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Magnetic Field Analysis of Permanent Magnet Array for Planar Motor Based on Equivalent Magnetic Charge Method*

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***Abstract* –Planar motor is an important research area in complex automation system. In this paper, a simple and efficient topological structure of Halbach magnet array for planar motor is proposed. The Equivalent Magnetic Charge (EMC) method is used to analyze the proposed array. The calculation results of EMC method, compared with the results of Finite Element Method (FEM) using ANSOFT, indicate that the EMC method is reasonable and accurate. So the EMC method in this paper is applicable to simplify 3D model of complex automation system in order to research the optimal decoupling method and control strategy.**

Index Terms – Planar motor, Halbach array, EMC method.

I. INTRODUCTION

With the development and the needs of complex automation systems, lots of large-scale and complicated automated platforms, which include all kinds of mechanical equipments, are manufactured. And complex automation systems advance greater demands on automated platforms, for example, high accuracy; accurate localization; wide speed range and automation of all the process. So the selection and design of the fundamental operating mechanisms are important. There are various operating mechanisms in mechanical equipments. Meanwhile, multi-axis motion is increased accordingly to meet requirements of the complex automation systems.

Early stage, rotary motor and the middle conversion device are used to realize the linear motion. With the emergence and development of linear motor, the x-y linear motors are used to realize the surface motion. But this device increases the complexity of the transmission system. So the research and development of the planar motor are urgently needed.

The planar motor, which is instead of x-y liner motor, can drive 2-D device directly. The planar motor has more advantages such as low friction, no backlash, easy packaging and high accuracy, so it has been applied in the various fields, such as, machining operation, electronic products production and even drive robot. Until now, some types of planar motors have been researched. According to their operation principles, most of the planar motors can be classified into three types, i.e., variable reluctance planar motor, induction planar motor, and the synchronous permanent magnet planar motor (SPMPM). Among them, the synchronous permanent magnet planar motor has the advantages of low cost, simple structure, high-power conversion ratio, etc [1]. It has been investigated by more and more researchers and engineers in the academic and engineering fields.

The electromagnetic thrust of SPMPM is generated by the interaction between the magnetic field of permanent magnet array and the current of coil array. The higher magnetic flux density of permanent magnet array is, the larger electromagnetic thrust is. So the topological structure and its analytical method of permanent magnet array are important for the SPMPM.

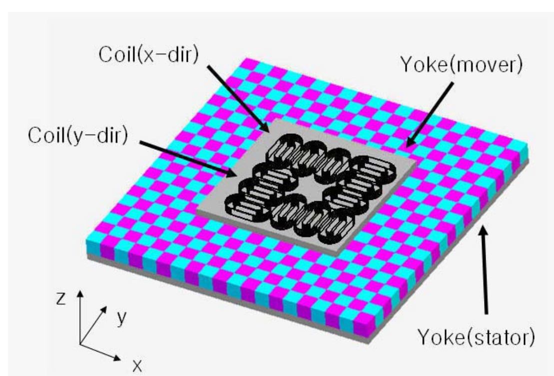

Fig. 1 Outline view of the 3-phase synchronous permanent magnet planar motor.

In this paper, a simple and efficient topological structure of permanent magnet array is proposed. The Equivalent

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Magnetic Charge (EMC) method is used to analyze proposed array. The EMC method is appropriate for not only 3D field solutions but also the complex geometries solutions, the modeling of complex automation systems is simplified, and the computational accuracy of EMC method is compared with the FEM (ANSOF).

II. MODELING

A. The structure of planar motor

Like other electronic motors, the planar motor consists of stator, mover, position detection device and supporting equipment. The SPMPM has two types in the mover and stator collocation. One is the moving-magnet-type which chooses the magnet as the mover and the coil as the stator and another is moving-coil-type which chooses the coil as the mover and the magnet as the stator. In this paper, we study the moving-coil-type SPMPM which is suitable for the long stroke applications since the movers can move freely. Fig. 1 shows the outline view of the 3-phase SPMPM which employs permanent magnet array as the stator. There are four coil sets in the air-gap, two coil sets are used for driving in the x-direction and the others for y-direction. To move in the x and y directions, the coil sets need the appropriate magnet array arrangement that is same shape when viewed from the x and y sides. Fig.2 shows the part-view of analytical model of the planar motor.

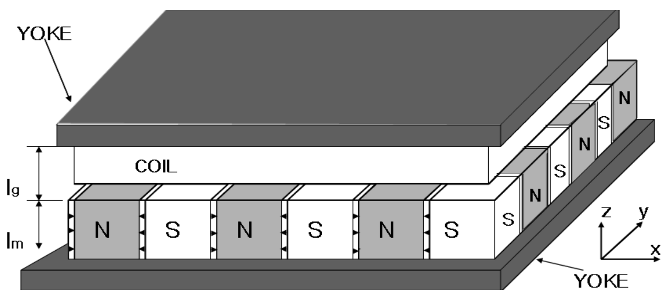

Fig. 2 Part-view of analytical model

B. The modeling of Halbach array

The topological structure of permanent magnet array is the key to planar motor. At present, many permanent magnet arrays have been designed to obtain the desired magnetic field distribution in the patents and papers in [2-5].

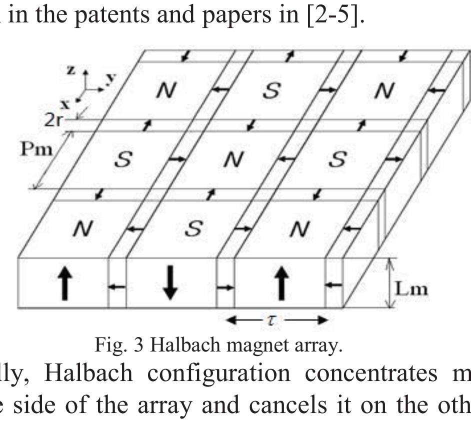

Fig. 3 Halbach magnet array.

Specially, Halbach configuration concentrates magnetic flux on one side of the array and cancels it on the others and generators for the advantages including minimized drag from eddy current effects, reduced power consumption, and good magnetic field distribution. Now, the different topological structures of Halbach array are designed to obtain higher the magnetic flux density in the air-gap. Reference [6] provided the new Halbach array, which has the higher magnetic flux density, but the triangle permanent magnet is too complex to fabricate and packaging. Reference [7] provided the magnet array, in which the angle of the adjacent permanent magnet unit is 45° . The structure is effective to suppress higher harmonics, but the manufacturing process is complex to realize the special angle. A simple and efficient topological structure of Halbach array is proposed in this paper. Fig.3 shows the Halbach magnet array. The arrow means the direction of magnetization intensity.

III. EQUIVALENT MAGNETIC CHARGE METHOD

In general, the FEM is used to calculate the characters of the different magnet arrays. But FEM method needs long time and is difficult to analyze and optimize other variables. So the efficient and accurate analytical method, which can build the modeling and realize decoupling control, is needed. Now there are three analysis methods to analyze the magnet array: equivalent current method, the Fourier series method, and the equivalent magnetic charge method. The equivalent current method is large computing and its result is complex. The Fourier analysis and the magnetic scalar potential were used to analyze magnetic field distribution [6-9]. When the topological structure of permanent magnet array is complex, the magnetic flux density was analyzed by multiple Fourier series [6].

But if the topological structures of magnet array are complex, such as 3D field or the complicated geometry, the above methods are difficult to analyze the magnetic field in general. In this paper, the equivalent magnetic charge (EMC) method is used to analyze the 3D permanent magnet field and to simplify the modelling.

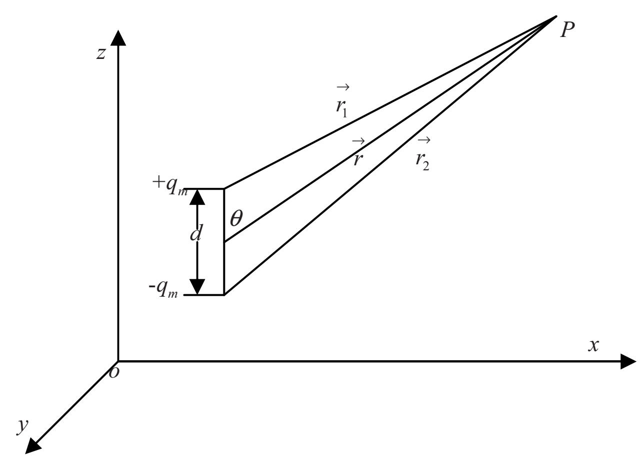

Fig. 4 The magnetic field by single magnetic dipole.

A. the magnetic field calculation by single magnetic dipole

In the EMC method, the permanent magnet can be regarded as the aggregation of a lot of magnetic dipole. So the magnetic field calculation of permanent magnet is divided into two steps: the magnetic field calculation of single magnetic dipole and the magnetic medium volume integral. Fig. 4 shows the magnetic dipole, which is generated by magnetic charge $\pm q_m$. And the little distance of the magnetic charge is d, the magnetic dipole moment is expressed as follow

$$\vec{m}=q_m \vec{d} . \tag{1}$$

The point P is at any point in the magnetic field space. So the magnetic scalar potential of the point P, which is generated by magnetic dipole, is expressed as

$$u_m=\frac{q_m}{4\pi}\left(\frac{1}{r_1}-\frac{1}{r_2}\right)=\frac{q_m}{4\pi}\frac{r_2-r_1}{r_1 r_2}, \tag{2}$$

where r_1 and r_2 are the respective distance between $+\mathrm{q}_m$, $-\mathrm{q}_m$ and the point P.

Compared with the radius vector r, which is from the magnetic dipole to the point P, d can be ignored. And these approximate relations can be obtained as: $r^2=r_1\cdot r_2$ and $r_2-r_1=d\cos\theta$. The equation (2) can be rewritten as follow

$$u_m=\frac{1}{4\pi}\frac{q_m d\cos\theta}{r^2}=\frac{1}{4\pi}\frac{\vec{m}\cdot\vec{r}}{r^3}=\frac{1}{4\pi}\vec{m}\cdot\nabla'\left(\frac{1}{r}\right), \tag{3}$$

where $\nabla'\left(\frac{1}{\mathrm{r}}\right)=\frac{\vec{r}}{r^3}$ represents differential operators of the source point, the direction of $\vec{r}$ is toward the point P, and the angle between magnetic moment of the magnetic dipole and radius vector r is θ.

The u_m is magnetic scalar potential, and the magnetic medium is isotropic and homogeneous, so the sum of magnetic scalar potential, which is caused by all magnetic dipole in the magnetic medium, can be expressed as

$$\mathrm{U}=\frac{1}{4\pi}\int_v \vec{M}\cdot\nabla'\left(\frac{1}{r}\right)dv, \tag{4}$$

where v is the volume of the magnetic medium, and $\vec{M}$ is the magnetic dipole moment volume density, magnetization intensity.

According to the vector operations, we can obtain the expression as

$$\nabla'\cdot\left(\frac{1}{r}\vec{M}\right)=\nabla'\left(\frac{1}{r}\right)\cdot\vec{M}+\frac{1}{r}\nabla'\cdot\vec{M} . \tag{5}$$

Substituting the equation (5) into the equation (4), we can yield

$$\mathrm{U}=\frac{1}{4\pi}\int_v -\frac{\nabla'\cdot\vec{M}}{r}dv+\frac{1}{4\pi}\int_v \nabla'\cdot\frac{\vec{M}}{r}dv . \tag{6}$$

And the Gauss integration formula is

$$\int_v \nabla'\cdot\vec{R}dv=\oint_s \vec{R}\cdot\vec{n}ds . \tag{7}$$

So the equation (6) can be rewritten as follow

$$\mathrm{U}=\frac{1}{4\pi}\int_v -\frac{\nabla'\cdot\vec{M}}{r}dv+\frac{1}{4\pi}\oint_s \frac{\vec{M}\cdot\vec{n}}{r}ds . \tag{8}$$

The electric potential, which is generated by the equivalent volume free electric charge density $\rho_m{}^e$ and the equivalent surface free electric charge density $\sigma_m{}^e$, can be expressed as

$$\phi=\frac{1}{4\pi\varepsilon_0}\int_v \frac{\rho_m{}^e}{r}dv+\frac{1}{4\pi\varepsilon_0}\oint_s \frac{\sigma_m{}^e}{r}ds . \tag{9}$$

With analogy of the equation (9), the equation (8) is described as

$$\mathrm{U}=\frac{1}{4\pi\mu_0}\int_v \frac{\rho_m}{r}dv+\frac{1}{4\pi\mu_0}\oint_s \frac{\sigma_m}{r}ds , \tag{10}$$

where $\rho_m=-\mu_0\nabla'\cdot\vec{M}$ is the equivalent volume magnetic charge density and $\sigma_m=\mu_0\vec{M}\cdot\vec{n}$ is the equivalent surface magnetic charge density.

Because the magnetic medium is isotropic and homogeneous, $\vec{M}$ is constant, and also $\rho_m=-\mu_0\nabla'\cdot\vec{M}=0$. The conclusions simplify the analysis model of 3D magnetic field. The equation (10) is simplified as follow

$$\mathrm{U}=\frac{1}{4\pi\mu_0}\oint_s \frac{\sigma_m}{r}ds . \tag{11}$$

B. Magnetic scalar potential in magnetic medium

For the magnetostatic field, the maxwell equation can be expressed as

$$\begin{cases}\nabla\times\vec{H}=\vec{J}\\ \nabla\cdot\vec{B}=0\\ \vec{B}=\mu_0(\vec{H}+\vec{M})\cdot\end{cases} \tag{12}$$

Form the second and third expressions of equation (12), we can deduce

$$\nabla\cdot\vec{H}=\text{-}\nabla\cdot\vec{M}={\rho_m}/{\mu_0} . \tag{13}$$

In certain regions of magnetostatic field, the current density $\vec{J}$ is zero. The first expression of equation (12) equals zero. In certain region where the current density equals zero, the field is rotational. So the magnetic scalar potential is used to express magnetic field strength. We have written magnetic field strength in terms of a scalar function as

$$\vec{H}=\text{-}\nabla U , \tag{14}$$

$$\nabla\cdot\vec{H}=\text{-}\nabla\cdot\nabla U=-\nabla^2 U={\rho_m}/{\mu_0} . \tag{15}$$

From equation (15), we can obtain that the magnetic scalar potential satisfies the Poisson's equation. And the equation (13)-(15) are accordingly agreed with the electric field by electric charge.

With the same analysis method of electric field generated by electric charge, the normal component of magnetic field strength, which is caused by the equivalent surface magnetic charge in the interface of two mediums, is mutational. On the other hand, the magnetic flux density is continuous. So the boundary conditions are expressed as

$$(\vec{B}_2 - \vec{B}_1)\cdot\vec{n} = B_{2n} - B_{1n} = 0\,, \qquad (16)$$

$$\mu_0(\vec{H}_2 - \vec{H}_1)\cdot\vec{n} = -\mu_0(\vec{M}_2 - \vec{M}_1)\cdot\vec{n} = \sigma_m\,. \qquad (17)$$

C. Permanent magnet model by EMC

In the permanent magnet, there is not only the magnetization intensity of magnet in operation point, which is expressed as $\vec{M}' = \chi\vec{H}$, but also the remnant magnetization intensity of permanent magnet related to the remnant flux density $\vec{M}_0 = \vec{B}_r / \mu_0$, where $\vec{B}_r$ is the remnant magnetic flux density. So the magnetic flux density is expressed as

$$\vec{B} = \mu_0(\vec{H} + \vec{M}_0 + \vec{M}') = \mu_0\left[(1+\chi)\vec{H} + \vec{M}_0\right] = \mu\vec{H} + \mu_0\vec{M}_0\,. \qquad (18)$$

Applying the divergence operator to both sides of equation (18), we can yield

$$\nabla\cdot\vec{B} = \nabla(\mu\vec{H}) + \nabla(\mu_0\vec{M}_0)\,. \qquad (19)$$

Applying the continuity of magnetic flux $\nabla\cdot\vec{B} = 0$ to the equation (19), we can obtain

$$\nabla(\mu\vec{H}) = -\mu_0\nabla\cdot\vec{M}_0 = \rho_m\,. \qquad (20)$$

Applying the Gauss' law to the equation (20), we can yield

$$\mu_0\vec{M}_0\cdot\vec{n} = \sigma_m\,. \qquad (21)$$

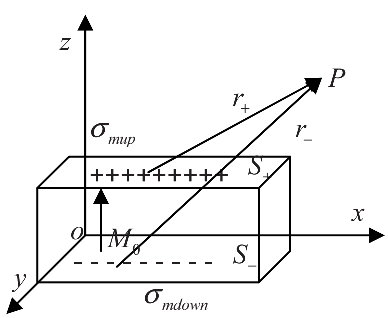

Fig. 5 The permanent magnet model by EMC

The permanent magnet model by EMC is described in Fig.5. The direction of magnetization intensity is z-axis. The σ_{mup}, σ_{mdown} and σ_{mside}, respectively, are the upper surface, the lower surface and the side surface equivalent surface magnetic charge density.

By the above analysis, the magnetic field strength is given as follow

$$\vec{H} = -\frac{\sigma_m}{4\pi\mu}\oint_S \nabla\frac{1}{r}ds = \frac{\sigma_m}{4\pi\mu}\oint_S \frac{\vec{r}}{r^3}ds\,. \qquad (22)$$

The equation (21) can be rewritten as

$$\sigma_m = \mu_0\vec{M}_0\cdot\vec{n} \Rightarrow \begin{cases}\sigma_{mup} = B_r\\ \sigma_{mdown} = -B_r\\ \sigma_{mside} = 0\end{cases}\,. \qquad (23)$$

So the magnetic field strength of the point P is given by

$$\vec{H}_p = \vec{H}_+ - \vec{H}_- = \frac{\sigma_m}{4\pi\mu}\oint_{S_+}\frac{\vec{r}_+}{r_+^3}ds - \frac{\sigma_m}{4\pi\mu}\oint_{S_-}\frac{\vec{r}_-}{r_-^3}ds\,. \qquad (24)$$

IV. Magnetic Field Analysis and Comparsion

By EMC method, the z- component of the magnetic field strength of the proposed Halbach array, which is shown in Fig.2, is expressed as follow

$$\vec{H}_z = \vec{H}_{z0} + \vec{H}_{z1} + \vec{H}_{z2} + \vec{H}_{z3} + \vec{H}_{z4}\,, \qquad (25)$$

where
$$\vec{H}_{z0} = k\begin{Bmatrix}\alpha(x-\tau_x+r, y-\tau_b+r, z-l_m)\\ -\alpha(x-\tau_x+r, y-r, z-l_m)\\ -\alpha(x-r, y-\tau_b+r, z-l_m)\\ +\alpha(x-r, y-r, z-l_m)\\ -\alpha(x-\tau_x+r, y-\tau_b+r, z)\\ +\alpha(x-\tau_x+r, y-r, z)\\ +\alpha(x-r, y-\tau_b+r, z)\\ -\alpha(x-r, y-r, z)\end{Bmatrix},$$

$$\vec{H}_{z1} = k\begin{Bmatrix}\beta(x-r, y-r, z-l_m)\\ -\beta(x-\tau_x+r, y-r, z-l_m)\\ -\beta(x-r, y, z-l_m)\\ +\beta(x-\tau_x+r, y, z-l_m)\end{Bmatrix},$$

$$\vec{H}_{z2} = k\begin{Bmatrix}\beta(x-r, y-\tau_y+r, z-l_m)\\ -\beta(x-\tau_x+r, y-\tau_y+r, z-l_m)\\ -\beta(x-r, y-r, z-l_m)\\ +\beta(x-\tau_x+r, y-r, z-l_m)\end{Bmatrix},$$

$$\vec{H}_{z3} = k\begin{Bmatrix}\lambda(x-r, y-r, z-l_m)\\ -\lambda(x-r, y-\tau_y+r, z-l_m)\\ -\lambda(x, y-r, z-l_m)\\ +\lambda(x, y-\tau_y+r, z-l_m)\end{Bmatrix},$$

$$\vec{H}_{z4} = k\begin{Bmatrix}\lambda(x-\tau_x+r, y-r, z-l_m)\\ -\lambda(x-\tau_x+r, y-\tau_y+r, z-l_m)\\ -\lambda(x-\tau_x, y-r, z-l_m)\\ +\lambda(x-\tau_x, y-\tau_y+r, z-l_m)\end{Bmatrix},$$

$$k = \frac{B_r}{4\pi},$$

$$\alpha(r_1,r_2,r_3)=\arcsin\frac{r_1 r_2}{\sqrt{r_1^2+r_3^2}\sqrt{r_2^2+r_3^2}},$$

$$\beta(r_1,r_2,r_3)=\ln\frac{r_1+\sqrt{r_1^2+r_2^2+r_3^2}}{r_1+\sqrt{r_1^2+r_2^2+\left(r_3+l_m\right)^2}},$$

$$\text{and }\ \lambda(r_1,r_2,r_3)=\ln\frac{r_2+\sqrt{r_1^2+r_2^2+r_3^2}}{r_2+\sqrt{r_1^2+r_2^2+\left(r_3+l_m\right)^2}}.$$

The dimensions and material properties of the proposed SPMPM are shown in Table I.

Table I
Dimensions and Parameters

Part	Item	Symbol	Quantity	Unit
Permanent Magnet	Relative permeability		1.05	
	Residual flux density	Br	1.3	T
	Pitch of x-direction	τ_x	60	mm
	Pitch of y-direction	τ_y	10	mm
	The gap of N-S pole	$2r_1$	2	mm
	Thickness	l_m	10	mm

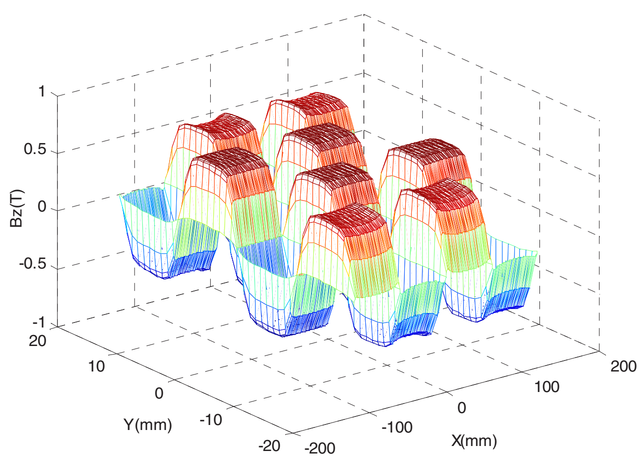

Fig. 6 Magnetic flux density with Halbach array

The Fig.6 shows the 3D air-gap magnetic flux density distribution of Halbach array on the x-y plane which is 1 mm apart from the magnet array surface.

The Fig.7 shows the magnetic flux density from magnet surface to mover yoke at magnet centre. And the results of equivalent magnetic charge method result are compared with the results of FEM calculation by using ANSOFT and the results of Fourier series method [10].

V. Conclusions

The topological structure of Halbach array proposed in this paper is easy to fabricate and high magnetic flux density. The equivalent magnetic charge method has been used to analyze the 3D air-gap flux density distribution of Halbach array based on planar motor. In consideration of the above analysis, the equivalent volume magnetic charge density is zero, which simplifies the analysis model of 3D magnetic field. So the equivalent magnetic charge method can be applied to structure the effective and simple model of complex planar motor control systems in order to research the optimal decoupling method and control strategy.

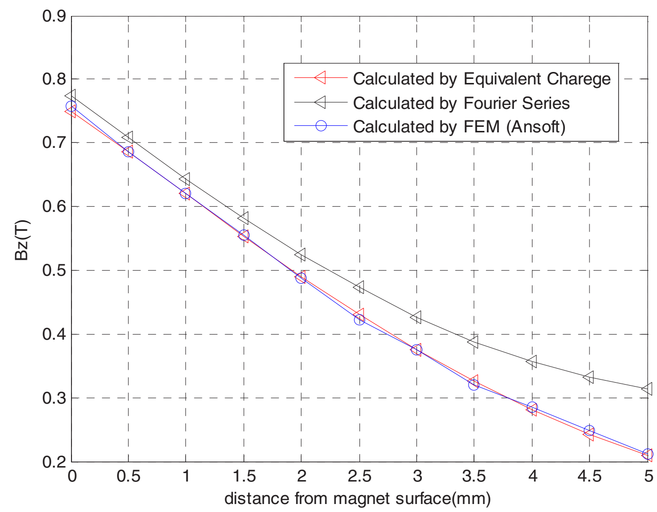

Fig. 7 Magnetic flux density along air-gap length

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