Is a 'Free Lunch' a Good Lunch? The Performance of Zero Wholesale Price–Based Supply-Chain Contracts[1]

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Abstract: In the real world, some manufacturers supply their products to retailers at a zero wholesale price (ZWP), and receive compensation by sharing the revenue or receiving some side payment from the retailers. In this paper, we analyse this kind of ZWP-based supply-chain contract. In the basic model, ordering is done either by the manufacturer or the retailer. For both of these cases, we explore ZWP-revenue-sharing (ZR) contracts, ZWP-side-payment (ZS) contracts, and ZWP-revenue-sharing-plus-side-payment (ZRS) contracts. We prove that, irrespective of the ordering scenario being retailer-led or manufacturer-led, only a ZRS contract can achieve win-win coordination. In the extended models, we first study the scenario with multiple products and discuss how a generalised ZRS contract can coordinate the supply chain efficiently. We then investigate greedy wholesale price (GWP)–based contracts, in which the manufacturer charges the retailer a wholesale price equal to the retail price. We find that a GWP-based revenue-sharing-plus-side-payment (GRS) contract and a ZRS contract can both achieve win-win coordination. However, the ZRS contract mean-variance dominates the GRS contract in bringing a lower level of risk to both the retailer and manufacturer if the unit production cost is sufficiently small. We further discuss cases of differences in the perception of ZWP versus non-ZWP contracts. Important managerial insights are derived from the findings.

***Keywords*:** Supply chain contracts; Nash bargaining; Multiple products; Supply chain management; Coordination.

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1. Introduction

1.1 Background and Motivation

Supply chain contracting is a well-established area in the operations management (OM) literature (Mobini et al. 2019; Niederhoff and Kouvelis 2019). The wholesale-pricing contract is the most common of the many forms of supply chain contracts. In recent years there has been an interesting emergence and discussion of zero wholesale price (ZWP)–based contracts for perishable and short-life products (Qiu and Xu 2015; Zhao and Zhu 2017). ZWP contracts are commonly used by manufacturers that are not yet well-established (e.g., new entrepreneurs and start-ups). For example, many start-up designer labels in the fashion industry provide products to well-established department stores on a ZWP basis, receiving a share of the revenue from sales. In the agricultural industry, some organic farmers supply their fresh organic agricultural products to supermarkets on a ZWP basis and receive compensation in the forms of revenue-sharing and a certain fixed payment. In addition to these two widely observed scenarios, ZWP contracts are also found in the electricity market[4].

ZWP-based contracts are attractive to retailers as the low cost of obtaining the products reduces the risks associated with purchasing. However, a series of questions remain in regard to these types of supply chain contracts, which we attempt to answer here, namely: Does a ZWP-based contract outperform a traditional supply chain contract, such as the non-zero wholesale pricing (NZWP) revenue sharing contract? Can a ZWP-based contract achieve win-win supply chain coordination? If so, which is the most efficient contractual format in the presence of multiple products? Does a ZWP-based contract outperform its opposite, a greedy wholesale price (GWP) contract in which the wholesale price is set equal to the retail price? What are the implications of a difference in the perception of ZWP versus non-ZWP contracts?

[4] https://www.bloomberg.com/news/articles/2018-08-06/negative-prices-in-power-market-as-wind-solar-cut-electricity (accessed 14 May 2019)

We pay specific attention to the following areas in exploring ZWP-based contracts: (1) Simplicity, which is especially important when the contract involves multiple products. (2) Coordination ability, which measures whether specific kinds of ZWP-based supply chain contracts can improve the performance of the supply chain through coordination. (3) Risk, which investigates whether ZWP-based contracts will bring a higher or lower level of risk to the retailer and the manufacturer. It is sometimes claimed that with a ZWP the retailer has no risk to bear, but this is not accurate; the retailer incurs costs in the handling, care and storage of these products . (4) Differences in perception, which refer to certain psychological effects that have been reported in relation to zero prices; these perceptions are critical but under-explored in the supply chain contracting literature.

A similar and related type of supply chain contract is a consignment contract. However, ZWP-based supply chain contracts and consignment contracts are somewhat different. For instance, under consignment the manufacturer retains ownership of the product and usually controls a number of parameters, such as the selling location and even the retail price. Under ZWP-based supply chain contracts, the manufacturer does not maintain ownership of the product once it is supplied to the retailer. As with classic and standard wholesale pricing contracts in newsvendor supply chains, under ZWP-based contracts the manufacturer only controls the contract parameters, such as the revenue sharing rate and/or the side payment (with the wholesale price set to be zero), if the retailer decides the quantity; if the manufacturer controls the quantity, the retailer usually determines the contract parameters. Thus, a ZWP-based supply chain contract is not equivalent to a consignment contract.

In this paper we explore ZWP-based supply chain contracts by building an analytical supply chain model. In the basic model, we consider the case where ordering is decided by the retailer and the case where ordering decisions are made by the manufacturer. For each of these cases, we investigate the ZWP-revenue-sharing (ZR) contract, the ZWP-side-payment (ZS) contract, and the ZWP-revenue-sharing-plus-side-payment (ZRS) contract. We analytically prove that, irrespective of the specific ordering scenario, only a ZRS contract (but not a ZR or ZS contract) can achieve win-win coordination, in which the supply chain is optimised and both the manufacturer and the retailer benefit from the implementation of the contract. In the extended models, we first analytically explore the case of multiple products and reveal how a generalised ZRS contract can assist to efficiently coordinate the supply chain. Second, we study GWP-based contracts, which are the opposite of ZWP-base contracts in that the manufacturer charges the retailer a wholesale price equal to the retail price. We show that GWP-based revenue-sharing-plus-side-payment (GRS) contracts and ZRS contracts can both achieve win-win coordination. However, the ZRS contract mean-variance (MV) dominates the GRS contract in bringing a lower level of risk for both the retailer and manufacturer when the unit production cost is sufficiently small. We further discuss psychological factors, exploring the cases of differences in perception between ZWP and non-ZWP contracts, and reveal why the ZWP based supply chain contracts may find a niche application. We further discuss the managerial implications.

1.2 Contribution Statement[5] and Paper's Structure

To the best of our knowledge, this is the first paper to explicitly and comprehensively explore ZWP-based supply chain contracts. The findings uncover the significance of ZWP contracts and highlight how an efficient ZRS contract can coordinate a supply chain with multiple products, "mean-variance (MV)-dominate" the GRS contract, and make a large psychological difference to perceptions. The results are robust, as we have examined a range of extended cases (e.g., different channel leadership, different numbers of products,

[5] Note that this paper's approach follows the standard 'closed-form analytical operational analyses' that are commonly seen in supply chain operations management literature (e.g., see the following most recently published *EJOR* papers (Guo et al. (2020); Zennyo (2020); Zhang and Zhang (2020); Zhao et al. (2020)). As the goal is to obtain theoretically proven closed-form results, the models are not overly complex and can be solved analytically. This is different from the mathematical programming-based operational research studies.

different balances of bargaining power, with/without psychological perception differences). The findings contribute to the supply-chain contracting literature while also helping to advance practice in the use of ZWP-based supply chain contracts. We also explain why ZWP contracts are useful and can even outperform GRS contracts.

The rest of this paper is arranged as follows. Section 2 presents a concise literature review of related studies. Section 3 introduces the basic model and some preliminaries. Section 4 explores the win-win coordination under different channel leadership. Section 5 extends the basic model analysis by considering several important cases, involving multiple products, GRS contracts, and psychological-perception differences. Section 6 concludes the paper with a discussion of the managerial implications, and ideas for future research.

2. Literature Review

This paper contributes to the supply chain management literature on supply-chain contracting and coordination. We examine several important areas within this well-established research area. A number of different types of supply chain contract, such as revenue sharing contracts (Cachon and Lariviere 2005; Becker-Peth and Thonemann 2016; Niederhoff and Kouvelis 2019), side-payment and two-part tariff contracts (Leng and Zhu 2009; Dong et al. 2017; Mobini et al. 2019), consignment contracts[6] (Zhang et al. 2010; De Giovanni et al. 2019; Lu et al. 2019), options contracts (Zhao et al. 2018; Zhuo et al. 2018), buyback contracts (Choi et al. 2008), and wholesale pricing contracts (Lu et al. 2019), are explored extensively in the literature. Under the category of the revenue-sharing contract, for instance, Cachon and Lariviere (2005) comprehensively examine its strengths and weaknesses for different supply-chain configurations; Becker-Peth and Thonemann (2016) analyse the application of revenue-sharing contracts by addressing the influences of

[6] As mentioned in Section 1, although ZWP-based contracts appear similar to consignment contracts, there are fundamental differences between the two.

behavioural aspects associated with reference-dependent valuation in the inventory-ordering process; and Niederhoff and Kouvelis (2019) explore the conditions for improving the supply-chain system by implementing a revenue-sharing contract, finding that coordination by a revenue-sharing contract may not be optimal when decision makers (e.g., the suppliers) emphasise fairness concerns and are strongly risk averse. Within the literature on the supply-chain coordination of side-payment contracts, Leng and Zhu (2009) apply both the Cournot and Bertrand games to explore the design of side-payment schemes for achieving channel coordination, demonstrating the high performance of side-payment contracts in solving the forward-buying problem in a two-echelon single-supplier single-retailer supply chain. Mobini et al. (2019) study asymmetric information in the supply chain and discuss the supplier's optimal side-payment contract design for influencing the retailer's ordering decision, showing that small local-incentive constraints can be helpful in improving a side-payment contract's performance. For consignment contracts, Zhang et al. (2010) examine the consignment coordination between the supplier and the retailer by comparing the performances of different contractual schemes; De Giovanni et al. (2019) analyse the suitability of consignment contracts for coordinating a vendor managed inventory–based supply chain system, finding that firms can improve the performance of a consignment contract under such a system by adjusting the revenue sharing rate; and Lu et al. (2019) compare the effectiveness of consignment contracts with traditional wholesale-price contracts in a consideration of dynamic advertising.

It is common knowledge in the field that when incentive-alignment contracts are in use, the presence of the double marginalisation effect (Chen et al. 2010; Li et al. 2013) means that the supply chain will not be automatically optimal (i.e. coordinated). In standard newsvendor product–based supply chains with only a quantity decision, such supply-chain contracts as revenue-sharing, buyback, two-part tariff, and options can be successful in enticing risk-neutral supply-chain agents to produce and order at the supply chain's optimal quantity, and arbitrarily allocate the optimal supply-chain profit among the seller and the buyer. These kinds of contracts are therefore called supply-chain coordination contracts. However, pure wholesale-pricing contracts cannot coordinate the supply chain when the agents are risk-neutral and will not accept a situation of zero profit. In this paper we follow this investigative stream of the literature, while focusing on the ZWP-based contract as a particular type of supply-chain contract.

In the literature, the use of ZWP has been touched upon in a number of papers. For example, Lan et al. (2013) mention ZWP in explaining why a full-refund returns policy makes sense; Qiu and Xu (2015) propose ZWP as a way for a retailer to pass the supplier's granted discount on to consumers; and Wang and Shin (2015) explore the role of the ZWP in supply-side innovations. Zhao and Zhu (2017) also mention the presence of a ZWP in the context of reverse-supply chains. In each of these studies, ZWP is mentioned as special case of supply-chain contracting, but it is not a main focus. In particular, the supply-chain coordination capabilities of ZWP are not highlighted; however, this is the core topic of this paper. Furthermore, we also consider the special psychological effects wrought by zero pricing; to the best of our knowledge, these have never been examined in the supply-chain contracting literature.

We also examine the supply-chain coordination problem under different channel-leadership scenarios. This involves two sub-areas: push and pull contracts (Cachon 2004; Granot and Yin 2008), and the role played by different channel leaders (e.g., Choi et al. 2013). We examine the separate cases of the quantity decision being made by the retailer and by the manufacturer. More details regarding push and pull contracts can be found in the recent studies of Wang et al. (2014) and Yang et al. (2018).

Our extended model also explores the supply-chain coordination challenge when multiple products are involved. Coordinating a supply chain with multiple products can be viewed as a simple extension of the single product case, but with the key being to determine how to derive an efficient contract that is simple to implement. In the recent literature, Shen et al. (2019) examine the pricing-coordination issue in the context of a two-product supply chain with a linear price-dependent demand model. In this paper, we also discuss supply chain coordination with multiple products while generalising it in an $N(>2)$ product supply chain.

3. Basic Model: Preliminaries

In the basic model, following the standard literature on supply-chain coordination (e.g., Li and Liu 2019, Sainathan and Groenevelt 2019), we consider a supply chain for a newsvendor type of product. It has two members, namely the manufacturer (he) and the retailer (she). For the manufacturer, the unit production cost is m. Under a ZWP-based contact, the wholesale price is set to zero with the manufacturer compensated by other means. For the retailer, the unit retail price is p, and there is a per unit operations cost (e.g., manpower) ξ incurred for each unit of the product kept during the selling season. At the end of the selling season, each leftover unit of product incurs a net cost of h. The product quantity is denoted by q. These are quite standard settings for such a model. Note that in one of the extended models, in accordance with the results of relevant empirical studies, we consider the case of ZWP-based contracts having different psychological effects on the retailer and hence potentially giving rise to an additional benefit (see Section 5.3.2). Demand over the selling season, denoted by x, is uncertain and follows a density function $f(\cdot)$ and a cumulative distribution function $F(\cdot)$.

We consider three types of ZWP-based contracts in the basic model. First, in ZR contracts, the retailer shares a proportion of its revenue, α, for each product sold to the manufacturer, and keeps $1-\alpha$ proportion to herself. Notice that the revenue-sharing contract is widely adopted in real-world practices for channel coordination, with one famous instance being the arrangement between Blockbuster (a video rental store in the United States) and the Hollywood studios (Cachon and Lariviere 2005). Under this specific revenue-sharing contract, Blockbuster incentivises the Hollywood studios to reduce their wholesale prices by agreeing to transfer a part of her sale revenue at the end of the selling season. Another common application of the revenue sharing contract is to prevent damages within a supply chain involving perishable products (e.g., food supply chains), which is becoming increasingly popular in recent years (Leng and Zhu 2009). In this case, the manufacturers will quantify the level of damage and use this to determine their reimbursement rate for the retailers. Given the popularity and the significance of the revenue-sharing contract in practice, the ZR contract is explored in this paper.

Second, in ZS contracts, the retailer gives a side-payment T to the manufacturer per season, where T is independent of the order quantity. Such constant side-payment contracts are supported by existing supply chain coordination research, such as that of Leng and Zhu (2009) and other two-part tariff studies.

Third, by combining the ZR contract and the ZS contract we arrive at the ZRS contract type. Under a ZRS contract, the retailer offers a side-payment T to the manufacturer in addition to sharing with the manufacturer a proportion of revenue, α, for each product sold, keeping $1-\alpha$ to herself.

Under a ZR contract, the retailer's profit function and the manufacturer's profit are given as follows:

$$\pi_R^{ZR} = (1-\alpha)p\min(q,x) - h\max(q-x,0) - \xi q, \tag{3.1}$$

$$\pi_M^{ZR} = \alpha p\min(q,x) - mq. \tag{3.2}$$

From (3.1) and (3.2), it is easy to find the expected profits of the retailer and the manufacturer under a ZR contract as follows:

$$E[\pi_R^{ZR}] = [(1-\alpha)p - \xi]q - [(1-\alpha)p + h]\int_0^q F(x)dx, \tag{3.3}$$

$$E[\pi_M^{ZR}] = (\alpha p - m)q - \alpha p\int_0^q F(x)dx. \tag{3.4}$$

From (3.3) and (3.4), we can find the expected profit function for the whole supply chain:

$$E[\pi_{SC}] = (p - \xi - m)q - (p+h)\int_0^q F(x)dx. \tag{3.5}$$

Note that the supply chain's expected profit function (3.5) is contract-type-independent (across ZR, ZS, and ZRS contracts) and therefore we can remove the superscript indicating a ZR contract.

Similarly, we can express the profit and expected profit functions for the retailer and the manufacturer under a ZS contract as follows:

$$\pi_R^{ZS} = p\min(q,x) - h\max(q-x,0) - \xi q - T, \tag{3.6}$$

$$E[\pi_R^{ZS}] = (p-\xi)q - (p+h)\int_0^q F(x)dx, \tag{3.7}$$

$$E[\pi_M^{ZS}] = \pi_M^{ZS} = T - mq. \tag{3.8}$$

Combining the results derived above, it is also straightforward to find the expected profits for the retailer and the manufacturer under a ZRS contract, as follows:

$$E[\pi_R^{ZRS}] = [(1-\alpha)p - \xi]q - [(1-\alpha)p + h]\int_0^q F(x)dx - T, \tag{3.9}$$

$$E[\pi_M^{ZRS}] = (\alpha p - m)q - \alpha p\int_0^q F(x)dx + T. \tag{3.10}$$

Define $q_k^{l*} = \arg\max_q E[\pi_k^l]$, where $l \in (ZR, ZS, ZRS)$ represents the contract type and $k \in (R, M)$ represents whether the optimal quantity is determined from the perspective of the retailer (R) or the manufacturer (M). The optimal quantities can then be derived as follows (see Appendix): (a) $q_R^{ZR*} = F^{-1}\left(\frac{(1-\alpha)p-\xi}{(1-\alpha)p+h}\right)$ and $q_M^{ZR*} = F^{-1}\left(\frac{\alpha p - m}{\alpha p}\right)$; (b) $q_R^{ZS*} = F^{-1}\left(\frac{p-\xi}{p+h}\right)$ and $q_M^{ZS*} = 0$; (c) $q_R^{ZRS*} = F^{-1}\left(\frac{(1-\alpha)p-\xi}{(1-\alpha)p+h}\right)$ and $q_M^{ZRS*} = F^{-1}\left(\frac{\alpha p - m}{\alpha p}\right)$; (d) $q_{SC}^* = F^{-1}(\beta_{SC})$, where $\beta_{SC} = \frac{p-\xi-m}{p+h}$.

A few findings emerge from these optimal quantity expressions. First, the optimal quantities under ZR and ZRS contracts are the same for the retailer-decide (RD) and manufacturer-decide (MD) scenarios. For ZS contracts, the side-payment is independent of the quantity and thus has no effect on the optimal quantity. As the wholesale price is zero, we have $q_M^{ZS*} = 0$ and $q_R^{ZS*} = F^{-1}\left(\frac{p-\xi}{p+h}\right)$. Finally, for the supply chain, the optimal quantity is independent of the contract type and is given by $q_{SC}^* = F^{-1}(\beta_{SC})$.

4. Win-Win Coordination

With the model and results derived in Section 3, we now proceed to explore the channel coordination issue. In the following, we first define two concepts: quantity coordination and win-win coordination.

Definition 1 (quantity coordination). *In the supply chain, if the optimal order quantity determined by the supply chain members under a decentralised setting is equal to the optimal product quantity for the whole supply chain, quantity coordination is achieved.*

Definition 2 (win-win coordination). *In the supply chain, if quantity coordination is achieved and both the manufacturer and the retailer are better off than in the case without quantity coordination (with respect to their bargaining powers), win-win coordination is achieved.*

These definitions of quantity coordination and win-win coordination are consistent with existing literature, but are explicitly defined here to ensure that the terminology is clear.

Proposition 4.1. *Under the basic model, a ZR contract cannot achieve win-win coordination but can achieve quantity coordination, a ZS contract cannot achieve any coordination, and a ZRS contract can achieve win-win coordination.*

Proposition 4.1 shows the importance of developing the ZRS contract type. Contrary to some common beliefs expressed in the supply-chain contracting literature, a ZS contract is in fact incapable of coordinating the channel because there is no further control of the wholesale price. A ZR contract, despite having less flexibility than a more common revenue-sharing contract, can in fact achieve quantity coordination. To achieve win-win coordination, the side-payment scheme is incorporated into the ZR contract to create the ZRS contract type, which can guarantee the achievability of win-win coordination in the supply chain.

Our goal in this paper is to achieve win-win coordination, which we relate to the bargaining powers of the supply chain members. Specifically, we use the Nash bargaining model, in which the manufacturer's bargaining power is γ, and the retailer's bargaining power is $(1-\gamma)$. We use the Nash bargaining model because it is the most common in the literature for deriving bargaining outcomes (Yang et al. 2018). Moreover, in the basic model we assume the bargaining power is fixed and independent of whether there is a ZWP contract; in the extended model, we relax this assumption.

Under the Nash bargaining model (see Shi et al. 2018), we denote the Nash bargaining product under a contract *l* by $\Theta_{NB}^{l} \triangleq E[\pi_R^l]^{\gamma} E[\pi_M^l]^{1-\gamma}$. Defining $E[\pi_{SC}^{l*}]$ as the maximum expected profit achievable for the supply chain system with the contract *l*, the Nash bargaining solution can be obtained by solving Problem (N-B) in the following:

Problem (N-B) Max $\Theta_{NB}^{l} = E[\pi_R^l]^{\gamma} E[\pi_M^l]^{1-\gamma}$

Subject to $E[\pi_R^l] + E[\pi_M^l] \le E[\pi_{SC}^{l*}]$.

Solving Problem (N-B) yields the Nash bargaining solution (see Appendix for further analyses of the features of the Nash bargaining solution). To achieve win-win coordination under the basic model with one product, we can adopt a ZRS contract and set the contract parameters in accordance with Proposition 4.2.

Proposition 4.2. *(a) Under the basic model: (i) For the RD scenario, win-win coordination can be achieved by setting the following two critical contract parameters as:* $\alpha = \dfrac{p-\xi-\beta_{SC}(p+h)}{(1-\beta_{SC})p}$ *and*

$$T = ((\gamma - \left(\frac{p-\xi-\beta_{SC}(p+h)}{(1-\beta_{SC})p}\right))p - \gamma\xi + (1-\gamma)m)q_{SC}^{*} + ((\left(\frac{p-\xi-\beta_{SC}(p+h)}{(1-\beta_{SC})p}\right) - \gamma)p - \gamma h)\int_0^{q_{SC}^{*}} F(x)dx$$

. (ii) For the MD scenario, win-win coordination can be achieved by setting the following two critical contract parameters as: $\alpha = \dfrac{m}{(1-\beta_{SC})p}$ *and*

$$T = \left[\left(\left(\frac{m}{(1-\beta_{SC})p}\right)-\gamma\right)p+\gamma\xi-(1-\gamma)m\right]q_{SC}^{*}+\left[\left(\gamma-\left(\frac{m}{(1-\beta_{SC})p}\right)\right)p+h\right]\int_{0}^{q_{SC}^{*}}F(x)dx \quad . \qquad (b)$$

$$0<\frac{p-\xi-\beta_{SC}(p+h)}{(1-\beta_{SC})p}<1 \ \ and \ \ 0<\frac{m}{(1-\beta_{SC})p}<1.$$

In Proposition 4.2 we have two scenarios that respectively represent the case when the quantity decision is made by the retailer (the RD scenario) and the case when the quantity decision is decided by the manufacturer (the MD scenario). Proposition 4.2 (b) shows that the optimal revenue-sharing rates under the RD and MD scenarios are always bounded by 0 and 1, and hence the presence of an optimum is guaranteed. It is also important to note that under the Nash bargaining model the final profits achieved by the retailer and the manufacturer will be the same under the win-win coordination scenario. In other words, there is no difference in terms of the expected profits that the retailer and the manufacturer can achieve between the RD and the MD scenarios.

For notational purposes we define the following, with Table 4.1 showing the functions of each contract parameter: $\alpha_R^* = \frac{p-\xi-\beta_{SC}(p+h)}{(1-\beta_{SC})p}$, $T_R^* = ((\gamma-\alpha_R^*)p-\gamma\xi+(1-\gamma)m)q_{SC}^* + ((\alpha_R^*-\gamma)p-\gamma h)\int_0^{q_{SC}^*}F(x)dx,$

$\alpha_M^* = \frac{m}{(1-\beta_{SC})p}$, $T_M^* = ((\alpha_M^*-\gamma)p+\gamma\xi-(1-\gamma)m)q_{SC}^* + ((\gamma-\alpha_M^*)p+h)\int_0^{q_{SC}^*}F(x)dx.$

Table 4.1. Features of the ZRS contracts and functions of the contract parameters to achieve win-win coordination.

Scenarios	Details	Contract Parameters	Functions
RD	The manufacturer first sets α and *T*, then the retailer decides the ordering quantity	$\alpha = \alpha_R^*$	Achieve quantity coordination
		$T = T_R^*$	Ensure win-win with respect to the bargaining powers of supply chain members
MD	The retailer first sets α and *T*, then the manufacturer decides the product quantity	$\alpha = \alpha_M^*$	Achieve quantity coordination
		$T = T_M^*$	Ensure win-win with respect to the bargaining powers of supply chain members

5. Further Analyses

5.1. Multiple Products and Efficient Contracts

In the basic model we focused on the single product scenario. We now consider the case of multiple ($N > 1$) heterogeneous products, for which the key is to demonstrate how win-win coordination can be achieved by using the most efficient ZWP-based supply-chain contracts.

We follow the basic model and add a subscript $i = 1,..., N$ to denote the specific product. Under the ZWP-based contract, all products' wholesale prices are set to zero with the manufacturer compensated by other means, in the same fashion as the single product case. For product $i = 1,..., N$, the unit retail price is p_i, the unit operations cost for the retailer is ξ_i, the net unit product leftover cost is h_i, the unit product cost is m_i, and demand is x_i, which follows a density function $f_i(\cdot)$ and a cumulative distribution function $F_i(\cdot)$. We define the following and then present Proposition 5.1.

Proposition 5.1. *Under the case with N heterogeneous products:*

(a) (i) For the RD scenario, win-win coordination can be achieved using a ZRS contract by setting the following two critical contract parameters as follows: $\alpha_i = \alpha^*_{R,i} \triangleq \dfrac{p_i - \xi_i - \beta_{SC,i}(p_i + h_i)}{(1-\beta_{SC,i})p_i}$ *and*

$$T = \sum_{i=1}^{N} [((\gamma_i - \alpha^*_{R,i})p_i - \gamma\xi_i + (1-\gamma)m_i)q^*_{SC,i} + ((\alpha^*_{R,i} - \gamma)p_i - \gamma h_i)\int_0^{q^*_{SC,i}} F_i(x_i)dx_i],$$

for all $i = 1,..., N$. *(ii) For the MD scenario, win-win coordination can be achieved using the ZRS contract by setting the following two critical contract parameters as follows:* $\alpha_i = \alpha^*_{M,i} \triangleq \dfrac{m_i}{(1-\beta_{SC,i})p_i}$ *and*

$$T = \sum_{i=1}^{N} [((\alpha^*_{M,i} - \gamma)p_i + \gamma\xi_i - (1-\gamma)m_i)q^*_{SC,i} + ((\gamma - \alpha^*_{M,i})p_i + h_i)\int_0^{q^*_{SC,i}} F_i(x_i)dx_i],$$

for all $i = 1,..., N$. *(b) The win-win coordinating ZRS contract requires N + 1 contract parameters under both the RD and MD* *scenarios.*

Proposition 5.1 provides a few important insights. First, the format of the ZRS contract that can achieve win-win coordination is very similar to its single-product counterpart. Second, to guarantee the achievability of win-win coordination, $N + 1$ contract parameters are required in the case of all N products being different. Third, it can happen that, for some products, granting a ZWP and sharing a certain proportion of revenue may lead to a loss to the manufacturer for that specific group of products. However, in the presence of the side payment and other more profitable products, the manufacturer is guaranteed to benefit under win-win coordination. As a result, Proposition 5.1 clearly shows the good performance and effectiveness of ZRS contracts in coordinating a multi-product supply chain.

5.2. Greedy Wholesale Price–Based Contracts and Risk Analysis

In this paper we focus on ZWP-based supply chain contracts, which seem to present greater risk to the manufacturer and less risk to the retailer. It is then natural to consider GWP-based supply chain contracts, in which the manufacturer charges a wholesale price equal to the retail price and then compensates the retailer with a share of revenue and/or a side payment. In this extended analysis, we consider the case of GWP-based contracts. This is an important extension, given that firms in practice have in recent years begun to realise the significant influences of supply chain risks, and have started to emphasise the trade-off between profit and risk (Zhuo et al. 2018). Hewlett-Packard, for instance, has established a formal procurement risk-management system, through which it can manage supply chain risks using structured supply chain contracts (Nagali et al. 2008).

Similar to the analysis of ZWP-based contracts, we analyse three GWP-based contracts. First, in a GWP-revenue-sharing (GR) contract, the manufacturer charges a wholesale price equal to the retail price p, and grants λp for each product sold to the retailer, where $\lambda > 0$. Second, in a GWP-side-payment (GS) contract, the manufacturer charges a wholesale price equal to the retail price *p*, and grants a side-payment *J* to the retailer per season, where *J* is independent of the order quantity. Third, combining the GR contract and the GS contract, we arrive at the GRS contract.

Proposition 5.2. (a) *Using the GRS contract: (i) For the RD scenario, win-win coordination can be achieved by setting* $\lambda = \frac{p-m}{(1-\beta_{SC})p}$ *and* $J = ((1-\gamma-\lambda_R^*)p + \gamma\xi - (1-\gamma)m)q_{SC}^* + ((\lambda_R^*+\gamma)p+\gamma h)\int_0^{q_{SC}^*} F(x)dx$. *(ii) For the MD scenario, win-win coordination can be achieved by setting* $\lambda = \frac{p-m}{(1-\beta_{SC})p}$ *and* $J = ((1-\lambda_M^*-\gamma)p + \gamma\xi - (1-\gamma)m)q_{SC}^* + ((\gamma+\lambda_M^*)p+\gamma h)\int_0^{q_{SC}^*} F(x)dx$. *(b)* $\lambda_R^* \triangleq \frac{p-m}{(1-\beta_{SC})p} > 0$ [7] *and* $0 < \lambda_M^* \triangleq \frac{p-m}{(1-\beta_{SC})p} < 1$.

Proposition 5.2 is very similar to Proposition 4.2, indicating that GRS contracts can also facilitate win-win coordination. However, it is worthwhile to compare GRS and ZRS contracts for which one presents the most risk to the manufacturer and the retailer. Employing the MV criterion, which is now very commonly used in OR/OM analysis (Chiu and Choi 2016; Chiu et al. 2018; Choi et al. 2019), we conduct a risk analysis (Asian and Nie 2014) for the supply chain members. Under the MV analysis framework, we quantify the benefit by 'expected profit' (i.e., the mean) and the level of risk by 'variance of profit' (i.e., the variance). This approach is commonly used in finance, such as for portfolio management, and in supply chain analysis (see, for example, Choi et al. 2008; Chiu and Choi 2016). The MV approach is fundamental to risk hedging, and 'variance' is used as a standard risk-hedging measure in these fields, both in industry and academia.

To be specific, for the respective win-win coordinating contracts, the variance of profit (VP) of each

[7] Note that λ_R^* can be greater than 1 under GRS.

supply chain member can be derived as follows.

Define:

$Var[\cdot]$ as the variance operator,

$$\lambda_R^* = \frac{p-m}{(1-\beta_{SC})p}, \tag{5.1}$$

$$\lambda_M^* = \frac{p-m}{(1-\beta_{SC})p}, \tag{5.2}$$

$$J_R^* = ((1-\gamma-\lambda_R^*)p + \gamma\xi - (1-\gamma)m)q_{SC}^* + ((\lambda_R^* + \gamma)p + \gamma h)\int_0^{q_{SC}^*} F(x)dx, \tag{5.3}$$

$$J_M^* = ((1-\lambda_M^* - \gamma)p + \gamma\xi - (1-\gamma)m)q_{SC}^* + ((\gamma + \lambda_M^*)p + \gamma h)\int_0^{q_{SC}^*} F(x)dx. \tag{5.4}$$

$$Var[\max(q-x,0)] = \Phi(q), \tag{5.5}$$

$$\Omega_{SC}^* = \Phi(q_{SC}^*) > 0. \tag{5.6}$$

Under a ZRS contract, we have the following:

$$VP_R^{ZRS*} = [(1-\alpha_R^*)p + h]^2 \Omega_{SC}^*, \tag{5.7}$$

$$VP_M^{ZRS*} = (\alpha_M^* p)^2 \Omega_{SC}^*. \tag{5.8}$$

Under a GRS contract, we have the following:

$$VP_R^{GRS*} = [(1+\lambda_R^*)p + h]^2 \Omega_{SC}^*, \tag{5.9}$$

$$VP_M^{GRS*} = (\lambda_M^* p)^2 \Omega_{SC}^*. \tag{5.10}$$

With the results in (5.7) to (5.11), we have Lemma 5.1.

Lemma 5.1. (a) $VP_R^{ZRS*} < VP_R^{GRS*}$. *(b)* $VP_M^{ZRS*} \begin{pmatrix} > \\ = \\ < \end{pmatrix} VP_M^{GRS*}$ *if and only if* $m \begin{pmatrix} > \\ = \\ < \end{pmatrix} \frac{p}{2}$.

Lemma 5.1 shows the analytical comparison between VPs under a GRS contract and a ZRS contract for the manufacturer and the retailer. It is clear and intuitive that $VP_R^{ZRS*} < VP_R^{GRS*}$. However, for the manufacturer, whether the VP is greater under ZRS depends on the unit production cost.

With the derived expressions of VPs, we define the following:

Definition 3 (mean-variance (MV) domination): *Given two supply chain contracts, X and Y, both of which can achieve win-win coordination independently, Contract X is said to MV-dominate Contract Y if and only if the VPs of both the manufacturer and the retailer under Contract X are less than their respective VPs under Contract Y.*

From Lemma 5.1, together with the definition of the MV-dominating contract, we can directly derive Proposition 5.3.

Proposition 5.3. *The ZRS contract MV-dominates the GRS contract if and only if the unit production cost is sufficiently small. However, the GRS contract can never MV-dominate the ZRS contract as the retailer will always face a higher level of risk if she adopts GRS rather than ZRS.*

Proposition 5.3 can be explained as follows. Even though both the ZRS contract and the GRS contract can achieve win-win coordination (noting also that the same level of expected profits are attained under ZRS and GRS), we can see from Lemma 5.1 that the retailer suffers a higher level of risk under the GRS contract than under the ZRS contract. For the manufacturer, compared with GRS, Lemma 5.1 shows that he enjoys a lower level of risk under ZRS if and only if the unit production cost m is sufficiently low ($m < p/2$).

Proposition 5.3 thus confirms our intuition that a ZRS contract is less risky to the retailer. Indeed, adopting a GRS contract will be much riskier for the retailer than its ZRS counterpart, even though both will lead to the same expected profit. Thus, for the retailer, it is wise to select a ZRS contract and decline a GRS contract if possible. For the manufacturer, the situation is trickier because he can enjoy a lower or higher level of risk depending on the unit production cost that he faces. To be specific, if the unit production cost is sufficiently small, adopting the ZRS contract is, surprisingly, the lower-risk scenario for the manufacturer. This leads to an interesting result: when the unit production cost is sufficiently low, adopting a ZRS contract is a beneficial option for both the retailer and the manufacturer, as it yields a lower level of risk while achieving the same level of profit (i.e., an MV-dominating scenario when compared to the GRS contract).

5.3. Differences in Perception and Psychological Effects

5.3.1. Bargaining Power

Supply chain contracting involves behavioural factors (Li 2018; Liu et al. 2018) related to bargaining (Shi et al. 2018). One major difference between ZWP-based supply chain contracts and those that are NZWP-based is in perception. That is, ZWP-based supply chain contracts may be perceived as the contract setter doing a favour for its supply chain partner. For example, if the contract is determined by the manufacturer, granting a ZWP to the retailer might imply that the retailer has 'nothing to lose' (Ahmetoglu et al. 2014) or at least 'very little to lose'. This may make the retailer more willing to work with the manufacturer. If, when compared to NZWP-based supply chain contracts, granting a ZWP will make the supply chain partner happier as they are receiving a 'favour', we argue that it may increase the bargaining power for final supply-chain profit-sharing. Note that the presence of this psychological effect does not affect the whole supply chain system's maximum total profit (i.e., that under the centralised setting), as it only affects the behaviour of the supply chain agents. The total supply chain system's cost and revenue parameters are unchanged, as are its demand parameters. This will actually benefit the contract setter in offering a versatile ZWP contract, such as a ZRS contract. To summarise this finding, we denote the bargaining power of the contract setter $k \in (R, M)$ under scenario $l \in (ZWP, NZWP)$ by ψ_l^k. We therefore arrive at Proposition 5.4.

Proposition 5.4. *If* $\psi_{ZWP}^k > \psi_{NZWP}^k$ for all $k \in (R, M)$, *offering ZRS is always a wise strategy for the contract setter k.*

Proposition 5.4 is a very important result, as it shows why some supply chains may prefer ZWP-based supply chain contracts over their NZWP counterparts, because offering a 'free lunch' (i.e., a ZWP) is actually more beneficial than offering a ‘paid lunch’ (i.e., a NZWP).

5.3.2. Perceived Benefit

The pricing literature reveals that ‘zero price’ or ‘free’ products have a deeper meaning than just a ‘cost reduction’. To be specific, Shampanier et al. (2007) report from behavioural experiments that decision makers perceive the benefit that they can derive from ‘zero-price’ products as going beyond simply the ‘zero cost’. To be specific, they find that decision makers tend to associate a ‘zero price’ product with an additional ‘invisible’ perceived benefit. This is echoed by Ascarza et al. (2012), in which a three-part tariff contract is established. Based on this empirical finding in the literature, we now consider the situation of the manufacturer offering a ZRS contract to the retailer. The retailer perceives the ZWP offered under ZRS to carry an additional perceived benefit B per unit of product ordered. In this sense, the retailer’s expected profit function[8] is revised as follows (with ^ denoting the case of an invisible benefit perceived by the retailer when a ZWP is offered):

$$E[\hat{\pi}_R^{ZRS}] = [(1-\alpha)p - \xi + B]q - [(1-\alpha)p + h]\int_0^q F(x)dx - T, \quad (5.12)$$

Maximizing (5.12) shows the optimal ordering quantity made by the retailer: $q_R^{ZRS*} = F^{-1}\left(\frac{(1-\alpha)p - \xi + B}{(1-\alpha)p + h}\right)$. We then arrive at Proposition 5.5.

Proposition 5.5. *In the presence of the retailer’s perceived benefit B, offering ZRS is beneficial to the manufacturer.*

Proposition 5.5 is intuitive, yet important. It is intuitive because when the retailer perceives an additional benefit in her own decision-making process and ‘calculation’, the actual amount of side payment the manufacturer will receive will be higher compared to the case when the additional perceived benefit is zero. This is an important finding, as it may help to explain why some manufacturers are willing to offer

[8] Strictly speaking, it is better to denote this an ‘expected benefit’ function, as it includes an additional perceived benefit *B*. However, for simplicity. we denote it an ‘expected profit function’.

ZWP-based supply chain contracts to retailers; i.e., the manufacturer perceivers that these contract actually can generate additional benefits to themselves.

6. Conclusion and Implications

6.1. Summary

Motivated by various observed real-world practices, we have explored zero wholesale price (ZWP)–based supply chain contracts in this paper. In the basic model, we have considered two cases: that in which ordering is decided by the retailer (called the RD scenario) and that in which the ordering decision is made by the manufacturer (called the MD scenario). For each of these cases, we have examined three contract types: ZWP-revenue-sharing (ZR), ZWP-side-payment (ZS), and ZWP-revenue-sharing-plus-side-payment (ZRS). We have analytically proven that under both the RD and MD scenarios, only the ZRS contract (but not the ZR or ZS contract) can achieve win-win coordination, in which the supply chain is optimised and both the manufacturer and the retailer benefit from the implementation of the contract. This highlights the power of the ZRS contract.

In the extended models, we have considered three extended analyses. First, we have analysed the scenario with multiple products and analytically proven how a generalised ZRS contract can be applied to achieve win-win coordination efficiently. Second, we have studied greedy wholesale price (GWP)–based contracts, which are the opposite of ZWP-based contracts in that the manufacturer charges the retailer a wholesale price equal to the retail price. We have shown that a GWP-based revenue-sharing-plus-side-payment (GRS) contract and a ZRS contract can both achieve win-win coordination. However, with the concept of risk in mind, we have shown that the ZRS contract MV-dominates the GRS contract in yielding a smaller level of risk to both the retailer and manufacturer when the unit production cost is sufficiently small. Third, we have further discussed the cases of differences in perception between ZWP and NZWP contracts, and revealed how ZWP-based supply chain contracts can find a competitive niche.

6.2. Insights and Implications

Based on the findings derived from the analysis, we discuss below some important managerial insights and implications.

Choice of ZWP-based contracts: From Proposition 4.1, we note that only the ZRS contract, but not the ZR and ZR contracts, can achieve win-win coordination. This means the ZRS contract is a powerful option, especially because no matter whether the ordering decision is decided by the retailer (i.e., under the RD scenario) or by the manufacturer (i.e., under the MD scenario), the ZRS contract can still achieve win-win coordination. In terms of contract setting in the real world, the ZRS contract is also easy to implement, as indicated by Proposition 4.2.

Coordinating multiple-product supply chains: From Proposition 5.1, first, we can easily see that when the supply chain produces and sells N heterogeneous products, a generalised ZRS contract with only $N + 1$ contract parameters can efficiently achieve win-win coordination. In addition, the format of the ZRS contract that can achieve win-win coordination is very similar to its single product counterpart. Second, to guarantee the achievability of win-win coordination, only $N + 1$ contract parameters are needed, not the $2N$ contract parameters that might be expected to be required, based on the finding that two contract parameters can ensure win-win coordination for a single product. Third, for some products, granting ZWP and sharing a certain proportion of revenue may lead to a loss to the manufacturer for that specific group of products. However, the side-payment helps compensate for this, which is the merit and versatility of the generalised ZRS contract.

GWP-based contracts: From Proposition 5.2, we observe that a GRS contract, similar to a ZRS contract, can also achieve win-win coordination. However, Lemma 5.1 indicates that retailers suffer a higher level of risk under GRS contracts than ZRS contracts, and hence will prefer the latter. Proposition 5.3 provides further insight into MV domination, indicating that a GRS contract can never MV-dominate a ZRS contract, as the retailer will face a higher level of risk if she adopts GRS over ZRS. On the contrary, the ZRS contract will MV-dominate the GRS contract if and only if the unit production cost is sufficiently small. In other words, when the unit production cost is sufficiently small, adopting ZRS leaves both the retailer and the manufacturer better off as they both face a smaller level of risk under ZRS than under GRS. This is an interesting and important finding.

Differences in perception: ZWP-based supply chain contracts, by definition, involve the manufacturer offering ZWP and doing a favour to the retailer. If this has a positive psychological effect on the bargaining power of the contract setter[9], then it is in fact wise to adopt a ZWP-based contract type, such as a ZRS contract, because it can generate a higher profit level (see Proposition 5.4). Furthermore, if offering ZWP makes the retailer perceive they are receiving an additional benefit, as has been reported by empirical studies, then it will be beneficial to the manufacturer to offer the ZWP-based contract, compared to its NZWP counterpart. We argue that these two differences in perception are critical to explain why ZWP-based contracts may have a niche.

A 'free lunch' versus a 'paid lunch': Our discussions in this paper, including the comparisons between ZRS and GRS, highlight the capacity of ZRS contracts to outperform GRS contracts and other NZWP-based counterparts. This goes some way to explaining why ZWP-based contracts exist and may become increasingly common in the real world. A 'free lunch' can indeed outperform a 'paid lunch'.

6.3. Future Research

This paper does not consider the value of information in the supply chain (Teunter et al. 2018); future research

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[9] In this paper, we assumed that the contract can be set by either the manufacturer or the retailer, with respect to their bargaining powers.

could examine this critical issue. Competition is another important topic in supply chain contracting (Wang and Shin 2015), and in a future study we plan to analyse the comparative performance of ZWP-based supply chain contracts under different competitive scenarios.

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Appendix: All Proofs and Derivations

Derivations of the optimal quantities: (a) Under a ZR contract, we have:

$$E[\pi_R^{ZR}]=[(1-\alpha)p-\xi]q-[(1-\alpha)p+h]\int_0^q F(x)dx, \tag{A1.1}$$

$$E[\pi_M^{ZR}]=(\alpha p-m)q-\alpha p\int_0^q F(x)dx. \tag{A1.2}$$

It is easy to find that $E[\pi_R^{ZR}]$ and $E[\pi_M^{ZR}]$ are concave functions of q. Thus, solving $dE[\pi_R^{ZR}]/dq=0$ and $dE[\pi_M^{ZR}]/dq=0$ yields $q_R^{ZR*}=F^{-1}\left(\frac{(1-\alpha)p-\xi}{(1-\alpha)p+h}\right)$, and $q_M^{ZR*}=F^{-1}\left(\frac{\alpha p-m}{\alpha p}\right)$.

For other contracting scenarios, Parts (b), (c), and (d), the steps are similar. Solving the respective first order conditions gives: $q_R^{ZS}=F^{-1}\left(\frac{p-\xi}{p+h}\right)$, $q_M^{ZS*}=0$; $q_R^{ZRS*}=F^{-1}\left(\frac{(1-\alpha)p-\xi}{(1-\alpha)p+h}\right)$, and $q_M^{ZRS*}=F^{-1}\left(\frac{\alpha p-m}{\alpha p}\right)$; $q_{SC}^*=F^{-1}(\beta_{SC})$, where $\beta_{SC}=\frac{p-\xi-m}{p+h}$. (Q.E.D.)

Proof of Proposition 4.1: Under the basic model, the ZR contract can achieve quantity coordination by setting $q_R^{ZR*}=q_{SC}^*$, and $q_M^{ZR*}=q_{SC}^*$:

$$q_R^{ZR*}=q_{SC}^*\Leftrightarrow F^{-1}\left(\frac{(1-\alpha)p-\xi}{(1-\alpha)p+h}\right)=F^{-1}(\beta_{SC})\Leftrightarrow\alpha=\alpha_R^*,\text{ where }\alpha_R^*=\frac{p-\xi-\beta_{SC}(p+h)}{(1-\beta_{SC})p}.$$

$$q_M^{ZR*}=q_{SC}^*\Leftrightarrow F^{-1}\left(\frac{\alpha p-m}{\alpha p}\right)=F^{-1}(\beta_{SC})\Leftrightarrow\alpha=\alpha_M^*,\text{ where }\alpha_M^*=\frac{m}{(1-\beta_{SC})p}.$$

However, as controlling only one contract parameter does not permit flexible allocation of the supply chain profits between the manufacturer and the retailer, a win-win outcome cannot be guaranteed. Thus, a ZR contract cannot achieve win-win coordination. ZS contracts cannot achieve coordination because changing the side payment cannot affect the ordering quantity. A ZRS contract includes the power of the ZR contract in achieving quantity coordination, in which the supply chain's profit is maximised. With the addition of the side payment, it can also allocate the supply chain profit flexibly between the manufacturer and the retailer, and thus can achieve win-win coordination. (Q.E.D.)

Nash Bargaining Solution (i.e., Solution of Problem (N-B)):

Problem (N-B) Max $\Theta_{NB}^l=E[\pi_R^l]^\gamma E[\pi_M^l]^{1-\gamma}$

Subject to $E[\pi_R^l]+E[\pi_M^l]\le E[\pi_{SC}^{l*}]$.

For the members to receive the maximum expected profits, $E[\pi_R^l]+E[\pi_M^l]=E[\pi_{SC}^{l*}]$ holds. As a result, we have: $E[\pi_R^l]=E[\pi_{SC}^{l*}]-E[\pi_M^l]$ and $E[\pi_M^l]=E[\pi_{SC}^{l*}]-E[\pi_R^l]$. We check the first-order derivatives:

$$\begin{aligned}\frac{\partial\Theta_{NB}^l}{\partial E[\pi_R^l]} &= \frac{\partial}{\partial E[\pi_R^l]}\{E[\pi_R^l]^\gamma E[\pi_M^l]^{1-\gamma}\} = \frac{\partial}{\partial E[\pi_R^l]}\{E[\pi_R^l]^\gamma (E[\pi_{SC}^{l*}]-E[\pi_R^l])^{1-\gamma}\}\\ &= E[\pi_R^l]^\gamma \frac{\partial(E[\pi_{SC}^{l*}]-E[\pi_R^l])^{1-\gamma}}{\partial E[\pi_R^l]}+(E[\pi_{SC}^{l*}]-E[\pi_R^l])^{1-\gamma}\frac{\partial E[\pi_R^l]^\gamma}{\partial E[\pi_R^l]}\\ &= E[\pi_R^l]^{\gamma-1}(E[\pi_{SC}^{l*}]-E[\pi_R^l])^{-\gamma}(\gamma E[\pi_{SC}^{l*}]-E[\pi_R^l]).\end{aligned}$$

$$\begin{aligned}\frac{\partial\Theta_{NB}^l}{\partial E[\pi_M^l]} &= \frac{\partial}{\partial E[\pi_M^l]}\{E[\pi_R^l]^\gamma E[\pi_M^l]^{1-\gamma}\}\\ &= \frac{\partial}{\partial E[\pi_M^l]}\{(E[\pi_{SC}^{l*}]-E[\pi_M^l])^\gamma E[\pi_M^l])^{1-\gamma}\}\\ &= E[\pi_M^l]^{-1}(E[\pi_{SC}^{l*}]-E[\pi_M^l])^{\gamma-1}((1-\gamma)E[\pi_{SC}^{l*}]-E[\pi_M^l]).\end{aligned}$$

As the optimal decisions must satisfy the first order conditions, we have: $\frac{\partial\Theta_{NB}^l}{\partial E[\pi_R^l]}=0\Rightarrow E[\pi_R^l]=\gamma E[\pi_{SC}^{l*}]$,

and $\frac{\partial\Theta_{NB}^l}{\partial E[\pi_M^l]}=0\Rightarrow E[\pi_M^l]=(1-\gamma)E[\pi_{SC}^{l*}]$.

Thus, at the optimal Nash bargaining solution, the manufacturer shares $(1-\gamma)$ of the maximum supply chain profit and the retailer shares γ of the maximum supply chain profit under the contract *l*. For the contract *l* which can achieve supply chain coordination, the maximum supply chain profit under the contract *l* becomes the 'global' maximum supply chain profit $E[\pi_{SC}^*]$. (Q.E.D.)

Proof of Proposition 4.2:

(a) Under the basic model, define: $\alpha_R^*=\frac{p-\xi-\beta_{SC}(p+h)}{(1-\beta_{SC})p}$ and $\alpha_M^*=\frac{m}{(1-\beta_{SC})p}$.

(i) For the RD scenario, as we have shown in the proof of Proposition 4.1 above, quantity coordination can be achieved by setting $\alpha=\alpha_R^*$. To achieve win-win coordination, considering the Nash bargaining model, we need to set the side payment in such a way such that the retailer will receive $(1-\gamma)$ of the optimised supply chain's expected profit. The expected profit of the optimized supply chain is given as follows:

$$E[\pi_{SC}^*] = E[\pi_{SC}(q_{SC}^*)](p-\xi-m)q_{SC}^* - (p+h)\int_0^{q_{SC}^*} F(x)dx.$$

In other words, we have:

$$E[\pi_R^{ZRS}(q_{SC}^*;\alpha_R^*)] = E[\pi_R^{ZRS*}] = [(1-\alpha_R^*)p-\xi]q_{SC}^* - [(1-\alpha_R^*)p+h]\int_0^{q_{SC}^*} F(x)dx - T \; = (1-\gamma)E[\pi_{SC}^*]. \qquad \text{(A1.3)}$$

Solving (A1.3) yields:

$T = T_R^* = ((\gamma-\alpha_R^*)p - \gamma\xi + (1-\gamma)m)q_{SC}^* + ((\alpha_R^*-\gamma)p - \gamma h)\int_0^{q_{SC}^*} F(x)dx.$ Thus, under the RD scenario, win-win coordination can be achieved by setting $\alpha = \alpha_R^*$ and $T = T_R^*$.

(ii) For the MD scenario, similar to the steps for the RD scenario, we have (A1.4):

$$E[\pi_M^{ZRS}(q_{SC}^*;\alpha_M^*)] = E[\pi_M^{ZRS*}] = (\alpha_M^* p - m)q_{SC}^* - \alpha_M^* p\int_0^{q_{SC}^*} F(x)dx + T = \gamma E[\pi_{SC}^*]. \qquad \text{(A1.4)}$$

Solving (A1.4) yields: $T = T_M^* = ((\alpha_M^*-\gamma)p + \gamma\xi - (1-\gamma)m)q_{SC}^* + ((\gamma-\alpha_M^*)p + h)\int_0^{q_{SC}^*} F(x)dx.$ Thus, under the MD scenario, win-win coordination can be achieved by setting $\alpha = \alpha_M^*$ and $T = T_M^*$. (Q.E.D)

(b) By definition, we have: $\alpha_R^* = \dfrac{p-\xi-\beta_{SC}(p+h)}{(1-\beta_{SC})p}$. As $\beta_{SC} = \dfrac{p-\xi-m}{p+h}$ and $\beta_{SC} < 1$, it is straightforward to show that $(1-\beta_{SC})p > 0$ and $p-\xi-\beta_{SC}(p+h) = p-\xi-(p-\xi-m) = m > 0$. Thus, $\alpha_R^* > 0$. Moreover, as $-\xi-\beta_{SC}(p+h) < -\beta_{SC}p$, it is easy to see that $p-\xi-\beta_{SC}(p+h) < (1-\beta_{SC})p$ is always true. Thus, we have: $\alpha_R^* < 0$. As such, we have: $0 < \alpha_R^* < 1$.

For $0 < \alpha_M^* < 1$, note that $\alpha_M^* = \dfrac{m}{(1-\beta_{SC})p}$. With the same argument, it is easy to prove that $0 < \alpha_M^* < 1$ is also always true. (Q.E.D.)

Proof of Proposition 5.1: Define $\beta_{SC,i} = \dfrac{p_i-\xi_i-m_i}{p_i+h_i}$, $q_{SC,i}^* = F_i^{-1}(\beta_{SC,i})$,

$\alpha_{R,i}^* = \dfrac{p_i-\xi_i-\beta_{SC,i}(p_i+h_i)}{(1-\beta_{SC,i})p_i}$, $\alpha_{M,i}^* = \dfrac{m_i}{(1-\beta_{SC,i})p_i}$,

$$T_R^{N*} = \sum_{i=1}^{N} [((\gamma_i - \alpha_{R,i}^*) p_i - \gamma \xi_i + (1-\gamma) m_i) q_{SC,i}^* + ((\alpha_{R,i}^* - \gamma) p_i - \gamma h_i) \int_0^{q_{SC,i}^*} F_i(x_i) dx_i],$$

$$T_M^{N*} = \sum_{i=1}^{N} [((\alpha_{M,i}^* - \gamma) p_i + \gamma \xi_i - (1-\gamma) m_i) q_{SC,i}^* + ((\gamma - \alpha_{M,i}^*) p_i + h_i) \int_0^{q_{SC,i}^*} F_i(x_i) dx_i].$$

(a) This proof follows the same logic as in Proposition 4.2. In the case of N heterogeneous products: (i) For the RD scenario, win-win coordination can be achieved using a ZRS contract by setting $\alpha_i = \alpha_{R,i}^*$ and $T = T_R^{N*}$ for all $i = 1, \ldots, N$. Here, setting N contract parameters $\alpha_i = \alpha_{R,i}^*$ for all $i = 1, \ldots, N$ achieves quantity coordination. Setting $T = T_R^{N*}$ = $\sum_{i=1}^{N} [((\gamma_i - \alpha_{R,i}^*) p_i - \gamma \xi_i + (1-\gamma) m_i) q_{SC,i}^* + ((\alpha_{R,i}^* - \gamma) p_i - \gamma h_i) \int_0^{q_{SC,i}^*} F_i(x_i) dx_i]$ ensures that win-win coordination is achieved. (ii) For the MD scenario, similar to the RD scenario, win-win coordination can be achieved using a ZRS contract by setting $\alpha_i = \alpha_{M,i}^*$ for all $i = 1, \ldots, N$ (for quantity coordination for each product), and $T = T_M^{N*}$, where

$$T_M^{N*} = \sum_{i=1}^{N} [((\alpha_{M,i}^* - \gamma) p_i + \gamma \xi_i - (1-\gamma) m_i) q_{SC,i}^* + ((\gamma - \alpha_{M,i}^*) p_i + h_i) \int_0^{q_{SC,i}^*} F_i(x_i) dx_i].$$

(b) From Part (a), it is clear that a win-win coordinating ZRS contract requires N + 1 contract parameters under both the RD and MD scenarios. (Q.E.D.)

Proof of Proposition 5.2: This proof follows the same logic and procedure as in Proposition 4.2, with the only major change being the different contract type. We therefore omit the proof to save space. (Q.E.D.)

Proof of Lemma 5.1:

(a) From (5.7) and (5.9), we have: $VP_R^{ZRS*} = [(1-\alpha_R^*) p + h]^2 \Omega_{SC}^*$ and $VP_R^{GRS*} = [(1+\lambda_R^*) p + h]^2 \Omega_{SC}^*$. Since $0 < \alpha_R^* < 1$ (Proposition 4.2) and $0 < \lambda_R^*$ (Proposition 5.2), we have: $1 - \alpha_R^* < 1 + \lambda_R^*$, and hence:

$[(1-\alpha_R^*)p+h]^2\Omega_{SC}^* < [(1+\lambda_R^*)p+h]^2\Omega_{SC}^*$, which implies $VP_R^{ZRS*} < VP_R^{GRS*}$.

(b) From (5.8) and (5.10), we have: $VP_M^{ZRS*} = (\alpha_M^* p)^2\Omega_{SC}^*$ and $VP_M^{GRS*} = (\lambda_M^* p)^2\Omega_{SC}^*$. By definition, we have:

$\lambda_M^* = \dfrac{p-m}{(1-\beta_{SC})p}$ and $\alpha_M^* = \dfrac{m}{(1-\beta_{SC})p}$. Comparing the analytical expressions of VP_M^{ZRS*} and VP_M^{GRS*},

$VP_M^{ZRS*}\begin{pmatrix}>\\=\\<\end{pmatrix}VP_M^{GRS*}$ if and only if $m\begin{pmatrix}>\\=\\<\end{pmatrix}\dfrac{p}{2}$. (Q.E.D.)

Proof of Proposition 5.3: This proof is implied from the result of Lemma 5.1 and the definition of MV domination. Specifically, from Lemma 5.1 we note that $VP_M^{ZRS*}\begin{pmatrix}>\\=\\<\end{pmatrix}VP_M^{GRS*}$ if and only if $m\begin{pmatrix}>\\=\\<\end{pmatrix}\bar{m}$ and $VP_R^{ZRS*} < VP_R^{GRS*}$. Thus, a ZRS contract will MV-dominate a GRS contract if and only if the unit production cost is sufficiently small (i.e., $m < \bar{m}$). However, a GRS contract can never MV-dominate a ZRS contract, as the retailer will face a higher level of risk by adopting the former. (Q.E.D.)

Proof of Proposition 5.4: If $\psi_{ZWP}^k > \psi_{NZWP}^k$ for all $k \in (R, M)$, offering ZRS will mean the contract setter's ψ_{ZWP}^k is larger than for its NZWP-contract supply chain counterpart. As ZRS can achieve win-win coordination, under the Nash bargaining model the contract setter will receive a ψ_{ZWP}^k proportion of the optimal supply chain's expected profit, i.e., $\psi_{ZWP}^k E[\pi_{SC}^*]$. Under the NZWP counterpart, even if the respective contract can achieve win-win coordination, under the Nash bargaining model the contract setter will receive a ψ_{NZWP}^k proportion of the optimal supply chain's expected profit: $\psi_{NZWP}^k E[\pi_{SC}^*]$. As $\psi_{ZWP}^k > \psi_{NZWP}^k$, we have $\psi_{ZWP}^k E[\pi_{SC}^*] > \psi_{NZWP}^k E[\pi_{SC}^*]$. (Q.E.D.)

Proof of Proposition 5.5: In the presence of the retailer's perceived benefit *B*, we have

$E[\hat{\pi}_R^{ZRS}] = [(1-\alpha)p - \xi + B]q - [(1-\alpha)p + h]\int_0^q F(x)dx - T,$ and $q_R^{ZRS*} = F^{-1}\left(\frac{(1-\alpha)p - \xi + B}{(1-\alpha)p + h}\right)$. It is straightforward to prove that the manufacturer can offer a ZRS contract to coordinate the channel and obtain the respective proportion of the optimised supply chain's profit. Meanwhile, as the retailer perceives the presence of *B* in her own calculations, the amount of side-payment that the manufacturer can receive will be greater than the case when *B* is zero. Therefore, offering ZRS is beneficial to the manufacturer compared to NZRS because of the presence of *B*. (Q.E.D.)