2.1 Problem description

The structure of the logistics system under consideration is shown in Fig. 1. The products produced by suppliers will be shipped to customers via facilities to meet customers' demands. Each supplier can supply a wide variety of products to several facilities, and the supply capacity for each product is known. Our problem is to determine whether the existing facilities should be open or not, what the expansion size of each open facility should be, which new facilities should be established, and which customers should be served by an open facility so that the sum of the expansion costs, the savings from closing the existing facilities, the fixed setup costs, the facility operating costs and the transportation costs is minimized.

2.2 The model

Our problem is formulated as a mixed integer linear programming model (MILP), and the following parameters are used:

Parameters

- I Set of suppliers, $i \in I$
- J Set of customers, $j \in J$
- K Set of all facilities, $k \in K$. Note that $K_0 \subseteq K$ and $K_1 \subseteq K$ are the sets of the existing facilities and potential facilities, respectively
- L Set of products, $l \in L$
- $|\cdot|$ The total number of elements in set $\cdot$
- S_{il} Capacity of supplier i for product l
- D_{jl} Demand of customer j for product l
- f_k Fixed setup cost of facility k
- v_k Operating cost per unit of product at facility k
- R_k Maximum allowed expansion (additional) amount at facility k
- p_k Savings from closing facility k
- M_k^L Minimum required throughput at open facility k
- M_k^U Maximum allowed capacity before being expanded at facility k
- c_{ikjl} Unit cost of shipping product l from supplier i via facility k to customer j
- e_k Unit expansion cost of facility k

We define the following binary and continuous decision variables to determine whether facility k will be open or not, which customers will be served by facility k, the total throughput of facility k and the expansion size of each open facility. Here quadruply subscripted continuous variable x_{ikjl} is used to represent the logistics volume of a product from a supplier to a customer via a facility, which can easily track the origin of a product after it has arrived at a facility or a customer and can also give an advantage when Benders decomposition method is applied.

Decision variables

$$z_k = \begin{cases} 1 & \text{if facility } k \text{ is open} \\ 0 & \text{otherwise} \end{cases} \quad \forall k \in K_0 \cup K_1.$$

$$u_{kj} = \begin{cases} 1 & \text{if customer } j \text{ is served by facility } k \\ 0 & \text{otherwise} \end{cases} \quad \forall k \in K_0 \cup K_1; \forall j \in J.$$

x_{ikjl} Continuous variable which corresponds to the amount of product l shipped from supplier i via facility k to customer j

s_k Continuous variable which corresponds to the amount of capacity expansion of facility k

According to the above notations, the logistics facility location problem with capacity expansions of the existing facilities can be formulated as follows:

$$\text{Min} \sum_{k \in K_0 \cup K_1} e_k s_k + \sum_{i \in I} \sum_{k \in K_0 \cup K_1} \sum_{j \in J} \sum_{l \in L} c_{ikjl} x_{ikjl} + \sum_{k \in K_1} f_k z_k + \sum_{k \in K_0 \cup K_1} v_k \sum_{j \in J} \sum_{l \in L} D_{jl} u_{kj} - \sum_{k \in K_0} p_k (1 - z_k) \tag{1}$$

s.t.

$$\sum_{k \in K_0 \cup K_1} \sum_{j \in J} x_{ikjl} \le S_{il} \quad \forall i \in I; \forall l \in L. \tag{2}$$

$$\sum_{i \in I} x_{ikjl} = D_{jl} u_{kj} \quad \forall k \in K_0 \cup K_1; \forall j \in J; \forall l \in L. \tag{3}$$

$$M_k^L z_k \le \sum_{j \in J} \sum_{l \in L} D_{jl} u_{kj} \le M_k^U z_k + s_k \quad \forall k \in K_0 \cup K_1. \tag{4}$$

$$s_k \le R_k z_k \quad \forall k \in K_0 \cup K_1. \tag{5}$$

$$\sum_{k \in K_0 \cup K_1} u_{kj} = 1 \quad \forall j \in J. \tag{6}$$

$$x_{ikjl} \ge 0 \quad \forall i \in I; \forall k \in K_0 \cup K_1; \forall j \in J; \forall l \in L. \tag{7}$$

$$s_k \ge 0 \quad \forall k \in K_0 \cup K_1. \tag{8}$$

$$z_k \in \{0, 1\} \quad \forall k \in K_0 \cup K_1. \tag{9}$$

$$u_{kj} \in \{0, 1\} \quad \forall k \in K_0 \cup K_1; \forall j \in J. \tag{10}$$

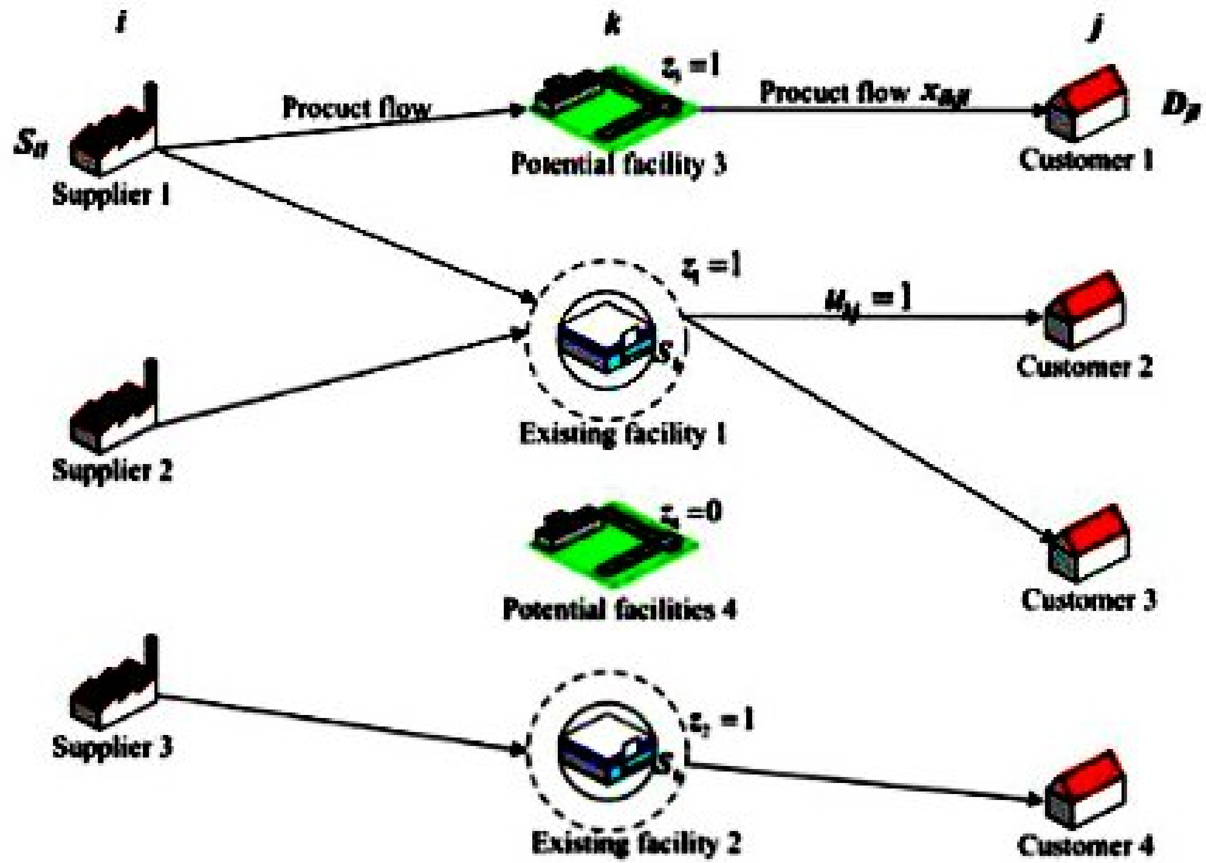

Fig. 1 The structure of the system under consideration

To facilitate describing and understanding, let D_l be the total demands for product l, and D be the total demands for all products. All parameters generated randomly obey the following uniform distributions. Transportation cost (c_{iljl}) is selected from set $\{1, \ldots, 10\}$. Demand (D_{jl}) is generated from interval [20, 100], and facility operating cost of unit product (v_k) is chosen from set $\{5, \ldots, 10\}$. In order to guarantee that a feasible solution can be obtained from the randomly generated data, the supply amount of each product (S_{il}) is generated from $[S_{1l}, S_{2l}]$, where $S_{1l} = D_l/|I|$, $S_{2l} = 2D_l/|I|$. Based on the rule that the total capacities of all the existing facilities are less than the total demands, the sum of the maximum allowed capacity before being expanded for all the existing facilities (defined as W_T) is generated randomly from $[2/5D, 2/3D]$, the maximum allowed capacity before being expanded for each existing facility (M_k^U ($\forall k \in K_0$)) is generated from $[4W_T/5|K_0|, 6W_T/5|K_0|]$, the minimum required throughput for each existing facility (M_k^L ($\forall k \in K_0$)) is generated from $[M_k^U/3, 2M_k^U/5]$. Let $N_T = D - W_T$, and the maximum allowed capacity before being expanded for each potential facility (M_k^U ($\forall k \in K_1$)) and the minimum required throughput for each potential facility (M_k^L ($\forall k \in K_1$)) are generated from $[N_T/|K_1|, N_T/2]$ and $[M_k^U/3, 2M_k^U/5]$, respectively. The fixed setup cost for each potential facility (f_k ($\forall k \in K_1$)) is generated from $[2M_k^U, 5M_k^U]$. Unit expansion cost (e_k) and the maximum allowed expansion amount (R_k) are generated from $[2f_k/M_k^U, 4f_k/M_k^U]$ and $[2M_k^U/5, M_k^U/2]$, respectively. The savings from closing each existing facility (p_k ($\forall k \in K_0$)) is generated from $[4f_k/5, f_k]$.

Table 1 Problem classes generated from the following uniform distributions

Class	$\lvert I \rvert$	$\lvert K_0 \rvert$	$\lvert K_1 \rvert$	$\lvert J \rvert$	$\lvert L \rvert$
C1	[2, 5]	[2, 5]	[2, 5]	[5, 10]	[3, 5]
C2	[2, 5]	[2, 5]	[2, 5]	[10, 30]	[3, 5]
C3	[2, 5]	[2, 5]	[5, 10]	[30, 50]	[3, 5]
C4	[5, 10]	[2, 5]	[5, 10]	[30, 50]	[3, 5]
C5	[2, 5]	[5, 10]	[5, 10]	[10, 30]	[5, 10]
C6	[5, 10]	[2, 5]	[5, 10]	[30, 50]	[5, 10]
C7	[5, 10]	[5, 10]	[5, 10]	[30, 50]	[5, 10]
C8	[10, 15]	[5, 10]	[5, 10]	[30, 50]	[10, 20]
C9	[10, 15]	[5, 10]	[15, 20]	[50, 80]	[10, 20]
C10	[15, 20]	[5, 10]	[5, 10]	[50, 100]	[30, 50]