Model Following Active Suspension System Using Inertial Delay Control

Geeta A. Bokil
2^{nd} Year M.E. Electronics & Telecommunications,
JSPM's College of Engineering Hadapsar, Pune, India,
geetakulkarni009@gmail.com

D. G. Ingale
Professor, Electronics & Telecommunications,
JSPM's College of Engineering, Hadapsar, Pune, India,
ingale.dg2@gmail.com

Dr. D. B. Salunke
Professor, Electronics & Telecommunications,
JSPM's College of Engineering, Hadapsar, Pune, India,
dbsalunke@rediffmail.com

S. B. Phadke
Professor, Instrumentation & Control,
College of Engineering Pune, India,
sbp.instru@coep.ac.in

***Abstract*—In this paper an active suspension system is designed using inertial delay control for estimating the effect of uncertainties in suspension parameters, sprung mass variation and unknown road profile. The proposed control is designed with the objective of improving the ride comfort without having to use sensors for measuring the road profile or sensing the displacement and velocity of the unsprung mass. The design is validated by simulation for two road profiles and the performance is compared with passive suspension systems.**

***Index Terms*—active suspension, inertial delay control, uncertainty estimation, model following**

I. Introduction

Active suspension for commercial cars is an attractive proposition for car designers interested in improving ride comfort and handling [1]–[3]. The passive suspension that currently predominates is able to isolate the car body from the road in only a limited way. From the control point of view the passive suspension system is a one- zero- two- pole system which is unable to fully suppress the effect of road disturbance in the frequency range of 4Hz to 8Hz. According to ISO 2631 [4], the human beings are more sensitive to vibrations in these frequencies which is a motivation for designing an active suspension system.

In the past, several control methods have been used to mitigate the ride discomfort coming from a rough road [5]. These methods include optimal control [6], H_∞ based control [7], fuzzy control [8], sliding mode control [9], [10], disturbance observer based sliding mode control [11]–[13] skyhook control [14], electromagnetic suspension [15], nonlinear adaptive control [16] among many others. The performance obtained with these methods is sensitive to uncertainties in suspension parameters and to unknown road profile or must use a discontinuous control. Some new control strategies [17]–[19] employ special road preview sensors to estimate the road profile and use the information so obtained to improve the ride comfort. Recent results [20] focus on multi-objective active suspension systems as well.

In this paper, a new controller is proposed to overcome some of the drawbacks of existing methods. The main strategy is to use the inertial delay control (IDC), originally developed in [21]–[23], to estimate the effect of road disturbance and uncertainties in the system parameters. Then a control is designed to make the system follow a model to assure satisfactory performance. The novelty of the proposed system is getting a response better than the skyhook suspension. The method of IDC has been used for designing active suspension before in [24] but there are important differences in the way IDC is used in the proposed paper. Unlike in [24], the present paper does not use skyhook model [25], does not use sliding mode control and does not need measurement of displacement and velocity of the unsprung mass thereby saving two sensors and simplifying the implementation considerably.

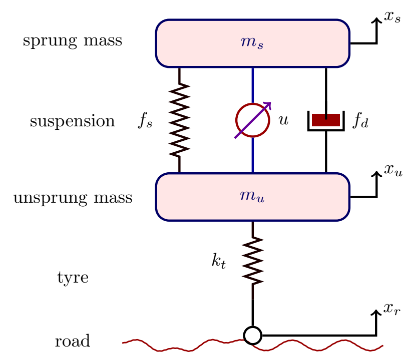


Fig. 1. Quarter car suspension system

The main contributions of the paper are:

(a) no need to employ costly preview sensors to estimate the road profile,
(b) no need to measure the unsprung mass position and velocity,
(c) robustness to suspension parameter uncertainties, mass variation and road profile, and
(d) model following control given with possibilities for further generalization

II. System description and problem statement

Consider a quarter car model of a typical car suspension system. This system consists of two masses called the sprung and unsprung masses connected by a spring and a damper. In the active system an actuator is placed in parallel with the spring and damper. The unsprung mass is basically the wheel which is commonly modelled as a spring. The basic system is shown in Fig. 1.

The dynamic equations of the quarter car system are given by,

$$m_{sa}\ddot{x}_s = -f_s - f_d + u \tag{1}$$

$$m_u\ddot{x}_u = f_s + f_d - f_t - u \tag{2}$$

978-1-5090-3704-9/17/$31.00 2017 IEEE

where m_{sa} is the sprung mass, m_u is the unsprung mass, x_s is the displacement of the sprung mass relative to its static position, x_u is the displacement of the unsprung mass relative to its static position, x_r is the uneven road profile relative to plane ground, f_s is the non-linear spring force, f_d is the non-linear damping force, f_t is the tyre force and u is the control force generated by the actuator. The spring force f_s is given by,

$$f_s = k_1\Delta_x + k_2\Delta_{x^2} + k_3\Delta_{x^3} \tag{3}$$

where the coefficients k_1, k_2 and k_3 are constants and $\Delta_x = x_s - x_u$ is the suspension deflection, and the damping force f_d is given by,

$$f_d = c_1\Delta_{\dot{x}} + c_1\Delta_{\dot{x}^2} \tag{4}$$

where the coefficients c_1 and c_2 are constants. If the tyre loses contact with the ground, the force exerted by the tyre becomes zero. There the tyre force f_t is given by

$$f_t = \begin{cases} k_t(x_u - x_r) & x_u - x_r < \frac{(m_s+m_u)g}{k_t} \\ 0 & x_u - x_r \geq \frac{(m_s+m_u)g}{k_t} \end{cases} \tag{5}$$

where g is the acceleration due to gravity and k_t is the tyre spring constant. Let the state variables be denoted as $x_1 = x_s$, $x_2 = \dot{x_s}$, $x_3 = x_u$ and $x_4 = \dot{x_u}$. The dynamics are expressed in the state variable form as,

$$\dot{x}_1 = x_2 \tag{6}$$

$$\dot{x}_2 = \frac{1}{m_{sa}}(-f_s - f_d + u) \tag{7}$$

$$\dot{x}_3 = x_4 \tag{8}$$

$$\dot{x}_4 = \frac{1}{m_u}(f_s + f_d - f_t - u) \tag{9}$$

Next the system equations are rewritten in a form more suitable for designing a model following control. Let k_s and c_s resp. be the spring stiffness and damping coefficients of the model to be followed. By adding and subtracting $\frac{1}{m_s}(-k_sx_1 - c_sx_2 + u)$ from equation (7), where m_s is the nominal sprung mass, the equation (6) and equation (7) can be rewritten as,

$$\dot{x}_1 = x_2$$

$$\dot{x}_2 = \frac{1}{m_{sa}}(-f_s - f_d + u) - \frac{1}{m_s}(-k_sx_1 - c_sx_2 + u) \tag{10}$$

$$+ \frac{1}{m_s}(-k_sx_1 - c_sx_2) + \frac{1}{m_s}u \tag{11}$$

Therefore,

$$\dot{x}_2 = \frac{1}{m_s}(-k_sx_1 - c_sx_2) + \frac{1}{m_s}u + \frac{1}{m_s}e \tag{12}$$

where

$$e = m_s[\frac{1}{m_{sa}}(-f_s - f_d + u) - \frac{1}{m_s}(-k_sx_1 - c_sx_2 + u)] \tag{13}$$

Notice from (12), the uncertainty can be expressed as

$$e = m_s\dot{x}_2 + k_sx_1 + c_sx_2 - u \tag{14}$$

The objective of control is to make the acceleration of the sprung mass $\dot{x}_2$ small by making the system follow a model and without measuring the unsprung mass displacement and the velocity and without using road preview sensors in spite of uncertainties in f_s, f_d and sprung mass m_{sa}.

III. Control design using IDC

In this section the uncertainty e is estimated using IDC and then the control u is designed to make the system effectively follow a model. Unlike the time delay control, the IDC estimates the uncertainty by considering a broadband filter $G_f(s)$ given by,

$$G_f(s) = \frac{1}{1+\tau s} \tag{15}$$

where τ is a small positive constant. If uncertainty e is passed through $G_f(s)$, the output $\hat{e}$ given by,

$$\hat{e} = G_f(s)e \tag{16}$$

can be considered as an estimate of e. Since e is an uncertainty (16) can not be implemented. However using (14) in (16),

$$\hat{e} = G_f(s)[m_s\dot{x}_2 + k_sx_1 + c_sx_2 - u] \tag{17}$$

If the control is selected as,

$$u = -\hat{e} \tag{18}$$

then,

$$\hat{e} = G_f(s)[m_s\dot{x}_2 + k_sx_1 + c_sx_2 + \hat{e}] \tag{19}$$

which can be written as,

$$\tau\dot{\hat{e}} + \hat{e} = m_s\dot{x}_2 + k_sx_1 + c_sx_2 + \hat{e} \tag{20}$$

Simplifying and integrating,

$$\hat{e} = \frac{m_s}{\tau}x_2 + \frac{1}{\tau}\int(k_sx_1 + c_sx_2)dt \tag{21}$$

Employing the control in (13), the sprung mass acceleration

$$\dot{x_2} = \frac{1}{m_s}(-k_sx_1 - c_sx_2) + \frac{1}{m_s}\tilde{e} \tag{22}$$

where $\tilde{e} = e - \hat{e}$, is the estimation error. The dynamics of the estimation error can be shown to be

$$\dot{\tilde{e}} = -\frac{1}{\tau}\tilde{e} + \dot{e} \tag{23}$$

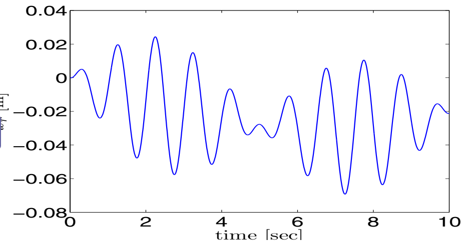
If τ is sufficiently small $\tilde{e}$ can be kept small and then the sprung mass system performs as prescribed by the designer. When $\tilde{e}$ becomes sufficiently small, the controlled system is approximately governed by

$$\dot{x}_2 = \frac{1}{m_s}(-k_sx_1 - c_sx_2) \tag{24}$$

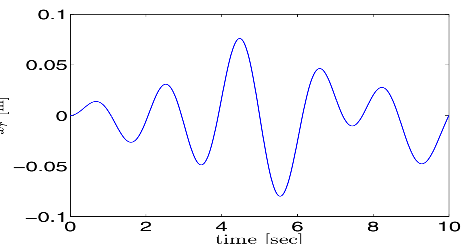
which is the desired model to be followed. The designer can choose any other model giving flexibility in performance without having to change the control methodology. Notice that the values of the suspension parameters or the sprung mass were not used in designing the control or estimating e, this proves the robustness of the design to uncertainties in suspension parameters or any variation in the sprung mass. Further the control law is free from x_3, x_4 and therefore there is no need to sense displacement and velocity of the unsprung mass.

IV. Simulation and discussion of results

To validate the control the system is simulated for two typical road profiles. The parameters of the suspension system are: $m_s = 250kg$, $m_{sa} = 300kg$, $200 \leq m_{sa} \leq 300$, $k_t = 160,000$ N/m, $k_1 = 12500$ N/m, $c_1 = 1400$ N sec/m , $m_u = 25kg$. The control parameters are selected as $k = 200$, $\tau = 0.01$, $k_s = 15000$ N/m and $c_s = 1680$ N sec/m The actual sprung mass was varied in the range considering three values $m_{sa} = 200kg$, $250kg$ and $300kg$ and the results for active and passive suspension systems are obtained for the two road

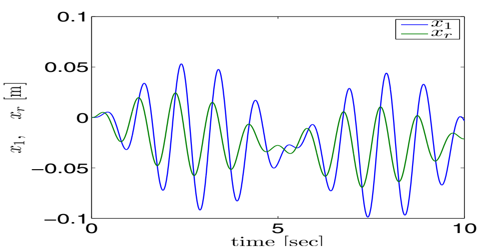


(a)

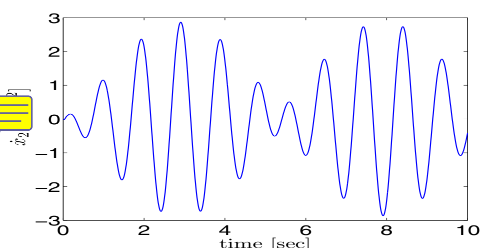


(b)

Fig. 2. Road profiles. (a) Case 1, (b) Case 2

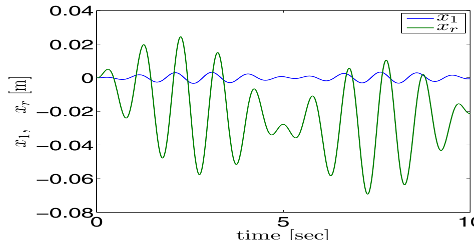


(a)

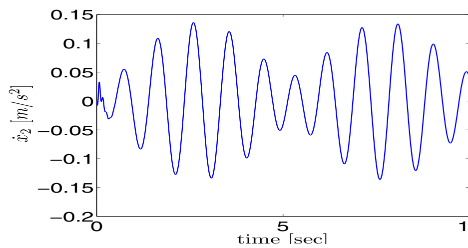


(b)

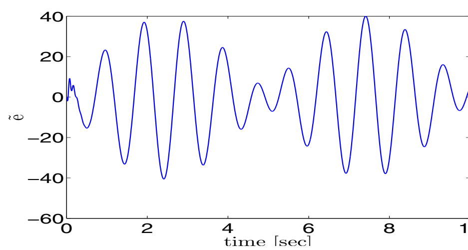
Fig. 3. Case 1 road profile, passive suspension



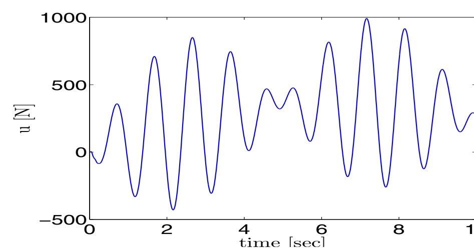
(a)



(b)



(c)



(d)

Fig. 4. Case 1 road profile, active suspension

profiles shown in Fig. 2. As for handling requirements, it is required that the relative suspension deflection defined as $\xi = \frac{x_1 - x_3}{x_R}$ and the relative tyre force defined as $\zeta = \frac{k_t(x_1 - x_3)}{(m_s + m_u)/g}$ remain less than 1 all the time.

A. Case 1

First consider the profile shown in Fig. 2a which is generated by,

$$x_r = 0.04\sin(2\pi t)\sin(0.2\pi t) + 0.03\cos(0.075\pi t + \frac{\pi}{2}) \quad (25)$$

The Fig. 3a shows the sprung mass deflection and the road profile x_r for the passive system while Fig. 4a shows the same results for the active case. It can be seen that the sprung mass deflection for the active suspension is much smaller for the active case. The Figs. 3b and 4b show the sprung mass acceleration for the passive and active cases.

It can be seen that the active system improves the ride comfort considerably. The improvement is brought out by the estimation of e through IDC. Fig. 4c shows the plot of $\tilde{e}$. It can be seen that $\hat{e}$ estimates e fairly accurately. The control effort is plotted in Fig. 4d.

Next the effect of the active control on handling, represented through the relative suspension deflection ξ and relative tyre force ζ is demonstrated. Fig. 5a shows ξ and Fig. 5b shows the plot of ζ which are less than 1 in magnitude as described. Similar results were obtained for other values of the sprung mass. The results for all the cases of sprung mass uncertainty are summarized in tabular form for the road profile 1 in Tables I and II.

B. Case 2

Next consider the profile shown in Fig. 2b which is generated by,

$$x_r = 0.06\sin(\pi t)\sin(0.1\pi t) + 0.05\sin(0.6\pi t)\sin(0.03\pi t) \quad (26)$$

Fig. 6a shows the sprung mass deflection and the road profile x_r for the passive system while Fig. 7a shows the same results for the

TABLE I
PASSIVE SUSPENSION ROAD PROFILE CASE 1

		m_{sa}		
		200 kg	250 kg	300 kg
$\dot{x}_2$	max	2.6280	2.8210	2.8670
$\dot{x}_2$	rms	1.3465	1.4365	1.4495
u	max	0	0	0
u	rms	0	0	0
ξ	max	0.4262	0.5726	0.6999
ζ	max	0.2087	0.2736	0.3286

TABLE II
ACTIVE SUSPENSION ROAD PROFILE CASE 1

		m_{sa}		
		200 kg	250 kg	300 kg
$\dot{x}_2$	max	0.1368	0.1347	0.1360
$\dot{x}_2$	rms	0.0701	0.0707	0.0714
u	max	987.3	984.6	991
u	rms	425.29	427.68	430.16
ξ	max	0.8490	0.8492	0.8494
ζ	max	0.0220	0.0242	0.0265

active case. It can be seen that the sprung mass deflection for the active suspension is much smaller for the active case. The Figs. 6b and 7b show the sprung mass acceleration for the passive and active cases. It can be seen that the active system improves the ride comfort considerably. The improvement is brought out by the estimation of e through IDC. Fig. 7c shows the plot of $\tilde{e}$. It can be seen that $\hat{e}$ estimates e fairly accurately. The control effort is plotted in Fig. 7d.

Similar results were obtained for other values of the sprung mass. Next the effect of the active control on handling, represented through the relative suspension deflection ξ and relative tyre force ζ is considered. It is seen that the peak values of ξ and ζ are within 1 as desired. The tabulated values of peak ξ which is very close to 1 show the inherent trade off between ride comfort and suspension space limitation. One can bring down the peak value of ξ by modifying the control which is a topic for future work. The plots are omitted to save space. The results for all the cases are summarized in tabular form for the road profile 2 in Tables III and IV.

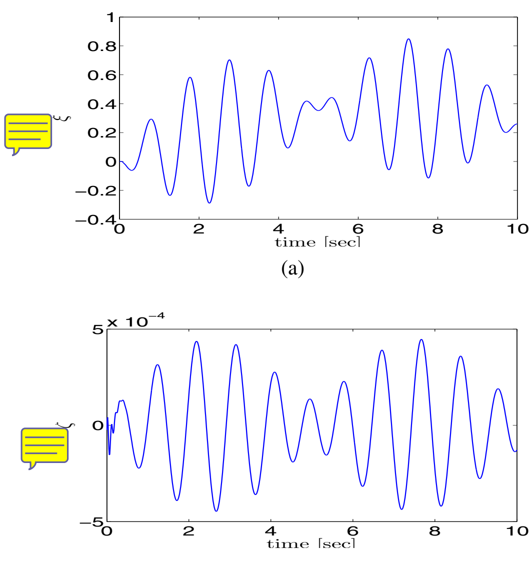


Fig. 5. Relative suspension deflection and relative tyre force for case 1 active suspension

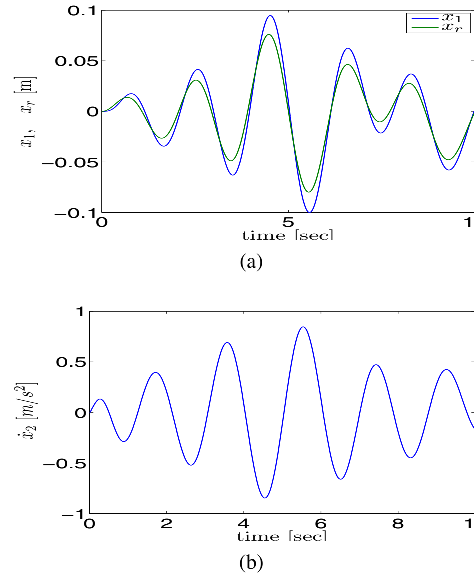


Fig. 6. Case 2 road profile, passive suspension

TABLE III
PASSIVE SUSPENSION ROAD PROFILE CASE 2

		m_{sa}		
		200 kg	250 kg	300 kg
$\dot{x}_2$	max	0.7740	0.8087	0.8464
$\dot{x}_2$	rms	0.3698	0.3880	0.4080
u	max	0	0	0
u	rms	0	0	0
ξ	max	0.1462	0.1908	0.2397
ζ	max	0.0636	0.0812	0.1004

It can be seen that for both road profiles, the proposed active scheme is successful in keeping the sprung mass acceleration $\hat{x_2}$ small compared to the passive system. The sprung mass variation affects the passive system more than the active system.

V. CONCLUSION

In this paper, an active suspension system using IDC is designed to enhance the ride comfort. The proposed control is successful in forcing the suspension system to follow a user defined model. The analysis of the designed system shows that it is robust to uncertainties in suspension parameters and sprung mass variation. No measurements of road profile or unsprung mass displacement and velocity were required. The proposed control meets the design objectives. The system is validated by simulation for two road profiles. It is seen that the proposed active system is successful in improving the ride comfort considerably compared to the passive system.

TABLE IV
ACTIVE SUSPENSION ROAD PROFILE CASE 2

		m_{sa}		
		200 kg	250 kg	300 kg
$\dot{x}_2$	max	0.0203	0.0203	0.0203
$\dot{x}_2$	rms	0.0101	0.0101	0.0101
u	max	1.0504e3	1.0507e3	1.0510e3
u	rms	448.2	448.3	448.6
ξ	max	0.9985	0.9985	0.9985
ζ	max	0.0065	0.0066	0.0067

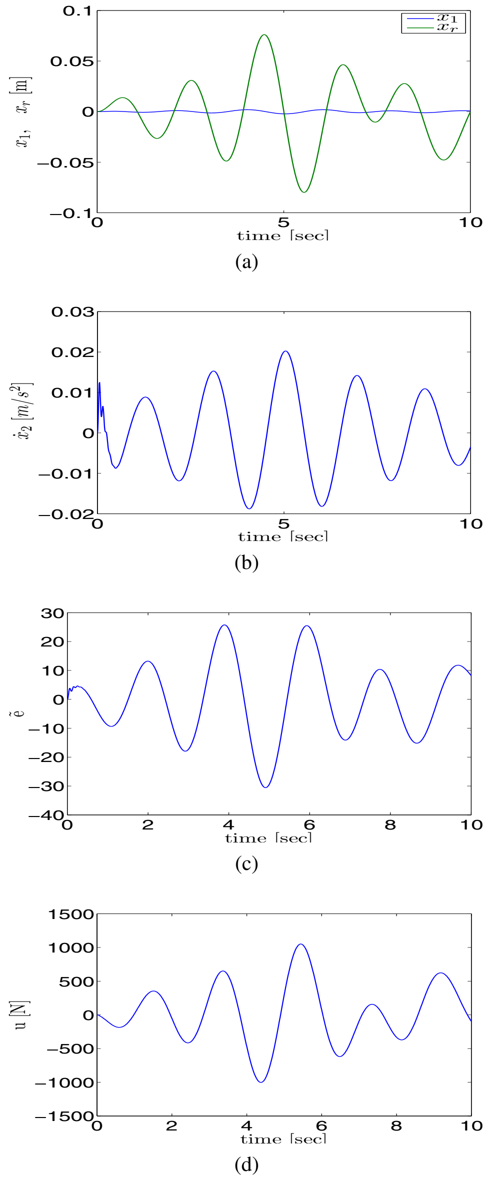


Fig. 7. Case 2 road profile, active suspension

REFERENCES

[1] H. E. Tseng and D. Hrovat, "State of the art survey: active and semi-active suspension control," *Vehicle System Dynamics*, vol. 53, no. 7, pp. 1034–1062, Jul. 2015.

[2] D. Hrovat, "Survey of advanced suspension developments and related optimal control applications," *Automatica*, vol. 33, no. 10, pp. 1781–1817, 1997.

[3] R. A. Williams, "Automotive active suspensions part 1: basic principles," *Proc. Instn Mech Engrs Part D*, vol. 211, pp. 415–426, 1997.

[4] ISO, "Mechanical vibration and shock: Evaluation of human exposure to whole-body vibration part 1, general requirements," no. 2631-1:(E):ISO, 1997.

[5] D. Cao, X. Song, and M. Ahmadian, "Editor's perspectives: road vehicle suspension design, dynamics and control," *Vehicle System Dynamics*, vol. 49, no. 1-2, pp. 3–28, Jan.-Feb. 2011.

[6] D. Hrovat, "Applications of optimal control to advance automotive suspension design," *Automatica*, vol. 115, pp. 328–342, Jun. 1993.

[7] H. Gao, W. Sun, and P. Shi, "Robust sampled-data H_∞ control for vehicle active suspension systems," *IEEE Trans. Contr. Syst. Technol.*, vol. 18, no. 1, pp. 238–245, 2010.

[8] R. J. Lian, "Enhanced adaptive self-organizing fuzzy sliding mode controller for active suspension systems," *IEEE Trans. Ind. Electr.*, vol. 60, no. 3, pp. 958–968, Mar. 2013.

[9] P. C. Chen and A. C. Huang, "Adaptive sliding control of active suspension systems with uncertain hydraulic actuator dynamics," *Vehicle System Dynamics*, vol. 44, no. 5, pp. 357–368, May 2006.

[10] C.-P. Cheng, C.-H. Chao, and T.-H. S. Li, "Design of observer- based fuzzy sliding -mode control for an active suspension system with full-car model." IEEE, 2010, pp. 1939–1944.

[11] V. S. Deshpande, B. Mohan, P. D. Shendge, and S. B. Phadke, "Disturbance observer based sliding mode control of active suspension systems," *J. Sound and Vibration*, vol. 333, no. 11, pp. 2281 – 2296, 2014.

[12] D. Ginoya, P. Shendge, and S. Phadke, "Sliding mode control for mismatched uncertain systems using an extended disturbance observer," *IEEE Trans. Ind. Electr.*, vol. 61, no. 4, pp. 1983–1992, Apr. 2014.

[13] V. S. Deshpande, P. D. Shendge, and S. B. Phadke, "Dual objective active suspension system based on a novel nonlinear disturbance compensator," *Vehicle System Dynamics*, vol. 54, no. 9, pp. 1269–1290, 2016. [Online]. Available: http://dx.doi.org/10.1080/00423114.2016.1198490

[14] C. Kim and P. I. Ro, "A sliding mode controller for vehivle active suspension systems with non-linearities," *Proc. IMechE, Part D: J Automobile Engineering*, vol. 212, pp. 79–92, 1998.

[15] L.J.Gysen and L.G.Janssen, "Active electromagnetic suspension system for improved vehicle dynamics," *IEEE Trans. Veh. Technol.*, vol. 59, no. 3, pp. 1156–1163, Mar. 2010.

[16] A. Alleyen and J. K. Hedrick, "Nonlinear adaptive control of active suspensions," *IEEE Trans. Contr. Syst. Technol.*, vol. 3, no. 1, pp. 845–860, Mar. 1995.

[17] A. Akbari and B. Lohmann, "Output feedback $H\infty/GH_2$ preview control of active vehicle suspensions: a comparison study of LQG preview," *Vehicle System Dynamics*, vol. 48, no. 12, pp. 1474–1494, Dec. 2010.

[18] M. Abdel-Hady, "Active suspension with preview control," *Vehicle System Dynamics*, pp. 1–13, 1994.

[19] M. Rahman and G. Rideout, "Using the lead vehicle as preview sensor in convoy vehicle active suspension control," *Vehicle System Dynamics*, pp. 1923–1948, 2012.

[20] V. S. Deshpande, P. D. Shendge, and S. B. Phadke, "Nonlinear control for dual objective active suspension systems," *IEEE Trans. Intell. Transp. Syst.*, vol. 18, no. 3, pp. 656–665, Mar. 2017.

[21] Q.-C. Zhong and D. Rees, "Control of uncertain lti systems based on an uncertainty and disturbance estimator," *ASME J. Dyn. Syst. Meas. Control*, vol. 126, no. 4, pp. 905–910, Dec. 2004.

[22] S. E. Talole and S. B. Phadke, "Model following sliding mode control based on uncertainty and disturbance estimator," *ASME J. Dyn. Syst. Meas. Control*, vol. 130, no. 3, p. 034501, 2008.

[23] V.S.Deshpande and S.B.Phadke, "Control of uncertain nonlinear systems using an uncertainty and disturbance estimator," *ASME J. Dyn. Syst. Meas. Control*, vol. 134, pp. 024 501–7, Mar. 2012.

[24] V. S. Deshpande, P. D. Shendge, and S. B. Phadke, "Active suspension systems for vehicles based on a sliding-mode controller in combination with inertial delay control," *Proc IMechE, Part D: J Automobile Engineering*, vol. 227, no. 5, pp. 675–690, May 2013.

[25] S. B. A. Kashem, M. Ektesabi, and R. Nagarajah, "Comparison between different sets of suspension parameters and introduction of new modified skyhook control strategy incorporating varying road condition," *Vehicle System Dynamics*, vol. 50, no. 7, pp. 1173–1190, Jul. 2012.