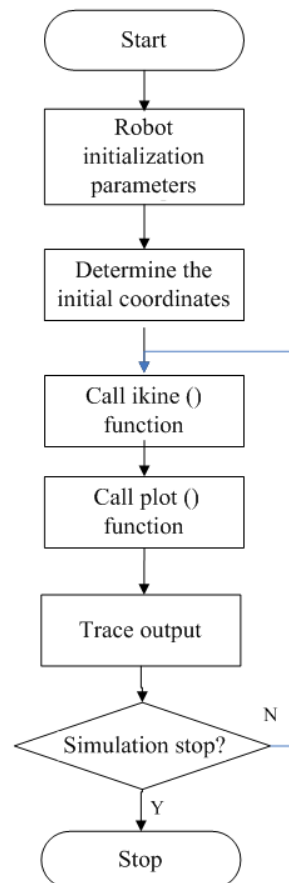


Figure 3: SCARA simulation flow chart

SCARA robot uses Matlab to generate a three-dimensional simulation graphics, simulation before the input parameters of the robot. The first four elements of connecting rod are  $\alpha$ 、 $A$ 、 $\theta$ 、 $d$ , and the last element of the joint type: 0 for the rotational joint, a mobile joints. Commands are as follows:

L1=link([0,200,0,0,0]) , L2=link([0,200,0,0,0]) ,  
 L3=link([0 0 0 0 1]) , L4=link([0 0 0 -15 0]) ,  
 r=robot({L1 L2 L3 L4}) , Predetermined trajectory simulation, the starting position [0 0 0 -15] means that each robot joints are in the zero position shown in Figure 4.

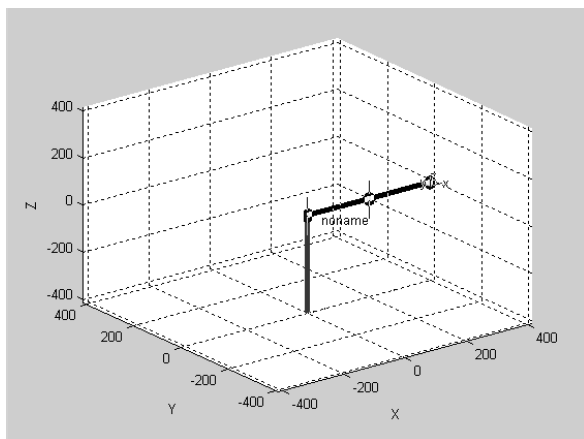


Figure 4: SCARA zero position of the three-dimensional simulation

Robot at point B and point C relative to the base coordinate system can pose homogeneous transformation matrix  $T_B$  and  $T_C$ . Then for the inverse kinematics solution to the problem, we can obtain from A to B, B to C, each joint variable. Here is the inverse kinematics with Robotics Toolbox command `ikine` to solve. The initial value of joint variables defined:  $q_A = [0\ 0\ 0\ -15]$ , obtained  $q_{AB} = [-64.4042\ 128.8085\ 15.1500\ -61.4099]$ , description SCARA robot from A to B, the joint 1 to be reverse rotation 64.4042 rad, joint 2 to be rotated forward 128.8085 rad, the joint 3 to be moved upward and the

joint 4 to be 15.1500 mm reverse rotation 61.4099 rad. Similarly obtained,  $q_{BC} = [52.6250\ -106.8207\ 0.0300\ 52.7509]$ , described SCARA robots from B to C, joints a forward rotation required 52.6250 rad, joints 2 to be reverse rotation 106.8207 rad, joints 3 to be moved up 0.0300 mm, joint 4 required Forward rotation 52.7509 rad.

The command plot ( ) of the robot movement from A to B simulation, the robot can see the movement of joints of the specific situation as shown.

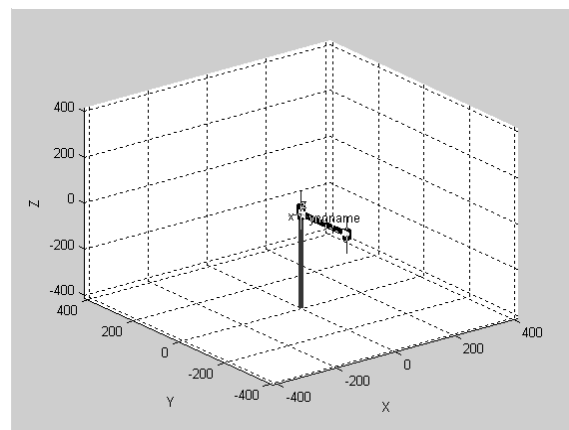


Figure 5: SCARA robot moves to point B

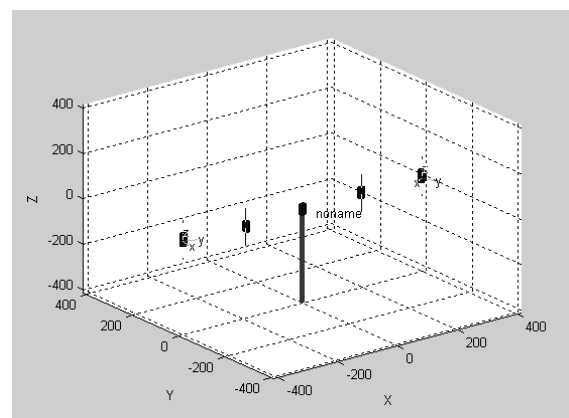


Figure 6: A-B dynamic process simulation diagram

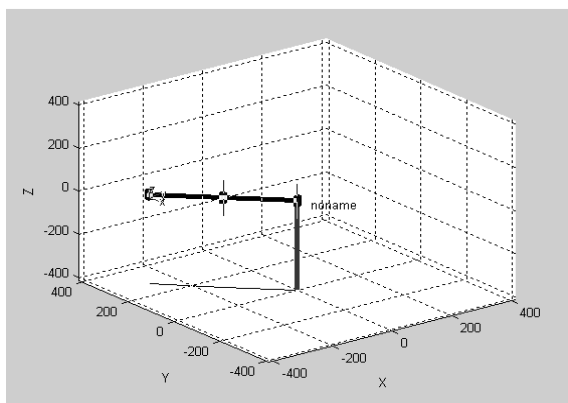


Figure7: SCARA robot moves to point C

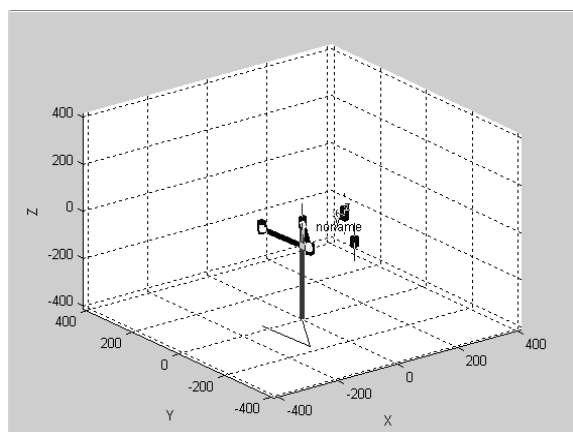


Figure 8: B-C diagram dynamic process simulation

### 4.2 Simulation results analysis

In the motion simulation process, we observe the robot from A to B, from B to C, the state of motion of each joint is normal, and the output trajectory data is shown in Table 2 and Table 3, which verifies the rationality of all connecting rod parameters, and describes the various design parameters to achieve the desired goal.

Table 2: Initial state A to point B the trajectory output data

Joint1	Joint2	Joint3	Joint4
0	0	0	-15.0000
-0.0144	0.0288	0.0034	-15.0104
-0.1101	0.2202	0.0259	-15.0793
-0.3552	0.7104	0.0836	-15.2560
-0.8041	1.6082	0.1892	-15.5794
-1.4983	2.9966	0.3525	-16.0797
-2.4675	4.9350	0.5804	-16.7781
-3.7303	7.4606	0.8775	-17.6881
-5.2951	10.5903	1.2456	-18.8157
-7.1612	14.3225	1.6846	-20.1604
-9.3194	18.6388	2.1922	-21.7156

Table 3: Initial state B to C output data of the trajectory

Joint 1	Joint 2	Joint 3	Joint 4
64.4042	128.8085	15.1500	61.4099
64.3781	128.7559	15.1466	61.3844
64.2041	128.4057	15.1242	61.2147
63.7587	127.5089	15.0666	60.7802
62.9431	125.8666	14.9612	59.9846
61.6816	123.3268	14.7982	58.7540
59.9205	119.7809	14.5707	57.0361
57.6259	115.1609	14.2742	54.7977
54.7824	109.4357	13.9069	52.0239
51.3915	102.6084	13.4688	48.7161

From trajectory data it can also be seen: taken in the simulation time. The displacement of rotational joints 1 by zero gradually change to 52.6250 rad. Rotational joints 2 displacement by zero gradually change to -106.8207 rad; Mobile joints 3 moved up 0.0300 mm; rotational joints 4 is also gradually changed to 52.7509 rad; description robot from A motion to B, from B to move to C, terminal joint along the x, y, z direction, the displacement from the initial positions are changed to -0.4821, 0.4821, 0.1800 mm. By the end of the joint position and posture change, it can be seen that the robot four joint movement can be achieved in different directions, and once again it shows its parameters is reasonable.



Jian fang received the B.S.degrees in Automation from Jilin Institute of Technology in 1998, M.S. degree in control engineering from Jilin University in 2007. In 1998, he joined the Jilin Teachers' Institute of Engineering &Technology in 2000,where he is cuttently a Associate Professor and associate dean in electric work academy. His current research interests are Motion control of a mobile robot, path planning, intelligent controland their applications.

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