JOURNAL OF OPTOELECTRONICS AND ADVANCED MATERIALS, Vol. 10, No. 10, October 2008, p. 2555 - 2563

Laser dressing effects on the energy spectra in different shaped quantum wells under an applied electric field

A. RADU, E. NICULESCU*, M. CRISTEA
[a]*Physics Department, University "Politehnica" of Bucharest, Splaiul Independenţei 313, Bucharest, 060042, Romania*

The influence of a laser field on the subband energy levels in finite V-shaped, parabolic and square GaAs-GaAlAs quantum wells under the electric field is studied by using a variational method in the effective mass approximation. The dependence of the interband optical absorption on the laser field amplitude, the geometric shape of the quantum wells, and the applied electric field is discussed.

(Received May 5, 2008; accepted August 14, 2008)

Keywords: Quantum wells, Laser effects, Static electric field, Absorption coefficient

1. Introduction

Quantum wells involving substantially inhomogeneous parts, rather than having a rectangular profile, have received increased interest because of their various applications. Such structures present desirable optical features for devices design and favorable "Stark shift" characteristics, which can be used to control and modulate the intensity output of the devices. For instance, the parabolic quantum wells (PQWs) have been used as the graded barrier part of the quantum-well lasers to improve the optical confinement factor and enhance the carrier collection into the thin quantum well, so as to reduce the threshold current density [1,2]. PQWs are also used to design infrared detectors with low leakage currents [3] and employed in resonant tunneling devices for their potential applications in high-speed circuits [4]. V-shaped quantum wells (VQWs) have been grown and experimentally studied, these structures being potentially interesting for designing millimeter wave communication devices [5], light-emitting devices [6] and MOSFET devices [7]. As a consequence, many theoretical studies of the electronic states in graded composition quantum wells (GCQWs) with different profiles have been performed [8-13]. The general picture indicates a substantial dependence of the electronic states on the main features of the well profile.

The study of the electrostatic field effects on the low-dimensional systems is of great importance from both fundamental and applicative points of view therefore many intensive researches concerning this subject have been reported [12-18].

More recently, these studies have been extended to quantum wells under intense electric fields created by high-intensity THz lasers. Neto *et al*. [19] have derived the laser-dressed quantum well potential for an electron in a square quantum well (SQW), in the frame of a non-perturbation theory and a variational approach. A simple scheme based on the inclusion of the effect of the laser interaction with the semiconductor through the renormalization of the effective mass has been proposed by Brandi and Jalbert [20]. Ozturk *et al* [21] have investigated the effect of the laser field on the intersubband optical transitions for a graded QW under external electric field. Optical transitions involving acceptor impurities in semiconductors under infrared laser radiation have been studied by Nunes *et al*. [22]. They found that the IR laser field has direct influence on the absorption coefficient in GaAs irradiated by an intense CO_2 laser. A systematic study on the influence of two intense, long-wavelength, nonresonant laser fields on the electron levels and density of states in GaAs/AlGaAs quantum wells have been performed by Enders *et al* [23] within a Green's function approach. It is shown that the laser induced blueshifts depend on the laser intensity and frequency as well as on the polarization direction.

We recently investigated [24] the binding energy of an off-center donor hydrogenic impurity in a cylindrical quantum wire, in the simultaneous presence of an intense high frequency laser field and a static magnetic field. In this work, we studied the effects of the laser field on the energy spectra in finite VQW, PQW and SQW GaAs quantum wells under an electric field. The absorption coefficient related to the interband transitions is also discussed as a function of the laser parameter, geometric shape of the wells and the applied electric field. To our knowledge, this is the first study of the effects of laser fields on the energy spectra in triangular and parabolic quantum wells.

2. Theory

For a particle moving in a quantum well under the combined forces of a high-frequency laser field and a static electric field, the Hamiltonian is given by

$$H = \frac{(\boldsymbol{p} + q\boldsymbol{A})^2}{2m^*} + V(z) - qFz \,. \tag{1}$$

Here q and m^* are the particle electric charge and effective mass, respectively, $\boldsymbol{A}$ is the vector potential of the laser field, $V(z)$ is the quantum well potential energy and F is the strength of the external electric field applied along the growth direction.

2.1. Zero electric field case

When no electric field is present, the electron and hole single-particle bound states in a finite quantum well under a high-frequency laser field are obtained in the frame of a non-perturbative theory and a variational approach [19]. We assume that the laser field is non-resonant with the semiconductor structure and can be represented by a monochromatic plane wave of frequency ω. For a linear polarization along the z-direction, the vector potential of the laser field is $\boldsymbol{A}(t) = A_0 \cos(\omega t)\boldsymbol{u}_z$. Under these conditions the semiclassical, time-independent Schrödinger equation in the momentum gauge reads

$$\left[\frac{(\mathbf{p}+q\mathbf{A})^2}{2m^*} + V(z)\right]\Psi(z,t) = i\hbar\frac{\partial\Psi(z,t)}{\partial t} \qquad (2)$$

By applying a translation of vector

$$\boldsymbol{\alpha}(t) = -\frac{e}{m^*}\int_0^t \mathbf{A}(t')dt' = \alpha(t)\mathbf{u}_z \qquad (3)$$

the equation (2) becomes [26]

$$\left[-\frac{\hbar^2}{2m^*}\frac{d^2}{dz^2} + V(z+\alpha(t))\right]\Psi(z,t) = i\hbar\frac{\partial\Psi(z,t)}{\partial t} \qquad (4)$$

In the high-frequency limit the bound electron and hole states $\Psi^\sigma(z)$, $\sigma = e,h$, with energies ε_k^σ, are the solutions of the single-particle Schrödinger equation

$$\left[-\frac{\hbar^2}{2m_\sigma^*}\frac{d^2}{dz^2} + V_d^\sigma(z,\alpha_0^\sigma)\right]\Psi_k^\sigma(z) = \varepsilon_k^\sigma\Psi_k^\sigma(z). \qquad (5)$$

Here $V_d^\sigma(z,\alpha_0^\sigma)$ is the laser "dressed" potential [19]

$$V_d^\sigma(z,\alpha_0^\sigma) = \frac{\omega}{2\pi}\int_0^{2\pi/\omega} V^\sigma\left[z - \alpha_0^\sigma \sin(\omega t)\right]dt, \qquad (6)$$

and $\alpha_0^\sigma = \frac{q A_0}{m_\sigma^* \omega}$ is laser parameter. For a finite QW, the carrier confinement potentials are given by

$$V^\sigma(z) = \begin{cases} V_0^\sigma, |z| \geq b \\ V_0^\sigma \left|\frac{z}{b}\right|^n, |z| < b \end{cases}, \qquad (7)$$

where V_0^σ is the conduction (valence) band offset, and $b = L/2$ is the half-width of the quantum well. n is equal to 1, 2, and ∞, corresponding to the V-shaped quantum well (VQW), parabolic quantum well (PQW) and square quantum well (SQW), respectively.

It is significant to emphasize that the approach (6) of the laser-dressed potential is valid for high-frequency regime, e.g., $\omega\tau >> 1$, where τ is the transit time of the electron in the quantum structure [19]. For a GaAs-$Ga_{0.7}Al_{0.3}As$ square quantum well with a width of about 100 Å, subjected to a monochromatic CO_2 laser beam ($\omega \approx 10^{14}\ s^{-1}$), this condition is very well satisfied [22]. Since, for a given barrier height V_0^σ, the quantum confinement increases as n decreases, one may expect that the high-frequency condition will be even better verified for PQWs and VQWs.

Instead of the laser dressed potential of the graded QW obtained in a square-wave approach [12], we used an exact analytical form obtained from Eq. (6). Thus for $n = 1$ and $n = 2$ the laser-dressed potential $V_d^\sigma(z,\alpha_0^\sigma)$ is given by:

$$V_d^\sigma(z,\alpha_0^\sigma)^{VQW} = \begin{cases} \frac{V_0^\sigma\alpha_0^\sigma}{\pi b}\left[\pi\frac{|z|}{\alpha_0^\sigma} - 2\frac{|z|}{\alpha_0^\sigma}\arccos\chi(z) + 2\sqrt{1-\chi^2(z)}\right], & |z| \in D_1; \\ V_0^\sigma + \frac{V_0^\sigma\alpha_0^\sigma}{\pi b}\left[-2\frac{|z|}{\alpha_0^\sigma}\arccos\chi(z) + 2\sqrt{1-\chi^2(z)} + \frac{|z|-b}{\alpha_0^\sigma}\arccos\frac{|z|-b}{\alpha_0^\sigma} - \sqrt{1-\left(\frac{|z|-b}{\alpha_0^\sigma}\right)^2}\right], & |z| \in D_2; \\ V_0^\sigma, & |z| \in D_3. \end{cases} \qquad (8)$$

$$V_d^\sigma(z,\alpha_0^\sigma)^{PQW} = \begin{cases} \frac{V_0^\sigma}{2b^2}\left[2z^2 + \alpha_0^{\sigma 2}\right], & |z| \in D_1; \\ V_0^\sigma + \frac{V_0^\sigma}{2\pi b^2}\left[\left(2z^2 + \alpha_0^{\sigma 2} - 2b^2\right)\arccos\frac{|z|-b}{\alpha_0^\sigma} - \alpha_0^\sigma(3|z|+b)\sqrt{1-\left(\frac{|z|-b}{\alpha_0^\sigma}\right)^2}\right], & |z| \in D_2; \\ V_0^\sigma, & |z| \in D_3. \end{cases} \qquad (9)$$

where $\chi(z) = \min\left(\frac{|z|}{\alpha_0^\sigma}, 1\right)$, $D_1 = [0, b - \alpha_0^\sigma)$, $D_2 = [b - \alpha_0^\sigma, b + \alpha_0^\sigma]$, and $D_3 = (b + \alpha_0^\sigma, +\infty)$.

As in Ref. [19], for the "dressed potential" of a SQW we obtain the expression

$$V_d^\sigma(z, \alpha_0^\sigma)^{SQW} = \begin{cases} 0, & |z| \in D_1; \\ \frac{V_0^\sigma}{\pi} \arccos \frac{b - |z|}{\alpha_0^\sigma}, & |z| \in D_2; \\ V_0^\sigma, & |z| \in D_3. \end{cases} \quad (10)$$

For $z \geq 0$, the wave functions $\phi_j^\sigma(z)$ of the non-perturbed quantum wells can be written as

$$\phi_j^\sigma(z)^{VQW} = \begin{cases} a_1^\sigma Ai\left[\left(\frac{2m_\sigma^* V_0^\sigma}{\hbar^2 b}\right)^{\frac{1}{3}}\left(z - b\frac{E_{0j}^\sigma}{V_0^\sigma}\right)\right] + a_2^\sigma Bi\left[\left(\frac{2m_\sigma^* V_0^\sigma}{\hbar^2 b}\right)^{\frac{1}{3}}\left(z - b\frac{E_{0j}^\sigma}{V_0^\sigma}\right)\right], & 0 \leq z < b \\ a_3^\sigma \exp\left\{-\left[\frac{2m_\sigma^*}{\hbar^2}\left(V_0^\sigma - E_{0j}^\sigma\right)\right]^{-1/2} z\right\}, & z \geq b \end{cases} \quad (11)$$

$$\phi_j^\sigma(z)^{PQW} = \begin{cases} a_1^\sigma \exp\left(-\sqrt{\frac{m_\sigma^* V_0^\sigma}{2\hbar^2}}\frac{z^2}{b}\right)\chi\left(\frac{1}{4} - \frac{E_{0j}^\sigma b}{4}\sqrt{\frac{2m_\sigma^*}{\hbar^2 V_0^\sigma}}, \frac{1}{2}, \sqrt{\frac{2m_\sigma^* V_0^\sigma}{\hbar^2}}\frac{z^2}{b}\right), & 0 \leq z < b \\ a_2^\sigma \exp\left\{-\left[\frac{2m_\sigma^*}{\hbar^2}\left(V_0^\sigma - E_{0j}^\sigma\right)\right]^{-1/2} z\right\}, & z \geq b \end{cases} \quad (12)$$

$$\phi_j^\sigma(z)^{SQW} = \begin{cases} a_1^\sigma \cos\left\{\sqrt{\frac{m_\sigma^* E_{0j}^\sigma}{2\hbar^2}} z - (j-1)\frac{\pi}{2}\right\}, & 0 \leq z < b \\ a_2^\sigma \exp\left\{-\left[\frac{2m_\sigma^*}{\hbar^2}\left(V_0 - E_{0j}^\sigma\right)\right]^{-1/2} z\right\}, & z \geq b \end{cases} \quad (13)$$

where a_i^σ and E_{0j}^σ are obtained from the continuity conditions of $\phi_j^\sigma(z)$ and its first derivative at the interface $z = b$. $Ai(x)$ and $Bi(x)$ are Airy functions, $\chi(a, c, x)$ is the confluent hypergeometric function $M(a, c, x)$ or $U(a, c, x)$, depending on the parity of the state [17].

The laser field determines a mixing of the eigenstates, so that in Eq. (5) the dressed wave functions are taken as linear combinations of the eigenfunctions ϕ_k^σ of the non-perturbed QW [19]

$$\Psi_k^\sigma(z) = \sum_j c_j^k \phi_j^\sigma(z), \quad (14)$$

where *j* denotes all bound states in the quantum well.

The dressed bound-state energies ε_k^σ and the coefficients c_j^k are determined using the system of linear equations

$$\left(\mathbf{H}_\mathbf{d}^\sigma - \varepsilon^\sigma \mathbf{S}^\sigma\right)\mathbf{C}^\sigma = 0, \quad (15)$$

where $\mathbf{H}_\mathbf{d}^\sigma$ and $\mathbf{S}^\sigma$ represent, respectively, the matrices of the particle energy in the laser field, and the norm with respect to the basis functions $\{\phi_j^\sigma(z)\}$. $\mathbf{C}^\sigma$ denotes the column matrix of the coefficients c_j^k.

2.2 Electric field effects

To account for the redistribution of the electronic charge density under an electric field parallel to the growth axis, we treat the qFz term of the Hamiltonian in a variational manner and we choose the wave functions as [16]

$$\Phi_k^\sigma(z) = \Psi_k^\sigma(z) e^{-\lambda_k^\sigma z}, \quad (16)$$

where λ_k^σ is the variational parameter. Note that in the presence of an electric field there is no true bound state for a finite QW of depth V_0^σ, since the potential energy is negative and large in absolute value, for large, negative values of z. If, however, the field is not excessively large, so that

$$F_k^\sigma < \frac{\sqrt{2m_\sigma^*}}{\hbar q}\left(V_k^\sigma - \varepsilon_k^\sigma\right)^{3/2}, \qquad (17)$$

where ε_k^σ represents the zero-field energy related to the $\Psi_k^\sigma(z)$ wave function, the states will have a long lifetime and can therefore be considered as quasi-bound states [16]. In the following we will consider the values of F such that Eq. (17) holds.

In the presence of the electric field energy levels of the electron in the laser "dressed" potential are given by

$$E_k^\sigma(\alpha_0, F) = \min_{\lambda_k^\sigma} \frac{\left\langle \Phi_k^\sigma(z) \middle| H\,\Phi_k^\sigma(z)\right\rangle}{\left\langle \Phi_k^\sigma(z) \middle| \Phi_k^\sigma(z)\right\rangle}. \qquad (18)$$

Once all bound state energy levels are determined, the absorption coefficient corresponding to the interband transitions $(h_j \to e_j)$ between the states of heavy-hole and electron can be evaluated. Neglecting the population effects for both initial and final states, the absorption coefficient reads

$$A(E) = A_0 \sum_j \left|\left\langle \Phi_k^h \middle| \Phi_k^e \right\rangle\right|^2 \Theta\left(E - E_j^t\right). \qquad (19)$$

Here $A_0 = \frac{\pi e^2 E_p}{n_0 c E m_0 l \hbar} \frac{m_e^* m_h^*}{m_e^* + m_h^*}$, E_p is the Kane matrix element, n_0 is the refractive index of the QW, l is the total thickness of the structure, and E_j^t is the interband transition energy [19]. By choosing A_0 as the "natural" unit for $A(E)$, Eq. (19) describes the usual ladder profile for the absorption coefficient, as expected from the joint density of states of the 2D subbands.

3. Results and discussion

In the above calculations, we consider an L=100 Å GaAs-Ga$_{1-x}$Al$_x$As quantum well. As in Ref. [19] we assume a constant effective mass throughout the entire structure and we use the following parameters: the aluminium concentration in the barrier material $x = 0.30$, $m_e^* = 0.0665\, m_0$, $m_h^* = 0.36\, m_0$ (m_0 is the free-electron mass) and the band gap $E_g = 1519$ meV.

For the electron (hole) states the barrier height V_0^σ is obtained from the 60% (40%) ($x \le 0.45$) rule of the band gap discontinuity, $\Delta E_g = 1247x$ (meV).

In order to show the effects of the potential-shape, n, the laser dressing parameter, α_0, and the electric field, F, on single-particle spectra in quantum wells, we have calculated the energy levels of the electron and heavy-hole in GaAs-Al$_x$Ga$_{1-x}$As QWs as a function of n, α_0, and F, respectively.

3.1 η and α$_0$ effects on spectra

Figs. 1(a)-3(a) present the laser-dressed potential V_d^σ as a function of the z position, for different values of the laser parameter, and for $n = 1$, $n = 2$, and $n = \infty$. The corresponding bound-state energy levels versus the laser-dressing parameter are shown in Figs. 1(b)-3(b). The numbers on the curves represent the indices of the dressed electron (hole) subbands. The zero point of energy is situated at the edge of the conduction (valence) band in the finite barrier GaAs QW. In our calculations, the energy of the hole is considered to be a positive value.

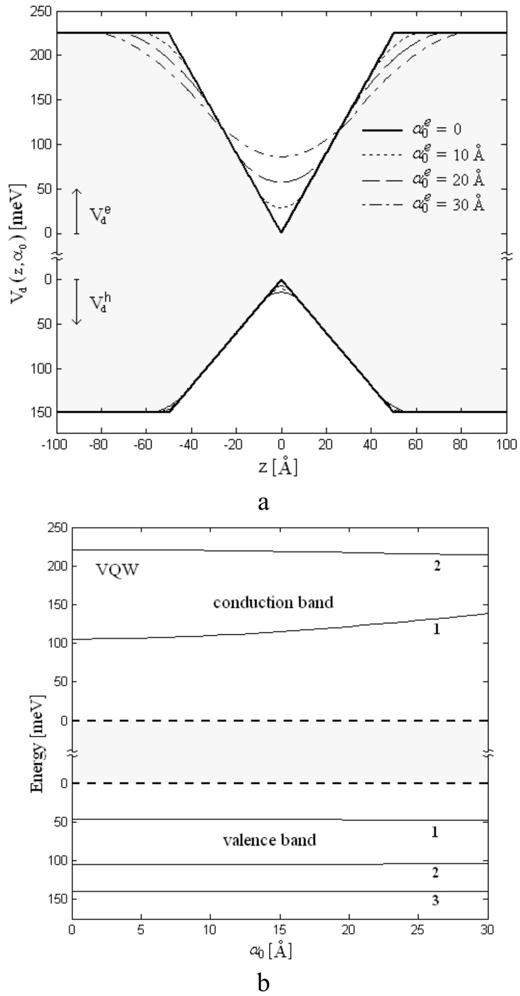


Fig. 1. VQW: a) Laser-dressed potential versus z position, for different values of the laser parameter; b) Bound-state energy dependence on the laser field parameter.

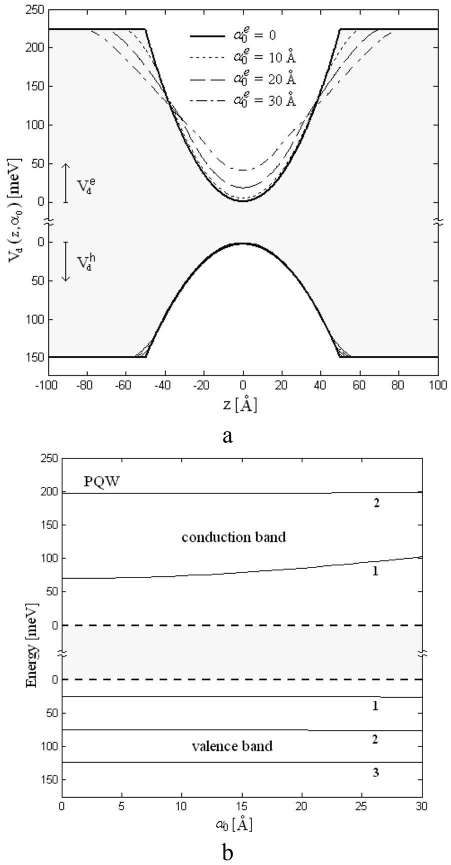


Fig. 2. The same as Fig.1, for PQW.

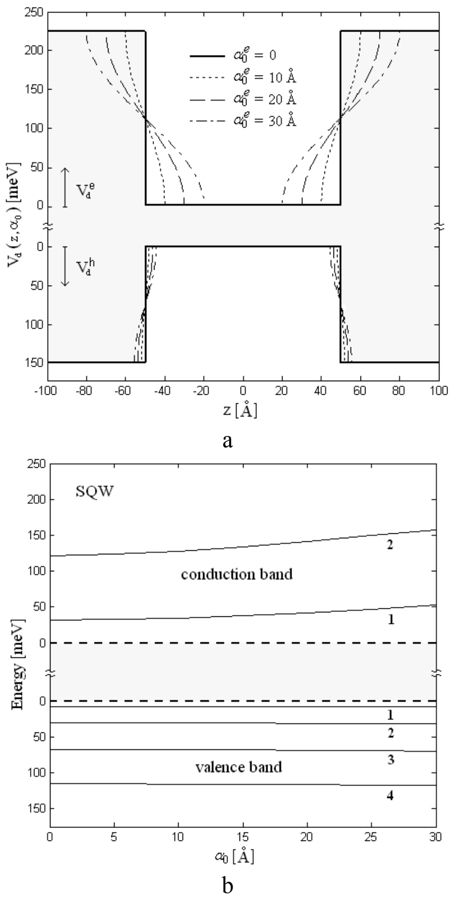


Fig. 3. The same as Fig. 1, for SQW.

It is seen from the figures that the ground state energy in the conduction band presents an increase with the laser intensity. Note that, at a given value of the laser parameter, the energy shift decreases with the increase of *n*, as a result of a weaker localization of the electron wave function in the structure. For the SQW, in agreement with Ref. [19], the energy levels are more laser-sensitive as the subband index increases. For the PQW and VQW, where the geometrical confinement overwhelms the laser effect, ε_2^e is less dependent on the α_0. Furthermore, in the VQW case, ε_2^e is slowly decreasing with the increase of α_0. The laser field less affects the energy of a heavy-hole than the energy of an electron, this behavior being a common feature of the quantum structures [19].

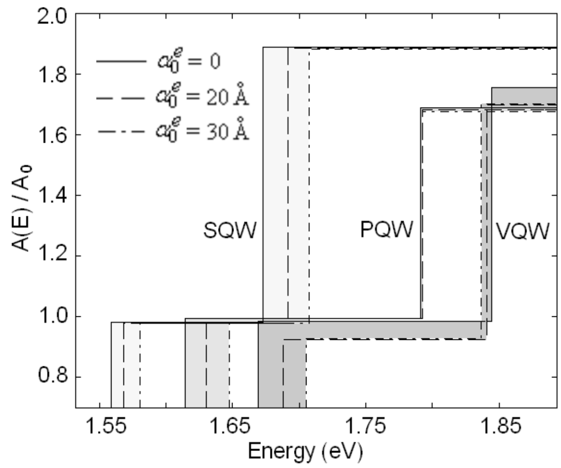


Fig. 4 The absorption coefficient for L=100 Å GaAs-$Ga_{0.7}Al_{0.3}As$ SQW, PQW and VQW, at α_0^e = 0, 20 and 30 Å.

We have also investigated the changes of the absorption coefficient $A(E)$, induced by the laser field. The staircase lineshape of the absorption coefficient in the GaAs-$Ga_{0.7}Al_{0.3}As$ VQW, PQW and SQW with $L = 100$ Å and α_0^e = 0, 10, 20 and 30 Å is shown in Fig. 4. As seen in this figure, the laser induced blue shift of the fundamental absorption edge for VQW and PQW is greater than for SQW, while for the 2*e*-2*h* interband transition the laser effect may be observed only for the square well.

3.2 Electric field effects

Figs. 5-7 depicts the single particle energy as a function of the applied electric field, for different values of *n* and α_0 parameters. As expected, for small values of F, as both electron and hole are strongly confined, the electric field has only a small effect on the wave functions. As a consequence, up to an electric field value, $F_c(n,\alpha_0)$, the Stark shift is given by the perturbation theory in terms of the confined well states, leading to a parabolic behavior of the energy levels [27]. The polarization effect is more pronounced as the electron wave function is more spread out in the barrier material. Consequently, as the graphs 5-7 show, the electric field $F_c(n,\alpha_0)$ (indicated by arrows for the ground states at $\alpha_0 = 0$ and $\alpha_0 = 30$ Å), decreases with α_0 and *n*. For higher fields such that the electric interaction overcomes the geometrical confinement potential, the Stark effect becomes linear. The effect of the electric field on the energy levels is more important for the excited states, where the transition between the low and high field behaviors becomes apparent at smaller values of the electric field (see Figs. 5(b)-7(b) and insets of Figs 5(c)-7(c)).

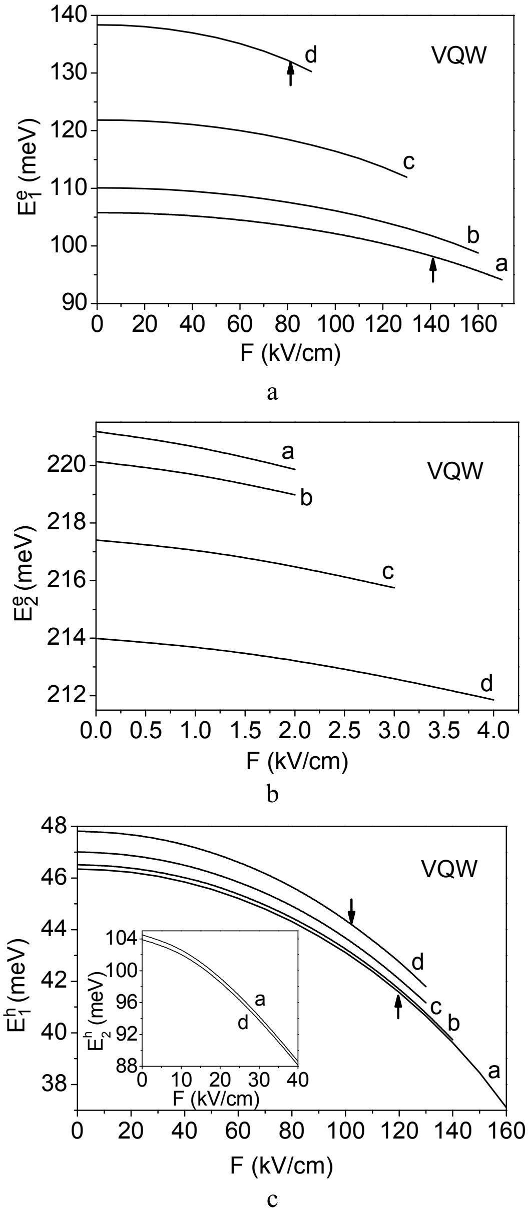


Fig. 5. Subband energy levels in a 100 Å thick SQW as functions of the electric field for different laser field parameter values: α_0^e = 0, 10, 20 and 30 Å, indicated on the lines by a, b, c and d, respectively: (a) first and (b) second electron state; (c) heavy hole states.

Note that for the V-shaped quantum well the energy of the second subband decreases with α_0, as shown in Fig. 5b. For this structure, E_2^e level lies substantially higher than for the SQW. As α_0 increases, the electron begins to localize in the upper part of the "dressed" well, with larger well widths. Thus the quantum confinement becomes weaker and consequently the E_2^e level decreases.

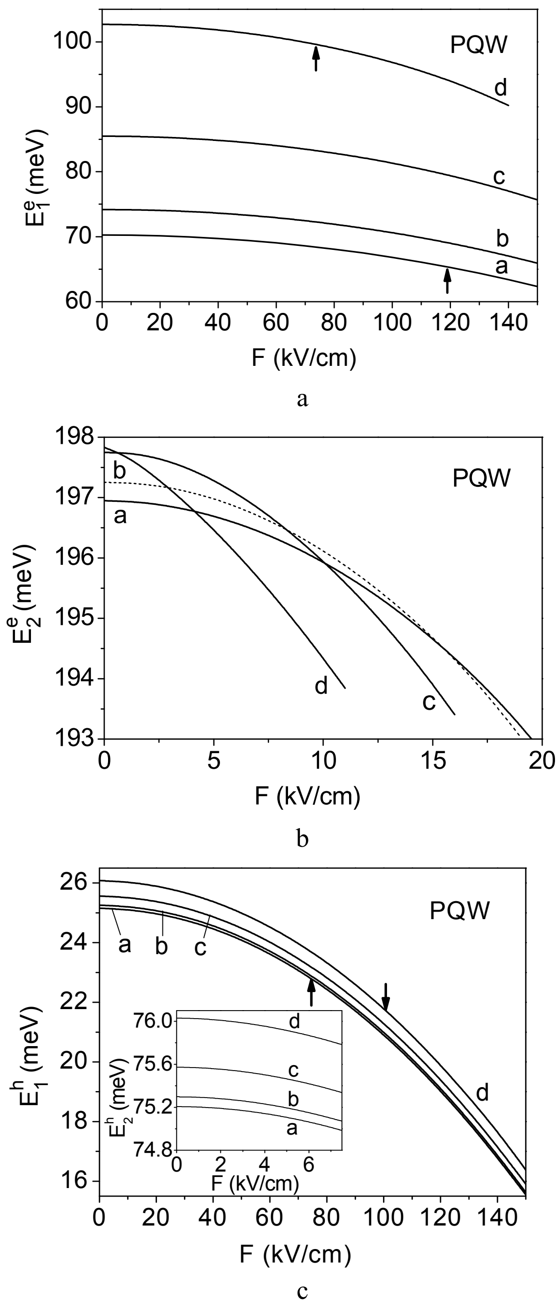
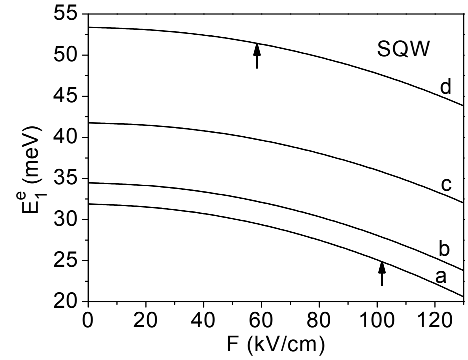
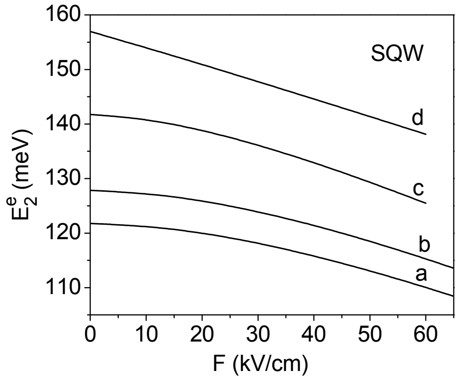


Fig. 6. The same as in Fig. 5, but for the PQW.

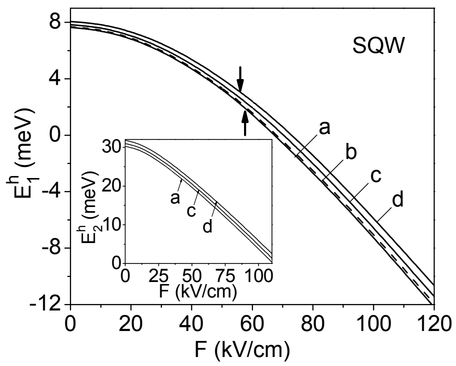
The same dependence on the laser parameter occurs in the case of E_2^h level. Therefore, for the interband recombination energies in a VQW in the intense, non-resonant laser field, a blue shift (for the $1e-1h$ transition) or a red shift (for the $2e-2h$ transition) is obtained. Such a potential shape effect can be useful in electronic device applications. We also observe that in the studied quantum structures the laser field has the effect of sharpening the transition from the low to the high electric-field behavior. This added degree of freedom may have applications in switching devices.



a



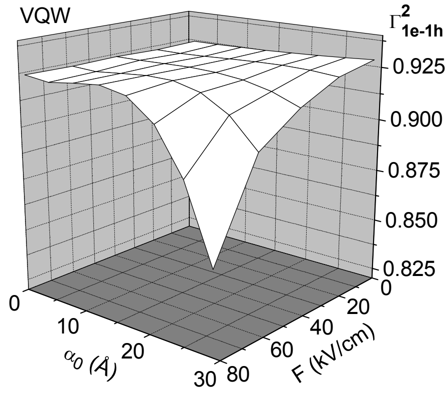
b



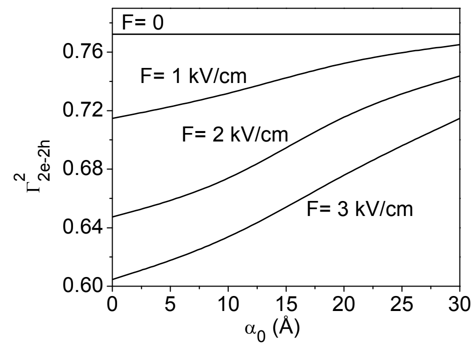
c

Fig. 7. The same as Fig. 5, but for the SQW.

The preceding discussion may be completed by studying the dependence of the overlap integral, Γ_{ke-kh}, $k=1,2$, on the laser parameter. The square of this quantity is plotted in Figs. 8-10, for different values of the electric field strength. As seen in these figures, in the absence of the electric field the overlap integrals are nearly constant. The presence of the static field induces a spatial shift of the hole (electron) along (opposite to) the field direction. As a result, the recombination rate decreases with the increase of the electric field strength.



a



b

Fig. 8. Square of the overlap integral between electron and heavy-hole states in a VQW as a function of the laser field parameter and the electric field strength: a) 1e-1h transition; b) 2e-2h transition.

Note that in the VQW case, the domain of F values used for the calculation of Γ_{1e-1h} corresponds to a "low-field regime", when the extent of the wave function $\Phi_1^{e,h}$ is determined by the competition of the laser field with the confinement potential. As α_0 increases, the electron (hole) level, initially localized in the lower part of the QW, is displaced in the upper part of the "dressed" well. As a result, the tunnelling probability of the electron (hole) increases with α_0 and Γ^2_{1e-1h} decreases (see Fig. 8a). As n increases, for the electron placed on the first level the effective width of the dressed well decreases with α_0 (Figs. 2, 3). This enhance the quantum confinement of the carriers and consequently the overlap integral of their wave functions. The same competition between the laser and the electric field effects on the quantum confinement determines the behavior of Γ_{2e-2h} integrals.

Note that the previous figures may be used to investigate the change induced by laser fields on the absorption coefficient corresponding to the interband transitions in quantum wells under an electric field.

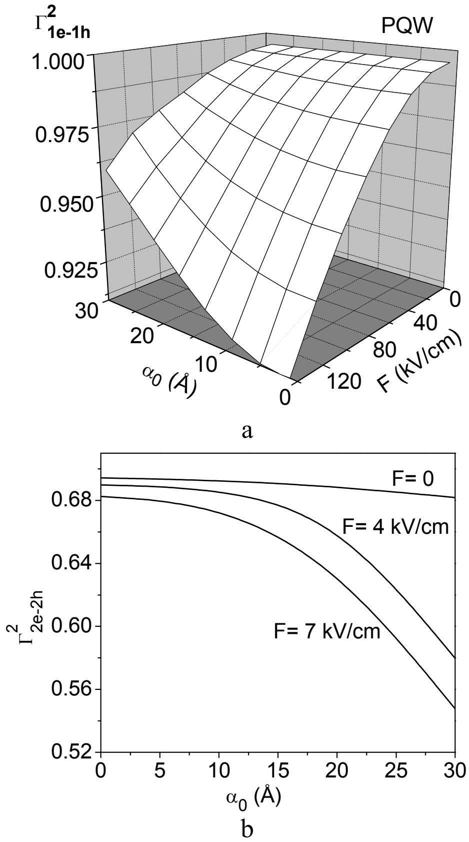


Fig. 9. The same as Fig. 8, but for the PQW.

…

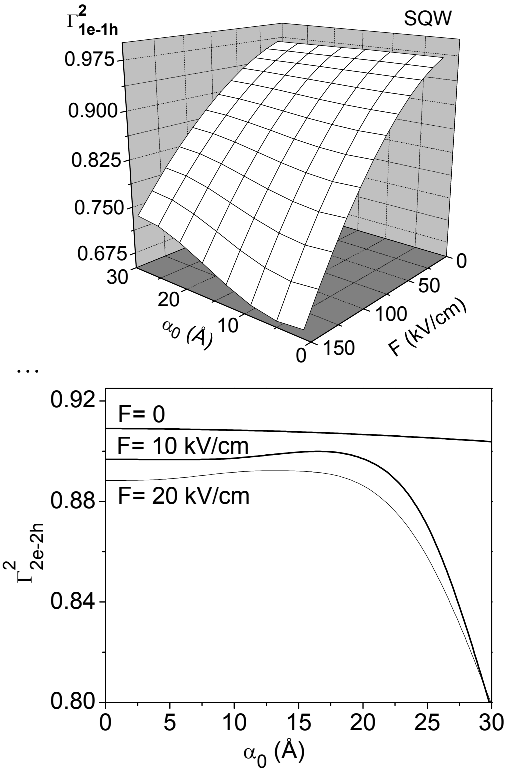


Fig. 10. The same as Fig. 8, but for SQW.

4. Conclusions

We have studied the effects of the high-frequency laser field on the intersubband transitions in different shaped quantum wells (VQW, PQW and SQW) in the presence of an external electric field. The dependence of the absorption coefficient on the laser parameter, geometric shape of the wells and the applied electric field was discussed. For SQW our results reasonably agree with the previous results [19]. We showed that for the graded GaAs-GaAlAs quantum well the laser field provides an important effect on the electronic and optical properties and the changes in the energy levels are dependent on shape of the confinement potential and the strength of the electric applied field. Such effects may provide methods to drive tuneable semiconductor lasers, which can be tailored by quantum wells parameters and by external applied fields.

References

[1] D. Kasamet, C. S. Hong, N. B. Patel, P. D. Dapkus, IEEE J. Quantum Electron **QE-19**, 1025 (1983).
[2] L. J. Mawst, M. E. Givens, C. A. Zmudzinski, M. A. Emanuel, J. J. Coleman, IEEE J. Quantum Electron **QE-23**, 696 (1987).
[3] R. P. G. Karunasiri and K. L. Wang, Superlatt. Microstruct. **4**, 661 (1988).
[4] A. C. Gossard, R. C. Miller, W. Wiegmann, Surf. Sci. **174**, 131 (1986).
[5] S. Ciugni, T. L. Tansley, G. L. Griffiths, J. Cryst. Growth **111**, 50 (1991).
[6] R. J. Choi, H. W. Shim, S. M. Jeong, H. S. Yoon, E. K. Suh, C. H. Hong, H. J. Lee, and Y. W. Kim, Phys. Stat. Sol. (a) **192**, 430 (2002).
[7] A. T. M. Fairus, V. K. Arora, Microelectronics Journal **32**, 679 (2001).
[8] E. Kasapoglu, H. Sari, and I. Sokmen, Physica B **339**, 17 (2003); idem, Physica B **392**, 213 (2007).
[9] E. C. Niculescu, Phys. Lett. A **213**, 85 (1996).
[10] G. H. Herling M. L. Rustgi, J. Appl. Phys. **69**, 2328 (1990).
[11] G. Z. Jiang, C. Z. Wen, Phys. Rev. B **50**, 2689 (1994).
[12] E. Kasapoglu, H. Sari, I. Sokmen, Physica B **315**, 261 (2002).
[13] S. E. Okan, H. Akbas, S. Aktas, Superlatt. Micro struct. **28**, 171 (2002).
[14] L. Zhang, Superlatt. Microstruct. **37**, 261 (2005).
[15] L. Zhang, H. J. Xie, Mod. Phys. Lett. B **17**, 347 (2003).
[16] G. Bastard, E. E. Mendez, L. L. Chang, L. Esaki, Phys. Rev. B **28**, 3241 (1983).
[17] W. P. Yuen, Phys. Rev. B **48**, 17316 (1993).
[18] H. Akbas, C. Dane, K. Kasapoglu, N. Talip, Physica E **40**, 627 (2008).
[19] O. O. D. Neto, Q. Fanyao, Superlatt. Microstruct. **35**, 1 (2004).
[20] H. S. Brandi, G. Jalbert, Solid State Commun. **113,** 207 (2000).

[21] E. Ozturk, H. Sari, and I. J. Sokmen, J. Phys. D **38**, 935 (2005).
[22] O. A. C. Nunes, A. L. A. Fonseca, F. M. S. Lima, D. A. Agrello, Solid State Commun **122**, 425 (2002).
[23] B. G. Enders, F. M. S. Lima, O. A. C. Nunes, A. L. A. Fonseca, D. A. Agrello, Q. Fanyao, E. F. Da Silva, Jr., and V. N. Freire, Phys. Rev. B **70**, 035307 (2004).
[24] E. C. Niculescu, L. Burileanu, J. Optoelectron. Adv. Mater. **9**, 2713 (2007).
[25] M. Gavrila, J. Z. Kaminski, Phys. Rev. Lett. **52,** 613 (1984).
[26] H. Kramers, Collected Scientific Papers (Amsterdam, North-Holland), 866 (1956).
[27] K. L. Janssens, B. Partoens and F. M. Peeters, Phys. Rev. B **65**, 233301 (2002).

*Corresponding author: niculescu@physics.pub.ro