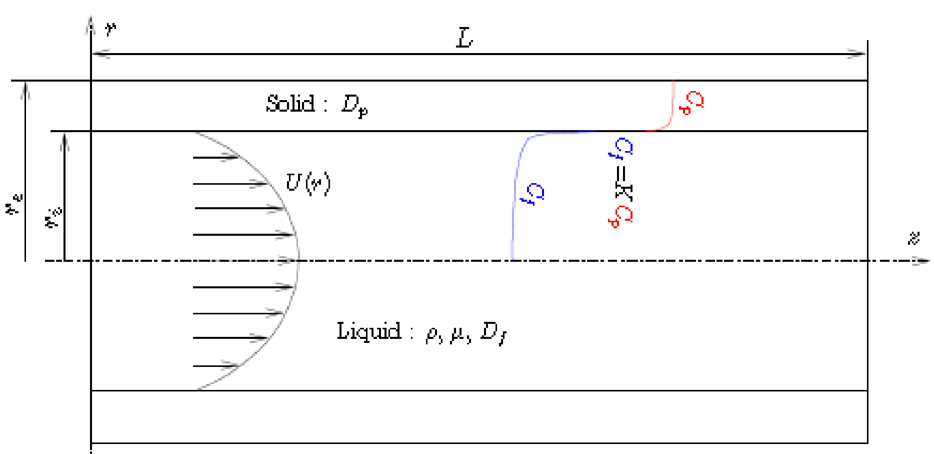
The following figure shows a PVC tube through which a liquid flows between two points. It has been noted that plasticizer that exists in the tube is transferred to the liquid through the internal surface of the tube and because of this, a mass transfer through solid (in the tube) and liquid take place. This transport process is modelled by the following equations for both tube and liquid zone. Since the flow rate of the liquid is not very large, one can assume that the flow regime is Laminar and for simplicity, it can be described by Hagen-Poiseuille equation. Based on these pieces of information, obtain the concentration profile of the plasticizer in both tube wall and liquid flowing inside the tube as a time dependent profile for two values of partition coefficient (i.e., $K = 10^{-3}$ *and* $K = 1$). Based on the obtained profiles determine the total amount of the plasticizer transferred to liquid as a function of time.



$$\begin{cases} \textit{For Solid Zone} & \dfrac{\partial C_p}{\partial t} = D_p \left[\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial C_p}{\partial r} \right) + \dfrac{\partial^2 C_p}{\partial z^2} \right] \\ \textit{For Liquid Zone} & \dfrac{\partial C_f}{\partial t} + U(r)\dfrac{\partial C_f}{\partial z} = D_f \left[\dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial C_f}{\partial r} \right) + \dfrac{\partial^2 C_f}{\partial z^2} \right] \end{cases}$$

Where $\boldsymbol{C_p}, \boldsymbol{C_f}$ are plasticizer concentrations in solid and liquid (fluid) zones, respectively and U(r) is liquid velocity that can be obtained by Hagen-Poiseuille equation:

$$\boldsymbol{U(r) = \frac{2Q}{\pi {r_i}^2}\left[1 - \left(\frac{r}{r_i}\right)^2\right]}$$

Initial and boundary conditions of the problem along with physical properties of the solid and liquid phase are as follows:

$$\begin{cases} \begin{cases} @r = r_e & \dfrac{\partial C_p}{\partial r} = 0 \\ @r = r_i & C_f = KC_p \textit{ where k is partition factor} \\ @z = 0 & \dfrac{\partial C_p}{\partial z} = 0 \\ @z = L & \dfrac{\partial C_p}{\partial z} = 0 \\ @t = 0 & C_p(r, z, 0) = C_{p0} \end{cases} & \textit{For Solid Zone} \\ \begin{cases} @r = 0 & \dfrac{\partial C_f}{\partial r} = 0 \\ @r = r_i & D_f\left(\dfrac{\partial C_f}{\partial r}\right) = D_p\left(\dfrac{\partial C_p}{\partial r}\right) \\ @z = 0 & C_f = 0 \\ @z = L & \dfrac{\partial C_f}{\partial z} = 0 \\ @t = 0 & C_f(r, z, 0) = 0 \end{cases} & \textit{For Liquid Zone} \end{cases}$$

Property	Value
D_f	$2.44 \times 10^{-10}\ \frac{m^2}{s}$
D_p	$1.1 \times 10^{-13}\ \frac{m^2}{s}$
ρ_f	$914\,\frac{kg}{m^3}$
Liquid Viscosity (μ_f)	$1.59 \times 10^{-3}\ Pa \cdot s$
C_{p0}	$605\,\frac{kg}{m^3}$
Tube Length	$8\ m$
Tube Inner Radius	$7.93\ mm$
Tube Outer Radius	$12.70\ mm$
Liquid Flowrate	$1\ \frac{lit.}{min.}$

Good luck