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Adaptive nonlinear observer–based sliding mode control of robotic manipulator for handling an unknown payload

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Abstract

This article presents the control synthesis of robotic manipulators with an unknown constant payload. A novel nonlinear disturbance observer with an adaptive scheme is designed to estimate the external force induced by the unknown constant payload. A general design procedure for designing the gain of the nonlinear observer is developed rather than the time-consuming trials and error to choose proper gain. The nonlinear observer gain is designed using an adaptive technique to extend the applicability of the disturbance observer. The stability of the proposed observer is established using Lyapunov method under certain conditions. The proposed nonlinear disturbance observer will be integrated with the sliding mode control to substantially alleviate the chattering problem. Also, simulation results are presented to verify the effectiveness of the proposed methods.

Keywords

Adaptive disturbance observer, observer-based control, sliding mode control, robotic manipulator, unknown payload

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Introduction

With the increasing demands for sophisticated operations, the task of robotic manipulators is becoming diversified and complicated, and the operation is not limited to repetitive positioning operation, such as the operation, assembling components, and capturing targets, play an increasingly significant role and has attracted more attention in recent years.[1] And the requirements for the control performances of robotic manipulators are ever-increasing. However, the performance of motion control for robotic manipulators system is very sensitive to the large payload variation.[2] For example, when the end effector of robotic manipulator grabs an object with the unknown mass, the physical properties of the dynamics are changed. This contributing factor will degrade the system performances severely.

Research on deal with the unknown payload begun with early work on single manipulator[3–6] has stated that the unknown physical parameters of robot arm were estimated using an adaptive coordinated control method. The augmented adaptive sliding mode controller with the online parameter estimation algorithm, which is designed to estimate the unknown physical parameters of the robotic manipulator such as mass and moment of inertia, is developed.[1] These researchers were dedicated to the identification schemes which can be used to identify the set of mass parameters. These techniques can apply to both geared and direct drive robots for capturing target with unknown payload.

The observer-based control techniques exploit the compensation control structure of manipulator, and effective techniques have been developed.[7–10] Meanwhile, the observer-based sliding mode control (OBSMC) chattering can be reduced by decreasing the switching gain without sacrifice of the disturbance rejection ability of sliding mode control (SMC). In reality, an excess of control effort will be expended for suppressing the transients due to system uncertainties, if the developed approaches rely heavily on this

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conservative upper bound. This will degrade SMC performance such as tracking precision and undesired chattering.[11]

This article reviews a number of widely used nonlinear uncertainty estimation techniques.[12] In this context, by estimating the unknown parameters of robotic manipulator via an adaptive nonlinear observer,[1] the SMC scheme has been designed without requiring the multiaxis force/torque sensors or information on payload. Chen and Guo[13] have described that the closed-loop system under observer-based control is input-to-state stable (ISS) with the assumption that the disturbance is bounded variation. Therefore, the influence of the uncertainties can only be attenuated to a specified level. Several works[14–17] have developed an SMC approach for systems with mismatched uncertainties via a nonlinear observer based on the assumption that the derivative of the disturbance is zero when time approaches to infinity. In general, these observers can only be available for slowly time-varying disturbance. However, it is not a reasonable assumption for practical systems, since many engineering systems may suffer from time-varying disturbance, such as the uncertainty, which consists of both high-frequency and low-frequency parts.

Furthermore, the gain matrix of observer is a crucial design parameter to approximate the unknown uncertain. However, according to Yang et al.,[14] for multi-input and multi-output (MIMO) system, there is no general design procedure for designing observer gain. The time-consuming trials and error are inevitable to choose proper gain.

In this article, we will further investigate the design of observer-based control techniques for the tracking trajectory of robotic manipulator, with the explicit consideration of the unknown external force. By designing a novel adaptive gain algorithm, the observer states can be driven to the desired equilibrium in the presence of the unknown constant payload. There are mainly two contributions of this article as follows:

1. A general design procedure for designing observer gain matrix is developed via a novel adaptive scheme. From this point of view, in comparison with the existing observer[18–20] in literature, such as nonlinear observer, this work is the extension of the nonlinear disturbance observer design approach to MIMO system.
2. The boundary assumption of disturbance in SMC is relaxed by the developed disturbance observer technique. Similar to the approach,[14] the chattering can be substantially alleviated by designing the switching gain to be greater than the bound of the estimation error rather than that of the disturbance.

The rest of this article is arranged as follows. Section "Problem formulation" presents the control problem formulation. In section "Basic control design," some results of basic control design for manipulator will be recalled. In section "Main results," as main results of this brief, the design and analysis of adaptive nonlinear observer for robotic manipulators is presented. Then, SMC with adaptive nonlinear observer is derived in section "SMC design with adaptive nonlinear observer." In section "Simulation example and comparisons," simulation example and comparisons are present. The conclusions are finally given in section "Conclusion."

Problem formulation

For the considered n-link rigid manipulator, its dynamic can be described by the Lagrange–Euler vector equation in the joint space

$$M(\theta)\ddot{\theta} + C(\theta,\dot{\theta})\dot{\theta} + G(\theta) = \boldsymbol{\tau} + J^{\mathrm{T}}\Delta \tag{1}$$

where $\theta \in \Re^n$, $\dot{\theta} \in \Re^n$, and $\ddot{\theta} \in \Re^n$ are the joint angle vector, the joint velocity vector, and the joint acceleration vector, respectively; $M(\theta) \in \Re^{n\times n}$ is the positive definite inertia matrix for all θ; $C(\theta,\dot{\theta}) \in \Re^{n\times n}$ is the centrifugal and Coriolis matrix; $G(\theta) \in \Re^n$ is the gravitational torque; and $\tau \in \Re^n$ is control torque acting on joints. The torque in joint space $J^{\mathrm{T}}\Delta \in \Re^n$ represents the disturbance torque vector generated by the unknown forces $\Delta \in \Re^n$ applied at the end effector. $J \in \Re^{n\times n}$ can also be the manipulator Jacobian matrix from the task space to joint space. The force in task space Δ represents the interaction of the end effector with environment such as the force exerted by the object on the end effector or external disturbances.

A nonlinear observer will be derived in this article to estimate the unknown external input Δ. The influences of Δ will be compensated in feedback control channel. Control object is to make the angle of joint follow the reference trajectory well.

Basic control design

Tracking error is given by $e(t) = \theta - \theta_d$, where $\theta_d \in \Re$ is the reference trajectory in kinematic coordination. The design task is to develop a control law for manipulator system such that tracking error $e(t)$ converges to zero in the presence of the unknown payload. To do that, the sliding variable is selected as

$$s = \dot{e} + \Lambda e \tag{2}$$

where Λ is the positive constant diagonal matrix. When the sliding mode $s = 0$ is reached, $e = 0$ is the attractor of system error dynamic $\dot{e} + \Lambda e = 0$. Substituting equation (2) into equation (1) yields

$$M(\theta)\left(\ddot{\theta} - \dot{s}\right) + C\left(\theta,\dot{\theta}\right)\left(\dot{\theta} - s\right) + G(\theta) = \Phi\left(\theta,\dot{\theta},\dot{\theta}_r,\ddot{\theta}_r\right) \tag{3}$$

where $\Phi(\theta,\dot{\theta},\dot{\theta}_r,\ddot{\theta}_r) = M(\theta)\ddot{\theta}_r + C(\theta,\dot{\theta})\dot{\theta}_r + G(\theta)$ and $\dot{\theta}_r = \dot{\theta}_d - \Lambda e$. Then, using equation (1), we can write equation (3) in the following form

$$M(\theta)\dot{s} + C(\theta,\dot{\theta})s = \tau - \Phi(\theta,\dot{\theta},\dot{\theta}_r,\ddot{\theta}_r) + J^{\mathrm{T}}\Delta \tag{4}$$

Consider the candidate Lyapunov function $V = 1/2(s^T M(\theta)s)$. Then, its time-derivation along the trajectory of equation (4) is $\dot{V} = s^T M(\theta)\dot{s} + 1/2(s^T\dot{M}(\theta)s)$. In view of fundamental property that the matrix $\dot{M}(\theta) - 2C(\theta,\dot{\theta})$ is a skew symmetric matrix and substituting equation (4), $\dot{V}$ can be written as

$$\dot{V} = s^T M(\theta)\dot{s} + s^T C(\theta,\dot{\theta})s = s^T \left(\tau - \Phi(\theta,\dot{\theta},\dot{\theta}_r,\ddot{\theta}_r) + J^{\mathrm{T}}\Delta\right) \tag{5}$$

Now, the basic sliding mode control (BSMC) law can be derived as

$$\tau = \Phi(\theta,\dot{\theta},\dot{\theta}_r,\ddot{\theta}_r) - \rho_0 \mathrm{sgn}(s) \tag{6}$$

where ρ_0 is a constant. It is common that $J^{\mathrm{T}}\Delta$ is assumed to be bounded, that is, $\|J^{\mathrm{T}}\Delta\| \leqslant \rho$. If we have obtained a bound ρ_0 by the priori knowledge such that $\rho_0 > \rho$, then $\dot{V}$ can be simplified as

$$\dot{V} = s^T\left(J^{\mathrm{T}}\Delta - \rho_0\mathrm{sgn}(s)\right) \leqslant -(\rho_0 - \rho)\|s\| \leqslant 0 \tag{7}$$

It is clear that if $s \neq 0$, then $\dot{V}$ is negative. Using Barbalat's Lemma, we can conclude that the trajectory of the system equation (1) can be driven onto sliding surface $s(t) = 0$ as $t \to +\infty$.

It is noted that even if the bound can be obtained sometimes, it is usually very conservative. For example, it is assumed that uncertainties are bounded and the upper bound is known.[21] And the conservative upper bound is usually used in control design.[22,23] Furthermore, it can be seen from equation (6) that the undesirable chattering is reduced by tuning the parameters ρ_0. A practical approach that replaces signum function by a continuous approximation $s/(s+\varepsilon)$, where ε is a small positive constant, will be applied in the experimental tests to reduce the chattering. Moreover, a smaller ε leads to a better approximation performance.[5] To remove the chattering, a saturation function $\mathrm{sat}(x)$ is used to replace the signum function.[11] However, the disadvantage of this method is that the original robustness and control performance will be degraded such as reducing tracking positional accuracy and increasing steady-state errors. In addition, the upper bound of uncertainty, in practical systems, may not be easily obtained due to the completely unknown targets. The gain ρ_0 in equation (6) needs to be selected large enough when the bound is not exactly settled. The large gain will result in violent chattering or control input saturation.

In addition, SMC based on disturbance observer is regarded as an effective scheme to reduce the chattering. If the observer cannot track the unknown disturbances, it can only guarantee the bounded motion around the sliding surface. The boundary layer of the sliding surface is determined by the estimation error.[11] Thus, the performances of the disturbance observer are more important, since it not only determines the behavior of the sliding surface but also impacts the decrease in undesired chattering. This method needs time-consuming trials and error to choose proper gains. From another point of view, the fixed gain implies that there would be still considerable room for control performance improvement by appropriately estimating them. In view of these situations, a new nonlinear observer will be introduced in the following section.

Main results

In this section, OBSMC is developed in the framework of the observer-based control design. The objective of OBSMC is twofold: (1) design a new nonlinear observer to estimate the unknown parameters of manipulators and (2) design OBSMC such that the system outputs can track accurately the reference trajectory during capturing target with unknown payload.

The design of nonlinear observer

We address the identification of manipulator mode parameters based on the nonlinear observer in this subsection. For this purpose, inspired by Chen and Chen,[18] the adaptive nonlinear disturbance observer (ANDO) for dynamic equation (1) is designed as

$$\begin{aligned}\dot{z} &= -l(t)J^{\mathrm{T}}z - l(t)\left(J^{\mathrm{T}}l(t)M(\theta)\dot{\theta} + f(t)\right) \\ &\quad - \left(p(t)M(\theta) + l(t)\dot{M}(\theta)\right)\dot{\theta} \\ \hat{\Delta} &= z + l(t)M(\theta)\dot{\theta}\end{aligned} \tag{8}$$

where $\hat{\Delta} \in \Re^n$ is the estimation of unknown term Δ, $z \in \Re^n$ is the state vector of the observer, $l(t) \in \Re^{n\times n}$ is the observer gain, $p(t) \in \Re^{n\times n}$ is the design parameter matrix, and $f(t) = \tau - C(\theta,\dot{\theta})\dot{\theta} - G(\theta)$.

Now, define the estimation error as

$$\tilde{\Delta} = \Delta - \hat{\Delta} \tag{9}$$

From equation (9), it can be obtained that

$$\dot{\tilde{\Delta}} = \dot{\Delta} - \dot{\hat{\Delta}} \tag{10}$$

Substituting Equations (1) and (8) into the above equation yields

$$\dot{\hat{\Delta}} = l(t)J^{\mathrm{T}}z + l(t)\left(J^{\mathrm{T}}l(t)\dot{\theta}\right) - l(t)J^{\mathrm{T}}\Delta + \dot{\Delta} \tag{11}$$

Applying $\mathrm{z} = \hat{\Delta} - l(t)M(\theta)\dot{\theta}$, it follows that

$$\dot{\hat{\Delta}} = l(t)J^T\hat{\Delta} - l(t)J^T\Delta + \dot{\Delta} \tag{12}$$

It can be shown that the estimation error is governed by

$$\dot{\tilde{\Delta}} = -l(t)J^T\tilde{\Delta} + \dot{\Delta} \tag{13}$$

Assumption 1. The derivative of the unknown external input in robotic manipulator system equation (1) is bounded, that is, $\dot{\Delta} < \infty$.

This is an assumption made for the continuous and bounded disturbances.

Lemma 1. Suppose that Assumption 1 is satisfied. The estimation error system equation (13) is locally ISS if the observer gain $l(t)J^{\mathrm{T}}$ is chosen such that

$$\dot{\tilde{\Delta}} = - l(t)J^{\mathrm{T}}\tilde{\Delta} \tag{14}$$

is asymptotically stable.

The proof of this Lemma 1 can also be derived by combining the result of Lemma 1 with the ISS definition in Khalil (1996).

Lemma 2. Consider a nonlinear system $\dot{x} = F(x, \omega)$ which is ISS.[14] If the input satisfies $\lim_{t\to\infty} \omega(t) = 0$, then the state $\lim_{t\to\infty} x(t) = 0$.

The design of the adaptive observer gain

It is more useful to develop a systematic method for the design of the observer gain.

Lemma 3. Let A be a constant matrix with $Re(\sigma(A)) < 0$. The all solution of

$$\dot{x}(t) = (A + C(t))x(t) \tag{15}$$

is globally asymptotically stable, if $C(t)$ is the continuous matrix valued function on the interval $[0, \infty)$ such that

$$C(t) < ce^{\delta t}, \forall t \geq 0 \tag{16}$$

for some constants, $c > 0$ and $\delta < 0$.

Proof. If we multiply equation (15) by the integrating factor e^{At} and integrating from 0 to t, we have

$$x(t) = e^{At}x_0 + \int_0^t e^{At}C(s)x(s)\mathrm{d}s \tag{17}$$

Since $Re(\sigma(A)) < 0$, there exists constants $\sigma < 0$ and $K > 0$, such that $e^{At} \leq Ke^{\sigma t}$ and $t \geq 0$. Using this estimate and taking norms in equation (17) yields

$$x(t) \leq Kx_0 e^{\sigma t} + \int_0^t Kx_0 e^{\sigma(t-s)}C(s)x(s)\mathrm{d}s \tag{18}$$

Applying Gronwall's inequality yields

$$e^{-\sigma t}x(t) \leq Kx_0 + \int_0^t K^2 x_0 C(s) exp\left[\int_s^t KC(u)\mathrm{d}u\right]\mathrm{d}s \tag{19}$$

We make the estimate

$$exp\left[\int_s^t KC(u)\mathrm{d}u\right] \leq exp\left[\int_s^\infty Kce^{\delta u}\mathrm{d}u\right] \leq e^{cKN} = c_0 \tag{20}$$

where $N = \int_s^\infty e^{\delta u}\mathrm{d}u$. Then, we get

$$e^{-\sigma t}x(t) \leq Kx_0 + c_0K^2x_0\int_0^t C(s)\mathrm{d}s \leq Kx_0 + c_0K^2Nx_0 \tag{21}$$

In short, $x(t) \leq be^{\sigma t}$ for some constant b. since σ is negative, we can conclude that $x(t) \to 0$, as $t \to \infty$. Thus, this completes the proof of Lemma 2.

The design of the observer gain $l(t)$ in equation (8) can be described by Theorem 1.

Theorem 1. The estimation error system equation (14) is asymptotically stable if the observer gain $l(t)$ in equation (8) is governed by

$$\begin{cases} \dot{l}(t) = p(t) \\ p(t)J^{\mathrm{T}} = - l(t)(\dot{J}^{\mathrm{T}} + \mu J^{\mathrm{T}}) + \mu\eta I \end{cases} \tag{22}$$

where η and μ are the positive constant and I is the identity matrix of order n.

Proof. Let $l^*(t)$ denotes the theoretical solution of the following equation $l(t)J^{\mathrm{T}} = \eta I$ such that

$$l^*(t)J^{\mathrm{T}} = \eta I \tag{23}$$

Define $\tilde{l}(t) = l(t) - l^*(t)$, where $\tilde{l}(t)$ denotes the error between $l(t)$ and $l^*(t)$. Taking derivative with respect to time, we have

$$\dot{\tilde{l}}(t) = \dot{l}(t) - \dot{l}^*(t) \tag{24}$$

It can be directly obtained from equation (22) that

$$p(t)J^{\mathrm{T}} + \mu l(t)J^{\mathrm{T}} - \mu\eta I = - l(t)\dot{J}^{\mathrm{T}} \tag{25}$$

Applying equations (22)–(25), we can obtain

$$\begin{aligned} (\dot{\tilde{l}}(t) + \mu\tilde{l}(t))J^{\mathrm{T}} &= p(t)J^{\mathrm{T}} + \mu l(t)J^{\mathrm{T}} - \mu l^*(t)J^{\mathrm{T}} \\ &\quad - \dot{l}^*(t)J^{\mathrm{T}} \\ &= p(t)J^{\mathrm{T}} + \mu l(t)J^{\mathrm{T}} - \mu\eta I - \dot{l}^*(t)J^{\mathrm{T}} \\ &= - l(t)\dot{J}^{\mathrm{T}} + l^*(t)\dot{J}^{\mathrm{T}} \\ &= - \tilde{l}(t)\dot{J}^{\mathrm{T}} \end{aligned} \tag{26}$$

It is immediately following that

$$\dot{\tilde{l}}(t)J^{\mathrm{T}} + \tilde{l}(t)\dot{J}^{\mathrm{T}} = - \mu\tilde{l}(t)J^{\mathrm{T}} \tag{27}$$

Define the energy function $E(\tilde{l}(t))$ as

$$E(\tilde{l}(t)) = \tilde{l}(t)J^{T2}_F = \mathrm{trace}\left\{\left[\tilde{l}(t)J^T\right]^T \times \left[\tilde{l}(t)J^T\right]\right\} \tag{28}$$

where $\|.\|_F$ is the Frobenius norm. Then

$$\frac{dE(\tilde{l}(t))}{dt} = \text{trace}\left\{\frac{\partial E(\tilde{l}(t))}{\partial(\tilde{l}(t)J^T)}\frac{\mathrm{d}(\tilde{l}(t)J^{\mathrm{T}})}{\mathrm{d}t}\right\} = \text{trace}\left\{2\left[\tilde{l}(t)J^{\mathrm{T}}\right]^{\mathrm{T}}\left(\dot{\tilde{l}}(t)J^{\mathrm{T}} + \tilde{l}(t)\dot{J}^{\mathrm{T}}\right)\right\} = -2\mu E(\tilde{l}(t))$$

Hence, for any initial state $\tilde{l}(0)$, there holds

$$E(\tilde{l}(t)) = e^{-2\mu t}E(\tilde{l}(0)) \tag{29}$$

Thus, $E(\tilde{l}(t))$ will converge to zero with decaying rate of $e^{-2\mu t}$. It implies that $\tilde{l}(t)J^{\mathrm{T}}$ globally converges exponentially to zero, as $t \to \infty$. Namely

$$\tilde{l}(t)J^{\mathrm{T}}{}_F = e^{-\mu t}l(0)J^{\mathrm{T}}(0) - \eta I_F \to 0, \text{as} t \to \infty \tag{30}$$

According to the definition of $\tilde{l}(t)$, we have

$$l(t)J^{\mathrm{T}} = \eta I + Ke^{-\mu t} \tag{31}$$

where $K = l(0)J^{\mathrm{T}}(0) - \eta I_F$.

Since $l(t)J^{\mathrm{T}}$ globally converges exponentially to ηI, the $l(t)J^{\mathrm{T}}$ converges into a residual set of ηI. According to Lemma 3, the estimation error system equation (14) is asymptotically stable. Thus, this completes the proof of Theorem 1.

With the result of Theorem 1, it can be derived from Lemma 1 that the estimation error system equation (13) is ISS.

Assumption 2. The derivative of the unknown external input in robotic manipulator system equation (1) is bounded and satisfies $\lim_{t\to\infty}\dot{\Delta} = 0$.

Based on Lemma 2 and the condition given in Assumption 2, the estimation errors in equation (13) satisfy $\lim_{t\to\infty}\tilde{\Delta} = 0$. This implies that the estimation error will slide to the desired equilibrium point asymptotically.

The structure of adaptive disturbance observer gain is specified in Figure 1.

Remark 1. As a further extension to the existing work,[18] a novel nonlinear observer is developed in this study in an effort to provide a new approach along the line of adaptive technique.

Remark 2. It is noted that an alternative approach is that the term J^T can be included in to-be-estimated lumped uncertainty, that is, $J^{\mathrm{T}}\Delta$ in equation (1). However, the lumped disturbances do not satisfy the assumption that its derivative is a constant steady-state value, that is

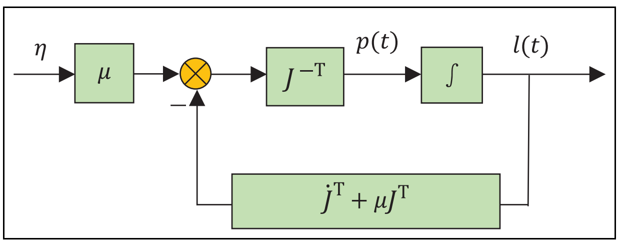

Figure 1. Block diagram for dynamic of adaptive nonlinear disturbance observer gain $l(t)$.

$$\lim_{t\to\infty}\frac{J^{\mathrm{T}}\Delta}{\mathrm{d}t} = 0 \tag{32}$$

This implies that the estimate error cannot be driven to the desired equilibrium point. The adverse effects of $J^{\mathrm{T}}\Delta$ on the estimate error will be shown in the latter simulation comparison studies.

SMC design with adaptive nonlinear observer

Assumption 3. The disturbance estimation error in equation (13) is bounded by $\gamma^* = \sup_{t>0}\|\Delta - \hat{\Delta}\|$.

Using the appropriate estimate of the disturbance, we obtain the following result presented in Theorem 2.

Theorem 2. Suppose that Assumptions 2 and 3 are satisfied for system equation (1). Considering the sliding variable equation (2) and nonlinear disturbance observer equation (8) under the proposed adaptive observer gain equation (22), state of the robotic manipulator system can be driven onto the sliding surface $s(t) = 0$ with the control law

$$\tau = \Phi(\theta, \dot{\theta}, \dot{\theta}_r, \ddot{\theta}_r) - J^{\mathrm{T}}\left(\hat{\Delta} + \gamma\text{sgn}(s^{\mathrm{T}}J^{\mathrm{T}})\right) \tag{33}$$

where $\gamma > \gamma^*$.

Proof. Consider the following Lyapunov function

$$V = \frac{1}{2}s^{\mathrm{T}}M(\theta)s \tag{34}$$

Taking the derivative V along the trajectory of equation (1) yields

$$\dot{V} = s^{\mathrm{T}}M(\theta)\dot{s} + \frac{1}{2}s^{\mathrm{T}}\dot{M}(\theta)s \tag{35}$$

For the dynamic system (1), the matrix $(1/2)\dot{M}(\theta) - C(\theta, \dot{\theta})$ is a skew symmetric matrix for all $\theta \in \Re^n$. Based on this fundamental property, it can be derived that

$$\dot{V} = s^{\mathrm{T}}M(\theta)\dot{s} + s^{\mathrm{T}}C(\theta, \dot{\theta})s \tag{36}$$

Substituting Equations (4) and (33) into equation (36) yields

$$\begin{aligned}\dot{V} &= s^{\mathrm{T}}(J^{\mathrm{T}}\Delta - J^{\mathrm{T}}(\hat{\Delta} + \gamma\text{sgn}(s^{\mathrm{T}}J^{\mathrm{T}}))) \\ &= -\gamma s^{\mathrm{T}}J^{\mathrm{T}}\text{sgn}(s^{\mathrm{T}}J^{\mathrm{T}}) + s^{\mathrm{T}}J^{\mathrm{T}}(\Delta - \hat{\Delta}) \\ &\leqslant -\gamma\|s^{\mathrm{T}}J^{\mathrm{T}}\| + \gamma^*\|s^{\mathrm{T}}J^{\mathrm{T}}\|\end{aligned} \tag{37}$$

By Theorem 1, the magnitude of the estimation error $\|\Delta - \hat{\Delta}\|$ asymptotically converges to zero in terms of the unknown constant payload Δ. Thus, there exists a sufficiently small positive control parameter γ such that $\gamma > \gamma^*$. According to equation (37), it holds that $\dot{V} < 0$ when $s \neq 0$. Thus, the system satisfies the Lyapunov stability theory. Then, the system state can reach the sliding mode $s = 0$ as $t \to +\infty$.

In the sliding mode $s = 0$, there is $\dot{e} + \Lambda e = 0$ with the positive constant diagonal matrix Λ. The state of system equation (1) satisfies $\lim_{t \to \infty} e(t) = 0$. Thus, the system state will converge to reference trajectory under the proposed control law equation (33).

Remark 3. Because the main cause of the chattering comes from the mandatory term $\gamma \mathrm{sgn}(s^{\mathrm{T}} J^{\mathrm{T}})$ in equation (33), which is used for suppression of the error of estimate, the chattering is reduced by tuning the controller parameter γ properly. Controller parameters selection is mainly conducted with empirical regularity. However, the following analysis can keep us on track amid controller parameter selection. Theoretically, an optimal controller parameters selection is sought to minimize γ subject to inequality constraint $\gamma > \|\Delta - \hat{\Delta}\|$. This means that controller parameter γ is usually selected to be a sufficiently small constant in practice. In addition, in terms of the unknown constant payload Δ, the magnitude of the estimation error $\|\Delta - \hat{\Delta}\|$ asymptotically converges to zero. Therefore, controller parameter γ can be also selected to be variable, which is asymptotically convergent in accordance with the convergence rate of observer.

Remark 4. The performances of the disturbance observer are more important, since it not only determines the behavior of the sliding surface but also impacts the decrease in undesired chattering. For the unknown constant payload, the proposed observer provides a new approach of the decrease in the chattering. From this point of view, the developed observer has distinctive qualities on OBSMC. Compared with BSMC law equation (6), the decrease in the chattering for the control law equation (33) is achieved when the inequality $\gamma \|J^{\mathrm{T}}\| < \rho_0$ is satisfied.

Remark 5. In this article, we do not consider some physical constraints of the manipulator, such as the dynamic of its actuators and the amplitude of the control signal, which are the potential causes of some performance degradation. However, it should be noted that we develop a general design procedure for designing the gain of the nonlinear observer. This new adaptive strategy is absolutely available for the general nonlinear control system design. The developed theoretical result and its applications are to provide a new research topic: the controller design with consideration of control input saturation or the dynamic of actuator.

Simulation example and comparisons

In this section, the simulations are conducted based on the robot manipulator with unknown parameters to validate the theoretical results and show the performance of the proposed strategies. It is reported that a drawback to nonlinear disturbance observer (NDO)[18] is that NDO has a poor performance to estimate the fast-varying disturbance due to the estimation error mainly depends on the frequency of disturbance.[16] To demonstrate the effectiveness of the proposed approach, NDO is employed for comparison.

The performance of ANDO

In this subsection, the numerical simulations with two-link robot manipulator are performed to investigate the performance of the proposed observer (equation (8)) when the mass of manipulator proceeds with abrupt changes. It is supposed that the unknown mass of the manipulator is the form which is shown in Figures 2 and 3. The parameters needed for observer are $\eta = 10$ and $\mu = 5$. The reference trajectory is given by $r = \sin(2t)$.

The model described in equation (1) for the two-link robot manipulator are given as follows

$$M(\theta) = \begin{bmatrix} (m_1 + m_2) r_1^2 + m_2 r_2^2 + 2 m_1 m_2 \cos\theta_2 & m_2 r_2^2 + 2 m_1 m_2 \cos\theta_2 \\ m_2 r_2^2 + 2 m_1 m_2 \cos\theta_2 & m_2 r_2^2 \end{bmatrix} \tag{38}$$

$$C(\theta, \dot{\theta}) = \begin{bmatrix} -m_2 r_1 r_2 \dot{\theta}_1 \sin\theta_2 & -m_2 r_1 r_2 \sin\theta_2 (\dot{\theta}_1 + \dot{\theta}_2) \\ m_2 r_1 r_2 \dot{\theta}_1 \sin\theta_2 & 0 \end{bmatrix} \tag{39}$$

$$G(\theta) = \begin{bmatrix} (m_1 + m_2) r_1 \cos\theta_2 + m_2 r_2 \cos(\theta_1 + \theta_2) \\ m_2 r_2 \cos(\theta_1 + \theta_2) \end{bmatrix} \tag{40}$$

The unknown mass is $\Delta = [\Delta_{m_1}\ \Delta_{m_2}]^{\mathrm{T}} \in \Re^2$. $J^{\mathrm{T}} = Y_m \Phi_m$. According to Slotine and Li,[24] the linear regression matrix Y_m and Φ_m can be expressed as

$$Y_m = \begin{bmatrix} y_{11} & y_{12} & y_{13} \\ y_{21} & y_{22} & y_{23} \end{bmatrix}, \Phi_m = \begin{bmatrix} r_1^2 & 0 & 0 \\ r_2^2 & r_2^2 & r_1 r_2 \end{bmatrix}^{\mathrm{T}} \tag{41}$$

with

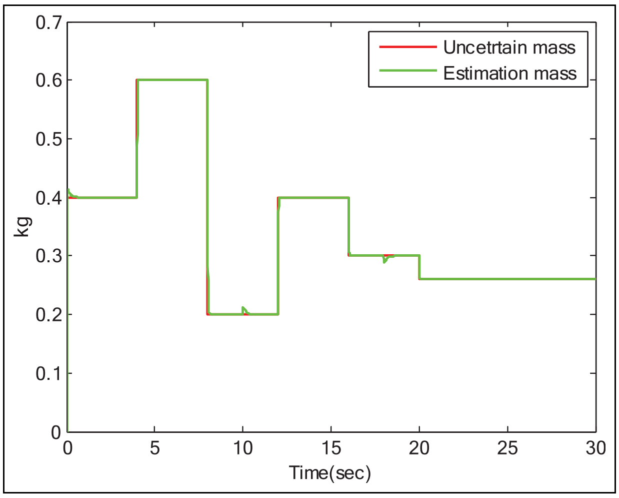

Figure 2. The unknown mass Δ_{m_1} and the estimation mass.

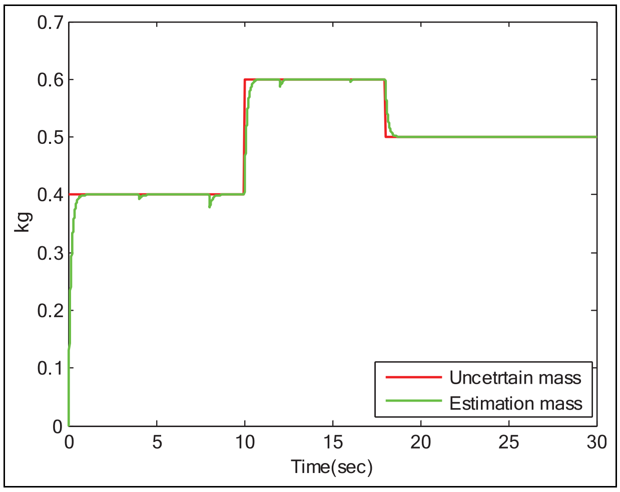

Figure 3. The unknown mass Δ_{m_2} and the estimation mass.

$$y_{11} = \ddot{\theta}_1 + \left(\frac{g}{r_1}\right)\cos\theta_2,\ y_{12} = y_{22} = \ddot{\theta}_1 + \ddot{\theta}_2,\ y_{21} = 0$$
$$y_{13} = 2\ddot{\theta}_1\cos\theta_2 + \ddot{\theta}_2\cos\theta_2 - \dot{\theta}_1\dot{\theta}_2\sin\theta_2 + \left(\frac{g}{r_1}\right)\cos(\theta_1 + \theta_2) - (\dot{\theta}_1 + \dot{\theta}_2)\dot{\theta}_2\sin\theta_2$$
$$y_{23} = \dot{\theta}_1^2\sin\theta_2 + \ddot{\theta}_1\cos\theta_2 + \left(\frac{g}{r_1}\right)\cos(\theta_1 + \theta_2) \tag{42}$$

The parameters of manipulator system are summarized as follows: $r_1 = 0.4\,\text{m}$ and $r_2 = 0.3\,\text{m}$. The gravitational acceleration are set to be $g = 9.81\,\text{m/s}^2$. The nominal values of the mass are $m_1 = 2.2\,\text{kg}$ and $m_2 = 1.6\,\text{kg}$.

The performances of the ANDO are shown in Figures 2 and 3. It is clear that the output of ANDO converges to the actual unknown mass Δ_{m_1} and Δ_{m_2} promptly even though the mass changes abruptly in some points. Form these results, the proposed ANDO provides a satisfactory estimation of the uncertain mass variation (Figure 4).

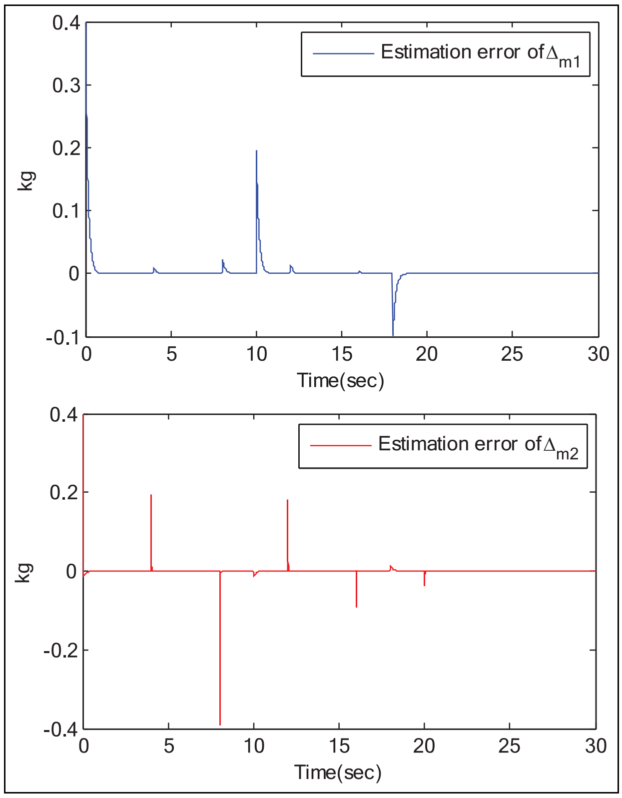

Figure 4. The estimation error of the unknown mass Δ_{m_1} and Δ_{m_2}.

Experimental results

To verify the effectiveness of the proposed OBSMC for fast reference trajectories, some experiments are performed in this section. For the considered single-link rigid manipulator, its dynamics can be described by

$$M\ddot{\theta} + mgL\cos\theta = \boldsymbol{\tau} + \Delta_m gL\sin\theta \tag{43}$$

where $\theta \in \Re$ is the joint angle. The parameters of the single-link manipulator are as follow. The load arm length is $L = 18.6\,\text{cm}$. The load arm mass is $m = 0.225\,\text{kg}$. The load mass is $m = 0.298\,\text{kg}$. The gravitational acceleration is set to be $g = 9.8\,\text{m/s}^2$. For the proposed control law, the parameters are selected by trial and error until an improved tracking performance is obtained. Control parameters and observer parameters in equation (8) are selected as $\gamma = 0.35$, $\eta = 6$, and $\mu = 2$ in the simulation. The unknown mass $\Delta_m \in \Re$ applied at the end effector.

The schematic of the experiment setup is depicted in Figure 5. The servo base unit is a geared servomechanism system. The plant consists of a DC motor in a solid frame. The DC motor on the rotary servo base unit is used to rotate the link from one end in the

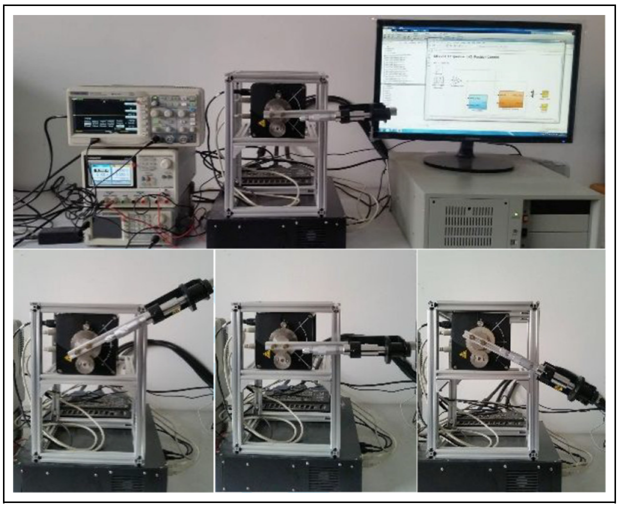

Figure 5. Test platform for manipulator to handle an unknown payload.

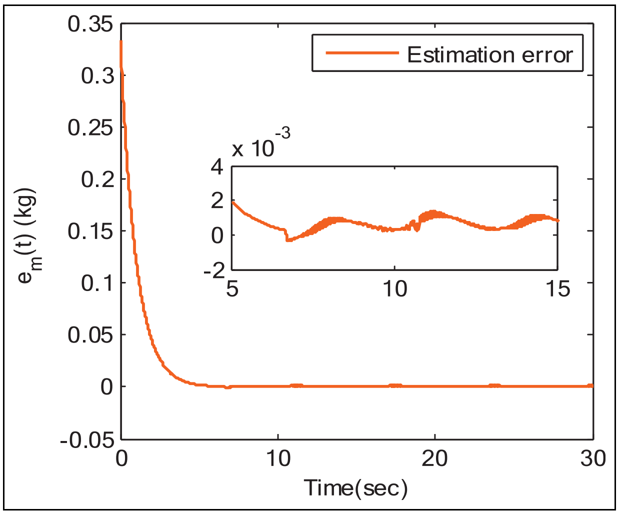

Figure 7. Estimation error $e_m(t)$ of the unknown payload Δ_m for $r_2 = \sin(2t)$.

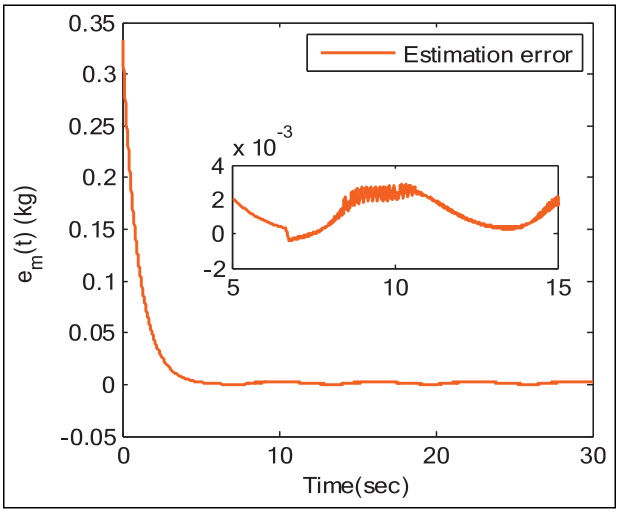

Figure 6. Estimation error $e_m(t)$ of the unknown payload Δ_m for $r_1 = \sin(t)$.

vertical plane. The position of the load shaft can be measured using a high-resolution encoder. The tachometer can measure the speed of the motor. The sensors are connected directly to the data-acquisition board. They provide the position feedback necessary to control the link. The data-acquisition board outputs a control voltage that is amplified and drives the motor.

Here, the unknown payload $\Delta_m = 0.15\,\text{kg}$, which is treated as an unknown term in simulation. The reference trajectories are given by $r_1 = \sin(t)$ and $r_2 = \sin(2t)$, respectively. First, we show the performance of the proposed nonlinear observer according to Equations (8) and (22) in Figures 6 and 7. It can be seen that the estimation error is ultimately bounded by 0.003. It shall be noticed that the proposed ANDO provides a satisfactory estimate for the unknown mass vector instead of the lumped uncertainty in this study. From Figures 6 and 7, we can know that the designed adaptive nonlinear observer is valid.

Figure 8 shows the position tracking with unknown moment inertia under OBSMC in equation (33). The proposed controller forces the position tracking error to the zero. The bounded control torque generated by OBSMC for manipulator system is shown in Figure 9. Meanwhile, it can be obviously seen that the OBSMC have very small vibration around the sliding mode surface. The reason behind this situation is that the proposed observer provides a proper control gain to ensure that the tracking error can reach into a small neighborhood of zero. Hence, the chattering problem is substantially alleviated. As stated in section "Problem formulation," the desired results for the OBSMC are obtained without the excessive controller chattering. The proposed ANDO is very effective in reducing chattering while achieving good tracking performance.

In summary, the proposed control scheme has good tracking performance and it works well for fast-varying references. Under a properly designed observer and proper sliding mode controller, the adverse impact caused by disturbance can be suppressed with the decrease in undesired control chattering.

Experimental comparisons

For the purpose of comparison, we evaluate the performance of the NDO in Chen and Chen[18] From equation (43), we can obtain the following form:

$$\ddot{\theta} = M^{-1}\boldsymbol{\tau} - M^{-1}mgL\cos\theta + d \tag{44}$$

where the lumped uncertainties $d = M^{-1}J^{\mathrm{T}}\Delta$; the load arm length $L = 19.8\text{cm}$. According to Chen and Chen,[18] for dynamic equation (44), the corresponding observer is reconstructed as follows

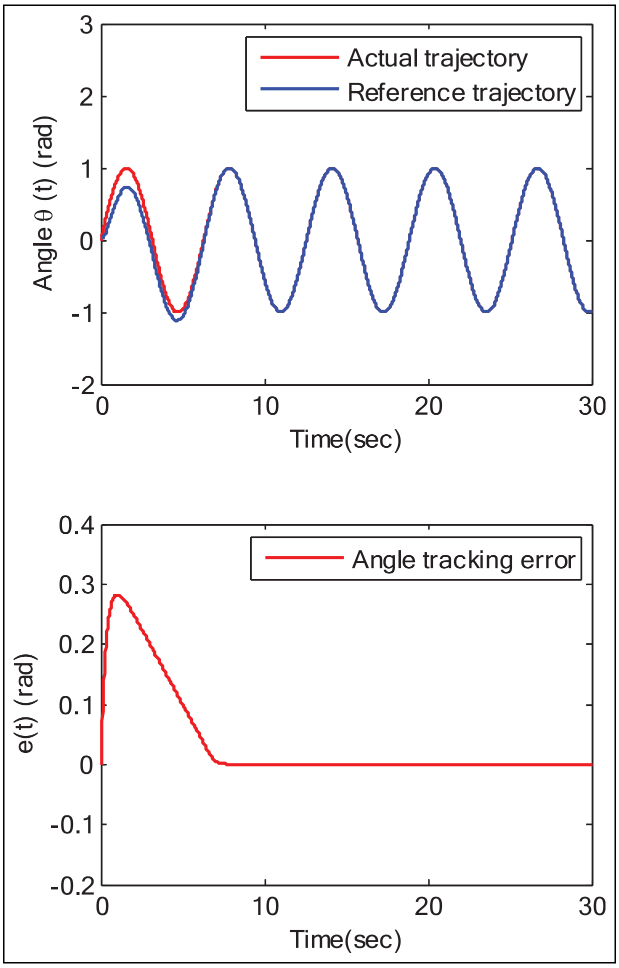

Figure 8. The response curves of the state $\theta(t)$ and its error $e(t)$ under OBSMC.

$$\begin{cases} \dot{z} = -lz - l\left(l\dot{\theta} + M^{-1}\tau - M^{-1}mgL\cos\theta\right) \\ \hat{d} = z + l\dot{\theta} \end{cases} \tag{45}$$

where $\hat{d}$ is the estimation of the disturbance d. The observer gain l is given by Chen and Guo.[13]

In order to investigate on the precise estimation problem of the observer equation (41) for the disturbance with different frequency, the following tested disturbances are considered to take the place of the uncertainty d in equation (44)

Case 1: $d_1(t) = \sin(0.2t)$
Case 2: $d_2(t) = \sin(0.6t)$
Case 3: $d_3(t) = \sin(t)$
Case 4: $d_4(t) = \sin(2t)$

The estimate errors for the four cases are illustrated by Figure 10. The variation disciplinarian of the maximum estimation error is also shown in Figure 10. It can be seen from these results that the change in the maximum estimation errors with the frequency of disturbance occurs, and the estimation accuracy mainly depends on the frequency of disturbance. The results

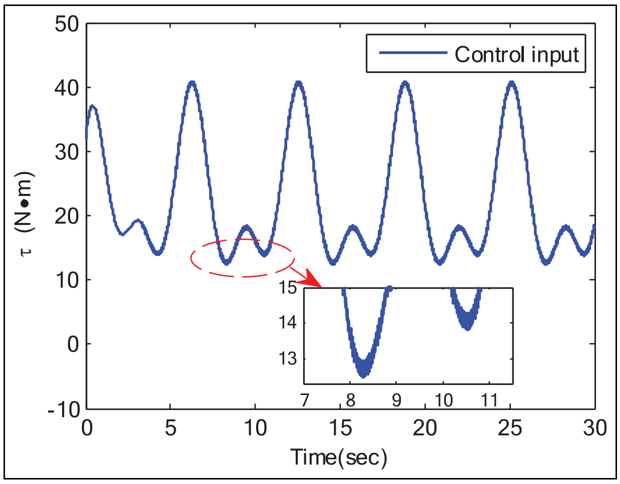

Figure 9. The response curves of the control input under OBSMC.

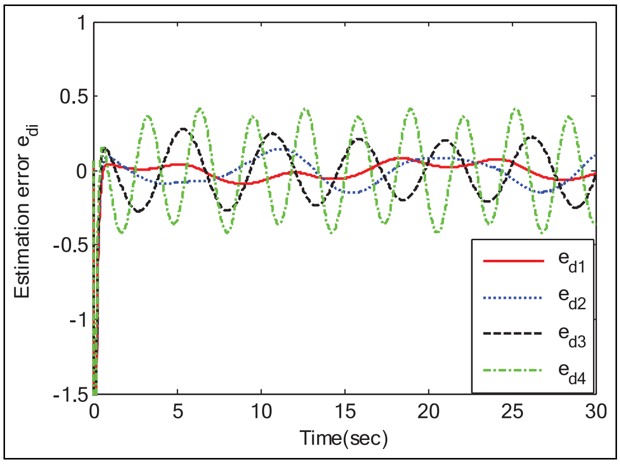

Figure 10. Comparisons of estimation error of NDO ($e_{\Delta i}$ is the estimation error for the disturbance d_i).

indicate that the frequency of disturbance is mainly attributed to this variation. It also reveals a fact that the observer[18] has to operate within the condition of its basic assumption that the estimated signal should be a slow-varying disturbance. In comparison with Chen et al.,[18] the proposed observer have a distinct advantage in this respect.

For BSMC, the controller parameter γ is selected by trial and error until an improved tracking performance is obtained. The values of $\gamma = 8$ are considered in equation (6). In order to evaluate the performance of the controller, the following indexes that include the integral of squared tracking error (ISTE) and the integral of squared control torque (ISCT) are considered as follows

$$\mathrm{ISTE} = \frac{1}{T}\int_0^T \|e(t)\|^2 \mathrm{d}t, \mathrm{ISCT} = \frac{1}{T}\int_0^T \|\tau(t)\|^2 \mathrm{d}t \tag{46}$$

Table 1. Performance indexes of controller.

Controller	ISTE	ISCT
Basic sliding mode control	0.0013	671.1907
Proposed sliding mode control	0.0035	582.4356

ISTE: integral of squared tracking error; ISCT: integral of squared control torque.

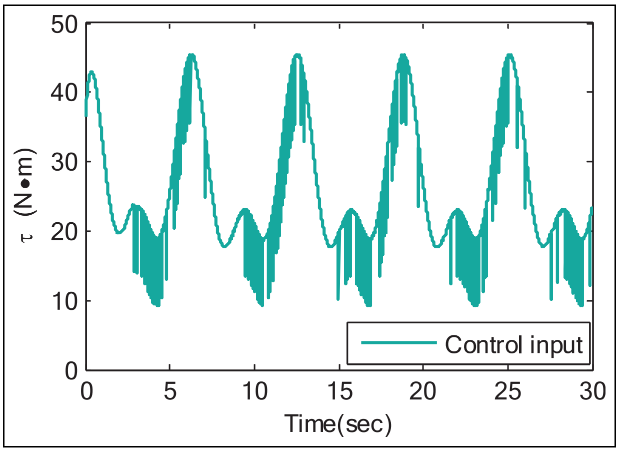

Figure 11. The response curves of the control input under BSMC.

Note that $T = 30$ s is selected for all cases. The values of these two indexes for each controller are summarized in Table 1.

Figure 11 depicts the response of control input for BSMC. As clearly shown in Figure 11, the controller has resulted in substantial chattering since a large switching gain. Furthermore, the performance index ISTE varies only from 0.0013 to 0.0035, whereas the performance ISTE decreases from 671.1907 to 582.4356 $N^2 m^2$. It shows that both approaches achieve similar performance in terms of control efficiency. However, it is clear that, when state trajectories cross the sliding surface, the undesired chattering can also be reduced effectively with the estimation of the disturbance by the ANDO in the presence of the unknown payload.

Conclusion

In this article, an OBSMC is developed for a class of robotic manipulator systems in the operating task such as target capturing task. To enhance the system performance, a novel nonlinear observer with adaptive scheme is designed, which can be used to approximate the corresponding uncertainties induced by the unknown payload of manipulators system. The proposed observer overcomes some restriction of the existing nonlinear disturbance observer. Asymptotic stability results are established when SMC is integrated with the proposed observer for manipulators with unknown payload. Finally, two examples are used to verify the effectiveness of the developed adaptive nonlinear observer and OBSMC scheme. The simulation results suggest that the proposed OBSMC is valid. The main advantages of this article is that a general design procedure for designing robust observer gain matrix is developed via a novel adaptive scheme, and the other is to extend the applicability of OBSMC to some engineering problem, such as nonlinear mechanical systems subject to variation of load and the vibration of mechanical structures.

Declaration of conflicting interests

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