IEEE Access
Multidisciplinary ⋮ Rapid Review ⋮ Open Access Journal

Received 14 October 2022, accepted 30 November 2022, date of publication 23 December 2022, date of current version 25 January 2023.

Digital Object Identifier 10.1109/ACCESS.2022.3231903

RESEARCH ARTICLE

Proposed Intuitionistic Fuzzy Entropy Measure Along With Novel Multicriteria Sorting Techniques

ASHUTOSH TIWARI, (Graduate Student Member, IEEE), AND Q. M. DANISH LOHANI, (Senior Member, IEEE)

Department of Mathematics, South Asian University, New Delhi 110021, India

Corresponding author: Q. M. Danish Lohani (danishlohani@cs.sau.ac.in)

The work of Q. M. Danish Lohani was supported by SERB, DST, Government of India, through the scheme of MATRICS Program, under Grant SERB/F/10728/2019-20.

ABSTRACT Recently, the major environmental change and a pandemic called COVID-19 have heavily impacted the economy, business, and health of each country. Moreover, the climatic changes and COVID-19 are calamities to human life. In other words, these two aspects threaten the existence of humans and the sustenance of the overall development of a country. These two factors particularly influence the tourism sector, so a strategy balancing environmental quality and dealing with the ill effects of COVID-19 is formulated to uplift the economic sectors. Atannasov's intuitionistic fuzzy domain is used to model the environmental quality and COVID-19 due to the involvement of hesitancy and uncertainty. The precise measurement of the imprecision in the information is obtained with the help of entropy measure. The paper analyzes the two aspects using a novel entropy measure based on multiple criteria sorting (MCS). Here, the two MCS problems are solved with the help of two proposed techniques: TOPSIS-GREY-sort and ENTROPY-TOPSIS-GREY-sort. A case study showing the impact of COVID-19 in the Philippines and the environmental quality of Tehran (the capital city of Iran) are considered to validate the functioning of the proposed techniques. We use "A novel sorting method TOPSIS-SORT: an application for Tehran environmental quality evaluation (2016), Ekonomica a management," and "Current Issues in Tourism 25.2(2022): 168-178, Taylor and Francis" for the comparative analysis.

INDEX TERMS Atanassov intuitionistic fuzzy set (AIFS), multicritria sorting (MCS), entropy, COVID-19, environmental quality, TOPSIS, GRA.

I. INTRODUCTION

In a decision-making problem, the solution is to provide a technique that selects the best alternative among the given alternatives. Generally, the multiple criteria of the alternatives are analyzed in a decision-making problem. Here, most decision-makers (DMs) employ a ranking of the alternatives to select the best alternative. The multiple criteria decision-making (MCDM) problems appear in the field of business, medical, and public health sectors. So it has been an active area of research during the last couple of decades (see: [1], [2], [3], [4], [5]). Various methods are proposed to solve the MCDM problems (see: [6], [7], [8]). Here, the novel extensions of the Technique for Order Preference by Similarity to Ideal Solution (TOPSIS) and Grey Relational Analysis (GRA) are proposed in the Atanassov intuitionistic fuzzy set (AIFS) environment to solve the MCDM problems. TOPSIS [9], and GRA [10] possess advantages over other MCDM methods due to: (i) computational simplicity, (ii) rationality, (iii) visualization, and (iv) a scalar value that accounts for best and worst alternatives. Moreover, TOPSIS and GRA do not demand any parameter from the DMs during the computation, so the decision-making becomes easy. The AIFS [11] characterizes a criterion based on the

The associate editor coordinating the review of this manuscript and approving it for publication was Muhammad Ali Babar.

degree of belongingness, non-belongingness, and hesitancy. The limitations of the fuzzy set (FS) [12] are dealt with in the AIFS due to the incorporation of an additional degree [13]. A measure-theoretic study of the AIFS is done in [14] and [15]. Some other well-known generalizations of FS are type-2 fuzzy set [16], vague set [17], interval-valued fuzzy set [18], etc. An interrelationship between interval-valued intuitionistic fuzzy set (IVIFS) and AIFS is recently discussed in [19].

Entropy measures the uncertainty within the information. The amalgamation of entropy and AIFS helps in better decision-making. The notion of fuzzy entropy was given by Zadeh [12], and it is further studied by Kaufman and Magens [20], Higashi and Klir [21], and Szmidt and Kacprzyk [22]. Joshi and Kumar [23], [24] introduced an (R, S) norm-based fuzzy information measure and entropy measure. Parkash et al. [25] proposed two new measures of weighted fuzzy entropy to study the maximum weighted fuzzy entropy principle. Bustince and Burillo [26] gave the notion of intuitionistic fuzzy entropy. Szmidt and Kacprzyk [22] proposed a non-probabilistic type of intuitionistic fuzzy entropy. Vlachos and Sergiadis [27] generalized De Luca and Termini's [28] notion of non-probabilistic entropy to AIFSs. Zhang and Jiang [29] suggested a similarity between AIFSs based upon intuitionistic fuzzy entropy. Chen and Li [30] proposed various entropies on AIFSs. In the same paper, they subdivided the attribute weights into two parts: subjective and objective weights. Subjective weights are determined as per the preference of DMs. The methods such as the AHP method [31], weighted least squares method [32], and Delphi method [33] usually use subjective weights. If subjective weights are not provided, the objective weights are determined using the entropy method, multi-objective programming [34], principle element analysis [35], etc. The entropy method is a commonly used approach that solves MCDM problems with partially known weight or unknown weight information. Zhang and Jiang [29] proposed several entropy methods with unknown weight information of attributes to solve MCDM problems.

The GRA technique computes similarities between the ideal option and given options, and here the increasing order of similarities is used for arranging options in the order of preferences. Moreover, realistic and effective processing of the inadequate information is carried out in GRA [36], [37]. The fuzzy variants of GRA are proposed (see: [38], [39], [40]) to deal with the imprecise information. AIFS is an application-rich and highly studied extension of the fuzzy set (see: [41], [42], [43]). In literature, we come across the proposals of AIFS-based variants of GRA (see: [44], [45], [46]). The reopening of the tourism industry and environmental quality improvisation are the two MCDM problems addressed in the paper. The MCDM method is the umbrella under which choosing, sorting, and ranking techniques are introduced, whereas Multiple criteria sorting (MCS) is a specific name allocated to sorting techniques [47]. The decision-making techniques employed to solve the tourism industry and environmental problems belong to MCS. A finite and pre-defined ordered set called a decision class is assigned to evaluate the multiple criteria in the MCS technique. This technique fuses the decisions and opinions of the DMs. The GRA motivated us to propose an IFGRA-sort [48]. Some interesting sorting techniques are VIKOR-sort [49], Flow-sort [50], AHP-sort [51], AHP-sort II [52], ELECTRE-TRI [53] and TOPSIS-sort [54], [55], IFGRA-sort [48].

TOPSIS selects an alternative out of the given alternatives nearest to the positive ideal solution and farthest to the negative ideal solution. The collection of best values of criteria is referred to as a positive-ideal solution, and the worst values across all criteria are collected in a set called a negative-ideal solution. It is claimed that the alternative chosen using TOPSIS is the best one. FS efficiently manages the uncertainties associated with the information. The fuzzy versions of the TOPSIS solving uncertainty possessing MCDM problems are introduced in (see: [56], [57]). In [58], a fuzzy TOPSIS involving the MCDM problem is remodeled in a classical domain. In classical TOPSIS, both the assessments and criteria weights of DMs are assigned specific numerical values. This is seldom the case in real-world situations because the criteria weight selection involves uncertainty, and the hundred percent precision in the DM perception is rarely achieved. We model the criteria weight selection and DM perception in the Atannasov intuitionistic fuzzy domain due to the presence of uncertainty and hesitancy in DM perceptions. The AIFS-based TOPSIS technique solves supplier selection problems [59], data envelopment analysis [60], multi-person MCDM problem [61], group decision-making problem [62], entropy-based public blockchain problem [63]. In [64], a survey of the research work related to the TOPSIS is given.

The notable contributions of the paper are as follows:

- A novel entropy measure compatible with the AIFS environment is proposed.
- The paper has proposed two novel MCS techniques, namely Intuitionistic fuzzy TOPSIS-GREY sorting (TOPSIS-GREY-sort) and Entropy-based intuitionistic fuzzy TOPSIS-GREY sorting with the proposed entropy method (ENTROPY-TOPSIS-GREY-sort).
- The MCS explores either subjective or objective weights, so we proposed a subjective weight-dependent technique called IFGRA-sort in [48]. Here, we have introduced the TOPSIS-GREY-sort technique, which uses entropy-driven objective weight. The proposed entropy measure introduces ENTROPY-TOPSIS-GREY-sort. The proposed entropy measure is used to calculate the objective weights in ENTROPY-TOPSIS-GREY-sort. A convex combination of subjective and objective weights has been used in the proposed ENTROPY-TOPSIS-GREY-sort to analyze the effect of both subjective and objective weights upon decision-making.

- We have successfully solved the problem of the COVID-19 pandemic concerning domestic tourism in the Philippines. Further, a novel methodology to improvise the environmental quality of Tehran has been suggested in the AIFS environment.

The paper is divided into six sections: Section II recalls the basic concepts. Section III mathematically proposes an entropy measure within the AIFS environment. In section IV, two MCS techniques, TOPSIS-GREY-sort and ENTROPY-TOPSIS-GREY-sort, are introduced. The proposed techniques are validated over case studies of domestic tourism in the Philippines and the environmental quality of Tehran in section V. Finally, the conclusion and future work is stated in section VI.

II. BASIC CONCEPTS

This section has three subsections. The basic definitions helpful in understanding the paper are provided in section II-A. The two algorithms, called Intuitionistic FUZZY TOPSIS and Intuitionistic fuzzy GREY RELATIONAL ANALYSIS, are given in section II-B and II-C, respectively.

A. ATANASSOV INTUITIONISTIC FUZZY SET

Definition 1 (Atanassov Intuitionistic Fuzzy Set [11]): Atanassov intuitionistic fuzzy set (AIFS) Q over the universe of discourse $\tilde{X}$ is of the form:

$$Q = \{(x, \mu_Q(x), \nu_Q(x)) \mid x \in \tilde{X}\}.$$

The membership function, $\mu_Q : \tilde{X} \rightarrow [0, 1]$ and non-membership function, $\nu_Q : \tilde{X} \rightarrow [0, 1]$ are defined in such a manner that:

1) $\mu_Q(x) \geq 0,\ \nu_Q(x) \geq 0\ \forall x \in \tilde{X}$.
2) $0 \leq \mu_Q(x) + \nu_Q(x) \leq 1\ \forall x \in \tilde{X}$.

The function $\pi_Q(x) = 1 - \mu_Q(x) - \nu_Q(x)$ calculates the hesitation degree whereas $\mu_Q(x)$ and $\nu_Q(x)$ assigns a membership degree and non-membership degree to $x \in \tilde{X}$, respectively. Here, the ordered pair $\tilde{x} = (\mu, \nu)$ is used to represent an AIFS.

Definition 2 (Operations Between AIFSs [65]): Let us take three AIFS, $\tilde{x} = (\mu, \nu)$, $\tilde{x_1} = (\mu_1, \nu_1)$, $\tilde{x_2} = (\mu_2, \nu_2)$ and $\zeta > 0$. The addition ($\oplus$), subtraction ($\ominus$), scaler multiplication (ζx), and law of indices (x^{ζ}) concerning to AIFS are defined as follows:

1) $\tilde{x_1} \oplus \tilde{x_2} = (\mu_1 + \mu_2 - \mu_1\mu_2, \nu_1\nu_2)$.
2) $\tilde{x_1} \ominus \tilde{x_2} = \begin{cases} \left(\frac{\mu_1-\mu_2}{1-\mu_2}, \frac{\nu_1}{\nu_2}\right), & \mu_1 \geq \mu_2, \nu_1 \leq \nu_2, \\ & \nu_2 \geq 0, \nu_1\pi_2 \leq \nu_2\pi_1 \\ (0, 1), & otherwise \end{cases}$
3) $\zeta\tilde{x} = (1 - (1 - \mu)^{\zeta}, \nu^{\zeta})$.
4) $\tilde{x}^{\zeta} = (\mu^{\zeta}, 1 - (1 - \nu)^{\zeta})$.

Definition 3 (Distance Measure Between AIFSs [66]): A function $D : AIFS(\tilde{X}) \times AIFS(\tilde{X}) \rightarrow$ [0,1] is a distance measure, if it satisfies following four properties:

1) $0 \leq D(\tilde{x_1}, \tilde{x_2}) \leq 1$.
2) $D(\tilde{x_1}, \tilde{x_2}) = 0$ if and only if $\tilde{x_1} = \tilde{x_2}$.
3) $D(\tilde{x_1}, \tilde{x_2}) = D(\tilde{x_2}, \tilde{x_1})$.
4) If $\tilde{x_1} \subseteq \tilde{x_2} \subseteq \tilde{x_3}$, where $\tilde{x_1}, \tilde{x_2}, \tilde{x_3} \in AIFSs(\tilde{X})$, then $D(\tilde{x_1}, \tilde{x_3}) \geq D(\tilde{x_1}, \tilde{x_2})$ and $D(\tilde{x_1}, \tilde{x_3}) \geq D(\tilde{x_2}, \tilde{x_3})$.

Let $\tilde{X_1} = (x_{1i})_{i=1}^{n}, \tilde{X_2} = (x_{2i})_{i=1}^{n}$ are the two AIFS with the universe of discourse as $\tilde{X} = \{x_1, x_2, \cdots x_n\}$, and an AIFS based normalized Euclidean distance is (see [67]):

$$D(\tilde{X_1}, \tilde{X_2}) = \left(\frac{1}{2n}\sum_{i=1}^{n}\left[\left(\mu_{1i} - \mu_{2i}\right)^2 + \left(\nu_{1i} - \nu_{2i}\right)^2 + \left(\pi_{1i} - \pi_{2i}\right)^2\right]\right)^{\frac{1}{2}} \quad (1)$$

Definition 4 (AIFSs Generator [68]): The membership ($\mu(x)$) is a fuzzy mapping $Z(x)$, tuning parameter $\alpha > 0$ and a constant $\zeta \in$ [0,1] dependent function. In the non-membership function ($\nu(x)$) and hesitancy function ($\pi(x)$) one more tuning parameter β is involved as follows:

$$\mu(x) = \zeta Z(x)^{\alpha} \quad (2)$$

$$\nu(x) = (1 - Z(x)^{\beta\alpha})^{1/\beta} \quad (3)$$

$$\pi(x) = 1 - \zeta Z(x)^{\alpha} - (1 - Z(x)^{\beta\alpha})^{1/\beta}. \quad (4)$$

This paper derives an AIFS $\Big(\mu(x), \nu(x)\Big)$ while considering $\zeta = 1, \alpha > 0$ and $0 < \beta \leq 1$ and $\pi(x) = 1 - \mu(x) - \nu(x)$.

B. INTUITIONISTIC FUZZY TOPSIS

Let $Q = \{Q_1, Q_2, \ldots Q_{p'}\}$ be the set of the alternatives and $\tilde{C} = \{\tilde{C_1}, \tilde{C_2}, \ldots \tilde{C_q}\}$ be the set of the criteria. The steps implementing Intuitionistic fuzzy TOPSIS method [59], [69] are given as follows:

Step1: The weights are assigned to G numbers of DMs based on their opinions. First grade the k^{th} DM with an AIFN $D_k = [\mu_k, \nu_k, \pi_k]$ and his opinion is given a weightage λ_k as follows:

$$\lambda_k = \frac{\left(\mu_k + \pi_k\left(\frac{\mu_k}{\mu_k+\nu_k}\right)\right)}{\sum_{k=1}^{G}\left(\mu_k + \pi_k\left(\frac{\mu_k}{\mu_k+\nu_k}\right)\right)} \quad \text{such that} \quad \sum_{k=1}^{G}\lambda_k = 1 \quad (5)$$

Step2: Evaluate criteria-dependent intuitionistic fuzzy decision matrices corresponding to the DMs and then aggregate them. Let the intuitionistic fuzzy decision matrix corresponding to k^{th} DM be $H^{(k)} = (h_{mn}^{(k)})_{p' \times q}$. The G DMs are allocated weight collection $\lambda = \{\lambda_1, \lambda_2, \ldots \lambda_G\}$ such that $\sum_{k=1}^{G}\lambda_k = 1, \lambda_k \in [0, 1]$. Here, intuitionistic fuzzy decision matrices, $H = (h_{mn})_{p' \times q}$, where $h_{mn} = (\mu_{Q_m}(x_n), \nu_{Q_m}(x_n), \pi_{Q_m}(x_n))(m = 1, 2, \ldots p'$; $n = 1, 2, \ldots q)$ are aggregated using the IFWA operator defined in [65] as follows:

$$\begin{aligned} h_{mn} &= IFWA_{\lambda}(h_{mn}^{(1)}, h_{mn}^{(2)}, \ldots .h_{mn}^{(G)}) \\ &= \lambda_1 h_{mn}^{(1)} \oplus \lambda_2 h_{mn}^{(2)} \oplus \lambda_3 h_{mn}^{(3)} \oplus \ldots .. \oplus \lambda_G h_{mn}^{(G)} \\ &= \left[1 - \prod_{k=1}^{G}(1 - \mu_{mn}^{(k)})^{\lambda_k}, \prod_{k=1}^{G}(\nu_{mn}^{(k)})^{\lambda_k}, \right. \\ &\qquad \left. \prod_{k=1}^{G}(1 - \mu_{mn}^{(k)})^{\lambda_k} - \prod_{k=1}^{G}(\nu_{mn}^{(k)})^{\lambda_k}\right] \end{aligned}$$

The aggregrated decision matrix H is obtained,

$$H = \begin{bmatrix} h_{11} & h_{12} & h_{13} & \dots & h_{1p'} \\ h_{21} & h_{22} & h_{23} & \dots & h_{2p'} \\ h_{31} & h_{32} & h_{33} & \dots & h_{3p'} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h_{q1} & h_{q2} & h_{q3} & \dots & h_{qp'} \end{bmatrix}$$

Step3: Calculation of the criteria weights, $W = [w_1, w_2, w_3, \dots .w_q]$, $w_n = (\mu_n, \nu_n, \pi_n)(n = 1, 2, \dots, q)$.

It is not necessary for all the criteria to be equally important in decision-making. So, a criteria weight matrix is calculated in which the perspectives of DMs regarding the criteria are merged. Here, the k^{th} DM rates criterion x_n with the help of an AIFN $w_n^{(k)} = [\mu_n^{(k)}, \nu_n^{(k)}, \pi_n^{(k)}]$. The IFWA operator combines the weights of entire criteria, and we obtain the criteria weight w_n of x_n as follows:

$$\begin{aligned} w_n &= IFWA_\lambda(w_n^{(1)}, w_n^{(2)}, \dots .w_n^{(G)}) \\ &= \lambda_1 w_n^{(1)} \oplus \lambda_2 w_n^{(2)} \oplus \lambda_3 w_n^{(3)} \oplus \dots . \oplus \lambda_G w_n^{(G)} \\ &= [1 - \prod_{k=1}^{G}(1-\mu_n^{(k)})^{\lambda_k}, \prod_{k=1}^{G}(\nu_n^{(k)})^{\lambda_k}, \\ &\qquad \prod_{k=1}^{G}(1-\mu_n^{(k)})^{\lambda_k} - \prod_{k=1}^{G}(\nu_n^{(k)})^{\lambda_k}] \end{aligned} \tag{6}$$

Step4: Aggregation of criteria weights and intuitionistic fuzzy decision matrices.

The criteria weights W and intuitionistic fuzzy decision matrix H are aggregated using the binary operation $\oplus$ given in [70]. Hence,

$$\begin{aligned} H \oplus W &= \Big\{x, \mu_{Q_m}(x).\mu_W(x), \nu_{Q_m}(x) + \nu_W(x) \\ &\qquad - \nu_{Q_m}(x).\nu_W(x) \mid x \in \tilde{X}\Big\} \\ \pi_{Q_m W(x)} &= \big[1 - \nu_{Q_m}(x) - \nu_w(x) - \mu_{Q_m}(x).\mu_w(x) \\ &\qquad + \nu_{Q_m}(x).\nu_w(x)\big] \end{aligned} \tag{7}$$

Then, the aggregated weighted intuitionistic fuzzy decision matrix H' is given as:

$$H' = \begin{bmatrix} h'_{11} & h'_{12} & h'_{13} & \dots & h'_{1p'} \\ h'_{21} & h'_{22} & h'_{23} & \dots & h'_{2p'} \\ h'_{31} & h'_{32} & h'_{33} & \dots & h'_{3p'} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h'_{q1} & h'_{q2} & h'_{q3} & \dots & h'_{qp'} \end{bmatrix}$$

where, $h'_{mn} = (\mu'_{mn}, \nu'_{mn}, \pi'_{mn}) = (\mu_{Q_m}(x_n), \nu_{Q_m}(x_n), \pi_{Q_m}(x_n))$ is the mn^{th} entry of H'.

Step5: Evaluation of intuitionistic fuzzy positive-ideal solution (Q^+) and intuitionistic fuzzy negative-ideal solution (Q^-).

Let us denote the benefit and cost criteria with J_1 and J_2, respectively. The following equations calculate Q^+ and Q^-:

$$\begin{aligned} Q^+ &= (\mu_{Q+W}(x_n), \nu_{Q+W}(x_n)) \text{ and} \\ Q^- &= (\mu_{Q-W}(x_n), \nu_{Q-W}(x_n)) \end{aligned} \tag{8}$$

$$\begin{aligned} \mu_{Q^+W}(x_n) &= ((max_m \mu_{Qm.W}(x_n) \mid n \in J_1), \\ &\qquad (min_m \mu_{Q_m.W}(x_n) \mid n \in J_2)) \end{aligned} \tag{9}$$

$$\begin{aligned} \nu_{Q^+W}(x_n) &= ((min_m \nu_{Qm.W}(x_n) \mid n \in J_1), \\ &\qquad (max_m \nu_{Q_m.W}(x_n) \mid n \in J_2)) \end{aligned} \tag{10}$$

$$\begin{aligned} \mu_{Q^-W}(x_n) &= ((min_m \mu_{Qm.W}(x_n) \mid n \in J_1), \\ &\qquad (max_m \mu_{Q_m.W}(x_n) \mid n \in J_2)) \end{aligned} \tag{11}$$

$$\begin{aligned} \nu_{Q^-W}(x_n) &= ((max_m \nu_{Qm.W}(x_n) \mid n \in J_1), \\ &\qquad (min_m \nu_{Q_m.W}(x_n) \mid n \in J_2)) \end{aligned} \tag{12}$$

Step6: Evaluation of the separation measures.

Q^+ and Q^- based separation measures S^+ and S^- for each alternative are calculated using the normalized euclidean distance measure [67] as follows:

$$\begin{aligned} S_m^+ = \Bigg(\frac{1}{2q}\sum_{m=1}^{q}\Big[\big(\mu_{Q_m}(x_m) - \mu_{Q+W}(x_m)\big)^2 \\ + \big(\nu_{Q_m}(x_m) - \nu_{Q+W}(x_m)\big)^2 \\ + \big(\pi_{Q_m}(x_m) - \pi_{Q+W}(x_m)\big)^2\Big]\Bigg)^{\frac{1}{2}} \end{aligned} \tag{13}$$

$$\begin{aligned} S_m^- = \Bigg(\frac{1}{2q}\sum_{m=1}^{q}\Big[\big(\mu_{Q_m}(x_m) - \mu_{Q-W}(x_m)\big)^2 \\ + \big(\nu_{Q_m}(x_m) - \nu_{Q-W}(x_m)\big)^2 \\ + \big(\pi_{Q_m}(x_m) - \pi_{Q-W}(x_m)\big)^2\Big]\Bigg)^{\frac{1}{2}} \end{aligned} \tag{14}$$

Step7: Evaluate relative closeness coefficient (RC_m) between intuitionistic ideal solution (Q^*) and alternative Q_m as follows:

$$RC_m = \frac{S_m^-}{S_m^+ + S_m^-}, \quad \text{where } 0 \leq RC_m \leq 1 \tag{15}$$

Step8: Select the best alternative, the alternative getting maximum (RC_m) value.

C. INTUITIONISTIC FUZZY GREY RELATIONAL ANALYSIS

Let $Q = \{Q_1, Q_2, \cdots, Q_{p'}\}$ be the set of alternatives with attribute set $\tilde{C} = \{\tilde{C}_1, \tilde{C}_2, \cdots, \tilde{C}_q\}$. Moreover, a set of weight vectors $W = \{w_1, w_2, \cdots, w_q\}^T$ is assigned to attribute set $\tilde{C}$ such that $w_n \in [0, 1]$, $\sum_{n=1}^{q} w_n = 1$. The steps implementing intuitionistic fuzzy GRA [71] method are as follows:

Step1: Standard decision matrix.

Let the standard decision matrix $H = [h_{mn}]_{p' \times q}$ is given in the form $h_{mn} = (\mu_{mn}, \nu_{mn})$.

Step2: Evaluation of the positive ideal solution and negative ideal solution.

The standard decision matrix based intuitionistic fuzzy positive ideal solution (PIS) Q^+ and intuitionistic fuzzy negative ideal solution (NIS) Q^- is calculated as follows:

$$\begin{aligned} Q^+ &= \Big((\mu_1^+, \nu_1^+), (\mu_2^+, \nu_2^+), \cdots, (\mu_n^+, \nu_n^+)\Big) \\ &= \Big(\max_m(\mu_{mn}), \min_m(\nu_{mn})\Big) \end{aligned} \tag{16}$$

$$Q^- = \left((\mu_1^-, \nu_1^-), (\mu_2^-, \nu_2^-), \cdots, (\mu_n^-, \nu_n^-)\right) = \left(\min_m(\mu_{mn}), \max_m(\nu_{mn})\right) \tag{17}$$

Step3: Calculation of the grey relational coefficient (GRC).

The PIS, NIS, and an alternative are used in the distance measure (see: Eq.(1)), which results in a positive GRC (ξ_{mn}^+) and negative GRC (ξ_{mn}^-) as follows:

$$\xi_{mn}^+ = \frac{\left(\min_{1\le m\le p'}\min_{1\le n\le q} D(h_{mn}, Q_n^+) + \eta \max_{1\le m\le p'}\max_{1\le n\le q} D(h_{mn}, Q_n^+)\right)}{\left(D(h_{mn}, Q_n^+) + \eta \max_{1\le m\le p'}\max_{1\le n\le q} D(h_{mn}, Q_n^+)\right)} \tag{18}$$

$$\xi_{mn}^- = \frac{\left(\min_{1\le m\le p'}\min_{1\le n\le q} D(h_{mn}, Q_n^-) + \eta \max_{1\le m\le p'}\max_{1\le n\le q} D(h_{mn}, Q_n^-)\right)}{\left(D(h_{mn}, Q_n^-) + \eta \max_{1\le m\le p'}\max_{1\le n\le q} D(h_{mn}, Q_n^-)\right)} \tag{19}$$

Here, we have an identification coefficient $\eta \in [0, 1]$, its commonly used value is $\eta = 0.5$.

Step4: Compute PIS based grey relational grade (GRG) and NIS based GRG corresponding to an alternative.

PIS based GRG (ξ_m^+) and NIS based GRG (ξ_m^-) of each criterion are calculated as follows:

$$\xi_m^+ = \sum_{n=1}^{q} w_n \xi_{mn}^+ \tag{20}$$

$$\xi_m^- = \sum_{n=1}^{q} w_n \xi_{mn}^- \tag{21}$$

We have assigned a weight vector W over the attribute $\tilde{C_n}$ such that $0 \le w_n \le 1$ and $\sum_{n=1}^{q} w_n = 1$.

Step5: Determination of the relative closeness coefficient.

For alternative Q_m, the relative closeness coefficient (RC_m) is given in terms of ξ_m^+ and ξ_m^- as follows:

$$RC_m = \frac{\xi_m^+}{\xi_m^+ + \xi_m^-} \tag{22}$$

Step6: Rank the alternatives.

Ranking of alternatives is based on the descending values of RC_m, that is, the most important alternative is assigned the maximum value of RC_m.

III. A NEW INTUITIONISTIC FUZZY ENTROPY MEASURE

In this section, a new entropy measure is proposed.

Definition 5 [72]: Suppose, $E : AIFS(\tilde{X}) \to$ [0,1] is a real-valued function. Then E is said to be entropy over $\tilde{X}$ if it satisfies the following properties:

1) $E(Q) = 0$ (minimum), iff Q is a crisp set, i.e. $\mu_Q(x_m) = 1$, $\nu_Q(x_m) = 0$ or vice-versa.
2) $E(Q) = 1$ (maximum), iff $\mu_Q(x_m) = \nu_Q(x_m)\ \forall\ x_m \in \tilde{X}$.
3) $E(Q_1) \le E(Q_2)$ iff $Q_1 \subseteq Q_2$, i.e. if $\mu_{Q_1}(x_m) \le \mu_{Q_2}(x_m)$ and $\nu_{Q_1}(x_m) \ge \nu_{Q_2}(x_m)$ for $\mu_{Q_1}(x_m) \le \nu_{Q_2}(x_m)$, or if $\mu_{Q_1}(x_m) \ge \mu_{Q_2}(x_m)$ and $\nu_{Q_1}(x_m) \le \nu_{Q_2}(x_m)$ for $\mu_{Q_1}(x_m) \ge \nu_{Q_2}(x_m)$.
4) $E(Q) = E(Q^c)$.

Theorem 1: Let $AIFS(\tilde{X})$ be a support of AIFS and Q is an AIFS, the function $E : AIFS(\tilde{X}) \to [0, 1]$ is an entropy measure over $AIFS(\tilde{X})$,

$$E(Q) = \frac{1}{2n}\sum_{x\in\tilde{X}}\left[\left\{cos\left(min\left(\Upsilon\right)\frac{\pi}{2}\right)\right\} \times \left\{\frac{1}{2^{max\left(\Upsilon\right)}}\right\} \times \left\{(2 + max\left(\Upsilon\right)^{max\left(\Upsilon\right)}\right\}\right] \tag{23}$$

where, $\Upsilon = \left(\mu(x) - \nu(x)\right), \left(\nu(x) - \mu(x)\right)$.

Proof 1: Let us verify the four properties of the Entropy measure in $E(Q)$ as follows:

1. Let Q is a crisp set i.e. $\mu_Q(x) = 1$, $\nu_Q(x) = 0$, or $\mu_Q(x) = 0$, $\nu_Q(x) = 1$ for any $x \in \tilde{X}$, then $E(Q) = 0$.

Conversely, $E(Q) = 0$, then

$$\frac{1}{2n}\sum_{x\in\tilde{X}}\left[\left\{cos\left(min\left(\Upsilon\right)\frac{\pi}{2}\right)\right\} \times \left\{\frac{1}{2^{max\left(\Upsilon\right)}}\right\} \times \left\{(2 + max\left(\Upsilon\right)^{max\left(\Upsilon\right)}\right\}\right] = 0.$$

as,

$$\left\{\frac{1}{2^{max\left(\Upsilon\right)}}\right\} \ne 0 \text{ and } \sum_{x\in\tilde{X}}\left\{(2 + max\left(\Upsilon\right)^{max\left(\Upsilon\right)}\right\} \ne 0.$$

then,

$$cos\left(min\left(\Upsilon\right)\frac{\pi}{2}\right) = 0 \text{ and} \tag{24}$$

$$-\frac{\pi}{2} \le \left(min\left(\Upsilon\right)\frac{\pi}{2}\right)\frac{\pi}{2} \le 0 \tag{25}$$

The Eq.(24)-(25) together yields $min\left(\Upsilon\right) = -1$ i.e., $min\left(\left(\mu(x) - \nu(x)\right), \left(\nu(x) - \mu(x)\right)\right) = -1$ and hence

$$\mu(x) = 1,\ \nu(x) = 0.$$

2. Let $\mu_Q(x) = \nu_Q(x)\ \forall\ x \in \tilde{X}$, then $E(Q) = 1$.

Conversely, let $E(Q) = 1$, so we have,

$$\frac{1}{2n}\sum_{x\in\tilde{X}}\left[\left\{cos\left(min\left(\Upsilon\right)\frac{\pi}{2}\right)\right\} \times \left\{\frac{1}{2^{max\left(\Upsilon\right)}}\right\} \times \left\{(2 + max\left(\Upsilon\right)^{max\left(\Upsilon\right)}\right\}\right] = 1$$

Thus,

$$cos\left(min\left(\Upsilon\right)\frac{\pi}{2}\right) = 1 \text{ and} \tag{26}$$

$$\left\{\frac{1}{2^{max\left(\Upsilon\right)}}\right\}\frac{1}{2n}\sum_{x\in\tilde{X}}\left\{(2 + max\left(\Upsilon\right)^{max\left(\Upsilon\right)}\right\} = 1. \tag{27}$$

The Eq.(26)-(27) are simultaneosly satisfied if

$$\mu(x) = \nu(x) \text{ and } x \in \tilde{X}.$$

3. $E(Q_1) \leq E(Q_2)$ iff $Q_1 \subseteq Q_2$,

We know that $Q_1 \subseteq Q_2$ is equivalent to $\mu_1(x) \leq \mu_2(x)$ and $\nu_1(x) \geq \nu_2(x)$ for $\mu_1(x) \leq \nu_2(x)$. Hence,

$$\mu_1(x) - \nu_1(x) \leq \mu_2(x) - \nu_2(x). \tag{28}$$

Let us assume that,

$$min\Big(\Big(\Upsilon_1\Big)\frac{\pi}{2}\Big) \leq min\Big(\Big(\Upsilon_2\Big)\frac{\pi}{2}\Big)$$

As $\cos\theta$ is an increasing function for $-\frac{\pi}{2} \leq \theta \leq 0$. So,

$$cos\Big(min\Big(\Upsilon_1\Big)\frac{\pi}{2}\Big) \leq cos\Big(min\Big(\Upsilon_2\Big)\frac{\pi}{2}\Big) \tag{29}$$

Now if,

$$max\Big(\Big(\Upsilon_1\Big)\frac{\pi}{2}\Big) \leq max\Big(\Big(\Upsilon_2\Big)\frac{\pi}{2}\Big) \tag{30}$$

Let $y(x) = \frac{1}{2^{\max\{x,-x\}}} \sum_{x\in\tilde{X}} \Big(2 + \max\{x, -x\}\Big)^{\max\{x,-x\}}$, so, $y(x) = \frac{1}{2^x} \sum_{x\in\tilde{X}} \Big(2 + x\Big)^x$.

As, $\frac{dy}{dx} = y\Big(\frac{2x}{2+x} + \log\Big(\frac{2+x}{2}\Big)\Big) > 0$, so due to the virtue of first derivative test $y(x)$ is an increasing function. Using $y(x)$ and Eq.(29)-(30), we have,

$$\begin{aligned} E(Q_1) &= \frac{1}{2n}\sum_{x\in\tilde{X}}\Bigg[\Big\{cos\Big(min\Big(\Upsilon_1\Big)\frac{\pi}{2}\Big)\Big\} \times \Big\{\frac{1}{2^{max(\Upsilon_1)}}\Big\} \\ &\quad \times \Big\{(2 + max\Big(\Upsilon_1\Big)^{max(\Upsilon_1)}\Big\}\Bigg] \\ &\leq \frac{1}{2n}\sum_{x\in\tilde{X}}\Bigg[\Big\{cos\Big(min\Big(\Upsilon_2\Big)\frac{\pi}{2}\Big)\Big\} \times \Big\{\frac{1}{2^{max(\Upsilon_2)}}\Big\} \\ &\quad \times \Big\{(2 + max\Big(\Upsilon_2\Big)^{max(\Upsilon_2))}\Big\}\Bigg] \\ &= E(Q_2) \implies E(Q_1) \leq E(Q_2). \end{aligned}$$

4. As we know that $Q^c = \{(x, \nu_Q(x), \mu_Q(x)) \mid x \in \tilde{X}\}$ i.e. $\mu_{Q^c}(x) = \nu_Q(x)$ and $\nu_{Q^c}(x) = \mu_Q(x)$. Thus we obtain,

$$\begin{aligned} E(Q) &= \frac{1}{2n}\sum_{x\in\tilde{X}}\Bigg[\Big\{cos\Big(min\Big(\Upsilon\Big)\frac{\pi}{2}\Big)\Big\} \times \Big\{\frac{1}{2^{max(\Upsilon)}}\Big\} \\ &\quad \times \Big\{(2 + max\Big(\Upsilon\Big)^{max(\Upsilon)}\Big\}\Bigg] \\ &= \frac{1}{2n}\sum_{x\in\tilde{X}}\Bigg[\Big\{cos\Big(min\Big(\nabla\Big)\frac{\pi}{2}\Big)\Big\} \times \Big\{\frac{1}{2^{max(\nabla)}}\Big\} \\ &\quad \times \Big\{(2 + max\Big(\nabla\Big)^{max(\nabla)}\Big\}\Bigg] = E(Q^c) \end{aligned}$$

where, $\nabla = \big(\nu(x) - \mu(x)\big), \big(\mu(x) - \nu(x)\big)$.

Hence, $E(Q)$ is a valid intuitionistic fuzzy entropy measure.

IV. PROPOSED TECHNIQUES

A. PROPOSED INTUITIONISTIC FUZZY TOPSIS-GREY SORTING (TOPSIS-GREY-SORT)

The proposed technique (TOPSIS-GREY-sort) is defined for a dataset with p' alternatives. The q numbers of criteria are involved to define the alternatives. The aim of the technique is the correct classification of p' alternatives in g classes $\{d_1, d_2, \ldots, d_g\}$. The TOPSIS-SGREY-sort technique is implemented in thirteen steps as follows:

Step1: Construct a decision matrix $H^k = (h^k_{mn})_{p'\times q}$ corresponding to k^{th} decision-maker (DM) $k = 1, 2, \ldots, G$. The g classes based information of a criterion forms a limit profile, and it is stored in the collection $Y = \{(y^1_l(d_1), y^1_u(d_1)), (y^2_l(d_2), y^2_u(d_2)), \cdots (y^t_l(d_g), y^t_u(d_g))\}$. In Y, the upper limit of the preceding class and lower limit of the succeeding class is same, i.e., $y^{j-1}_u(d_{j-1}) = y^j_l(d_j)$. Here, $y^j_l(d_j)$ is the lower limit of j^{th} class and $y^{j-1}_u(d_{j-1})$ is the upper limit of $(j-1)^{th}$ class. Let us aggregate the decision matrix $H^k = (h^k_{mn})_{p'\times q}$ into $H = (h_{mn})_{p'\times q}$ with the help of the pre-defined -aggregation operator.

Step2: Merge the limit profiles to the aggregated decision matrix (H) while computing $\Gamma = (H, X) = (\chi_{mn})_{p\times q}$.

Step3: Normalize the matrix Γ using Eq.(31) and its resultant is a matrix $\Lambda = (\lambda_{mn})_{p\times q}$ such that,

$$\begin{aligned} \lambda_{mn} = \Bigg(&\Big(\frac{\chi_{mn} - min_{mn}\chi_{mn}}{max_{mn}\chi_{mn} - min_{mn}\chi_{mn}} | n \in \tilde{C}_b\Big), \\ &\Big(1 - \frac{\chi_{mn} - min_{mn}\chi_{mn}}{max_{mn}\chi_{mn} - min_{mn}\chi_{mn}} | n \in \tilde{C}_c\Big)\Bigg). \end{aligned} \tag{31}$$

Step4: Use AIFS generator (see: def. 4) to transform the matrix $\Lambda = (\lambda_{mn})_{p\times q}$ into an intuitionistic fuzzy decision matrix $\tilde{\Lambda} = (\tilde{\lambda}_{mn} = (\mu_{mn}, \nu_{mn}))_{p\times q}$, where $(m = 1, 2, \cdots p; n = 1, 2, \cdots q)$.

Step5: Calculate intuitionistic fuzzy entropy in intuitionistic fuzzy decision matrix $\tilde{\Lambda} = (\tilde{\lambda}_{mn})_{p\times q}$ as follows:

$$E_{mn}(x_n) = \frac{\Big\{\min\big[\mu_{mn}(x_n), \nu_{mn}(x_n)\big] + \min\big[\overline{\nu_{mn}(x_n)}, \overline{\mu_{mn}(x_n)}\big]\Big\}}{\Big\{\max\big[\mu_{mn}(x_n), \nu_{mn}(x_n)\big] + \max\big[\overline{\nu_{mn}(x_n)}, \overline{\mu_{mn}(x_n)}\big]\Big\}} \tag{32}$$

where, $\overline{\nu_{mn}(x_n)} = 1 - \nu_{mn}(x_n)$, $\overline{\mu_{mn}(x_n)} = 1 - \mu_{mn}(x_n)$

$$\tilde{E} = \begin{bmatrix} E_{11} & E_{12} & E_{13} & \ldots & E_{1p} \\ E_{21} & E_{22} & E_{23} & \ldots & E_{2p} \\ E_{31} & E_{32} & E_{33} & \ldots & E_{3p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ E_{q1} & E_{q2} & E_{q3} & \ldots & E_{qp} \end{bmatrix}$$

The E_{mn} denotes entropy value at the mn^{th} position in intuitionistic fuzzy decision matrix.

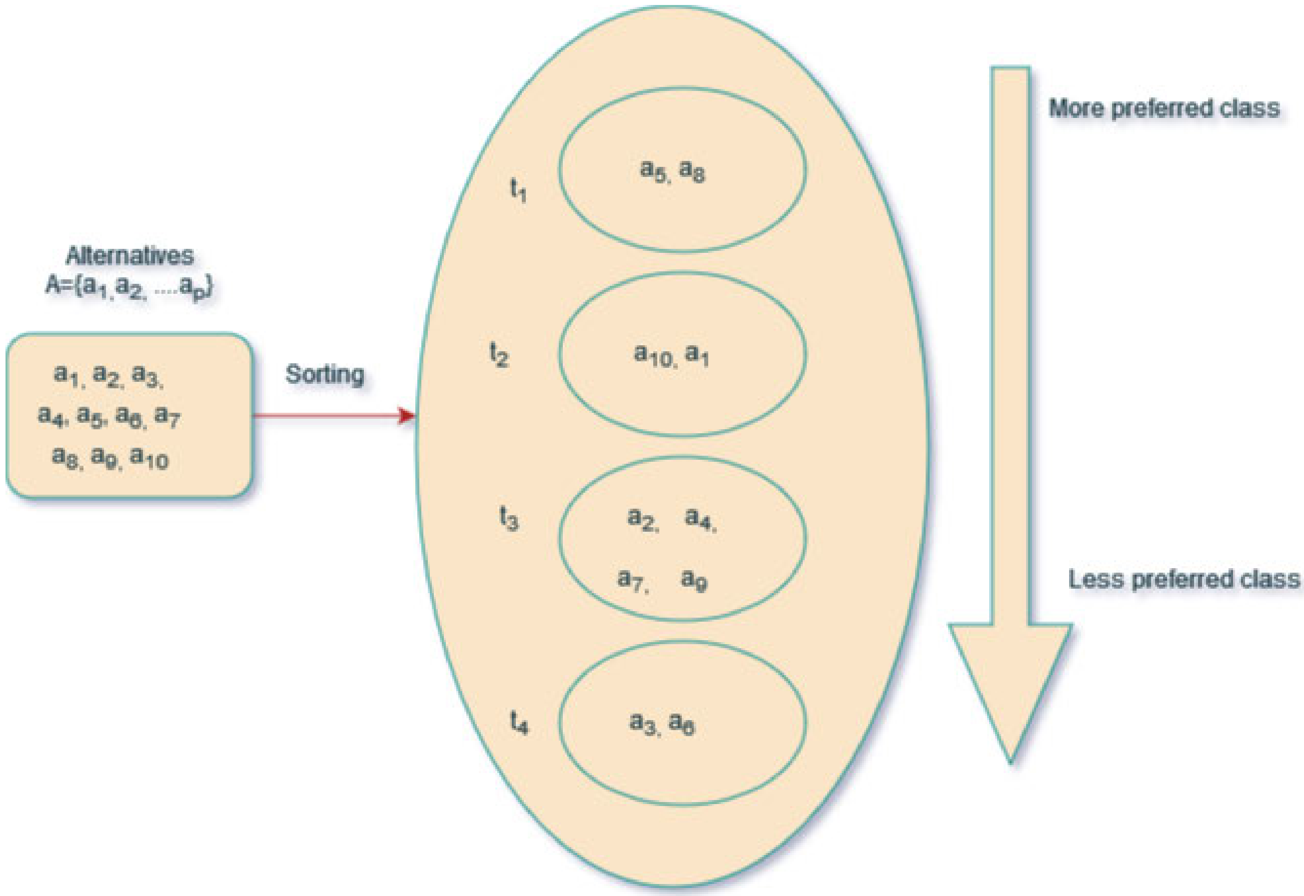

FIGURE 1. Schematic Representation of Sorting problem.

Step6: Normalize the intuitionistic fuzzy entropy values as follows:

$$e_{mn} = \frac{E_{mn}}{\max(E_{mn})} \tag{33}$$

The normalized intuitionistic fuzzy decision matrix is $\tilde{\tilde{E}}$, where,

$$\tilde{\tilde{E}} = \begin{bmatrix} e_{11} & e_{12} & e_{13} & \dots & e_{1p} \\ e_{21} & e_{22} & e_{23} & \dots & e_{2p} \\ e_{31} & e_{32} & e_{33} & \dots & e_{3p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ e_{q1} & e_{q2} & e_{q3} & \dots & e_{qp} \end{bmatrix}$$

Step7: Evaluate the objective weight of each criterion.

Here, we mainly employ the entropy weight method to calculate the initial objective weights. The matrix $\tilde{\tilde{E}} = \{e_{mn}, m = 1, 2 \cdots p, n = 1, 2 \cdots q\}$ is involved in the calculation of objective weight of a criterion:

$$W_n^1 = \frac{1 - e_{mn}}{\sum_{n=1}^{q}(1 - e_{mn})} \tag{34}$$

Step8: Derive the weighted matrix out of the intuitionistic fuzzy decision matrix.

The following formula proposed by Atanassov in [11] is used to construct a weighted matrix out of an intuitionistic fuzzy decision matrix ($\tilde{\Lambda}$):

$$W_n\tilde{\Lambda} = \left(1 - \left(1 - \mu_{\tilde{\Lambda}}\right)^{W_n}, \left(\nu_{\tilde{\Lambda}}\right)^{W_n}\right) \tag{35}$$
$$\text{where, } 0 < W_n < 1.$$

The aggregated weighted intuitionistic fuzzy decision matrix H' is shown in the equation at the bottom of the next page, $h'_{mn} = (\mu'_{mn}, \nu'_{mn}, \pi'_{mn}) = (\mu_{Q_m}(x_n), \nu_{Q_m}(x_n), \pi_{Q_m}(x_n))$ denotes the mn^{th} entry of aggregated weighted intuitionistic fuzzy decision matrix.

Step9: Calculate the intuitionistic fuzzy positive ideal solution (Q^+) and the intuitionistic fuzzy negative ideal solution (Q^-) from H', which is given as:

$$Q^+ = \left((\mu_1^+, \nu_1^+), (\mu_2^+, \nu_2^+), \cdots (\mu_q^+, \nu_q^+)\right) \tag{36}$$

$$Q_n^+ = \left(\mu_n^+, \nu_n^+\right) = \left(\max_m \mu_{mn}, \min_m \nu_{mn}\right)$$
$$\forall n = 1, 2, \cdots q. \tag{37}$$

$$Q^- = \left((\mu_1^-, \nu_1^-), (\mu_2^-, \nu_2^-), \cdots (\mu_q^-, \nu_q^-)\right) \tag{38}$$

$$Q_n^- = \left(\mu_n^-, \nu_n\right) = \left(\min_m \mu_{mn}, \max_m \nu_{mn}\right) \quad \forall n = 1, 2, \cdots q. \tag{39}$$

Step10: Determine the grey relational coefficient (GRC).

The use of Q_n^+ and Q_n^- in intuitionistic fuzzy normalized distance measure (1), calculates positive GRC (ξ_{mn}^+) and negative GRC (ξ_{mn}^-) which is as follows:

$$\xi_{mn}^+ = \frac{\left(\min_{1\leq m\leq p}\min_{1\leq n\leq q} D(h'_{mn}, Q_n^+)+\eta \max_{1\leq m\leq p}\max_{1\leq n\leq q} D(h'_{mn}, Q_n^+)\right)}{\left(D(h'_{mn}, Q_n^+) + \eta \max_{1\leq m\leq p}\max_{1\leq n\leq q} D(h'_{mn}, Q_n^+)\right)} \tag{40}$$

$$\xi_{mn}^- = \frac{\left(\min_{1\leq m\leq p}\min_{1\leq n\leq q} D(h'_{mn}, Q_n^-)+\eta \max_{1\leq m\leq q}\max_{1\leq n\leq p} D(h'_{mn}, Q_n^-)\right)}{\left(D(h'_{mn}, Q_n^-) + \eta \max_{1\leq m\leq p}\max_{1\leq n\leq q} D(h'_{mn}, Q_n^-)\right)} \tag{41}$$

Generally, during computation the identification coefficient $\eta = 0.5$, $\eta \in [0, 1]$ is used, and the same is taken here also.

Step11: Assign grey relational grade (GRG) to every alterative with the help of ξ_{mn}^+ and ξ_{mn}^- as follows:

$$\xi_m^+ = \sum_{n=1}^{q} W_n \xi_{mn}^+ \tag{42}$$

$$\xi_m^- = \sum_{n=1}^{q} W_n \xi_{mn}^- \tag{43}$$

here the n^{th} criterion is assigned weight W_n (obtained in Step7), and $0 \leq W_n \leq 1$; $\sum_{n=1}^{q} W_n = 1$.

Step12: Calculate the relative closeness coefficient.

The relative closeness coefficient (RC_m) is ξ_m^+ and ξ_m^- dependent:

$$RC_m = \frac{\xi_m^-}{\xi_m^+ + \xi_m^-} \tag{44}$$

Step13: Find the deviations of the upper limit profile and lower limit profile of the j^{th} class from Q^+ and Q^-. These deviations are $RC_n^{x_u^j}$ and $RC_n^{x_l^j}$, respectively. If RC_m values of the alternatives satisfy eq. 45, then such alternatives belong to the j^{th} interval class.

$$RC_n^{x_l^j} < RC_m < RC_n^{x_u^j} \quad \forall m, j \tag{45}$$

B. PROPOSED ENTROPY-BASED INTUITIONISTIC FUZZY TOPSIS-GREY SORTING (ENTROPY-TOPSIS-GREY-SORT)

Here, Steps 1-4 are the same as proposed in TOPSIS-GREY-sort, and from Step 5, a different approach is adopted.

Step5: Determine the intuitionistic fuzzy entropy values of $\tilde{\Lambda} = (\tilde{\lambda}_{mn})_{p\times q}$ using the Eq.(23).

$$\tilde{E} = \begin{bmatrix} E_{11} & E_{12} & E_{13} & \ldots & E_{1p} \\ E_{21} & E_{22} & E_{23} & \ldots & E_{2p} \\ E_{31} & E_{32} & E_{33} & \ldots & E_{3p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ E_{q1} & E_{q2} & E_{q3} & \ldots & E_{qp} \end{bmatrix}$$

where, E_{mn} represents each entropy value in the intuitionistic fuzzy decision matrix.

Step6: Normalization of intuitionistic fuzzy entropy values by repeating the Step-6 of TOPSIS-GREY-sort.

Step7: To aggregate and then evaluate data, some series of moderate and accurate weights as much as possible is undoubtedly needed. The weights are considered in two aspects. On the one hand, the objective weights are calculated through the entropy weight method such that its influence on data can be felt. On the other hand, the experts give subjective weights to express the decision preference. Finally, the weighted average value of objective and subjective weights is taken.

Step7.1 Evaluate the objective weights of criteria.

For objective weight (say W_n^1) calculation repeat the Step-7 of TOPSIS-GREY.

Step7.2 Obtain the subjective weights of criteria

The subjective weights $W_n^2 = [W_1^2, W_2^2, W_3^2 \cdots W_q^2]$ are given by the experts.

Step7.3 Combine objective and surjective weights

$$W = \rho W_n^1 + (1-\rho) W_n^2 \tag{46}$$

$$H' = \begin{bmatrix} (\mu_{11W}(x_1), \nu_{11W}(x_1), \pi_{11W}(x_1)) & (\mu_{12W}(x_2), \nu_{12W}(x_2), \pi_{12W}(x_2)) & \ldots & (\mu_{1qW}(x_q), \nu_{1qW}(x_q), \pi_{1qW}(x_q)) \\ (\mu_{21W}(x_1), \nu_{21W}(x_1), \pi_{21W}(x_1)) & (\mu_{22W}(x_2), \nu_{22W}(x_2), \pi_{22W}(x_2)) & \ldots & (\mu_{2qW}(x_q), \nu_{2qW}(x_q), \pi_{2qW}(x_q)) \\ \vdots & \vdots & \ddots & \vdots \\ (\mu_{p1W}(x_1), \nu_{p1W}(x_1), \pi_{p1W}(x_1)) & (\mu_{p2W}(x_2), \nu_{p2W}(x_2), \pi_{p2W}(x_2)) & \ldots & (\mu_{pqW}(x_q), \nu_{pqW}(x_q), \pi_{pqW}(x_q)) \end{bmatrix}$$

$$H' = \begin{bmatrix} h'_{11} & h'_{12} & h'_{13} & \ldots & h'_{1p} \\ h'_{21} & h'_{22} & h'_{23} & \ldots & h'_{2p} \\ h'_{31} & h'_{32} & h'_{33} & \ldots & h'_{3p} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ h'_{q1} & h'_{q2} & h'_{q3} & \ldots & h'_{qp} \end{bmatrix}$$

Step8 Repeat the steps 8-13 of TOPSIS-GREY-sort, by using the combined weights obtained in Step-7.3 for GRG calculation in this algorithm, for final sorting.

V. EXPERIMENTAL VALIDATION OF PROPOSED TECHNIQUES OVER TWO MCDM PROBLEMS

In this section, we solve two types of MCDM problems with the help of the proposed sorting techniques. The impact of COVID-19 on the tourism sector of the Philippines is analyzed, and a strategy to reopen domestic tourism is suggested. Further, the environmental quality of Tehran is studied in the second problem, and a remedy for improving environmental quality is recommended. Here, Tehran's tourist sites and regions are sorted into different classes as per the exposure to COVID-19 and their environmental quality.

A. DOMESTIC TOURISM PROBLEM OF PHILIPPINES [73]

The countries have taken vital initiatives for economic recovery amidst the COVID-19 pandemic. Yet, the tourism industry is fully at the fag-end in the sectors soon to be reopened. Generally, the strategies to restart the tourism industry do not incorporate the perceptions of the tourists. It is clear that there is fear amongst tourists because of the pandemic outbreak, so the strategies to restart the tourism industry should give due importance to the perceptions of the tourists also.

The tourism industry and COVID-19 infection rate are correlated [74], [75], [76]. As per the exposure to COVID-19, the tourist sites of the Philippines [73] are categorized into three classes, namely "Low exposure interval class (LEIC)," "Moderate exposure interval class (MEIC)," and "High exposure interval class (HEIC)." The two sorting techniques, TOPSIS-GREY-sort and ENTROPY-TOPSIS-GREY-sort, are used to study a six criteria-based dataset of twenty identified tourist sites in the central Philippine province to promote domestic tourism (see: Table 3). Initially, the limit profile of each criterion is known, and aggregated decision matrix is provided in Table 1. The detailed plan of the Philippines has a dataset with which the threshold of each class is identified.

1) TOPSIS-GREY

In this technique, the subjective weights of the experts are not used, rather entropy weight determination technique assigns the objective weights.

Step1: Construct the decision matrix $H^k = (h^k_{mn})_{p' \times q}$ using 9-point Likert scale scheme. Here z^k_{mn} is the assessment of k^{th} decision-maker $k = 1, 2, \ldots, G$ regarding n^{th} criterion $(n = 1, 2, \ldots, q)$ of the m^{th} tourist site $(m = 1, 2, \ldots, p')$. The expert group provides limit profiles of 3 classes $(g = 3)$, namely, 'LEIC,' 'MEIC,' and 'HEIC.' It is given as $Y = \{(0, 3), (3, 6.5), (6.5, 9)\}$ such that $d_1 \equiv (0, 3)$, $d_2 \equiv (3, 6.5)$ and $d_3 \equiv (6.5, 9)$. The well-known aggregation function called weighted mean aggregates $H^k, 1 \leq k \leq G$ and it results an aggregated decision matrix $H = (h_{mn})_{p' \times q}$, where $h_{mn} = \frac{1}{G} \sum_{k=1}^{G} h^k_{mn}$.

TABLE 1. The aggregated decision matrix with limit profiles.

Tourists sites	Criteria					
	$\tilde{C}_1$	$\tilde{C}_2$	$\tilde{C}_3$	$\tilde{C}_4$	$\tilde{C}_5$	$\tilde{C}_6$
d_1	5.529	6.500	5.904	6.135	5.990	6.846
d_2	5.760	6.365	5.957	6.111	6.125	6.798
d_3	5.875	6.442	6.106	6.226	6.202	6.837
d_4	5.981	6.476	6.154	6.255	6.202	6.913
d_5	6.288	6.548	6.192	6.365	6.707	7.183
d_6	6.159	6.466	6.082	6.317	6.663	7.024
d_7	6.274	6.519	6.202	6.380	6.740	7.063
d_8	5.813	6.293	5.856	6.163	6.005	6.659
d_9	5.875	6.380	6.014	6.226	6.154	6.740
d_{10}	5.750	6.216	5.788	6.024	5.933	6.543
d_{11}	6.188	6.572	6.298	6.639	6.558	7.298
d_{12}	6.231	6.644	6.394	6.615	6.572	7.260
d_{13}	5.923	6.351	6.053	6.317	6.197	6.938
d_{14}	6.115	6.567	5.269	6.692	6.452	7.091
d_{15}	6.582	6.803	6.750	6.885	7.111	7.486
d_{16}	5.990	6.303	5.971	6.322	6.082	6.702
d_{17}	5.933	6.346	5.928	6.183	6.149	6.784
d_{18}	6.447	6.615	6.558	7.010	6.942	7.394
d_{19}	5.990	6.226	5.947	6.106	6.313	6.976
d_{20}	6.091	6.284	6.135	6.034	6.365	6.947
l_1	3	3	3	3	3	3
l_2	6.5	6.5	6.5	6.5	6.5	6.5

TABLE 2. The normalized decision matrix with limit profiles.

Tourists sites	Attributes					
	$\tilde{C}_1$	$\tilde{C}_2$	$\tilde{C}_3$	$\tilde{C}_4$	$\tilde{C}_5$	$\tilde{C}_6$
d_1	0.4362	0.2198	0.3527	0.3012	0.3335	0.1427
d_2	0.3848	0.2499	0.3408	0.3065	0.3034	0.1534
d_3	0.3591	0.2327	0.3076	0.2809	0.2862	0.1447
d_4	0.3355	0.2251	0.2969	0.2744	0.2862	0.1277
d_5	0.2671	0.2091	0.2885	0.2499	0.1737	0.0675
d_6	0.2958	0.2274	0.3130	0.2606	0.1835	0.1030
d_7	0.2702	0.2156	0.2862	0.2465	0.1663	0.0943
d_8	0.3729	0.2659	0.3634	0.2949	0.3301	0.1844
d_9	0.3591	0.2465	0.3281	0.2809	0.2969	0.1663
d_{10}	0.3870	0.2831	0.3785	0.3259	0.3462	0.2102
d_{11}	0.2893	0.2037	0.2648	0.1888	0.2069	0.0419
d_{12}	0.2798	0.1877	0.2434	0.1942	0.2037	0.0504
d_{13}	0.3484	0.2530	0.3194	0.2606	0.2873	0.1222
d_{14}	0.3056	0.2049	0.2713	0.1770	0.2305	0.0881
d_{15}	0.2015	0.1523	0.1641	0.1340	0.0836	0.0000
d_{16}	0.3335	0.2637	0.3377	0.2595	0.3130	0.1748
d_{17}	0.3462	0.2541	0.3473	0.2905	0.2980	0.1565
d_{18}	0.2316	0.1942	0.2069	0.1061	0.1213	0.0205
d_{19}	0.3335	0.2809	0.3431	0.3076	0.2615	0.1137
d_{20}	0.3110	0.2679	0.3012	0.3237	0.2499	0.1202
l_1	1.0000	1.0000	1.0000	1.0000	1.0000	1.0000
l_2	0.2198	0.2198	0.2198	0.2198	0.2198	0.2198

TABLE 3. The priority weights of the decision attributes.

Code	Criteria	Priority weights
$\tilde{C}_1$	Proximity	0.203
$\tilde{C}_2$	Available modes of transportation	0.161
$\tilde{C}_3$	Duration of stay	0.153
$\tilde{C}_4$	Tourist activities	0.156
$\tilde{C}_5$	Area of the site's premises	0.156
$\tilde{C}_6$	The volume of tourist arrivals	0.171

Step2: The weighted aggregated decision matrix $\Gamma = (H, X) = (\chi_{mn})_{p\times q}$ takes into account the role of both limit profile and $H = (h_{mn})_{p'\times q}$ during decision making (see: Table 1).

Step3: Normalize matrix Γ and use Eq.(31) for the normalization process (use the minimized criterion value, as small-scale values are preferable). In Table 2, the normalized matrix $\Lambda = (\lambda_{mn})_{p\times q}$ is given.

Step4: The aggregated decision matrix is transformed to intuitionistic fuzzy decision matrix considering $\beta = 0.9$ and $\alpha = 0.5$ (see: Table 4).

Step5: The matrix $\tilde{E}$ containing intuitionistic fuzzy entropy values (E_{mn}) is calculated using $\tilde{\Lambda} = (\tilde{\lambda}_{mn})_{p\times q}$ and Eq.(32) as follows:

$$\tilde{E} = \begin{bmatrix} 0.4422 & 0.9767 & 0.5896 & 0.7092 & 0.6309 & 0.7064 \\ 0.5274 & 0.8629 & 0.6147 & 0.6954 & 0.7034 & 0.7486 \\ 0.5764 & 0.9252 & 0.6926 & 0.7650 & 0.7498 & 0.7143 \\ 0.6264 & 0.9549 & 0.7204 & 0.7841 & 0.7498 & 0.648 \\ 0.8066 & 0.9776 & 0.7435 & 0.8629 & 0.8300 & 0.4127 \\ 0.7234 & 0.9460 & 0.6792 & 0.8272 & 0.8700 & 0.5521 \\ 0.7969 & 0.9946 & 0.7498 & 0.8745 & 0.8003 & 0.5183 \\ 0.5493 & 0.8101 & 0.5680 & 0.7258 & 0.6385 & 0.8737 \\ 0.5764 & 0.8745 & 0.6431 & 0.7650 & 0.7204 & 0.8003 \\ 0.5233 & 0.7586 & 0.5389 & 0.6483 & 0.6032 & 0.9823 \\ 0.7410 & 0.9548 & 0.8136 & 0.8921 & 0.9680 & 0.3050 \\ 0.7683 & 0.8875 & 0.8856 & 0.9144 & 0.9548 & 0.3418 \\ 0.5984 & 0.8522 & 0.6635 & 0.8272 & 0.7466 & 0.6264 \\ 0.6977 & 0.9595 & 0.7935 & 0.8436 & 0.9338 & 0.4940 \\ 0.9453 & 0.7442 & 0.7913 & 0.6724 & 0.4765 & 0.0000 \\ 0.6309 & 0.8171 & 0.6215 & 0.8308 & 0.6792 & 0.8345 \\ 0.6032 & 0.8485 & 0.6008 & 0.7380 & 0.7175 & 0.7610 \\ 0.9294 & 0.9144 & 0.9680 & 0.5642 & 0.6229 & 0.2008 \\ 0.6309 & 0.7650 & 0.6098 & 0.6926 & 0.8243 & 0.5935 \\ 0.6842 & 0.8038 & 0.7092 & 0.6535 & 0.8629 & 0.6186 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 0.9767 & 0.9767 & 0.9767 & 0.9767 & 0.9767 & 0.9767 \end{bmatrix}$$

$$\tilde{\tilde{E}} = \begin{bmatrix} 0.4527 & 0.9820 & 0.6036 & 0.7261 & 0.6460 & 0.7191 \\ 0.5399 & 0.8675 & 0.6293 & 0.7120 & 0.7202 & 0.7621 \\ 0.5901 & 0.9302 & 0.7091 & 0.7833 & 0.7676 & 0.7271 \\ 0.6414 & 0.9600 & 0.7376 & 0.8028 & 0.7676 & 0.6597 \\ 0.8258 & 0.9828 & 0.7612 & 0.8834 & 0.8497 & 0.4201 \\ 0.7406 & 0.9511 & 0.6954 & 0.8468 & 0.8908 & 0.5620 \\ 0.8159 & 1.0000 & 0.7676 & 0.8953 & 0.8193 & 0.5276 \\ 0.5624 & 0.8145 & 0.5815 & 0.7431 & 0.6537 & 0.8894 \\ 0.5901 & 0.8792 & 0.6584 & 0.7833 & 0.7376 & 0.8146 \\ 0.5358 & 0.7627 & 0.5517 & 0.6637 & 0.6175 & 1.0000 \\ 0.7587 & 0.9599 & 0.8330 & 0.9134 & 0.9911 & 0.3105 \\ 0.7866 & 0.8923 & 0.9067 & 0.9362 & 0.9775 & 0.3479 \\ 0.6127 & 0.8568 & 0.6793 & 0.8468 & 0.7644 & 0.6376 \\ 0.7143 & 0.9647 & 0.8124 & 0.8637 & 0.9560 & 0.5028 \\ 0.9678 & 0.7482 & 0.8101 & 0.6884 & 0.4878 & 0.0000 \\ 0.6460 & 0.8216 & 0.6363 & 0.8506 & 0.6954 & 0.8495 \\ 0.6175 & 0.8531 & 0.6151 & 0.7555 & 0.7345 & 0.7747 \\ 0.9516 & 0.9193 & 0.9911 & 0.5776 & 0.6377 & 0.2044 \\ 0.6460 & 0.7692 & 0.6244 & 0.7091 & 0.8439 & 0.6042 \\ 0.7005 & 0.8082 & 0.7261 & 0.6690 & 0.8834 & 0.6297 \\ 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 & 0.0000 \\ 1.0000 & 0.9820 & 1.0000 & 1.0000 & 1.0000 & 0.9943 \end{bmatrix}$$

Step6: The normalized intuitionistic fuzzy entropy matrix $\tilde{\tilde{E}}$ is calculated using Eq.(33).

Step7: The criteria weights are calculated using $\tilde{\tilde{E}}$ and Eq.(34). The resultant is, {0.1543, 0.1995, 0.1614, 0.1763, 0.1740, 0.1345}

Step8: In Table-5, the weighted intuitionistic fuzzy decision matrix is provided.

Step9: The values of Q^+ and Q^- as determined using Eq.(37)-(39), respectively is given below,

$$\begin{aligned} Q^+ &= \{(1.0000, 0.0000), (1.0000, 0.0000), (1.0000, 0.0000), \\ &\quad (1.0000, 0.0000), (1.0000, 0.0000), (1.0000, 0.0000)\} \\ Q^- &= \{(0.0878, 0.8921), (0.0940, 0.8833), (0.0804, 0.9002), \\ &\quad (0.0671, 0.9150), (0.0576, 0.9262), (0.0000, 1.0000)\} \end{aligned}$$

Step10: To evaluate the GRC of each criteria from Q^+ and Q^-, Eq.(40)-(41) is used.

$$\xi^+_{mn} = \begin{bmatrix} 0.3751 & 0.3653 & 0.3699 & 0.3687 & 0.3711 & 0.3495 \\ 0.3708 & 0.3683 & 0.3689 & 0.3692 & 0.3684 & 0.3503 \\ 0.3687 & 0.3666 & 0.3662 & 0.3669 & 0.3669 & 0.3497 \\ 0.3669 & 0.3658 & 0.3653 & 0.3663 & 0.3669 & 0.3485 \\ 0.3616 & 0.3642 & 0.3646 & 0.3642 & 0.3571 & 0.3438 \\ 0.3638 & 0.3661 & 0.3666 & 0.3651 & 0.3580 & 0.3466 \\ 0.3618 & 0.3649 & 0.3644 & 0.3639 & 0.3565 & 0.3459 \\ 0.3698 & 0.3699 & 0.3708 & 0.3681 & 0.3708 & 0.3524 \\ 0.3687 & 0.3680 & 0.3678 & 0.3669 & 0.3678 & 0.3512 \\ 0.3710 & 0.3716 & 0.3720 & 0.3709 & 0.3722 & 0.3541 \\ 0.3633 & 0.3637 & 0.3627 & 0.3588 & 0.3600 & 0.3413 \\ 0.3626 & 0.3621 & 0.3610 & 0.3593 & 0.3598 & 0.3422 \\ 0.3679 & 0.3686 & 0.3671 & 0.3651 & 0.3670 & 0.3480 \\ 0.3645 & 0.3638 & 0.3632 & 0.3577 & 0.3621 & 0.3455 \\ 0.3566 & 0.3584 & 0.3546 & 0.3538 & 0.3486 & 0.3333 \\ 0.3667 & 0.3697 & 0.3686 & 0.3650 & 0.3692 & 0.3517 \\ 0.3677 & 0.3687 & 0.3694 & 0.3677 & 0.3679 & 0.3505 \\ 0.3589 & 0.3627 & 0.3581 & 0.3511 & 0.3523 & 0.3388 \\ 0.3667 & 0.3714 & 0.3691 & 0.3693 & 0.3648 & 0.3474 \\ 0.3650 & 0.3701 & 0.3657 & 0.3707 & 0.3638 & 0.3479 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.3580 & 0.3653 & 0.3591 & 0.3615 & 0.3612 & 0.3548 \end{bmatrix}$$

$$\xi^-_{mn} = \begin{bmatrix} 0.8775 & 0.9498 & 0.8949 & 0.8795 & 0.8509 & 0.8759 \\ 0.9021 & 0.9300 & 0.9008 & 0.8768 & 0.8652 & 0.8712 \\ 0.9147 & 0.9412 & 0.9175 & 0.8899 & 0.8737 & 0.8750 \\ 0.9265 & 0.9462 & 0.9230 & 0.8933 & 0.8737 & 0.8827 \\ 0.9623 & 0.9572 & 0.9274 & 0.9065 & 0.9352 & 0.9147 \\ 0.9469 & 0.9447 & 0.9148 & 0.9007 & 0.9292 & 0.8948 \\ 0.9606 & 0.9527 & 0.9286 & 0.9083 & 0.9398 & 0.8994 \\ 0.9079 & 0.9199 & 0.8897 & 0.8827 & 0.8524 & 0.8582 \\ 0.9147 & 0.9321 & 0.9071 & 0.8899 & 0.8684 & 0.8657 \\ 0.9010 & 0.9094 & 0.8824 & 0.8671 & 0.8449 & 0.8481 \\ 0.9503 & 0.9610 & 0.9400 & 0.9419 & 0.9156 & 0.9324 \\ 0.9554 & 0.9725 & 0.9518 & 0.9386 & 0.9174 & 0.9261 \\ 0.9200 & 0.9280 & 0.9115 & 0.9007 & 0.8731 & 0.8853 \\ 0.9417 & 0.9602 & 0.9366 & 0.9493 & 0.9025 & 0.9027 \\ 1.0000 & 1.0000 & 1.0000 & 0.9785 & 1.0000 & 1.0000 \\ 0.9275 & 0.9213 & 0.9023 & 0.9013 & 0.8606 & 0.8622 \\ 0.9211 & 0.9273 & 0.8976 & 0.8850 & 0.8679 & 0.8698 \\ 0.9821 & 0.9678 & 0.9730 & 1.0000 & 0.9701 & 0.9520 \\ 0.9275 & 0.9107 & 0.8997 & 0.8762 & 0.8862 & 0.8894 \\ 0.9390 & 0.9186 & 0.9208 & 0.8682 & 0.8922 & 0.8863 \\ 0.3566 & 0.3584 & 0.3546 & 0.3511 & 0.3486 & 0.3333 \\ 0.9890 & 0.9498 & 0.9654 & 0.9234 & 0.9084 & 0.8444 \end{bmatrix}$$

TABLE 4. The aggregated IF decision matrix.

Tourists sites	Criteria					
	$\tilde{C}_1$	$\tilde{C}_2$	$\tilde{C}_3$	$\tilde{C}_4$	$\tilde{C}_5$	$\tilde{C}_6$
d_1	(0.6605,0.2737)	(0.4688,0.4571)	(0.5938,0.3357)	(0.5488,0.3787)	(0.5775,0.3512)	(0.3777,0.5498)
d_2	(0.6203,0.3108)	(0.4999,0.4263)	(0.5838,0.3452)	(0.5536,0.3740)	(0.5508,0.3767)	(0.3916,0.5354)
d_3	(0.5993,0.3306)	(0.4824,0.4435)	(0.5546,0.3730)	(0.5300,0.3969)	(0.5350,0.3920)	(0.3804,0.5470)
d_4	(0.5792,0.3495)	(0.4745,0.4514)	(0.5449,0.3824)	(0.5238,0.4028)	(0.5350,0.3920)	(0.3574,0.5710)
d_5	(0.5168,0.4097)	(0.4573,0.4686)	(0.5371,0.3900)	(0.4999,0.4263)	(0.4167,0.5096)	(0.2599,0.6756)
d_6	(0.5439,0.3834)	(0.4768,0.4491)	(0.5594,0.3684)	(0.5105,0.4159)	(0.4283,0.4978)	(0.3209,0.6095)
d_7	(0.5198,0.4068)	(0.4643,0.4616)	(0.5350,0.3920)	(0.4965,0.4296)	(0.4078,0.5187)	(0.3071,0.6243)
d_8	(0.6107,0.3198)	(0.5157,0.4108)	(0.6028,0.3272)	(0.5431,0.3842)	(0.5746,0.3539)	(0.4294,0.4968)
d_9	(0.5993,0.3306)	(0.4965,0.4296)	(0.5728,0.3556)	(0.5300,0.3969)	(0.5449,0.3824)	(0.4078,0.5187)
d_{10}	(0.6221,0.3092)	(0.5321,0.3948)	(0.6152,0.3156)	(0.5709,0.3575)	(0.5884,0.3408)	(0.4585,0.4674)
d_{11}	(0.5379,0.3892)	(0.4514,0.4745)	(0.5146,0.4118)	(0.4345,0.4915)	(0.4548,0.4711)	(0.2047,0.7373)
d_{12}	(0.5289,0.3979)	(0.4332,0.4928)	(0.4934,0.4327)	(0.4406,0.4853)	(0.4514,0.4745)	(0.2245,0.7150)
d_{13}	(0.5903,0.3391)	(0.5030,0.4232)	(0.5652,0.3629)	(0.5105,0.4159)	(0.5360,0.3910)	(0.3495,0.5792)
d_{14}	(0.5528,0.3748)	(0.4526,0.4733)	(0.5209,0.4057)	(0.4207,0.5056)	(0.4801,0.4458)	(0.2967,0.6355)
d_{15}	(0.4489,0.4770)	(0.3902,0.5368)	(0.4051,0.5216)	(0.3660,0.5619)	(0.2891,0.6437)	(0.0000,1.0000)
d_{16}	(0.5775,0.3512)	(0.5135,0.4129)	(0.5811,0.3477)	(0.5094 ,0.4170)	(0.5594,0.3684)	(0.4181,0.5083)
d_{17}	(0.5884,0.3408)	(0.5041 ,0.4221)	(0.5893,0.3399)	(0.5389,0.3882)	(0.5459,0.3814)	(0.3956,0.5313)
d_{18}	(0.4813,0.4447)	(0.4406,0.4853)	(0.4548,0.4711)	(0.3257,0.6044)	(0.3482,0.5806)	(0.1432,0.8087)
d_{19}	(0.5775,0.3512)	(0.5300,0.3969)	(0.5857,0.3434)	(0.5546,0.3730)	(0.5114,0.4150)	(0.3372,0.5923)
d_{20}	(0.5576,0.3701)	(0.5176,0.4089)	(0.5488,0.3787)	(0.5689,0.3593)	(0.4999,0.4263)	(0.3466,0.5823)
l_1	(1.0000,0.0000)	(1.0000,0.0000)	(1.0000,0.0000)	(1.0000,0.0000)	(1.0000,0.0000)	(1.0000,0.0000)
l_2	(0.4688,0.4571)	(0.4688,0.4571)	(0.4688,0.4571)	(0.4688,0.4571)	(0.4688,0.4571)	(0.4688,0.4571)

TABLE 5. The weighted aggregated IF decision matrix.

Tourists sites	Criteria					
	$\tilde{C}_1$	$\tilde{C}_2$	$\tilde{C}_3$	$\tilde{C}_4$	$\tilde{C}_5$	$\tilde{C}_6$
d_1	(0.1535,0.8188)	(0.1186,0.8554)	(0.1354,0.8384)	(0.1309,0.8426)	(0.1392,0.8336)	(0.0618,0.9227)
d_2	(0.1388,0.8350)	(0.1291,0.8436)	(0.1320,0.8422)	(0.1326,0.8408)	(0.1300,0.8438)	(0.0647,0.9194)
d_3	(0.1316,0.8430)	(0.1231,0.8503)	(0.1224,0.8528)	(0.1246,0.8496)	(0.1247,0.8497)	(0.0623,0.9221)
d_4	(0.1250,0.8503)	(0.1204,0.8533)	(0.1194,0.8562)	(0.1226,0.8519)	(0.1247,0.8497)	(0.0577,0.9274)
d_5	(0.1062,0.8714)	(0.1148,0.8597)	(0.1169,0.8590)	(0.1150,0.8604)	(0.0895,0.8893)	(0.0397,0.9486)
d_6	(0.1141,0.8625)	(0.1212,0.8524)	(0.1240,0.8511)	(0.1183,0.8567)	(0.0927,0.8857)	(0.0507,0.9356)
d_7	(0.1070,0.8704)	(0.1171,0.8571)	(0.1163,0.8597)	(0.1140,0.8616)	(0.0871,0.8921)	(0.0481,0.9386)
d_8	(0.1355,0.8387)	(0.1347,0.8374)	(0.1385,0.8350)	(0.1290,0.8448)	(0.1382,0.8347)	(0.0727,0.9102)
d_9	(0.1316,0.8430)	(0.1279,0.8449)	(0.1283,0.8463)	(0.1246,0.8496)	(0.1280,0.8460)	(0.0680,0.9155)
d_{10}	(0.1394,0.8343)	(0.1406,0.8308)	(0.1429,0.8301)	(0.1386,0.8341)	(0.1431,0.8292)	(0.0792,0.9028)
d_{11}	(0.1123,0.8645)	(0.1129,0.8618)	(0.1101,0.8666)	(0.0956,0.8823)	(0.1002,0.8773)	(0.0303,0.9598)
d_{12}	(0.1097,0.8674)	(0.1071,0.8684)	(0.1040,0.8735)	(0.0974,0.8803)	(0.0992,0.8784)	(0.0336,0.9559)
d_{13}	(0.1286,0.8463)	(0.1302,0.8424)	(0.1258,0.8490)	(0.1183,0.8567)	(0.1251,0.8493)	(0.0562,0.9292)
d_{14}	(0.1168,0.8595)	(0.1133,0.8614)	(0.1120,0.8645)	(0.0918,0.8867)	(0.1076,0.8689)	(0.0462,0.9408)
d_{15}	(0.0878,0.8921)	(0.0940,0.8833)	(0.0804,0.9002)	(0.0772,0.9034)	(0.0576,0.9262)	(0.0000,1.0000)
d_{16}	(0.1245,0.8509)	(0.1339,0.8382)	(0.1311,0.8432)	(0.1180 ,0.8571)	(0.1329,0.8405)	(0.0702,0.9130)
d_{17}	(0.1280,0.8470)	(0.1306 ,0.8420)	(0.1338,0.8401)	(0.1276,0.8463)	(0.1283,0.8456)	(0.0655,0.9185)
d_{18}	(0.0963,0.8825)	(0.1094,0.8657)	(0.0933,0.8856)	(0.0671,0.9150)	(0.0718,0.9097)	(0.0206,0.9719)
d_{19}	(0.1245,0.8509)	(0.1398,08316.)	(0.1326,0.8415)	(0.1329,0.8404)	(0.1171,0.8581)	(0.0538,0.9320)
d_{20}	(0.1183,0.8578)	(0.1353,0.8366)	(0.1206,0.8549)	(0.1379,0.8349)	(0.1136,0.8621)	(0.0556,0.9298)
l_1	(1.0000,0.0000)	(1.0000,0.0000)	(1.0000,0.0000)	(1.0000,0.0000)	(1.0000,0.0000)	(1.0000,0.0000)
l_2	(0.0930,0.8862)	(0.1186,0.8554)	(0.0971,0.8813)	(0.1056,0.8711)	(0.1042,0.8727)	(0.0816,0.9001)

Step11: With the help of Eq. (42)-(43), the GRG (ξ^+ and ξ^-) are calculated. Refer Table 6 for GRG values.

Step12: RC_m values are calculated using Eq.(44) for each alternative and limit profile and these values are in Table 6.

Step13: Eq.(45) is used to assign alternatives to appropriate classes. The final sorting of tourist sites as per their exposure to COVID-19 is shown in Fig. 2.

2) ENTROPY-TOPSIS-GREY-SORT

In this technique, the subjective weights provided by the experts and the objective weights calculated using the entropy weight determination technique are merged, and their cumulative effects on decision-making are studied. Here **Step 1-4** are same as in TOPSIS-GREY-sort, so the same dataset, normalized matrix, same value of $\beta = 0.9$ and $\alpha = 0.5$ and IF aggregated matrix is obtained.

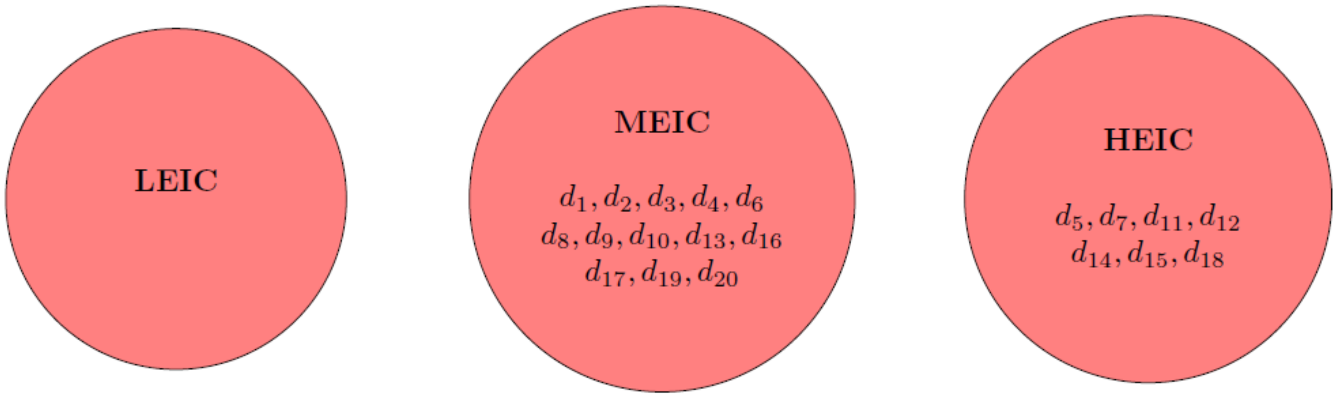

FIGURE 2. **The assignment of tourist sites to different interval classes.**

Step5: The intuitionistic fuzzy entropy values (E_{mn}) in $\tilde{E}$ are obtained using $\tilde{\Lambda} = (\tilde{\lambda}_{mn})_{p\times q}$ and using Eq.(23). Each value given in $\tilde{E}$ is of the form 1.0e-16.

$$\tilde{E} = \begin{bmatrix} 0.2056 & 0.0071 & 0.1403 & 0.0951 & 0.1240 & 0.0961 \\ 0.1662 & 0.0431 & 0.1303 & 0.1000 & 0.0972 & 0.0812 \\ 0.1456 & 0.0232 & 0.1011 & 0.0756 & 0.0808 & 0.0933 \\ 0.1258 & 0.0139 & 0.0911 & 0.0691 & 0.0808 & 0.1175 \\ 0.0615 & 0.0069 & 0.0830 & 0.0431 & 0.0538 & 0.2205 \\ 0.0901 & 0.0167 & 0.1059 & 0.0547 & 0.0407 & 0.1557 \\ 0.0648 & 0.0016 & 0.0808 & 0.0393 & 0.0636 & 0.1702 \\ 0.1568 & 0.0604 & 0.1491 & 0.0892 & 0.1212 & 0.0396 \\ 0.1456 & 0.0393 & 0.1194 & 0.0756 & 0.0911 & 0.0636 \\ 0.1680 & 0.0778 & 0.1613 & 0.1175 & 0.1349 & 0.0054 \\ 0.0839 & 0.0140 & 0.0592 & 0.0337 & 0.0098 & 0.2830 \\ 0.0745 & 0.0351 & 0.0358 & 0.0266 & 0.0140 & 0.2600 \\ 0.1367 & 0.0465 & 0.1117 & 0.0547 & 0.0819 & 0.1258 \\ 0.0992 & 0.0125 & 0.0659 & 0.0493 & 0.0205 & 0.1810 \\ 0.0169 & 0.0828 & 0.0667 & 0.1085 & 0.1891 & 0.6123 \\ 0.1240 & 0.0580 & 0.1277 & 0.0535 & 0.1059 & 0.0523 \\ 0.1349 & 0.0477 & 0.1358 & 0.0849 & 0.0922 & 0.0770 \\ 0.0219 & 0.0266 & 0.0098 & 0.1506 & 0.1271 & 0.3608 \\ 0.1240 & 0.0756 & 0.1322 & 0.1011 & 0.0557 & 0.1387 \\ 0.1041 & 0.0625 & 0.0951 & 0.1155 & 0.0431 & 0.1288 \\ 0.6123 & 0.6123 & 0.6123 & 0.6123 & 0.6123 & 0.6123 \\ 0.0071 & 0.0071 & 0.0071 & 0.0071 & 0.0071 & 0.0071 \end{bmatrix}$$

$$\tilde{\tilde{E}} = \begin{bmatrix} 0.3357 & 0.0117 & 0.2291 & 0.1553 & 0.2026 & 0.1569 \\ 0.2714 & 0.0703 & 0.2129 & 0.1634 & 0.1587 & 0.1327 \\ 0.2378 & 0.0379 & 0.1651 & 0.1234 & 0.1320 & 0.1523 \\ 0.2054 & 0.0227 & 0.1488 & 0.1128 & 0.1320 & 0.1920 \\ 0.1005 & 0.0113 & 0.1356 & 0.0703 & 0.0878 & 0.3601 \\ 0.1471 & 0.0273 & 0.1730 & 0.0894 & 0.0665 & 0.2542 \\ 0.1058 & 0.0027 & 0.1320 & 0.0642 & 0.1039 & 0.2779 \\ 0.2561 & 0.0986 & 0.2434 & 0.1457 & 0.1979 & 0.0646 \\ 0.2378 & 0.0642 & 0.1950 & 0.1234 & 0.1488 & 0.1039 \\ 0.2743 & 0.1270 & 0.2634 & 0.1918 & 0.2202 & 0.0089 \\ 0.1370 & 0.0228 & 0.0967 & 0.0550 & 0.0161 & 0.4622 \\ 0.1216 & 0.0574 & 0.0584 & 0.0434 & 0.0228 & 0.4246 \\ 0.2233 & 0.0760 & 0.1825 & 0.0894 & 0.1388 & 0.2054 \\ 0.1620 & 0.0204 & 0.1076 & 0.0806 & 0.0335 & 0.2957 \\ 0.0276 & 0.1352 & 0.1089 & 0.1771 & 0.3088 & 1.0000 \\ 0.2026 & 0.0948 & 0.2085 & 0.0874 & 0.1730 & 0.0854 \\ 0.2202 & 0.0779 & 0.2218 & 0.1387 & 0.1505 & 0.1257 \\ 0.0357 & 0.0434 & 0.0161 & 0.2460 & 0.2076 & 0.5892 \\ 0.2026 & 0.1234 & 0.2159 & 0.1651 & 0.0909 & 0.2265 \\ 0.1700 & 0.1020 & 0.1553 & 0.1886 & 0.0703 & 0.2104 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.0117 & 0.0117 & 0.0117 & 0.0117 & 0.0117 & 0.0117 \end{bmatrix}$$

TABLE 6. **Corresponding values of ξ_m^+, ξ_m^-, RC_m.**

Tourists sites	ξ_m^+	ξ_m^-	RC_m
d_1	0.3670	0.8902	0.7081
d_2	0.3665	0.8924	0.7089
d_3	0.3647	0.9036	0.7125
d_4	0.3638	0.9089	0.7141
d_5	0.3599	0.9347	0.7220
d_6	0.3616	0.9230	0.7185
d_7	0.3602	0.9328	0.7214
d_8	0.3675	0.8866	0.7069
d_9	0.3656	0.8979	0.7107
d_{10}	0.3692	0.8768	0.7037
d_{11}	0.3590	0.9408	0.7238
d_{12}	0.3584	0.9447	0.7250
d_{13}	0.3646	0.9040	0.7126
d_{14}	0.3600	0.9338	0.7218
d_{15}	0.3516	0.9962	0.7391
d_{16}	0.3657	0.8971	0.7104
d_{17}	0.3659	0.8960	0.7100
d_{18}	0.3543	0.9748	0.7334
d_{19}	0.3655	0.8983	0.7108
d_{20}	0.3646	0.9043	0.7127
l_1	1.0000	0.3511	0.2599
l_2	0.3604	0.9323	0.7212

Step6: The normalized intuitionistic fuzzy entropy values (e_{mn}) in $\tilde{\tilde{E}}$ are obtained using Eq.(33).

Step7: The objective weights of the criteria are calculated using $\tilde{\tilde{E}}$ and the entropy method (see: Eq.(34)) and we obtain weight values as

$$\{0.1967,\ 0.0661,\ 0.1751,\ 0.1346,\ 0.1425,\ 0.2850\}.$$

Subjective weights provided by experts for each criteria is given in Table- 3. The combined weight W is given as follows:

$$\{0.1999,\ 0.1135,\ 0.1641,\ 0.1453,\ 0.1492,\ 0.2280\}.$$

Step8: The weighted intuitionistic fuzzy decision matrix is in Table-7.

TABLE 7. The weighted aggregated IF decision matrix.

Tourists sites	Criteria					
	$\tilde{C}_1$	$\tilde{C}_2$	$\tilde{C}_3$	$\tilde{C}_4$	$\tilde{C}_5$	$\tilde{C}_6$
d_1	(0.1942,0.7718)	(0.0693,0.9149)	(0.1374,0.8360)	(0.1092,0.8684)	(0.1206,0.8554)	(0.1025,0.8725)
d_2	(0.1760,0.7917)	(0.0757,0.9077)	(0.1340,0.8399)	(0.1106,0.8668)	(0.1126,0.8644)	(0.1071,0.8672)
d_3	(0.1670,0.8015)	(0.0721,0.9118)	(0.1243,0.8506)	(0.1039,0.8743)	(0.1080,0.8696)	(0.1034,0.8715)
d_4	(0.1589,0.8105)	(0.0705,0.9136)	(0.1212,0.8541)	(0.1022,0.8762)	(0.1080,0.8696)	(0.0959,0.8801)
d_5	(0.1353,0.8367)	(0.0670,0.9175)	(0.1187,0.8568)	(0.0958,0.8835)	(0.0773,0.9043)	(0.0663,0.9145)
d_6	(0.1452,0.8256)	(0.0709,0.9131)	(0.1258,0.8489)	(0.0986,0.8803)	(0.0801,0.9011)	(0.0845,0.8933)
d_7	(0.1364,0.8355)	(0.0684,0.9160)	(0.1181,0.8576)	(0.0949,0.8845)	(0.0752,0.9067)	(0.0802,0.8982)
d_8	(0.1718,0.7963)	(0.0790,0.9039)	(0.1406,0.8325)	(0.1076,0.8702)	(0.1197,0.8564)	(0.1201,0.8526)
d_9	(0.1670,0.8015)	(0.0750,0.9085)	(0.1302,0.8440)	(0.1039,0.8743)	(0.1108,0.8664)	(0.1126,0.8610)
d_{10}	(0.1767,0.7909)	(0.0826,0.8999)	(0.1450,0.8276)	(0.1157,0.8612)	(0.1241,0.8516)	(0.1305,0.8408)
d_{11}	(0.1430,0.8281)	(0.0659,0.9188)	(0.1118,0.8646)	(0.0795,0.9019)	(0.0866,0.8937)	(0.0509,0.9329)
d_{12}	(0.1397,0.8318)	(0.0624,0.9228)	(0.1056,0.8716)	(0.0810,0.9003)	(0.0857,0.8947)	(0.0563,0.9264)
d_{13}	(0.1633,0.8056)	(0.0763,0.9070)	(0.1277,0.8468)	(0.0986,0.8803)	(0.1083,0.8692)	(0.0934,0.8829)
d_{14}	(0.1486,0.8219)	(0.0661,0.9186)	(0.1137,0.8624)	(0.0763,0.9056)	(0.0930,0.8864)	(0.0771,0.9018)
d_{15}	(0.1123,0.8625)	(0.0546,0.9318)	(0.0817,0.8987)	(0.0641,0.9197)	(0.0497,0.9364)	(0.0000,1.0000)
d_{16}	(0.1582,0.8113)	(0.0786,0.9044)	(0.1330,0.8409)	(0.0983 ,0.8806)	(0.1151,0.8616)	(0.1161,0.8570)
d_{17}	(0.1626,0.8064)	(0.0766 ,0.9067)	(0.1358,0.8378)	(0.1064,0.8715)	(0.1111,0.8660)	(0.1084,0.8657)
d_{18}	(0.1229,0.8505)	(0.0638,0.9212)	(0.0947,0.8838)	(0.0557,0.9294)	(0.0619,0.9221)	(0.0346,0.9527)
d_{19}	(0.1582,0.8113)	(0.0822,0.9004)	(0.1346,0.8391)	(0.1109,0.8665)	(0.1014,0.8770)	(0.0895,0.8874)
d_{20}	(0.1504,0.8199)	(0.0794,0.9034)	(0.1224,0.8527)	(0.1151,0.8618)	(0.0982,0.8805)	(0.0925,0.8840)
l_1	(1.0000,0.0000)	(1.0000,0.0000)	(1.0000,0.0000)	(1.0000,0.0000)	(1.0000,0.0000)	(1.0000,0.0000)
l_2	(0.1188,0.8552)	(0.0693,0.9149)	(0.0986,0.8795)	(0.0878,0.8925)	(0.0901,0.8897)	(0.1343,0.8365)

Step9: Use Eq.(37)-(39) to calculate Q^+ and Q^-. The values are:

$$Q^+ = \{(1.0000, 0.0000), (1.0000, 0.0000), (1.0000, 0.0000), (1.0000, 0.0000), (1.0000, 0.0000)\}$$

$$Q^- = \{(0.1123, 0.8625), (0.0546, 0.9318), (0.0817, 0.8987), (0.0557, 0.9294), (0.0497, 0.9364), (0.0000, 1.0000)\}$$

Step10: To evaluate the GRC of each criteria from Q^+ and Q^-, Eq.(40)-(41) is used.

$$\xi_{mn}^{+} = \begin{bmatrix} 0.3878 & 0.3514 & 0.3705 & 0.3624 & 0.3656 & 0.3609 \\ 0.3822 & 0.3531 & 0.3695 & 0.3628 & 0.3633 & 0.3622 \\ 0.3794 & 0.3521 & 0.3667 & 0.3609 & 0.3621 & 0.3612 \\ 0.3770 & 0.3517 & 0.3658 & 0.3604 & 0.3621 & 0.3591 \\ 0.3701 & 0.3508 & 0.3651 & 0.3587 & 0.3537 & 0.3511 \\ 0.3730 & 0.3518 & 0.3672 & 0.3594 & 0.3544 & 0.3560 \\ 0.3704 & 0.3512 & 0.3650 & 0.3584 & 0.3531 & 0.3548 \\ 0.3809 & 0.3540 & 0.3714 & 0.3619 & 0.3653 & 0.3659 \\ 0.3794 & 0.3529 & 0.3684 & 0.3609 & 0.3628 & 0.3638 \\ 0.3824 & 0.3549 & 0.3727 & 0.3642 & 0.3666 & 0.3689 \\ 0.3723 & 0.3505 & 0.3632 & 0.3543 & 0.3562 & 0.3470 \\ 0.3714 & 0.3496 & 0.3615 & 0.3547 & 0.3560 & 0.3484 \\ 0.3783 & 0.3532 & 0.3677 & 0.3594 & 0.3621 & 0.3584 \\ 0.3740 & 0.3506 & 0.3637 & 0.3534 & 0.3579 & 0.3540 \\ 0.3635 & 0.3475 & 0.3549 & 0.3502 & 0.3464 & 0.3333 \\ 0.3768 & 0.3538 & 0.3692 & 0.3594 & 0.3640 & 0.3648 \\ 0.3781 & 0.3533 & 0.3700 & 0.3616 & 0.3629 & 0.3626 \\ 0.3665 & 0.3500 & 0.3585 & 0.3480 & 0.3496 & 0.3427 \\ 0.3768 & 0.3548 & 0.3697 & 0.3628 & 0.3602 & 0.3574 \\ 0.3745 & 0.3541 & 0.3662 & 0.3640 & 0.3594 & 0.3582 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 & 1.0000 \\ 0.3654 & 0.3514 & 0.3595 & 0.3565 & 0.3571 & 0.3700 \end{bmatrix}$$

$$\xi_{mn}^{-} = \begin{bmatrix} 0.8524 & 0.9692 & 0.8936 & 0.8966 & 0.8674 & 0.8103 \\ 0.8811 & 0.9565 & 0.8995 & 0.8942 & 0.8805 & 0.8039 \\ 0.8959 & 0.9637 & 0.9164 & 0.9058 & 0.8881 & 0.8091 \\ 0.9099 & 0.9669 & 0.9220 & 0.9087 & 0.8881 & 0.8198 \\ 0.9532 & 0.9738 & 0.9265 & 0.9202 & 0.9432 & 0.8655 \\ 0.9345 & 0.9660 & 0.9137 & 0.9152 & 0.9379 & 0.8368 \\ 0.9511 & 0.9710 & 0.9277 & 0.9218 & 0.9473 & 0.8433 \\ 0.8878 & 0.9500 & 0.8883 & 0.8994 & 0.8688 & 0.7864 \\ 0.8959 & 0.9579 & 0.9059 & 0.9058 & 0.8833 & 0.7964 \\ 0.8798 & 0.9431 & 0.8809 & 0.8857 & 0.8619 & 0.7728 \\ 0.9386 & 0.9762 & 0.9392 & 0.9508 & 0.9258 & 0.8918 \\ 0.9448 & 0.9833 & 0.9512 & 0.9480 & 0.9274 & 0.8823 \\ 0.9022 & 0.9553 & 0.9103 & 0.9152 & 0.8876 & 0.8235 \\ 0.9283 & 0.9757 & 0.9357 & 0.9572 & 0.9141 & 0.8481 \\ 1.0000 & 1.0000 & 1.0000 & 0.9820 & 1.0000 & 1.0000 \\ 0.9111 & 0.9509 & 0.9011 & 0.9157 & 0.8762 & 0.7916 \\ 0.9035 & 0.9548 & 0.8963 & 0.9014 & 0.8828 & 0.8020 \\ 0.9777 & 0.9804 & 0.9726 & 1.0000 & 0.9739 & 0.9219 \\ 0.9111 & 0.9440 & 0.8984 & 0.8937 & 0.8994 & 0.8292 \\ 0.9249 & 0.9492 & 0.9198 & 0.8866 & 0.9048 & 0.8248 \\ 0.3635 & 0.3475 & 0.3549 & 0.3480 & 0.3464 & 0.3333 \\ 0.9863 & 0.9692 & 0.9649 & 0.9349 & 0.9193 & 0.7680 \end{bmatrix}$$

Step11: The GRG (ξ^+ and ξ^-) is calculated with the help of Eq. (42)-(43). Refer Table 8 for GRG values.

Step12: With the help of Eq. (42)-(43), the GRG (ξ^+ and ξ^-) are calculated. These values are given in Table 8.

Step13: Eq.(45) is used to assign alternatives to appropriate classes. The final sorting of tourist sites as per their exposure to COVID-19 is shown in Fig. 3.

B. RESULT DISCUSSION AND COMPARATIVE ANALYSIS

- In comparison to [73], the results given here are of generalized nature, and the tourist sites are categorized as a 'low exposure interval class (LEIC)', a 'moderate exposure interval class (MEIC)', and the 'high exposure

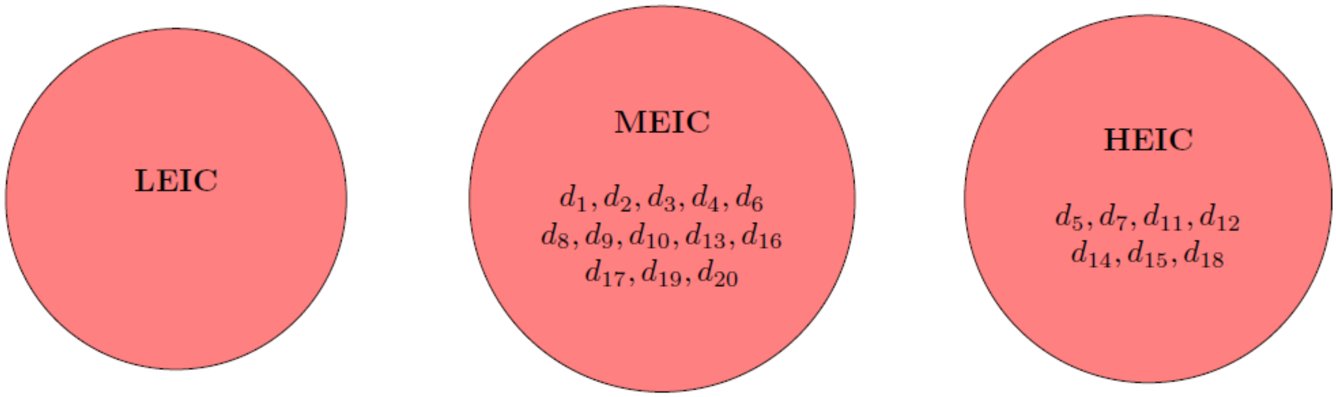

FIGURE 3. The assignment of tourist sites to different interval classes.

TABLE 8. Corresponding values of ξ_m^+, ξ_m^-, RC_m.

Tourists sites	ξ_m^+	ξ_m^-	RC_m
d_1	0.3677	0.8715	0.7033
d_2	0.3666	0.8769	0.7052
d_3	0.3648	0.8874	0.7087
d_4	0.3636	0.8944	0.7110
d_5	0.3586	0.9249	0.7206
d_6	0.3610	0.9101	0.7160
d_7	0.3595	0.9201	0.7191
d_8	0.3678	0.8707	0.7030
d_9	0.3659	0.8814	0.7067
d_{10}	0.3696	0.8610	0.6997
d_{11}	0.3575	0.9322	0.7228
d_{12}	0.3573	0.9338	0.7233
d_{13}	0.3640	0.8913	0.7100
d_{14}	0.3597	0.9187	0.7186
d_{15}	0.3489	0.9974	0.7408
d_{16}	0.3658	0.8822	0.7069
d_{17}	0.3658	0.8816	0.7068
d_{18}	0.3527	0.9671	0.7328
d_{19}	0.3642	0.8898	0.7096
d_{20}	0.3633	0.8954	0.7114
l_1	1.0000	0.3486	0.2585
l_2	0.3614	0.9136	0.7166

interval class (HEIC)'. The sorting techniques proposed in the paper assign ranges to LEIC, MEIC, and HEIC instead of a precise value. TOPSIS-GREY-sort assigns ranges to LEIC = [0,0], MEIC = [13,18], HEIC = [2,7], and ENTROPY-TOPSIS-GREY-sort assigns LEIC = [0,0], MEIC = [12,18], HEIC = [2,8]. The variations in TOPSIS-GREY-sort and ENTROPY-TOPSIS-GREY-sort outcomes are because of the variation of the parameters β and α are figuratively discussed (see Fig. 4, 5). The paper [73] shows that twelve sites are in the MEIC, eight sites are in the HEIC, and none belong to the LEIC. Both the techniques proposed in the paper overall ranges to LEIC = [0,0], MEIC = [12,18] and HEIC = [2,8].

It is possible to obtain a uniquely valued LEIC, MEIC, and HEIC. The TOPSIS-GREY-sort assigns, LEIC = [0,0], MEIC = [13,13], and HEIC = [7,7] at $\beta = 0.9$ and $\alpha = 0.5$. These specific valued results are shown in Fig. 2), the figure can be analyzed as, on average, COVID-19 highly impacts seven tourist sites, and thirteen are moderately affected. ENTROPY-TOPSIS-GREY-sort assigns, LEIC = [0,0], MEIC = [13,13], and HEIC = [7,7]. These specific valued results are depicted in Fig. 3), the figure can be analyzed as, on average, COVID-19 highly impacts seven tourist sites, and thirteen are moderately affected. So, to resume the tourism industry for economic purposes, authorities have excellent insight into decision-making and resource allocation for these tourist sites. In different political boundaries, the standard COVID-19 guidelines and public health measures are followed differently. Hence, we suggest that authorities first consider reopening domestic tourism in place of international tourism. The authorities find it easy to monitor, supervise, and control domestic tourism, motivating the governments to reopen international tourism.

- It is not always the case that subjective weights (weights allocated by DMs) are known. In such a case, objective weights are calculated using a data-driven weight determination scheme. Here, we have carried out the individual study of objective and subjective weights, and their cumulative effect is also analyzed. TOPSIS-GREY-sort uses objective weights (calculated by entropy method) which is (0.1543, 0.1995, 0.1614, 0.1763, 0.1740, 0.1345). Moreover, the convex combination of subjective weight (see: Table-3) and objective weight calculated by the proposed entropy method (see: 0.1967, 0.0661, 0.1751, 0.1346, 0.1425, 0.2850) is utilized in ENTROPY-TOPSIS-GREY-sort with combined weightage equal to (0.1999, 0.1135, 0.1641, 0.1453, 0.1492, 0.2280). The obtained results here with subjective and objective weights show the same results. Hence, we can say that the decision-making techniques used here give stable results.

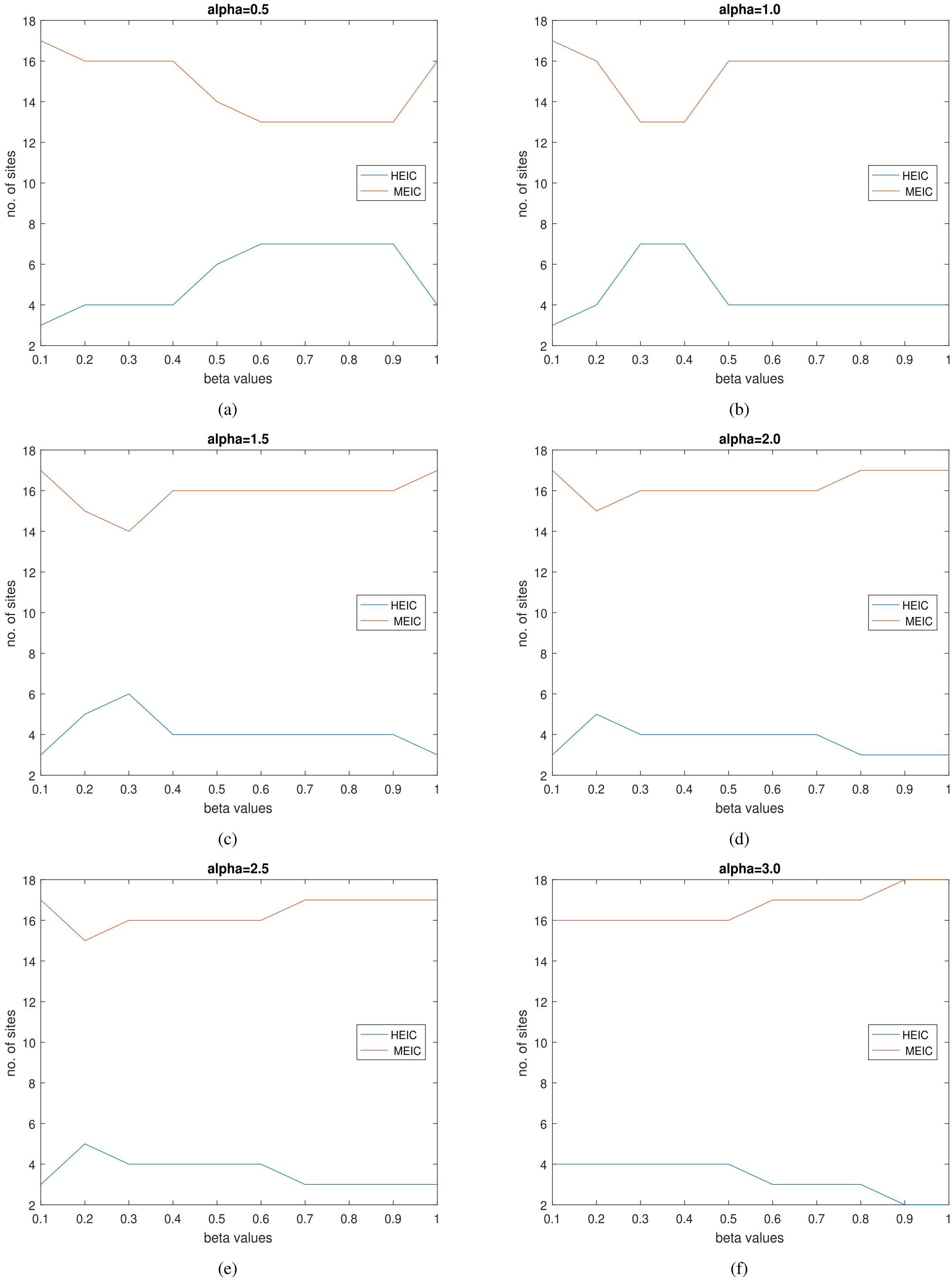

FIGURE 4. Study of allocation of tourist sites to different interval classes with TOPSIS-GREY tuning different values of β, α.

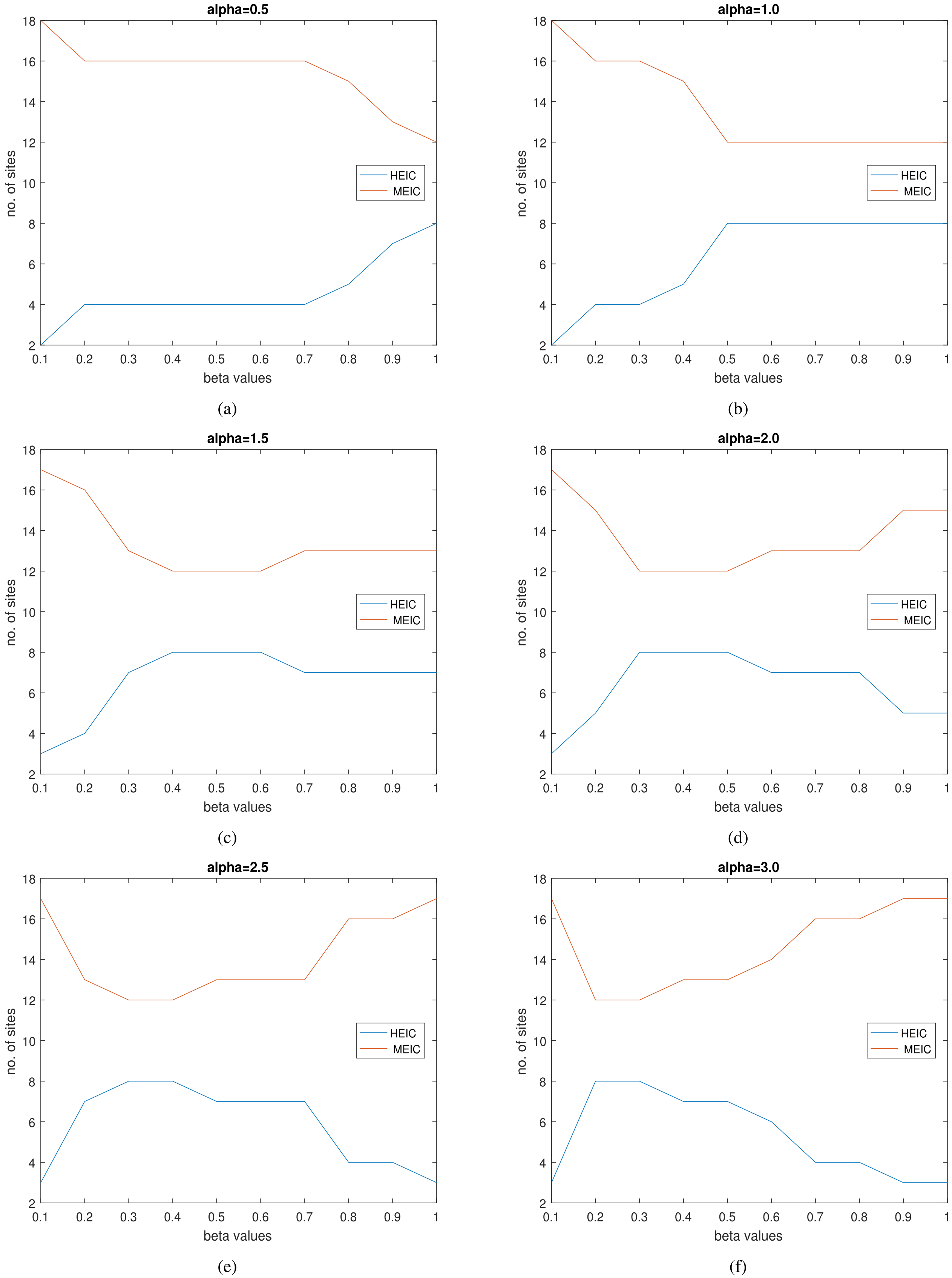

FIGURE 5. Study of allocation of tourist sites to different interval classes with ENTROPY-TOPSIS-GREY tuning different values of β, α.

TABLE 9. Weights provided by experts.

	Criteria				
	$\tilde{C}_1$	$\tilde{C}_2$	$\tilde{C}_3$	$\tilde{C}_4$	$\tilde{C}_5$
Weight	0.2	0.2	0.2	0.2	0.2
l_1	25	80	40	80	150
l_2	20	75	30	60	300
l_3	10	70	20	30	400
l_4	0	65	10	20	700

TABLE 10. The aggregated decision matrix with the limit profiles.

Districts	Criteria				
	$\tilde{C}_1$	$\tilde{C}_2$	$\tilde{C}_3$	$\tilde{C}_4$	$\tilde{C}_5$
d_1	15.50	82.40	40.10	70.66	313.69
d_2	9.90	90.80	22.00	82.21	130.61
d_3	15.20	82.30	25.20	65.57	269.37
d_4	23.40	88.30	60.20	69.28	119.03
d_5	16.60	76.80	22.70	69.28	116.82
d_6	12.40	78.60	22.60	50.28	199.55
d_7	4.00	85.00	27.80	51.40	188.94
d_8	4.40	78.20	25.60	56.59	126.37
d_9	5.20	80.20	27.60	86.29	108.82
d_{10}	2.30	80.70	24.20	79.88	97.95
d_{11}	4.90	77.70	26.30	61.28	122.58
d_{12}	5.70	80.90	26.80	70.75	82.75
d_{13}	33.40	85.30	28.10	67.68	126.14
d_{14}	5.90	82.50	29.80	82.61	149.32
d_{15}	14.00	88.50	24.60	90.71	335.24
d_{16}	11.40	81.80	26.50	61.04	139.62
d_{17}	3.50	78.90	22.40	60.82	99.42
d_{18}	12.96	78.40	25.20	67.25	180.18
d_{19}	28.00	82.40	25.30	56.89	102.78
d_{20}	15.70	82.10	27.70	24.64	193.52
d_{21}	25.00	91.30	67.10	79.81	916.67
d_{22}	48.00	85.10	49.20	86.71	173.79
l_1	25	80	40	80	150
l_2	20	75	30	60	300
l_3	10	70	20	30	400
l_4	0	65	10	20	700

C. ENVIRONMENTAL QUALITY PROBLEM OF TEHRAN [54]

Tehran is the capital city of Iran, situated in the northern part of Iran and on the southern slopes of the Alborz Mountains. It is the biggest city in Iran and is divided into 22 districts. It is the most important city in Iran. It hosts many national and international events and is the land of ministries, government institutions, and private organizations. The better air, rail, and road connectivities make it a transport hub and an economic driver across the country. Almost one-fourth of the urban population of Iran lives in Tehran, which has caused a significant expansion and escalation of pressures on the environment (Research and Planning Center of Tehran, 2012).

The high-altitude mountains surround the city in three directions, blocking the air passage. As a result, Tehran faces varying environmental problems and issues, such as air pollution mitigation and management, waste management, water and wastewater management, urban safety, and greening.

TABLE 11. The normalized decision matrix with the limit profiles.

Districts	Criteria				
	$\tilde{C}_1$	$\tilde{C}_2$	$\tilde{C}_3$	$\tilde{C}_4$	$\tilde{C}_5$
d_1	0.9831	0.9101	0.9563	0.9229	0.6578
d_2	0.9892	0.9009	0.9760	0.9103	0.8575
d_3	0.9834	0.9102	0.9725	0.9285	0.7061
d_4	0.9745	0.9037	0.9343	0.9244	0.8701
d_5	0.9819	0.9162	0.9752	0.9244	0.8726
d_6	0.9865	0.9143	0.9753	0.9451	0.7823
d_7	0.9956	0.9073	0.9697	0.9439	0.7939
d_8	0.9952	0.9147	0.9721	0.9383	0.8621
d_9	0.9943	0.9125	0.9699	0.9059	0.8813
d_{10}	0.9975	0.9120	0.9736	0.9129	0.8931
d_{11}	0.9947	0.9152	0.9713	0.9331	0.8663
d_{12}	0.9938	0.9117	0.9708	0.9228	0.9097
d_{13}	0.9636	0.9069	0.9693	0.9262	0.8624
d_{14}	0.9936	0.9100	0.9675	0.9099	0.8371
d_{15}	0.9847	0.9035	0.9732	0.9010	0.6343
d_{16}	0.9876	0.9108	0.9711	0.9334	0.8477
d_{17}	0.9962	0.9139	0.9756	0.9337	0.8915
d_{18}	0.9859	0.9145	0.9725	0.9266	0.8034
d_{19}	0.9695	0.9101	0.9724	0.9379	0.8879
d_{20}	0.9829	0.9104	0.9698	0.9731	0.7889
d_{21}	0.9727	0.9004	0.9268	0.9129	0.0000
d_{22}	0.9476	0.9072	0.9463	0.9054	0.8104
l_1	0.9727	0.9127	0.9564	0.9127	0.8364
l_2	0.9782	0.9182	0.9673	0.9345	0.6727
l_3	0.9891	0.9236	0.9782	0.9673	0.5636
l_4	1.0000	0.9291	0.9891	0.9782	0.2364

In other words, the geographical conditions of Tehran have a cascading effect on its environment.

The environmental quality problem of Tehran studied in [54] is dealt with in this paper also. Districts of Tehran are classified into five classes according to their environmental conditions: high suitable interval class (HSIC), suitable interval class (SIC), moderately suitable interval class (MSIC), low suitable interval class (LSIC), and very low suitable interval class (VLSIC). The study involves 22 identified districts of Tehran for environmental quality under five criteria, 'urban green space per capita ($\tilde{C}_1$)', 'waste and soil pollution management ($\tilde{C}_2$)', ' quality of environmental management of water and waste water projects ($\tilde{C}_3$)', 'air pollution management ($\tilde{C}_4$)' and 'ratio of incidents to the station ($\tilde{C}_5$)' (see: Table 9). The limit profiles of criteria are initially known. The basis of this criteria was the annual report of the Tehran municipality so that the quality of the environment could be assessed accordingly. For the threshold values of each class, data from the environmental plan, detailed plan, the comprehensive goal of greenery, plan of environment, plan of water and wastewater, and the experts opinion were used. So, a proper study and analysis help us to recognize the districts having poor environmental quality. These districts must be prioritized to establish a better environment and, thus, better living conditions. The proposed sorting techniques are implemented in the upcoming subsections. Moreover, the geographic location of the city makes the air pollution in Tehran even worse. The sorting techniques TOPSIS-GRA-sort and ENTROPY-TOPSIS-GREY-sort are implemented

TABLE 12. The aggregated IF decision matrix.

Districts	Criteria				
	$\tilde{C}_1$	$\tilde{C}_2$	$\tilde{C}_3$	$\tilde{C}_4$	$\tilde{C}_5$
d_1	(0.9747,0.0051)	(0.8682,0.0468)	(0.9351,0.0180)	(0.8866,0.0381)	(0.5335,0.2712)
d_2	(0.9838,0.0028)	(0.8552,0.0532)	(0.9642,0.0081)	(0.8685,0.0466)	(0.7941,0.0860)
d_3	(0.9752,0.0049)	(0.8684,0.0467)	(0.9590,0.0097)	(0.8946,0.0345)	(0.5934,0.2225)
d_4	(0.9620,0.0088)	(0.8590,0.0512)	(0.9031,0.0308)	(0.8888,0.0371)	(0.8117,0.0761)
d_5	(0.9730,0.0056)	(0.8770,0.0426)	(0.9631,0.0084)	(0.8888,0.0371)	(0.8151,0.0742)
d_6	(0.9798,0.0038)	(0.8742,0.0439)	(0.9632,0.0084)	(0.9189,0.0243)	(0.6919,0.1503)
d_7	(0.9935,0.0008)	(0.8642,0.0487)	(0.9549,0.0110)	(0.9171,0.0250)	(0.7074,0.1399)
d_8	(0.9928,0.0009)	(0.8748,0.0436)	(0.9584,0.0099)	(0.9088,0.0284)	(0.8005,0.0823)
d_9	(0.9915,0.0012)	(0.8717,0.0451)	(0.9552,0.0109)	(0.8622,0.0497)	(0.8273,0.0676)
d_{10}	(0.9962,0.0004)	(0.8709,0.0455)	(0.9607,0.0092)	(0.8722,0.0449)	(0.8441,0.0588)
d_{11}	(0.9920,0.0011)	(0.8756,0.0433)	(0.9573,0.0103)	(0.9014,0.0316)	(0.8063,0.0791)
d_{12}	(0.9907,0.0013)	(0.8706,0.0456)	(0.9565,0.0105)	(0.8865,0.0382)	(0.8677,0.0470)
d_{13}	(0.9458,0.0141)	(0.8637,0.0489)	(0.9544,0.0112)	(0.8913,0.0360)	(0.8009,0.0821)
d_{14}	(0.9904,0.0014)	(0.8681,0.0468)	(0.9516,0.0121)	(0.8679,0.0469)	(0.7659,0.1026)
d_{15}	(0.9772,0.0044)	(0.8587,0.0514)	(0.9600,0.0094)	(0.8553,0.0531)	(0.5052,0.2955)
d_{16}	(0.9814,0.0034)	(0.8692,0.0463)	(0.9570,0.0104)	(0.9018,0.0314)	(0.7805,0.0939)
d_{17}	(0.9943,0.0007)	(0.8737,0.0441)	(0.9636,0.0083)	(0.9021,0.0313)	(0.8418,0.0600)
d_{18}	(0.9789,0.0040)	(0.8745,0.0438)	(0.9590,0.0097)	(0.8920,0.0357)	(0.7202,0.1314)
d_{19}	(0.9545,0.0111)	(0.8682,00468.)	(0.9589,0.0097)	(0.9084,0.0286)	(0.8366,0.0626)
d_{20}	(0.9744,0.0052)	(0.8687,0.0465)	(0.9550,0.0110)	(0.9600,0.0094)	(0.7007,0.1443)
d_{21}	(0.9594,0.0096)	(0.8544,0.0536)	(0.8922,0.0356)	(0.8723,0.0448)	(0.0000,1.0000)
d_{22}	(0.9225,0.0228)	(0.8640,0.0488)	(0.9206,0.0236)	(0.8615,0.0500)	(0.7296,0.1253)
l_1	(0.9594,0.0096)	(0.8720,0.0450)	(0.9353,0.0179)	(0.8720,0.0450)	(0.7649,0.1032)
l_2	(0.9675,0.0071)	(0.8798,0.0413)	(0.9513,0.0122)	(0.9034,0.0307)	(0.5518,0.2559)
l_3	(0.9837,0.0028)	(0.8877,0.0377)	(0.9675,0.0071)	(0.9513,0.0122)	(0.4232,0.3710)
l_4	(1.0000,0.0000)	(0.8955,0.0341)	(0.9837,0.0028)	(0.9675,0.0071)	(0.1149,0.7459)

here, categorizing the districts as per their environmental quality. The aggregated decision matrix, along with the limit profile is calculated (see: Table 10).

1) TOPSIS-GREY-SORT

In this technique, the subjective weights of the experts are not used, rather entropy weight determination technique assigns the objective weights.

Step1: Construct the decision matrix $H^k = (h^k_{mn})_{p'\times q}$ using the data provided by Tehran environmental management. Here h^k_{mn} is the assessment of k^{th} decision-maker $k = 1, 2, \ldots, G$ regarding n^{th} criterion ($n = 1, 2, \ldots, q$) of the m^{th} district ($m = 1, 2, \ldots, p'$). The expert group provides limit profiles of 5 classes ($g = 5$), namely; HSIC, SIC, MSIC, LSIC, and VLSIC. The limit profiles of each class are shown in Table 9. The well-known aggregation function called weighted mean aggregates $H^k, 1 \le k \le G$ and it results an aggregated decision matrix $H = (h_{mn})_{p'\times q}$, where $h_{mn} = \frac{1}{G}\sum_{k=1}^{G} h^k_{mn}$.

Step2: The aggregated decision matrix $\Gamma = (H, X) = (\chi_{mn})_{p\times q}$ takes into account the role of both limit profile and $H = (h_{mn})_{p'\times q}$ during decision making (see: Table 10).

Step3: Normalize matrix Γ and use Eq. (31) for the normalization process (use the minimized criterion value, as small-scale values are preferable). In Table 11, the normalized matrix $\Lambda = (\lambda_{mn})_{p\times q}$ is given.

Step4: The aggregated decision matrix is transformed to intuitionistic fuzzy decision matrix considering $\beta = 0.75$ and $\alpha = 1.5$ (see: Table 12).

Step5: The matrix $\tilde{E}$ containing intuitionistic fuzzy entropy values (E_{mn}) is calculated using $\tilde{\Lambda} = (\tilde{\lambda}_{mn})_{p\times q}$ and Eq.(32) as follows:

$$\tilde{E} = \begin{bmatrix} 0.0154 & 0.0980 & 0.0432 & 0.0820 & 0.5844 \\ 0.0096 & 0.1099 & 0.0224 & 0.0977 & 0.1709 \\ 0.0151 & 0.0979 & 0.0260 & 0.0752 & 0.4589 \\ 0.0240 & 0.1063 & 0.0682 & 0.0801 & 0.1523 \\ 0.0166 & 0.0903 & 0.0232 & 0.0801 & 0.1489 \\ 0.0121 & 0.0927 & 0.0231 & 0.0556 & 0.2973 \\ 0.0037 & 0.1016 & 0.0289 & 0.0570 & 0.2759 \\ 0.0041 & 0.0922 & 0.0264 & 0.0636 & 0.1640 \\ 0.0049 & 0.0949 & 0.0287 & 0.1035 & 0.1365 \\ 0.0021 & 0.0956 & 0.0249 & 0.0945 & 0.1203 \\ 0.0046 & 0.0915 & 0.0272 & 0.0696 & 0.1579 \\ 0.0054 & 0.0959 & 0.0278 & 0.0821 & 0.0985 \\ 0.0353 & 0.1021 & 0.0292 & 0.0780 & 0.1636 \\ 0.0056 & 0.0981 & 0.0312 & 0.0983 & 0.2024 \\ 0.0138 & 0.1066 & 0.0253 & 0.1098 & 0.6534 \\ 0.0111 & 0.0972 & 0.0274 & 0.0693 & 0.1858 \\ 0.0032 & 0.0932 & 0.0229 & 0.0690 & 0.1224 \\ 0.0127 & 0.0925 & 0.0260 & 0.0774 & 0.2588 \\ 0.0291 & 0.0980 & 0.0261 & 0.0640 & 0.1274 \\ 0.0156 & 0.0976 & 0.0288 & 0.0254 & 0.2850 \\ 0.0258 & 0.1106 & 0.0772 & 0.0944 & 0.0000 \\ 0.0528 & 0.1018 & 0.0543 & 0.1041 & 0.2467 \\ 0.0258 & 0.0947 & 0.0431 & 0.0947 & 0.2036 \\ 0.0202 & 0.0878 & 0.0314 & 0.0679 & 0.5434 \\ 0.0097 & 0.0811 & 0.0202 & 0.0314 & 0.9008 \\ 0.0000 & 0.0745 & 0.0097 & 0.0202 & 0.2262 \end{bmatrix}$$

TABLE 13. The weighted aggregated IF decision matrix.

Districts	Criteria				
	$\tilde{C}_1$	$\tilde{C}_2$	$\tilde{C}_3$	$\tilde{C}_4$	$\tilde{C}_5$
d_1	(0.3111,0.5855)	(0.5137,0.3364)	(0.3523,0.5282)	(0.4505,0.4073)	(0.0799,0.8671)
d_2	(0.3415,0.5512)	(0.4970,0.3522)	(0.4107,0.4653)	(0.4276,0.4304)	(0.1586,0.7648)
d_3	(0.3124,0.5840)	(0.5139,0.3362)	(0.3979,0.4789)	(0.4614,0.3963)	(0.0936,0.8485)
d_4	(0.2819,0.6190)	(0.5019,0.3476)	(0.3097,0.5755)	(0.4534,0.4043)	(0.1668,0.7547)
d_5	(0.3063,0.5910)	(0.5254,0.3254)	(0.4078,0.4684)	(0.4534,0.4043)	(0.1684,0.7526)
d_6	(0.3264,0.5682)	(0.5216,0.3290)	(0.4082,0.4680)	(0.4987,0.3597)	(0.1207,0.8129)
d_7	(0.3992,0.4877)	(0.5084,0.3414)	(0.3885,0.4889)	(0.4957,0.3626)	(0.1256,0.8066)
d_8	(0.3934,0.4940)	(0.5224,0.3282)	(0.3964,0.4805)	(0.4824,0.3756)	(0.1615,0.7612)
d_9	(0.3830,0.5053)	(0.5182,0.3322)	(0.3892,0.4882)	(0.4201,0.4380)	(0.1746,0.7450)
d_{10}	(0.4319,0.4526)	(0.5172,0.3331)	(0.4018,0.4748)	(0.4320,0.4259)	(0.1838,0.7337)
d_{11}	(0.3867,0.5012)	(0.5235,0.3272)	(0.3939,0.4832)	(0.4712,0.3866)	(0.1642,0.7579)
d_{12}	(0.3773,0.5116)	(0.5168,0.3335)	(0.3921,0.4851)	(0.4503,0.4075)	(0.1983,0.7160)
d_{13}	(0.2557,0.6494)	(0.5078,0.3419)	(0.3875,0.4900)	(0.4568,0.4009)	(0.1617,0.7610)
d_{14}	(0.3751,0.5140)	(0.5135,0.3366)	(0.3818,0.4961)	(0.4269,0.4311)	(0.1467,0.7798)
d_{15}	(0.3181,0.5775)	(0.5015,0.3479)	(0.4002,0.4764)	(0.4123,0.4461)	(0.0740,0.8753)
d_{16}	(0.3321,0.5618)	(0.5149,0.3353)	(0.3931,0.4840)	(0.4717,0.3861)	(0.1527,0.7722)
d_{17}	(0.4073,0.4790)	(0.5210,0.3296)	(0.4090,0.4671)	(0.4723,0.3856)	(0.1825,0.7353)
d_{18}	(0.3234,0.5716)	(0.5220,0.3286)	(0.3979,0.4789)	(0.4577,0.4000)	(0.1299,0.8011)
d_{19}	(0.2688,0.6341)	(0.5137,0.3364)	(0.3976,0.4793)	(0.4817,0.3763)	(0.1796,0.7388)
d_{20}	(0.3102,0.5865)	(0.5143,0.3358)	(0.3889,0.4885)	(0.5872,0.2771)	(0.1235,0.8094)
d_{21}	(0.2771,0.6245)	(0.4961,0.3531)	(0.2979,0.5888)	(0.4322,0.4258)	(0.0000,1.0000)
d_{22}	(0.2282,0.6819)	(0.5082,0.3416)	(0.3312,0.5515)	(0.4194,0.4388)	(0.1331,0.7970)
l_1	(0.2771,0.6245)	(0.5186,0.3318)	(0.3525,0.5280)	(0.4318,0.4261)	(0.1463,0.7803)
l_2	(0.2931,0.6060)	(0.5293,0.3218)	(0.3812,0.4968)	(0.4742,0.3837)	(0.0839,0.8616)
l_3	(0.3409,0.5519)	(0.5405,0.3115)	(0.4195,0.4560)	(0.5644,0.2978)	(0.0583,0.8973)
l_4	(1.0000,0.0000)	(0.5522,0.3008)	(0.4798,0.3939)	(0.6101,0.2567)	(0.0132,0.9685)

$$\tilde{\tilde{E}} = \begin{bmatrix} 0.2915 & 0.8861 & 0.5597 & 0.7467 & 0.6487 \\ 0.1811 & 0.9935 & 0.2904 & 0.8905 & 0.1897 \\ 0.2855 & 0.8848 & 0.3364 & 0.6852 & 0.5094 \\ 0.4539 & 0.9612 & 0.8833 & 0.7299 & 0.1691 \\ 0.3137 & 0.8162 & 0.3004 & 0.7299 & 0.1653 \\ 0.2299 & 0.8385 & 0.2990 & 0.5069 & 0.3300 \\ 0.0701 & 0.9190 & 0.3743 & 0.5196 & 0.3063 \\ 0.0774 & 0.8335 & 0.3422 & 0.5793 & 0.1821 \\ 0.0921 & 0.8585 & 0.3714 & 0.9427 & 0.1516 \\ 0.0395 & 0.8647 & 0.3219 & 0.8611 & 0.1335 \\ 0.0866 & 0.8273 & 0.3524 & 0.6342 & 0.1753 \\ 0.1014 & 0.8672 & 0.3597 & 0.7478 & 0.1093 \\ 0.6689 & 0.9228 & 0.3787 & 0.7106 & 0.1817 \\ 0.1051 & 0.8874 & 0.4038 & 0.8956 & 0.2247 \\ 0.2615 & 0.9638 & 0.3277 & 1.0000 & 0.7253 \\ 0.2103 & 0.8785 & 0.3553 & 0.6313 & 0.2063 \\ 0.0610 & 0.8422 & 0.2961 & 0.6287 & 0.1359 \\ 0.2409 & 0.8360 & 0.3364 & 0.7054 & 0.2873 \\ 0.5516 & 0.8861 & 0.3378 & 0.5828 & 0.1414 \\ 0.2956 & 0.8823 & 0.3728 & 0.2310 & 0.3164 \\ 0.4876 & 1.0000 & 1.0000 & 0.8602 & 0.0000 \\ 1.0000 & 0.9203 & 0.7031 & 0.9481 & 0.2738 \\ 0.4876 & 0.8560 & 0.5582 & 0.8626 & 0.2260 \\ 0.3832 & 0.7940 & 0.4067 & 0.6191 & 0.6032 \\ 0.1830 & 0.7330 & 0.2620 & 0.2862 & 1.0000 \\ 0.0000 & 0.6732 & 0.1252 & 0.1844 & 0.2512 \end{bmatrix}$$

Step6: The normalized intuitionistic fuzzy entropy matrix $\tilde{\tilde{E}}$ is calculated using Eq.(33).

Step7: The criteria weights are calculated using $\tilde{\tilde{E}}$ and Eq.(34). The resultant is

$$\{0.1013,\ 0.3557,\ 0.1588,\ 0.2750,\ 0.1093\}.$$

Step8: In Table-13, the weighted intuitionistic fuzzy decision matrix is provided.

Step9: The values of Q^+ and Q^- as determined using Eq.(37)-(39), respectively is given below,

$$\begin{aligned} Q^+ &= \{(1.0000, 0.0000), (0.5522, 0.3008), (0.4798, 0.3939), \\ &\quad (0.6101, 0.2567), (0.1983, 0.7160)\} \\ Q^- &= \{(0.2282, 0.6819), (0.4961, 0.3531), (0.2979, 0.5888), \\ &\quad (0.4123, 0.4461), (0.0000, 1.0000)\} \end{aligned}$$

Step10: To evaluate the GRC of each criteria from Q^+ and Q^-, Eq.(40)-(41) is used.

$$\xi^+_{mn} = \begin{bmatrix} 0.3622 & 0.9077 & 0.7360 & 0.7018 & 0.7264 \\ 0.3740 & 0.8726 & 0.8387 & 0.6722 & 0.8904 \\ 0.3628 & 0.9081 & 0.8141 & 0.7169 & 0.7513 \\ 0.3516 & 0.8825 & 0.6748 & 0.7058 & 0.9112 \\ 0.3605 & 0.9341 & 0.8329 & 0.7058 & 0.9155 \\ 0.3681 & 0.9253 & 0.8337 & 0.7729 & 0.8045 \\ 0.3979 & 0.8963 & 0.7968 & 0.7681 & 0.8148 \\ 0.3954 & 0.9272 & 0.8113 & 0.7475 & 0.8977 \\ 0.3910 & 0.9177 & 0.7981 & 0.6630 & 0.9319 \\ 0.4127 & 0.9154 & 0.8213 & 0.6777 & 0.9572 \\ 0.3925 & 0.9297 & 0.8065 & 0.7308 & 0.9045 \\ 0.3885 & 0.9145 & 0.8032 & 0.7016 & 1.0000 \\ 0.3425 & 0.8950 & 0.7950 & 0.7105 & 0.8981 \\ 0.3876 & 0.9072 & 0.7849 & 0.6713 & 0.8618 \\ 0.3649 & 0.8817 & 0.8184 & 0.6536 & 0.7160 \\ 0.3703 & 0.9104 & 0.8052 & 0.7316 & 0.8760 \\ 0.4015 & 0.9239 & 0.8354 & 0.7324 & 0.9536 \\ 0.3669 & 0.9263 & 0.8141 & 0.7118 & 0.8240 \\ 0.3470 & 0.9077 & 0.8134 & 0.7464 & 0.9455 \\ 0.3619 & 0.9090 & 0.7974 & 0.9439 & 0.8102 \\ 0.3499 & 0.8706 & 0.6595 & 0.6779 & 0.5916 \\ 0.3333 & 0.8959 & 0.7044 & 0.6621 & 0.8310 \\ 0.3499 & 0.9187 & 0.7364 & 0.6775 & 0.8609 \\ 0.3556 & 0.9432 & 0.7837 & 0.7352 & 0.7335 \\ 0.3737 & 0.9701 & 0.8565 & 0.8935 & 0.6895 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 0.6175 \end{bmatrix}$$

TABLE 14. Corresponding values of ξ_m^+, ξ_m^-, RC_m.

Districts	ξ_m^+	ξ_m^-	RC_m
d_1	0.7488	0.8898	0.5430
d_2	0.7635	0.8839	0.5365
d_3	0.7682	0.8662	0.5300
d_4	0.7503	0.9065	0.5472
d_5	0.7951	0.8481	0.5161
d_6	0.7992	0.8298	0.5094
d_7	0.7859	0.8361	0.5155
d_8	0.8023	0.8241	0.5067
d_9	0.7769	0.8678	0.5276
d_{10}	0.7888	0.8496	0.5186
d_{11}	0.7983	0.8305	0.5099
d_{12}	0.7943	0.8459	0.5157
d_{13}	0.7728	0.8788	0.5321
d_{14}	0.7653	0.8740	0.5332
d_{15}	0.7385	0.9124	0.5527
d_{16}	0.7861	0.8475	0.5188
d_{17}	0.8075	0.8229	0.5047
d_{18}	0.7816	0.8532	0.5219
d_{19}	0.7957	0.8513	0.5169
d_{20}	0.8347	0.8083	0.4920
d_{21}	0.7009	0.9727	0.5812
d_{22}	0.7371	0.9337	0.5588
l_1	0.7595	0.8932	0.5405
l_2	0.7782	0.8567	0.5240
l_3	0.8400	0.7945	0.4861
l_4	0.9582	0.7295	0.4322

$$\xi^-_{mn} = \begin{bmatrix} 0.8018 & 0.9552 & 0.8636 & 0.9047 & 0.7593 \\ 0.7483 & 0.9974 & 0.7552 & 0.9593 & 0.6376 \\ 0.7992 & 0.9547 & 0.7763 & 0.8808 & 0.7341 \\ 0.8613 & 0.9847 & 0.9666 & 0.8982 & 0.6275 \\ 0.8109 & 0.9276 & 0.7599 & 0.8982 & 0.6255 \\ 0.7739 & 0.9364 & 0.7592 & 0.8088 & 0.6899 \\ 0.6655 & 0.9681 & 0.7927 & 0.8141 & 0.6826 \\ 0.6730 & 0.9344 & 0.7788 & 0.8386 & 0.6340 \\ 0.6867 & 0.9443 & 0.7914 & 0.9788 & 0.6181 \\ 0.6268 & 0.9467 & 0.7698 & 0.9482 & 0.6075 \\ 0.6817 & 0.9320 & 0.7833 & 0.8607 & 0.6306 \\ 0.6945 & 0.9477 & 0.7864 & 0.9052 & 0.5916 \\ 0.9234 & 0.9696 & 0.7945 & 0.8907 & 0.6338 \\ 0.6976 & 0.9557 & 0.8049 & 0.9612 & 0.6530 \\ 0.7887 & 0.9858 & 0.7724 & 1.0000 & 0.7709 \\ 0.7640 & 0.9522 & 0.7846 & 0.8596 & 0.6452 \\ 0.6555 & 0.9379 & 0.7579 & 0.8585 & 0.6090 \\ 0.7792 & 0.9354 & 0.7763 & 0.8887 & 0.6763 \\ 0.8912 & 0.9552 & 0.7769 & 0.8401 & 0.6123 \\ 0.8035 & 0.9537 & 0.7920 & 0.6800 & 0.6858 \\ 0.8720 & 1.0000 & 1.0000 & 0.9479 & 1.0000 \\ 1.0000 & 0.9686 & 0.9116 & 0.9808 & 0.6717 \\ 0.8720 & 0.9433 & 0.8631 & 0.9488 & 0.6535 \\ 0.8873 & 0.9188 & 0.8061 & 0.8547 & 0.7517 \\ 0.7494 & 0.8946 & 0.7413 & 0.7087 & 0.8038 \\ 0.3333 & 0.8706 & 0.6595 & 0.6536 & 0.9302 \end{bmatrix}$$

Step11: With the help of Eq. (42)-(43), the GRG (ξ^+ and ξ^-) is calculated. Refer Table 14 for GRG values.

Step12: RC_m values are calculated using Eq.(44) for each alternative and limit profile, and these values are in Table 14.

Step13: Eq.(45) is used to assign alternatives to appropriate classes. The final sorting of tourist sites as per their exposure to COVID-19 is shown in Fig. 6.

2) ENTROPY-TOPSIS-GREY-SORT

In this technique, the subjective weights provided by the experts and the objective weights calculated using the entropy weight determination technique are merged, and their cumulative effects on decision-making are studied. Here **Step 1-4** are same, so same dataset, normalized matrix, same value of $\beta = 0.75$ and $\alpha = 1.5$ and IF aggregated matrix are obtained.

Step5: The intuitionistic fuzzy entropy values (E_{mn}) in $\tilde{E}$ are obtained using $\tilde{\Lambda} = (\tilde{\lambda}_{mn})_{p\times q}$ and using Eq.(23). Each value given in $\tilde{E}$ is of the form 1.0e-16.

$$\tilde{E} = \begin{bmatrix} 0.5850 & 0.4658 & 0.5402 & 0.4860 & 0.1424 \\ 0.5951 & 0.4517 & 0.5732 & 0.4662 & 0.3877 \\ 0.5856 & 0.4660 & 0.5673 & 0.4948 & 0.1974 \\ 0.5706 & 0.4559 & 0.5043 & 0.4884 & 0.4058 \\ 0.5830 & 0.4754 & 0.5719 & 0.4884 & 0.4093 \\ 0.5906 & 0.4723 & 0.5721 & 0.5219 & 0.2881 \\ 0.6056 & 0.4614 & 0.5626 & 0.5199 & 0.3026 \\ 0.6049 & 0.4730 & 0.5666 & 0.5106 & 0.3943 \\ 0.6035 & 0.4696 & 0.5630 & 0.4593 & 0.4221 \\ 0.6085 & 0.4687 & 0.5692 & 0.4701 & 0.4398 \\ 0.6040 & 0.4738 & 0.5653 & 0.5024 & 0.4002 \\ 0.6026 & 0.4684 & 0.5644 & 0.4858 & 0.4652 \\ 0.5524 & 0.4609 & 0.5620 & 0.4911 & 0.3947 \\ 0.6023 & 0.4657 & 0.5589 & 0.4655 & 0.3594 \\ 0.5877 & 0.4555 & 0.5684 & 0.4518 & 0.1155 \\ 0.5924 & 0.4668 & 0.5650 & 0.5028 & 0.3740 \\ 0.6065 & 0.4718 & 0.5724 & 0.5032 & 0.4374 \\ 0.5896 & 0.4726 & 0.5673 & 0.4919 & 0.3148 \\ 0.5622 & 0.4658 & 0.5672 & 0.5101 & 0.4319 \\ 0.5847 & 0.4663 & 0.5628 & 0.5684 & 0.2963 \\ 0.5677 & 0.4509 & 0.4922 & 0.4702 & 0.6123 \\ 0.5260 & 0.4613 & 0.5238 & 0.4585 & 0.3238 \\ 0.5677 & 0.4699 & 0.5404 & 0.4699 & 0.3584 \\ 0.5768 & 0.4785 & 0.5586 & 0.5046 & 0.1593 \\ 0.5950 & 0.4871 & 0.5768 & 0.5586 & 0.0309 \\ 0.6123 & 0.4958 & 0.5950 & 0.5768 & 0.3397 \end{bmatrix}$$

$$\tilde{\tilde{E}} = \begin{bmatrix} 0.9554 & 0.9395 & 0.9080 & 0.8425 & 0.2325 \\ 0.9719 & 0.9110 & 0.9634 & 0.8084 & 0.6332 \\ 0.9563 & 0.9399 & 0.9536 & 0.8578 & 0.3225 \\ 0.9319 & 0.9194 & 0.8475 & 0.8466 & 0.6628 \\ 0.9521 & 0.9588 & 0.9612 & 0.8466 & 0.6685 \\ 0.9646 & 0.9526 & 0.9616 & 0.9047 & 0.4705 \\ 0.9890 & 0.9306 & 0.9456 & 0.9013 & 0.4942 \\ 0.9879 & 0.9539 & 0.9523 & 0.8853 & 0.6440 \\ 0.9856 & 0.9471 & 0.9462 & 0.7962 & 0.6894 \\ 0.9938 & 0.9453 & 0.9566 & 0.8150 & 0.7183 \\ 0.9864 & 0.9557 & 0.9502 & 0.8709 & 0.6536 \\ 0.9841 & 0.9446 & 0.9487 & 0.8422 & 0.7598 \\ 0.9021 & 0.9296 & 0.9447 & 0.8515 & 0.6446 \\ 0.9836 & 0.9392 & 0.9395 & 0.8070 & 0.5869 \\ 0.9599 & 0.9188 & 0.9554 & 0.7833 & 0.1887 \\ 0.9675 & 0.9416 & 0.9496 & 0.8716 & 0.6107 \\ 0.9904 & 0.9515 & 0.9622 & 0.8723 & 0.7143 \\ 0.9629 & 0.9532 & 0.9536 & 0.8528 & 0.5141 \\ 0.9182 & 0.9395 & 0.9533 & 0.8843 & 0.7054 \\ 0.9548 & 0.9405 & 0.9459 & 0.9853 & 0.4839 \\ 0.9271 & 0.9093 & 0.8272 & 0.8152 & 1.0000 \\ 0.8590 & 0.9303 & 0.8804 & 0.7949 & 0.5289 \\ 0.9271 & 0.9477 & 0.9083 & 0.8146 & 0.5853 \\ 0.9420 & 0.9650 & 0.9388 & 0.8748 & 0.2602 \\ 0.9716 & 0.9824 & 0.9695 & 0.9684 & 0.0505 \\ 1.0000 & 1.0000 & 1.0000 & 1.0000 & 0.5548 \end{bmatrix}$$

Step6: The normalized intuitionistic fuzzy entropy values (e_{mn}) in $\tilde{\tilde{E}}$ are obtained using Eq.(33).

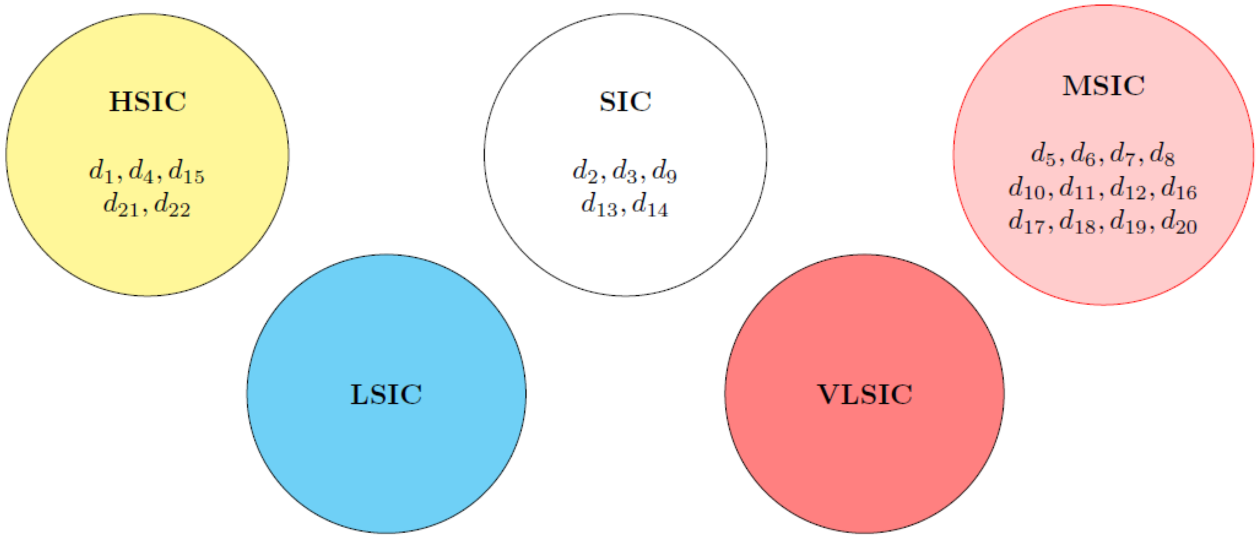

FIGURE 6. The assignment of districts to different interval classes as per their environmental quality.

TABLE 15. The weighted aggregated IF decision matrix.

Districts	Criteria				
	$\tilde{C}_1$	$\tilde{C}_2$	$\tilde{C}_3$	$\tilde{C}_4$	$\tilde{C}_5$
d_1	(0.5436,0.3241)	(0.3485,0.5233)	(0.4382,0.4285)	(0.3547,0.5182)	(0.1171,0.8081)
d_2	(0.5850,0.2854)	(0.3354,0.5377)	(0.5045,0.3621)	(0.3352,0.5396)	(0.2274,0.6699)
d_3	(0.5455,0.3223)	(0.3487,0.5231)	(0.4902,0.3762)	(0.3642,0.5080)	(0.1367,0.7824)
d_4	(0.5019,0.3643)	(0.3392,0.5336)	(0.3887,0.4802)	(0.3572,0.5155)	(0.2386,0.6566)
d_5	(0.5369,0.3305)	(0.3579,0.5131)	(0.5012,0.3653)	(0.3572,0.5155)	(0.2409,0.6540)
d_6	(0.5647,0.3042)	(0.3548,0.5164)	(0.5017,0.3649)	(0.3967,0.4732)	(0.1749,0.7338)
d_7	(0.6578,0.2206)	(0.3443,0.5279)	(0.4796,0.3867)	(0.3941,0.4760)	(0.1818,0.7252)
d_8	(0.6508,0.2266)	(0.3555,0.5157)	(0.4885,0.3779)	(0.3824,0.4884)	(0.2314,0.6651)
d_9	(0.6382,0.2377)	(0.3521,0.5194)	(0.4804,0.3859)	(0.3289,0.5466)	(0.2493,0.6440)
d_{10}	(0.6959,0.1885)	(0.3513,0.5203)	(0.4945,0.3719)	(0.3390,0.5355)	(0.2618,0.6295)
d_{11}	(0.6427,0.2337)	(0.3564,0.5148)	(0.4856,0.3807)	(0.3726,0.4989)	(0.2351,0.6608)
d_{12}	(0.6310,0.2439)	(0.3510,0.5206)	(0.4836,0.3827)	(0.3546,0.5184)	(0.2813,0.6070)
d_{13}	(0.4630,0.4030)	(0.3439,0.5284)	(0.4784,0.3879)	(0.3602,0.5123)	(0.2317,0.6649)
d_{14}	(0.6283,0.2463)	(0.3484,0.5235)	(0.4720,0.3943)	(0.3346,0.5403)	(0.2111,0.6895)
d_{15}	(0.5533,0.3149)	(0.3389,0.5339)	(0.4927,0.3736)	(0.3223,0.5539)	(0.1085,0.8195)
d_{16}	(0.5724,0.2970)	(0.3495,0.5222)	(0.4848,0.3815)	(0.3731,0.4984)	(0.2193,0.6796)
d_{17}	(0.6674,0.2124)	(0.3543,0.5170)	(0.5026,0.3640)	(0.3736,0.4979)	(0.2600,0.6316)
d_{18}	(0.5606,0.3081)	(0.3552,0.5161)	(0.4902,0.3762)	(0.3610,0.5114)	(0.1878,0.7179)
d_{19}	(0.4826,0.3834)	(0.3485,0.5233)	(0.4898,0.3766)	(0.3818,0.4891)	(0.2561,0.6361)
d_{20}	(0.5423,0.3253)	(0.3490,0.5228)	(0.4800,0.3863)	(0.4766,0.3910)	(0.1788,0.7290)
d_{21}	(0.4949,0.3712)	(0.3346,0.5386)	(0.3748,0.4950)	(0.3391,0.5353)	(0.0000,1.0000)
d_{22}	(0.4203,0.4467)	(0.3442,0.5281)	(0.4138,0.4538)	(0.3282,0.5473)	(0.1923,0.7124)
l_1	(0.4949,0.3712)	(0.3525,0.5190)	(0.4385,0.4282)	(0.3388,0.5357)	(0.2105,0.6902)
l_2	(0.5182,0.3484)	(0.3611,0.5097)	(0.4712,0.3950)	(0.3752,0.4961)	(0.1228,0.8005)
l_3	(0.5842,0.2862)	(0.3701,0.4999)	(0.5143,0.3526)	(0.4557,0.4121)	(0.0859,0.8505)
l_4	(1.0000,0.0000)	(0.3797,0.4896)	(0.5801,0.2902)	(0.4980,0.3697)	(0.0197,0.9533)

Step7: The objective weights of the criteria are calculated using $\tilde{\tilde{E}}$ and the entropy method (see: Eq.(34)) and we obtain weight values as

$$\{0.2264,\ 0.2228,\ 0.2217,\ 0.2025,\ 0.1266\}.$$

Subjective weights provided by experts for each criterion are given in Table 9. The combined weight W is given as follows:

$$\{0.2132,\quad 0.2114,\quad 0.2108,\quad 0.2012,\quad 0.1633\}.$$

Step8: The weighted intuitionistic fuzzy decision matrix is in Table-15.

Step9: Use Eq.(37)-(39) to calculate Q^+ and Q^-. The values are:

$$Q^+ = \{(1.0000, 0.0000), (0.3797, 0.4896), (0.5801, 0.2902), (0.4980, 0.3697), (0.2813, 0.6070)\}$$

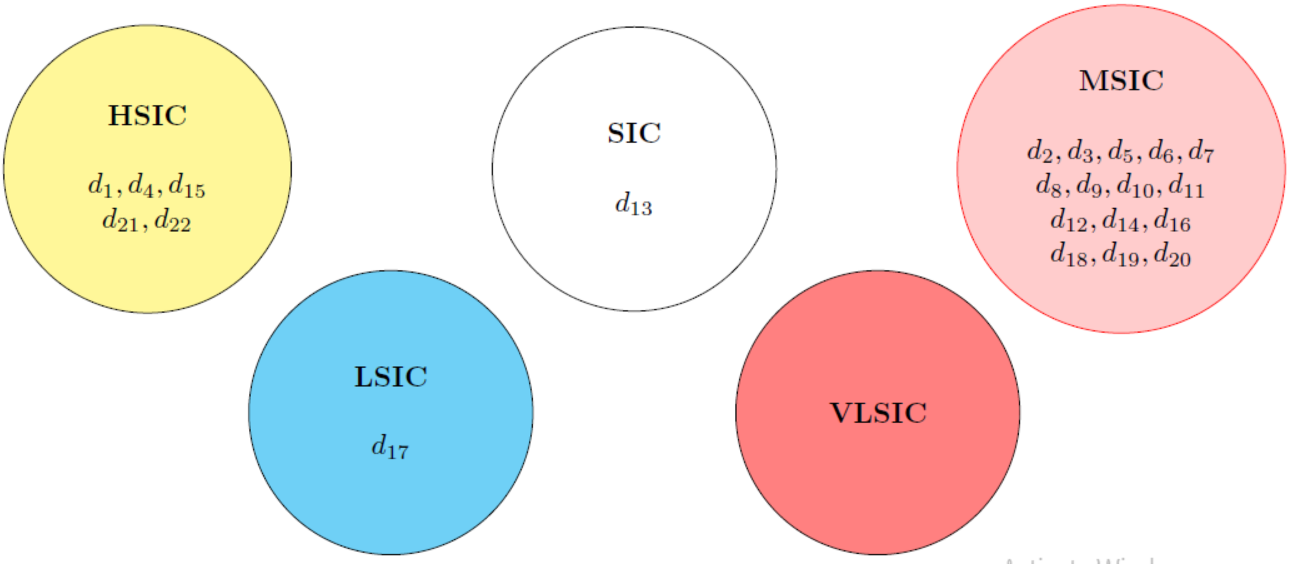

FIGURE 7. The assignment of districts to different interval classes as per their environmental quality.

$$
\begin{aligned}
Q^{-} &= \{(0.4203, 0.4467), (0.3346, 0.5386), (0.3748, 0.4950),\\
&\quad (0.3223, 0.5539), (0.0000, 1.0000)\}
\end{aligned}
$$

Step10: To evaluate the GRC of each criteria from Q^+ and Q^-, Eq.(40)-(41) is used.

$$
\xi^{+}_{mn} = \begin{bmatrix}
0.3927 & 0.8900 & 0.6524 & 0.6431 & 0.5865\\
0.4170 & 0.8501 & 0.7808 & 0.6124 & 0.8170\\
0.3937 & 0.8905 & 0.7493 & 0.6589 & 0.6185\\
0.3707 & 0.8613 & 0.5797 & 0.6472 & 0.8497\\
0.3890 & 0.9206 & 0.7734 & 0.6472 & 0.8565\\
0.4047 & 0.9104 & 0.7744 & 0.7197 & 0.6904\\
0.4668 & 0.8769 & 0.7274 & 0.7144 & 0.7049\\
0.4615 & 0.9126 & 0.7457 & 0.6918 & 0.8284\\
0.4523 & 0.9016 & 0.7290 & 0.6030 & 0.8832\\
0.4973 & 0.8989 & 0.7585 & 0.6181 & 0.9254\\
0.4556 & 0.9154 & 0.7396 & 0.6738 & 0.8391\\
0.4472 & 0.8978 & 0.7355 & 0.6428 & 1.0000\\
0.3520 & 0.8755 & 0.7251 & 0.6521 & 0.8290\\
0.4453 & 0.8895 & 0.7124 & 0.6114 & 0.7733\\
0.3982 & 0.8604 & 0.7547 & 0.5935 & 0.5734\\
0.4093 & 0.8931 & 0.7380 & 0.6746 & 0.7948\\
0.4741 & 0.9087 & 0.7765 & 0.6754 & 0.9192\\
0.4023 & 0.9115 & 0.7493 & 0.6535 & 0.7179\\
0.3612 & 0.8900 & 0.7484 & 0.6906 & 0.9057\\
0.3920 & 0.8915 & 0.7282 & 0.9250 & 0.6985\\
0.3672 & 0.8480 & 0.5619 & 0.6182 & 0.4285\\
0.3333 & 0.8764 & 0.6145 & 0.6020 & 0.7281\\
0.3672 & 0.9026 & 0.6528 & 0.6178 & 0.7719\\
0.3790 & 0.9313 & 0.7110 & 0.6785 & 0.5956\\
0.4164 & 0.9635 & 0.8039 & 0.8612 & 0.5407\\
1.0000 & 1.0000 & 1.0000 & 1.0000 & 0.4569
\end{bmatrix}
$$

$$
\xi^{-}_{mn} = \begin{bmatrix}
0.6815 & 0.9473 & 0.8018 & 0.8849 & 0.6108\\
0.6173 & 0.9970 & 0.6670 & 0.9505 & 0.4733\\
0.6783 & 0.9467 & 0.6918 & 0.8565 & 0.5799\\
0.7624 & 0.9821 & 0.9482 & 0.8772 & 0.4631\\
0.6932 & 0.9149 & 0.6725 & 0.8772 & 0.4612\\
0.6471 & 0.9252 & 0.6717 & 0.7717 & 0.5288\\
0.5313 & 0.9625 & 0.7116 & 0.7779 & 0.5207\\
0.5384 & 0.9229 & 0.6949 & 0.8067 & 0.4696\\
0.5518 & 0.9345 & 0.7101 & 0.9741 & 0.4539\\
0.4959 & 0.9374 & 0.6842 & 0.9371 & 0.4436\\
0.5469 & 0.9200 & 0.7002 & 0.8326 & 0.4663\\
0.5597 & 0.9386 & 0.7040 & 0.8854 & 0.4285\\
0.8590 & 0.9643 & 0.7138 & 0.8682 & 0.4694\\
0.5628 & 0.9479 & 0.7265 & 0.9528 & 0.4891\\
0.6651 & 0.9832 & 0.6872 & 1.0000 & 0.6256\\
0.6354 & 0.9438 & 0.7018 & 0.8313 & 0.4810\\
0.5218 & 0.9270 & 0.6701 & 0.8301 & 0.4450\\
0.6535 & 0.9241 & 0.6918 & 0.8658 & 0.5139\\
0.8072 & 0.9473 & 0.6926 & 0.8083 & 0.4482\\
0.6836 & 0.9455 & 0.7108 & 0.6234 & 0.5242\\
0.7781 & 1.0000 & 1.0000 & 0.9367 & 1.0000\\
1.0000 & 0.9631 & 0.8676 & 0.9765 & 0.5089\\
0.7781 & 0.9333 & 0.8011 & 0.9378 & 0.4897\\
0.7284 & 0.9046 & 0.7280 & 0.8255 & 0.6013\\
0.6185 & 0.8761 & 0.6510 & 0.6562 & 0.6693\\
0.3333 & 0.8480 & 0.5619 & 0.5935 & 0.8661
\end{bmatrix}
$$

Step11: With the help of Eq. (42)-(43), the GRG (ξ^+ and ξ^-) is calculated. Refer Table 16 for GRG values.

Step12: Using Eq.(44), the RC_m values of each alternative based on their limit profile are calculated. These values are given in Table 16.

Step13: Eq.(45) is used to assign alternatives to appropriate classes. The final sorting of tourist sites as per their exposure to COVID-19 is shown in Fig. 7.

D. RESULT DISCUSSION AND COMPARATIVE ANALYSIS

- In comparison to [54], the results given here are of generalized nature, and twenty-two districts of Tehran

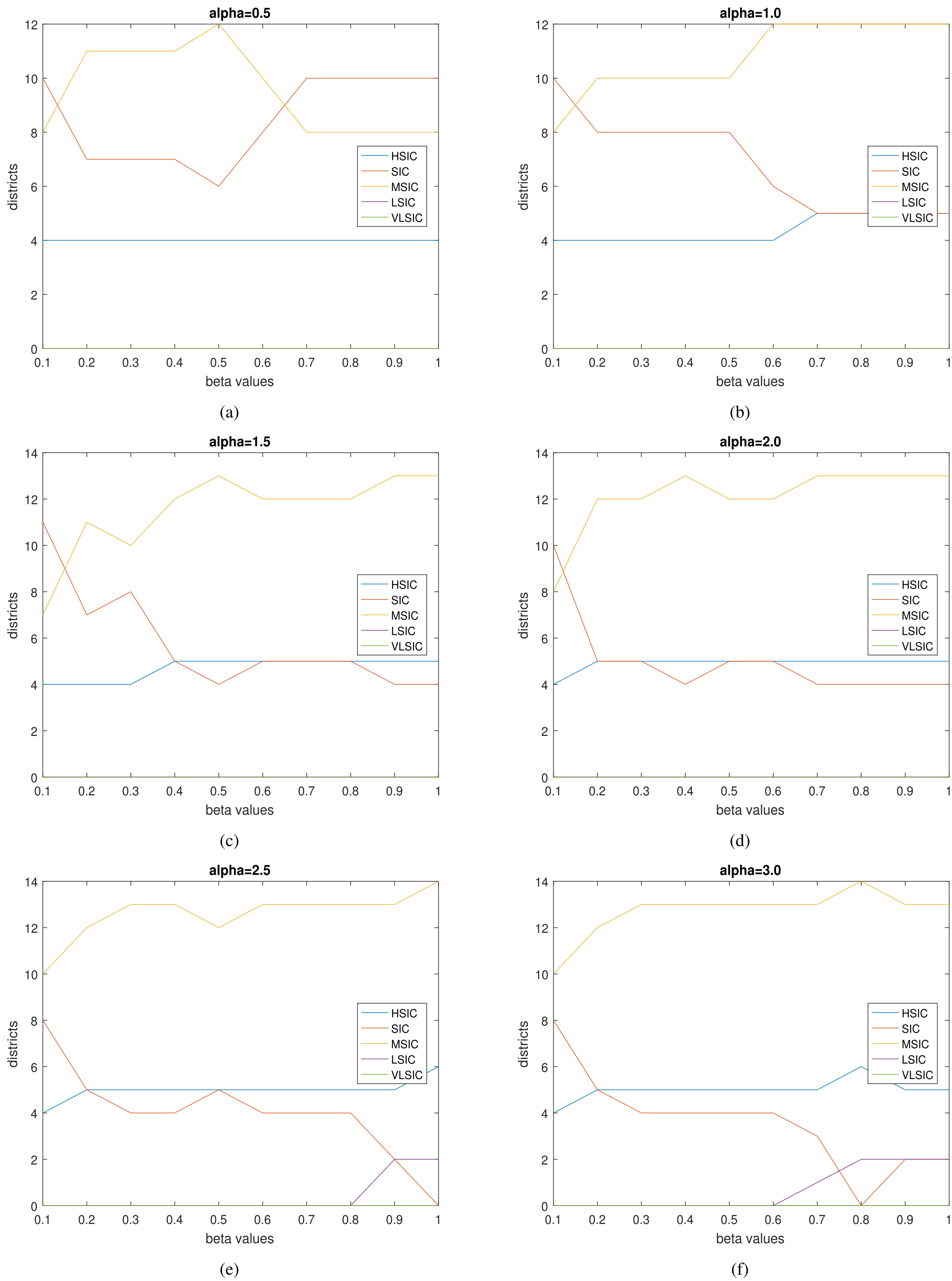

FIGURE 8. Study of allocation of districts to different interval classes with TOPSIS-GREY tuning different values of β and α.

are categorized as 'high suitable as *HSIC*', 'suitable as *SIC*', 'moderately suitable as *MSIC*', 'low suitable as *LSIC*' and 'very low suitable as *VLSIC*'. The sorting techniques proposed in the paper assign ranges to HSIC,

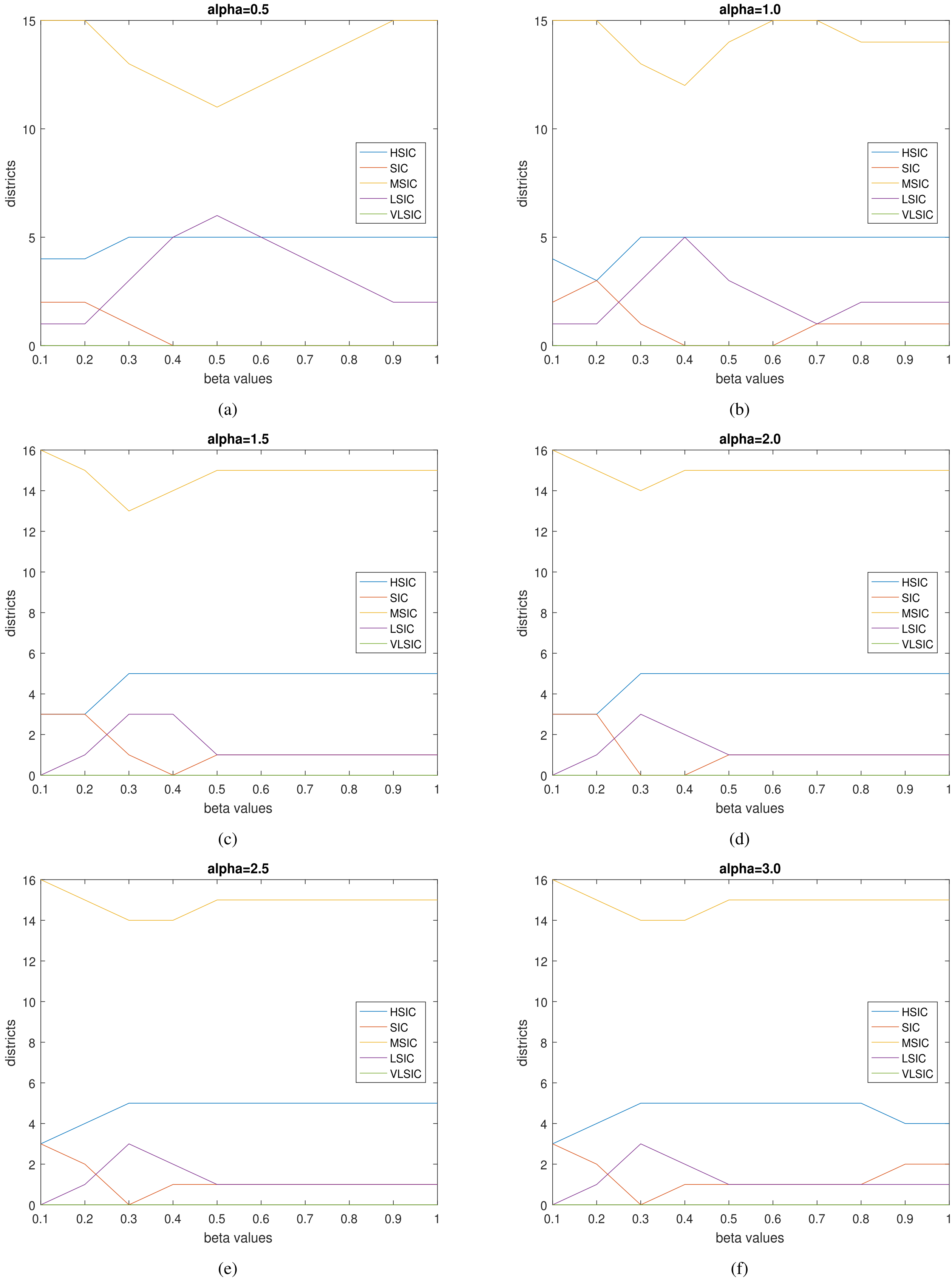

FIGURE 9. Study of allocation of districts to different interval classes with ENTROPY-TOPSIS-GREY tuning different values of β and α.

TABLE 16. Corresponding values of ξ_m^+, ξ_m^-, RC_m.

Districts	ξ_m^+	ξ_m^-	RC_m
d_1	0.6346	0.7925	0.5553
d_2	0.6899	0.7516	0.5214
d_3	0.6638	0.7577	0.5330
d_4	0.6523	0.8222	0.5576
d_5	0.7107	0.7348	0.5083
d_6	0.6996	0.7168	0.5061
d_7	0.6972	0.7084	0.5040
d_8	0.7230	0.6954	0.4903
d_9	0.7063	0.7351	0.5100
d_{10}	0.7315	0.7092	0.4923
d_{11}	0.7192	0.7025	0.4941
d_{12}	0.7329	0.7144	0.4936
d_{13}	0.6796	0.7889	0.5372
d_{14}	0.6825	0.7452	0.5219
d_{15}	0.6390	0.7980	0.5553
d_{16}	0.6972	0.7288	0.5111
d_{17}	0.7430	0.6882	0.4809
d_{18}	0.6852	0.7387	0.5188
d_{19}	0.7098	0.7543	0.5152
d_{20}	0.7258	0.7066	0.4933
d_{21}	0.5704	0.9399	0.6223
d_{22}	0.6260	0.8794	0.5842
l_1	0.6571	0.8008	0.5493
l_2	0.6614	0.7644	0.5361
l_3	0.7236	0.6957	0.4902
l_4	0.9113	0.6297	0.4086

SIC, MSIC, LSIC, and VLSIC instead of a precise value. TOPSIS-GREY-sort assigns ranges to HSIC = [4,6], SIC = [0,10], MSIC = [7,14], LSIC = [0,2], and VLSIC = [0,0]. ENTROPY-TOPSIS-GREY-sort assigns HSIC = [3,5], SIC = [0,3], MSIC = [12,16], LSIC = [0,3], and VLSIC = [0,0]. The variations in TOPSIS-GREY-sort and ENTROPY-TOPSIS-GREY-sort outcomes are because of the variation of the parameters β and α are figuratively discussed (see Fig. 8, 9). The paper [54] shows exactly two districts have high HSIC environment, nine districts have a SIC environment, and eleven districts have a MSIC environment. In contrast, none of the districts have a LSIC or VLSIC environment. The two techniques proposed in the paper overall ranges to HSIC = [1,6], SIC = [0,10], MSIC = [0,16], LSIC = [0,7], and VLSIC = [0,16].

It is possible to obtain a uniquely valued HSIC, SIC, MSIC, LSIC, and VLSIC. TOPSIS-GREY-sort assigns, HSIC = [5,5], SIC = [5,5], MSIC = [12,12], LSIC = [0,0], and VLSIC = [0,0] at β = 0.75 and α = 1.5. These specific valued results are depicted in Fig. 6), the figure shows that five districts have HSIC environment, five districts have a SIC environment, twelve districts have a MSIC environment, whereas none of the districts have a LSIC or VLSIC environment. ENTROPY-TOPSIS-GREY-sort assigns, HSIC = [5,5], SIC = [1,1], MSIC = [15,15], LSIC = [1,1], and VLSIC = [0,0]. These specific valued results are depicted in Fig. 7), the figure can be explained that five districts have HSIC environment, only one district has a SIC environment, fifteen districts have a MSIC environment, whereas one district has a LSIC environment and none of the districts have a VLSIC environment. So, to improve the environmental quality of the entire Tehran, authorities need to focus on only one district of the LSIC environment (as per the ENTROPY-TOPSIS-GREY-sort). The results of TOPSIS-GREY-sort and ENTROPY-TOPSIS-GREY-sort are similar to those obtained in [54]. We suggest the maximum number of districts belonging to the LSIC environment is three (as per ENTROPY-TOPSIS-GREY-sort) and two (as per TOPSIS-GREY-sort), and nearly the results are identical. Thus three districts need to be looked upon to improve the environmental quality of Tehran.

- It is not always the case that subjective weights (weights allocated by DMs) are known. In such a case, objective weights are calculated using a data-driven weight determination scheme. Here, we have carried out the individual study of objective and subjective weights, and both weights' cumulative effect is also analyzed. TOPSIS-GREY-sort uses objective weights (calculated by entropy method) which is (0.1013, 0.3557, 0.1588, 0.2750, 0.1093). Moreover, the convex combination of subjective weight (see: Table-9) and objective weight calculated by the proposed entropy method (see: 0.2264, 0.2228, 0.2217, 0.2025, 0.1266) is utilized in ENTROPY-TOPSIS-GREY-sort with combined weightage equal to (0.2132, 0.2114, 0.2108, 0.2012, 0.1633). The obtained results here with subjective and objective weights show the same results. Hence, we can say that the decision-making techniques here give stable results.

VI. CONCLUSION

In the AIFS environment, the hesitancy and uncertainty involved have been suitably dealt with due to the use of the MCS method. Here ratings at the attributes and alternatives level have been expressed in linguistic terms, and it is characterized by a generalized fuzzy set called AIFS. The domestic tourism problem concerning the reopening of the tourist sites in the Philippines amidst COVID-19 and the environmental quality of Tehran has been satisfactorily solved, which demonstrates the efficacy of our proposed novel sorting technique. Further, it is worth noting that the proposed methods and entropy measures have a generic structure; hence, they can also be applied to other MCS application domains. Some of the contributions and insights of proposed MCS techniques are as follows: (1) the proposed entropy measure in the AIFS environment models the uncertainty involved in MCS problems in a more generalized manner without much increasing the computational cost, (2) it profitably sorts tourist sites into three classes (i.e., LEIC, MEIC, and HEIC) and sorts districts of Tehran into five classes (i.e., HSIC, SIC, MSIC, LSIC, and VLSIC). It remains efficient with the increase in the number of classes by adding the limit profiles, and (3) the proposed MCS techniques offer considerable potential for other domain applications. The case study results provide a vision to stakeholders in the tourism and public health sectors in appropriately designing and implementing measures as the industry is slowly recovering amidst the pandemic and implementing a strategy in different districts of Tehran to improve their environmental quality as per their sorting into other classes. Because of the variation of the tuning parameters instead of crisp values, we are obtaining ranges through the proposed sorting techniques, which in general, helps the stakeholders and authorities. A brief study of subjective and objective weights is also carried out in this work. This work offers an AIFS extension of the TOPSIS-Sort. In a similar manner, type-2 fuzzy sets (T2FS) and IVIFS-based models of TOPSIS-sort can be proposed.

CONFLICT OF INTEREST

On behalf of both the authors, the corresponding author states that there is no conflict of interest. In the preparation of the manuscript, both the authors have contributed.

REFERENCES

[1] Z. Ashraf, M. S. Khan, A. Tiwari, and Q. M. D. Lohani, "Difference sequence-based distance measure for intuitionistic fuzzy sets and its application in decision making process," *Soft Comput.*, vol. 25, no. 14, pp. 9139–9161, Jul. 2021.

[2] Z. Stevic, D. Pamucar, A. Puska, and P. Chatterjee, "Sustainable supplier selection in healthcare industries using a new MCDM method: Measurement of alternatives and ranking according to compromise solution (MARCOS)," *Comput. Ind. Eng.*, vol. 140, Feb. 2020, Art. no. 106231.

[3] G. N. Zanon, A. L. Szejka, and E. De F. R. Loures, "Towards an integrated MCDM and BSC method to support the digital transformation strategy in railway companies," *Transdisciplinary Eng. Resilience, Responding Syst. Disruptions. Adv. Transdisciplinary Eng.*, vol. 16, pp. 302–311, Nov. 2021.

[4] A. E. Torkayesh and S. E. Torkayesh, "Evaluation of information and communication technology development in G7 countries: An integrated MCDM approach," *Technol. Soc.*, vol. 66, Aug. 2021, Art. no. 101670.

[5] J. S. Chai, G. Selvachandran, F. Smarandache, V. C. Gerogiannis, L. H. Son, Q.-T. Bui, and B. Vo, "New similarity measures for single-valued neutrosophic sets with applications in pattern recognition and medical diagnosis problems," *Complex Intell. Syst.*, vol. 7, no. 2, pp. 703–723, Apr. 2021.

[6] G. Petrović, J. Mihajlović, V. Z. Cojbasić, M. Madić, and D. Marinković, "Comparison of three fuzzy MCDM methods for solving the supplier selection problem," *Facta Universitatis, Ser., Mech. Eng.*, vol. 17, no. 3, pp. 455–469, 2019.

[7] M. Abdel-Basset, W. Ding, R. Mohamed, and N. Metawa, "An integrated plithogenic MCDM approach for financial performance evaluation of manufacturing industries," *Risk Manag.*, vol. 22, no. 3, pp. 192–218, Sep. 2020.

[8] A. Sotoudeh-Anvari, "The applications of MCDM methods in COVID-19 pandemic: A state of the art review," *Appl. Soft Comput.*, vol. 126, Sep. 2022, Art. no. 109238.

[9] C.-L. Hwang and K. Yoon, "Methods for multiple attribute decision making," in *Multiple Attribute Decision Making*. Berlin, Germany: Springer, 1981, pp. 58–191.

[10] J. L. Deng, "Introduction to grey system theory," *J. Grey Syst.*, vol. 1, no. 1, pp. 1–24, 1989.

[11] K. T. Atanassov, "Intuitionistic fuzzy sets," *Fuzzy Sets Syst.*, vol. 20, no. 1, pp. 87–96, 1986.

[12] L. A. Zadeh, "Fuzzy sets," *Inf. Control*, vol. 8, no. 3, pp. 338–353, Jun. 1965.

[13] K. Atanassov, "Review and new results on intuitionistic fuzzy sets," *Preprint Im-MFAIS-1-88, Sofia*, vol. 5, no. 1, 1988.

[14] L. Fei, H. Wang, L. Chen, and Y. Deng, "A new vector valued similarity measure for intuitionistic fuzzy sets based on OWA operators," *Iranian J. Fuzzy Syst.*, vol. 16, no. 3, pp. 113–126, 2019.

[15] F. Xiao, "A distance measure for intuitionistic fuzzy sets and its application to pattern classification problems," *IEEE Trans. Syst., Man, Cybern. Syst.*, vol. 51, no. 6, pp. 3980–3992, Jun. 2019.

[16] L. A. Zadeh, "Linguistic variables, approximate reasoning and dispositions," *Med. Informat.*, vol. 8, no. 3, pp. 173–186, Jan. 1983.

[17] H. Bustince and P. Burillo, "Vague sets are intuitionistic fuzzy sets," *Fuzzy Sets Syst.*, vol. 79, no. 3, pp. 403–405, 1996.

[18] P. Grzegorzewski, "Distances between intuitionistic fuzzy sets and/or interval-valued fuzzy sets based on the Hausdorff metric," *Fuzzy Sets Syst.*, vol. 148, no. 2, pp. 319–328, 2004.

[19] A. Tiwari, Q. M. D. Lohani, and P. K. Muhuri, "Interval-valued intuitionistic fuzzy TOPSIS method for supplier selection problem," in *Proc. IEEE Int. Conf. Fuzzy Syst. (FUZZ-IEEE)*, Jul. 2020, pp. 1–8.

[20] A. Kaufmann and N. Magens, *Introduction to the Theory of Fuzzy Subsets: Fundamental Theoretical Elements*. Cambridge, MA, USA: Academic Press, 1975.

[21] M. Higashi and G. J. Klir, "On measures of fuzziness and fuzzy complements," *Int. J. Gen. Syst.*, vol. 8, no. 3, pp. 169–180, 1982.

[22] E. Szmidt and J. Kacprzyk, "Entropy for intuitionistic fuzzy sets," *Fuzzy Sets Syst.*, vol. 118, no. 3, pp. 467–477, 2001.

[23] R. Joshi and S. Kumar, "(R, S)-norm information measure and a relation between coding and questionnaire theory," *Open Syst. Inf. Dyn.*, vol. 23, no. 3, Sep. 2016, Art. no. 1650015.

[24] R. Joshi and S. Kumar, "An (R, S)-norm fuzzy information measure with its applications in multiple-attribute decision-making," *Comput. Appl. Math.*, vol. 37, no. 3, pp. 2943–2964, Jul. 2018.

[25] O. Parkash, P. Sharma, and R. Mahajan, "New measures of weighted fuzzy entropy and their applications for the study of maximum weighted fuzzy entropy principle," *Inf. Sci.*, vol. 178, no. 11, pp. 2389–2395, Jun. 2008.

[26] P. Burillo and H. Bustince, "Entropy on intuitionistic fuzzy sets and on interval-valued fuzzy sets," *Fuzzy Sets Syst.*, vol. 78, no. 3, pp. 305–316, 1996.

[27] I. K. Vlachos and G. D. Sergiadis, "Intuitionistic fuzzy information–applications to pattern recognition," *Pattern Recognit. Lett.*, vol. 28, pp. 197–206, Jan. 2007.

[28] A. De Luca and S. Termini, "A definition of a nonprobabilistic entropy in the setting of fuzzy sets theory," in *Readings in Fuzzy Sets for Intelligent Systems*. Amsterdam, The Netherlands: Elsevier, 1993, pp. 197–202.

[29] Q.-S. Zhang and S.-Y. Jiang, "A note on information entropy measures for vague sets and its applications," *Inf. Sci.*, vol. 178, no. 21, pp. 4184–4191, Nov. 2008.

[30] T.-Y. Chen and C.-H. Li, "Determining objective weights with intuitionistic fuzzy entropy measures: A comparative analysis," *Inf. Sci.*, vol. 180, no. 21, pp. 4207–4222, Nov. 2010.

[31] P. T. Harker and L. G. Vargas, "The theory of ratio scale estimation: Saaty's analytic hierarchy process," *Manag. Sci.*, vol. 33, no. 11, pp. 1383–1403, Nov. 1987.

[32] A. T. W. Chu, R. E. Kalaba, and K. Spingarn, "A comparison of two methods for determining the weights of belonging to fuzzy sets," *J. Optim. Theory Appl.*, vol. 27, no. 4, pp. 531–538, Apr. 1979.

[33] C.-L. Hwang and M.-J. Lin, *Group Decision Making Under Multiple Criteria: Methods and Applications*. Berlin, Germany: Springer, 2012, vol. 281.

[34] E. U. Choo and W. C. Wedley, "Optimal criterion weights in repetitive multicriteria decision-making," *J. Oper. Res. Soc.*, vol. 36, no. 11, pp. 983–992, Nov. 1985.

[35] J. Ma, Z.-P. Fan, and L.-H. Huang, "A subjective and objective integrated approach to determine attribute weights," *Eur. J. Oper. Res.*, vol. 112, no. 2, pp. 397–404, Jan. 1999.

[36] D. Golmohammadi and M. Mellat-Parast, "Developing a grey-based decision-making model for supplier selection," *Int. J. Prod. Econ.*, vol. 137, no. 2, pp. 191–200, Jun. 2012.

[37] C. Bai and J. Sarkis, "Integrating sustainability into supplier selection with grey system and rough set methodologies," *Int. J. Prod. Econ.*, vol. 124, no. 1, pp. 252–264, 2010.

[38] Z. Ayag and F. Samanlioglu, "Fuzzy AHP-GRA approach to evaluating energy sources: A case of Turkey," *Int. J. Energy Sector Manag.*, vol. 14, no. 1, pp. 40–58, Jan. 2020.

[39] A. Gumus, A. Yayla, E. Çelik, and A. Yildiz, "A combined fuzzy-AHP and fuzzy-GRA methodology for hydrogen energy storage method selection in Turkey," *Energies*, vol. 6, no. 6, pp. 3017–3032, Jun. 2013.

[40] D. Panchal and P. Srivastava, "Qualitative analysis of CNG dispensing system using fuzzy FMEA–GRA integrated approach," *Int. J. Syst. Assurance Eng. Manag.*, vol. 10, no. 1, pp. 44–56, Feb. 2019.

[41] M. S. Khan, A. Tiwari, and Q. M. D. Lohani, "Necessary and sufficient condition for the existence of Atanassov's intuitionistic fuzzy based additive definite integral," in *Proc. IEEE Int. Conf. Fuzzy Syst. (FUZZ-IEEE)*, Jul. 2021, pp. 1–6.
[42] M. S. Khan, Q. M. D. Lohani, and M. Mursaleen, "A novel intuitionistic fuzzy similarity measure based on double sequence by using modulus function with application in pattern recognition," *Cogent Math.*, vol. 4, no. 1, Jan. 2017, Art. no. 1385374.
[43] R. Solanki, Q. M. D. Lohani, and P. K. Muhuri, "A novel clustering algorithm based on a new similarity measure over intuitionistic fuzzy sets," in *Proc. IEEE Int. Conf. Fuzzy Syst. (FUZZ-IEEE)*, Aug. 2015, pp. 1–8.
[44] C. Jana and M. Pal, "A dynamical hybrid method to design decision making process based on GRA approach for multiple attributes problem," *Eng. Appl. Artif. Intell.*, vol. 100, Apr. 2021, Art. no. 104203.
[45] S. Ashraf, S. Abdullah, and T. Mahmood, "GRA method based on spherical linguistic fuzzy Choquet integral environment and its application in multi-attribute decision-making problems," *Math. Sci.*, vol. 12, no. 4, pp. 263–275, Dec. 2018.
[46] C. H. Chen, "A new multi-criteria assessment model combining GRA techniques with intuitionistic fuzzy entropy-based TOPSIS method for sustainable building materials supplier selection," *Sustainability*, vol. 11, no. 8, p. 2265, Apr. 2019.
[47] B. Roy, "The optimisation problem formulation: Criticism and overstepping," *J. Oper. Res. Soc.*, vol. 32, no. 6, pp. 427–436, Jun. 1981.
[48] A. Tiwari, Q. M. D. Lohani, and P. K. Muhuri, "Intuitionistic fuzzy grey relational analysis sorting technique," in *Proc. IEEE Int. Conf. Fuzzy Syst. (FUZZ-IEEE)*, Jul. 2022, pp. 1–7.
[49] L. Fei, Y. Deng, and Y. Hu, "DS-VIKOR: A new multi-criteria decision-making method for supplier selection," *Int. J. Fuzzy Syst.*, vol. 21, no. 1, pp. 157–175, 2019.
[50] A. C. S. M. Campos, B. Mareschal, and A. T. de Almeida, "Fuzzy FlowSort: An integration of the FlowSort method and fuzzy set theory for decision making on the basis of inaccurate quantitative data," *Inf. Sci.*, vol. 293, pp. 115–124, Feb. 2015.
[51] A. Ishizaka, C. Pearman, and P. Nemery, "AHPSort: An AHP-based method for sorting problems," *Int. J. Prod. Res.*, vol. 50, no. 17, pp. 4767–4784, 2012.
[52] F. Miccoli and A. Ishizaka, "Sorting municipalities in Umbria according to the risk of wolf attacks with AHPSort II," *Ecol. Indicators*, vol. 73, pp. 741–755, Feb. 2017.
[53] V. Mousseau, R. Slowinski, and P. Zielniewicz, "A user-oriented implementation of the ELECTRE-TRI method integrating preference elicitation support," *Comput. Oper. Res.*, vol. 27, nos. 7–8, pp. 757–777, Jun. 2000.
[54] H. F. Sabokbar, A. Hosseini, A. Banaitis, and N. Banaitiene, "A novel sorting method TOPSIS-SORT: An applicaiton for Tehran environmental quality evaluation," *E & M Ekonomie a Manage.*, vol. 19, no. 2, pp. 87–104, 2016.
[55] D. F. D. L. Silva and A. T. D. A. Filho, "Sorting with TOPSIS through boundary and characteristic profiles," *Comput. Ind. Eng.*, vol. 141, Mar. 2020, Art. no. 106328.
[56] G. R. Jahanshahloo, F. H. Lotfi, and M. Izadikhah, "Extension of the TOPSIS method for decision-making problems with fuzzy data," *Appl. Math. Comput.*, vol. 181, no. 2, pp. 1544–1551, Oct. 2006.
[57] C.-T. Chen, "Extensions of the TOPSIS for group decision-making under fuzzy environment," *Fuzzy Sets Syst.*, vol. 114, no. 1, pp. 1–9, Aug. 2000.
[58] S.-H. Tsaur, T.-Y. Chang, and C.-H. Yen, "The evaluation of airline service quality by fuzzy MCDM," *Tourism Manag.*, vol. 23, no. 2, pp. 107–115, Apr. 2002.
[59] E. F. Boran, S. Genç, M. Kurt, and D. Akay, "A multi-criteria intuitionistic fuzzy group decision making for supplier selection with TOPSIS method," *Exp. Syst. Appl.*, vol. 36, no. 8, pp. 11363–11368, 2009.
[60] B. D. Rouyendegh, "The DEA and intuitionistic fuzzy TOPSIS approach to departments' performances: A pilot study," *J. Appl. Math.*, vol. 2011, pp. 1–16, Jan. 2011.
[61] F. Ye, "An extended TOPSIS method with interval-valued intuitionistic fuzzy numbers for virtual enterprise partner selection," *Exp. Syst. Appl.*, vol. 37, no. 10, pp. 7050–7055, Oct. 2010.
[62] S. Mijalkovski, D. Peltechki, K. Zeqiri, J. Kortnik, and D. Mirakovski, "Risk assessment at workplace in underground lead and zinc mine with application of fuzzy topsis method," *J. Inst. Electron. Comput.*, vol. 2, no. 1, pp. 121–141, 2020.
[63] H. Tang, Y. Shi, and P. Dong, "Public blockchain evaluation using entropy and TOPSIS," *Exp. Syst. Appl.*, vol. 117, pp. 204–210, Mar. 2019.
[64] K. Palczewski and W. Sałabun, "The fuzzy TOPSIS applications in the last decade," *Proc. Comput. Sci.*, vol. 159, pp. 2294–2303, 2019.
[65] Z. Xu, "Intuitionistic fuzzy aggregation operators," *IEEE Trans. Fuzzy Syst.*, vol. 15, no. 6, pp. 1179–1187, Dec. 2007.
[66] W. Wang and X. Xin, "Distance measure between intuitionistic fuzzy sets," *Pattern Recognit. Lett.*, vol. 26, no. 13, pp. 2063–2069, 2005.
[67] E. Szmidt and J. Kacprzyk, "Distances between intuitionistic fuzzy sets," *Fuzzy Sets Syst.*, vol. 114, no. 3, pp. 505–518, 2000.
[68] M. Kaushal, R. Solanki, Q. M. D. Lohani, and P. K. Muhuri, "A novel intuitionistic fuzzy set generator with application to clustering," in *Proc. IEEE Int. Conf. Fuzzy Syst. (FUZZ-IEEE)*, Jul. 2018, pp. 1–8.
[69] R. Solanki, G. Gulati, A. Tiwari, and Q. M. D. Lohani, "A correlation based intuitionistic fuzzy TOPSIS method on supplier selection problem," in *Proc. IEEE Int. Conf. Fuzzy Syst. (FUZZ-IEEE)*, Jul. 2016, pp. 2106–2112.
[70] K. T. Atanassov, *Intuitionistic Fuzzy Sets*. Berlin, Germany: Springer, 1999, pp. 1–137.
[71] J. Hou, "Grey relational analysis method for multiple attribute decision making in intuitionistic fuzzy setting," *J. Converg. Inf. Technol.*, vol. 5, no. 10, pp. 194–199, Dec. 2010.
[72] E. Szmidt and J. Kacprzyk, "Using intuitionistic fuzzy sets in group decision making," *Control Cybern.*, vol. 31, no. 4, pp. 1055–1057, 2002.
[73] K. Yamagishi and L. Ocampo, "Utilizing TOPSIS-sort for sorting tourist sites for perceived COVID-19 exposure," *Current Issues Tourism*, vol. 25, no. 2, pp. 168–178, Jan. 2022.
[74] T. Baum and N. T. T. Hai, "Hospitality, tourism, human rights and the impact of COVID-19," *Int. J. Contemp. Hospitality Manag.*, vol. 32, no. 7, pp. 2397–2407, May 2020.
[75] M. Sigala, "Tourism and COVID-19: Impacts and implications for advancing and resetting industry and research," *J. Bus. Res.*, vol. 117, pp. 312–321, Sep. 2020.
[76] J. Wen, C. C. Wang, and M. Kozak, "Post-COVID-19 Chinese domestic tourism market recovery: Potential influence of traditional Chinese medicine on tourist behaviour," *Anatolia*, vol. 32, no. 1, pp. 121–125, Jan. 2021.

ASHUTOSH TIWARI (Graduate Student Member, IEEE) received the master's degree in mathematics from the Department of Mathematics, South Asian University, New Delhi, India, in 2016, where he is currently pursuing the Ph.D. degree in mathematics. He made an international visit to Padua, Italy, to present his paper in FUZZ IEEE 2022, which was funded by South Asian University. He received the IEEE-CIS Grant for the conference paper presentation. He has published six papers in reputed journals/conferences. His current research interests include intuitionistic fuzzy sets, decision making, and ranking techniques. He is a Student Member of the IEEE Computational Intelligence Society.

Q. M. DANISH LOHANI (Senior Member, IEEE) received the Ph.D. degree from the Department of Mathematics, Aligarh Muslim University, India, in 2009. He is currently working as an Associate Professor with the Department of Mathematics, South Asian University, India. He has published more than 50 scientific articles in well-known journals, including IEEE TRANSACTIONS ON FUZZY SYSTEMS, *Applied Soft Computing*, and *Chaos, Solitons and Fractals*; and reputed conferences, such as Fuzz-IEEE, IEEE-CEC, and IJCNN. His current research interests include study of both theoretical and application aspects of fuzzy, intuitionistic fuzzy, type-2 fuzzy sets, operators, and summability theory. He is a member of the IEEE Computational Intelligence Society and the IEEE Computer Society.