به نام خدا

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برای معادلات دیفرانسیل مسائل ۱-۱ الی ۱-۴ ابتدا رابطه انتگرالی وزن دار و سپس فرم ضعیف آن را بدست آورده و در صورت امکان فرم تابعی درجه دوم آنها را بنویسید.

-۱-۱

A linear differential equation:

$$-\frac{d}{dx}\left[(1+2x^2)\frac{du}{dx}\right]+u=x^2$$

$$u(0)=1\,,\quad \left(\frac{du}{dx}\right)_{x=1}=2$$

-۱-۲

A nonlinear equation:

$$-\frac{d}{dx}\left(u\frac{du}{dx}\right)+f=0 \quad \text{for} \quad 0<x<1$$

$$\left.\left(u\frac{du}{dx}\right)\right|_{x=0}=0 \quad u(1)=\sqrt{2}$$

-۱-۳

The Euler-Bernoulli-von Kármán nonlinear theory of beams:

$$-\frac{d}{dx}\left\{EA\left[\frac{du}{dx}+\frac{1}{2}\left(\frac{dw}{dx}\right)^2\right]\right\}=f \quad \text{for} \quad 0<x<L$$

$$\frac{d^2}{dx^2}\left(EI\frac{d^2w}{dx^2}\right)-\frac{d}{dx}\left\{EA\frac{dw}{dx}\left[\frac{du}{dx}+\frac{1}{2}\left(\frac{dw}{dx}\right)^2\right]\right\}=q$$

$$u=w=0 \quad \text{at} \quad x=0,\ L;\quad \left.\left(\frac{dw}{dx}\right)\right|_{x=0}=0;\quad \left.\left(EI\frac{d^2w}{dx^2}\right)\right|_{x=L}=M_0$$

where *EA, EI, f,* and *q* are functions of *x* and M_0 is a constant. Here *u* denotes the axial displacement and *w* the transverse deflection of the beam.

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A general second-order equation:

$$-\frac{\partial}{\partial x}\left(a_{11}\frac{\partial u}{\partial x}+a_{12}\frac{\partial u}{\partial y}\right)-\frac{\partial}{\partial y}\left(a_{21}\frac{\partial u}{\partial x}+a_{22}\frac{\partial u}{\partial y}\right)+f=0 \quad \text{in} \quad \Omega$$

$$u=u_0 \text{ on } \Gamma_1, \quad \left(a_{11}\frac{\partial u}{\partial x}+a_{12}\frac{\partial u}{\partial y}\right)n_x+\left(a_{21}\frac{\partial u}{\partial x}+a_{22}\frac{\partial u}{\partial y}\right)n_y=t_0 \text{ on } \Gamma_2$$

where $a_{ij} = a_{ji}$ $(i, j = 1, 2)$ and f are given functions of position (x, y) in a two-dimensional domain Ω and u_0 and t_0 are known functions on portions $\Gamma 1$ and $\Gamma 2$ of the boundary Γ: $(\Gamma 1 + \Gamma 2) = \Gamma$.

۱-۵-

Compute the coefficient matrix and the right-hand side of the n-parameter Ritz approximation of the equation

$$-\frac{d}{dx}\left[(1+x)\frac{du}{dx}\right]=0 \quad \text{for} \quad 0<x<1$$

$$u(0)=0, \quad u(1)=1$$

Use algebraic polynomials for the approximation functions. Specialize your result for $n = 2$ and compute the Ritz coefficients.

راهنمایی : با توجه به شرایط مرزی مسئله شکل توابع تقریب (توانی) را بدست آورید.

۱-۶- مسئله قبل را با توابع تقریب مثلثاتی حل کنید.

راهنمایی :

$$\phi_0 = \sin\frac{\pi x}{2}, \quad \phi_i = \sin i\pi x$$

۱-۷-

Set up the equations for the n-parameter Ritz approximation of the following equations associated with a simply supported beam and subjected to a uniform transverse load $q = q_0$:

$$\frac{d^2}{dx^2}\left(EI\frac{d^2w}{dx^2}\right)=q \quad \text{for} \quad 0<x<L$$

$$w=EI\frac{d^2w}{dx^2}=0 \quad \text{at} \quad x=0,\ L$$

Identify (a) algebraic polynomials and (b) trigonometric functions for ϕ_0 and ϕ_i. Compute and compare the two-parameter Ritz solutions with the exact solution for uniform load of intensity q_0.

-١-٨

Consider the differential equation

$$-\frac{d^2u}{dx^2} = \cos \pi x \quad \text{for} \quad 0 < x < 1$$

subject to the following three sets of boundary conditions:

(1) $u(0) = 0, \quad u(1) = 0$

(2) $u(0) = 0, \quad \left(\frac{du}{dx}\right)_{x=1} = 0$

(3) $\left(\frac{du}{dx}\right)_{x=0} = 0, \quad \left(\frac{du}{dx}\right)_{x=1} = 0$

Determine a three-parameter solution, with trigonometric functions, using (a) the Ritz method, (b) the least-squares method, and (c) collocation at $x = \frac{1}{4}, \frac{1}{2}$, and $\frac{3}{4}$, and compare with the exact solutions:

(1) $u_0 = \pi^{-2} (\cos \pi x + 2x - 1)$

(2) $u_0 = \pi^{-2} (\cos \pi x - 1)$

(3) $u_0 = \pi^{-2} \cos \pi x$