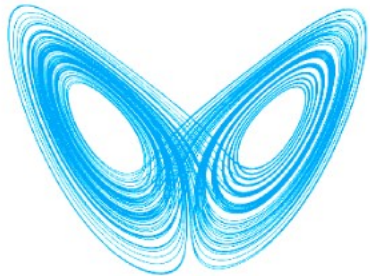
The set of equations has three stationary solutions

The approach to these solutions from an arbitrary starting point are dependent on the regime of **ρ**.

- If **ρ >1** the only real stationary point is the origin.
- If **ρ >1** all three stationary points exist. However, the origin is unstable and so you will never reach it unless you start exactly on it.
- As **ρ** increases, there is a higher degree of uncertainty in the system.
- If **ρ<1** is less than about 24 the system will converge to one of the two stationary points not found at the origin.
- If **ρ >1** is greater than about 24, the system will not converge to either solution and will move between the two fixed points in space. The behavior is bounded but non-periodic.

The Butterfly Effect

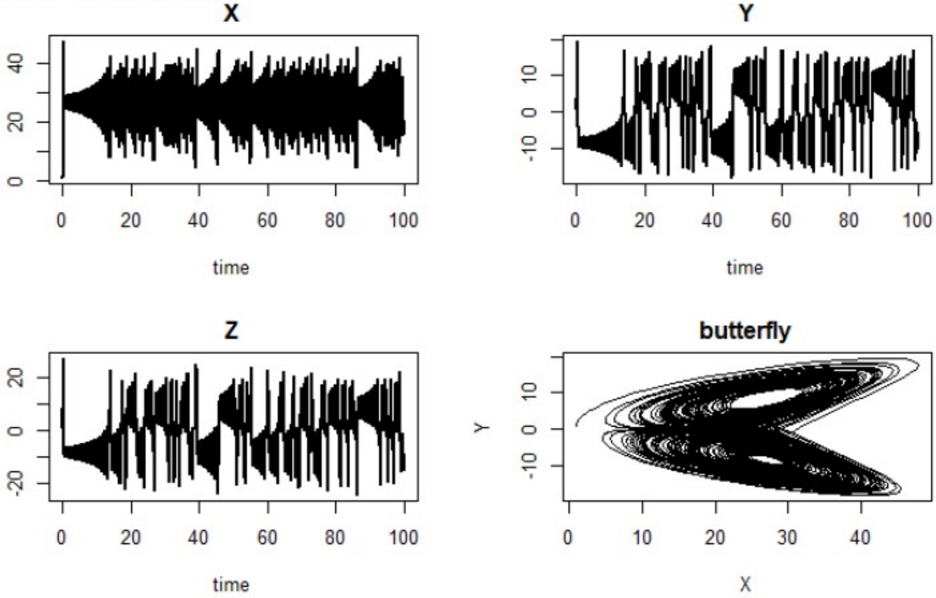
- The solution of Lorenz's equations resembles a butterfly.



- The Butterfly Effect means that two identical weather systems could produce two very different weather systems in the near future. A butterfly flapping its wings could alter the atmosphere.

R Plotting

beta <- -8/3; sigma <- -10; row <- 28



Runge-Kutta Fourth Order (RK4) Method

- We previously discussed the 4th order Runge Kutta method as a simple method to solve initial value problems. The RK4 method provides the approximate value of y for a given point x
- It computes next value y_{n+1} using current y_n plus weighted average of four increments
- $y_1 = y_0 + (1/6)(k_1 + 2k_2 + 2k_3 + k_4)$

Where,

$k_1 = hf(x_0, y_0)$

$k_2 = hf[x_0 + (1/2)h, y_0 + (1/2)k_1]$

$k_3 = hf[x_0 + (1/2)h, y_0 + (1/2)k_2]$

$k_4 = hf(x_0 + h, y_0 + k_3)$

k_1 is the increment based on the slope at the beginning of the interval using y
k_2 -- at the midpoint of the interval using $y + hk_1/2$
k_3 -- at the midpoint using $y + hk_2/2$
k_4 -- at the end of the interval using $y + hk_3$

The Lorenz Equations

$$\dot{x} = \sigma(y - x)$$
$$\dot{y} = x(\rho - z) - y$$
$$\dot{z} = xy - \beta z$$

The Lorenz equations are an example of a three-dimensional ODE system. These equations are a simplified description of thermal convection in the atmosphere.

where σ, ρ and $\beta > 0$ are parameters they depend on conditions like the fluid, the heat input, the size of the pot, etc.

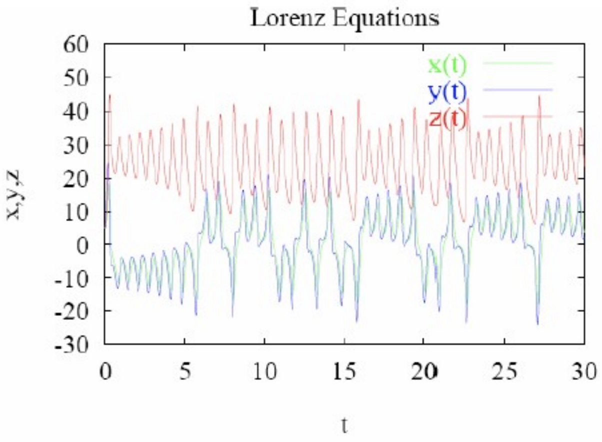
As per Lorenz, there are three quantities that characterize the state of the fluid:

x: the rate of convective motion i.e. how fast the rolls are rotating,

y: the temperature difference between the ascending and descending currents, and

z: the temperature difference between the top and the bottom of the fluid.

The set of equations has three stationary solutions



$\sigma = 10,\ b = \frac{8}{3},\ r = 28$; initial values: $(x, y, z) = (5, 5, 5)$

$$\frac{dx}{dt} = 0 \quad \text{and} \quad \frac{dy}{dt} = 0 \quad \text{and} \quad \frac{dz}{dt} = 0 \ \ ?$$

$$x = y = \pm\sqrt{\beta(\rho - 1)}$$

$$z = (\rho - 1)$$

$$x = y = z = 0$$