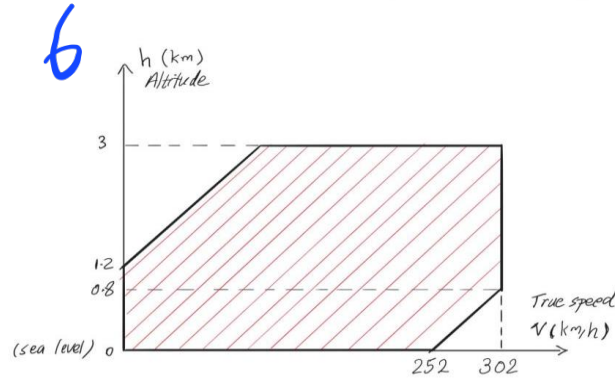


## BACKGROUND

Figure 1 shows the flight envelope (i.e. shaded region) for a general aviation airplane being designed by a Canadian company for operation in Canada and the United States. The airplane has a maximum take-off weight of 2200 lbs, it is equipped with a 145 hp piston engine and is expected to have 4 hours of endurance. The service ceiling for the airplane is 3 km, and the maximum flight speed at sea level is 252 km/h, and it is 302 km/h at  $h = 3$  km. The empty mass of the entire wing (excluding fuel tanks) in kg may be approximated by  $m_e = 26.91S$ , where  $S$  is the wing surface area in  $m^2$  (Anderson, 1999). Assume that the wing has a rectangular planform. The wing has 5 different variants, each assigned to a team; please see Table 1.



**Figure 1:** Flight envelope for a general aviation airplane

**Table 1:** Information on different variants of the wing.

Team #	Wing area $S$ ( $m^2$ )	Aspect ratio $AR$
1	15.0	7.10
2	15.2	7.15
3	15.4	7.20
4	15.6	7.25
5	15.8	7.30
6	16.0	7.35

## PART 1

Considering either the left or the right wing of the airplane, the mass moment of inertia about the elastic axis and the chordwise location of the center of mass (measured from the leading edge) change from  $I_p = 7 \text{ kg.m}^2$  and  $0.35c$  at the maximum take-off weight to  $I_p = 4 \text{ kg.m}^2$  and  $0.45c$  at the empty weight, respectively;  $c$  being the chord length. Nevertheless, the chordwise location of the elastic axis remains unchanged at  $0.4c$ . Each wing carries 80 kg of fuel. The wing has a bending rigidity of  $\overline{EI} = 2 \times 10^5 \text{ N.m}^2$  and a torsional rigidity of  $\overline{GJ} = 10^5 \text{ N.m}^2$ .

Considering the flight envelope shown in Figure 1, verify if the current design meets the certification requirements set by the aviation regulatory bodies. If there is no specific advisory on the flutter margin, take it 15% as mentioned in (Hodges and Pierce, 2011, page 220).

If the current design does not meet the requirements, propose reasonable aeroelastic design modifications, and for one of the proposed modifications, through a flutter analysis, show that it leads to an acceptable design. Discuss about the viability (pros and cons) of your design modifications.

Discuss the environmental impacts of the proposed design modifications. Identify data and

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tools you need to measure impacts of your design on the environment.

tools you need to measure impacts of your design on the environment.

## PART 2

The company is also considering the possibility of making an all-electric version of the airplane. The all-electric version, if successful, will make up to 50% of the company's 400-airplane fleet.

Identify the required design changes, and, if possible, through a simplified aeroelastic analysis, show that the all-electric version meets the certification requirements.

In addition, discuss the short-term (1 year) and long-term (10 years) environmental and economical impacts if the all-electric version of the airplane is successful. Support your discussions with some calculations.

## MODEL VALIDATION

Prior to performing the above studies, verify your computer programs and methods by comparing the numerical results against published results. Ideally, the numerical results from a theoretical model are validated against experimental results. This is something you do during your labs. Here, however, use the example shown in the textbook, where the dimensionless parameters are:  $a = -1/5$ ,  $e = -1/10$ ,  $\mu = 20$ ,  $r^2 = 6/25$ , and  $\sigma = 2/5$ . Find the variation of the dimensionless modal frequency and damping (i.e. for both pitch and plunge degrees-of-freedom) as a function of the reduced velocity. See (Hodges and Pierce, 2011, Figures 5.3 and 5.4) for the  $p$ -method plots and (Hodges and Pierce, 2011, Figures 5.20 and 5.21) for the  $p - k$ -method plots.

## DELIVERABLE

A report (20 pages, maximum) has to be submitted by each student, along with all computer programs developed. The team captains are responsible to submit the files on the Moodle. Use Elsevier's template in either [MS Word](#) or [L<sup>A</sup>T<sub>E</sub>X](#). The report should include the following sections:

1. Abstract (2%)
2. Brief theoretical background (3%)
3. Model validation results which include comparison of numerical values, plots and discussion (10%)
4. Part 1 (40%)
  - 4.1 Problem statement
  - 4.2 Summary of the relevant design requirements and constraints
  - 4.3 Flutter analysis for the original design, including design inputs, calculations, justification of the flutter analysis method, plots used to identify flutter speed, and discussions
  - 4.4 List of design modifications (if needed) and discussion of pros and cons
  - 4.5 Flutter analysis for the modified design (if needed)
  - 4.6 Discussion of the environmental impacts and identification of required data and tools to measure those impacts
5. Part 2 (35%)
  - 5.1 Problem statement
  - 5.2 Summary of the relevant design requirements and constraints
  - 5.2 List of design modifications
  - 5.3 Flutter analysis (see 4.3 for the parts to be included)
  - 5.4 Discussion of environmental and economical impacts
6. Conclusions (5%)
7. Statement of contributions (explain the tasks done by each member) (3%)
8. References (2%)



## BONUS

You will receive a 25% bonus, if in addition to the flutter analysis based on the typical section model, you also perform flutter analysis based on the uniform flexible wing model.

## REFERENCES

- J. D. Anderson, Aircraft performance & design, McGraw-Hill Science Engineering, 1999.
- D. H. Hodges, G. A. Pierce, Introduction to Structural Dynamics and Aeroelasticity, 2nd ed., Cambridge University Press, 2011.

## APPENDIX

You may approximate the dynamics of a uniform flexible wing, which in principle has an infinite number of degrees-of-freedom (DOFs), using a two-DOF (i.e. pitching and plunging) typical section model. This will allow you to perform a much simpler flutter analysis. Use the following expressions to approximate the uncoupled, circular natural frequencies for plunge and pitch DOFs (Hodges and Pierce, 2011):

$$\omega_h = (1.8751)^2 \sqrt{\frac{EI}{m\ell^3}}, \quad \omega_\theta = \frac{\pi}{2} \sqrt{\frac{GJ}{I_p \ell}}. \quad (1)$$

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The above expressions have been obtained by assuming that the first natural frequency of bending dynamics of the wing is equal to the natural frequency of plunging of the typical section and that the first natural frequency of torsional dynamics of the wing is equal to the natural frequency of pitching of the typical section.