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ROBUST CONTROL OF ROBOTIC MANIPULATORS WITHOUT VELOCITY FEEDBACK

JING YUAN AND YURY STEPANENKO

Department of Mechanical Engineering, University of Victoria, Victoria, BC V8W 2Y2, Canada

SUMMARY

This study concerns the problem of robust control of robotic manipulators without joint velocity feedback. A robust lead + bias controller is studied. The bias signal is intended to compensate the nonlinear dynamics of the robot. The focus of this study is robustness when the nonlinear compensation is not perfect and the external disturbances are not negligible.

A conservative polynomial bound is introduced to describe the worst feedback effect of the compensation error and the external disturbances. The polynomial bound covers a class of possible bias signals, synthesized according to the available knowledge about the robot dynamics. Based on the polynomial bound, the tracking errors of a lead + bias controller are proved to be uniformly bounded. They can be minimized by a proper design of the bias signal. In the ideal case where the bias signal compensates the robot dynamics perfectly, the tracking errors will converge to zero.

1. INTRODUCTION

Robust tracking control of robotic manipulators has been studied by many researchers.[1] Most of the reported controllers require complete state feedback to provide stable tracking for the closed-loop system, which means that both position and velocity must be measured at each joint. While the joint positions can be measured very accurately by encoders, the joint velocity measurements are often contaminated by noise, due to the less accurate nature of tachometers. To overcome this problem, some researchers proposed nonlinear observers for joint velocity estimation. A sliding observer for general nonlinear systems was studied by Slotine *et al.* (1987);[2] the first observer for robotic systems was proposed by Canudas de Wit and Slotine (1989);[3] Nicosia *et al.*[4–6] studied a number of nonlinear observers for nonlinear systems and elastic robots; observers plus controllers were studied by Nicosia and Tomei (1990)[7] and Canudas de Wit *et al.* (1990).[8] More recently, robust nonlinear smooth observers have been reported by Canudas de Wit *et al.*[9,10] All these works have a common objective: robust control of robotic manipulators without direct measurement of joint velocities.

In this paper, a different approach is investigated. Instead of trying to estimate the velocity by observers, the high-pass filtered position feedback is used as a substitute for the velocity feedback. In other words, a lead + bias controller is applied to robotic manipulators. The lead compensator is synthesized by the traditional technique for linear time-invariant systems while the bias signal is synthesized by feed-forward dynamics. When the robot parameters are not correct, the bias signal will be inaccurate. In order to cover a large class of admissible bias

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signals, a polynomial bound is introduced to describe the worst possible effect of the compensation error. Based on the polynomial bound, the tracking errors of a lead + bias controller are proved to be uniformly bounded if the feedback gain is sufficiently large. The area of attraction can be enlarged and the tracking error bound can be reduced simultaneously by adjusting a single design constant.

The paper is organized as follows: a detailed discussion on the system model, the polynomial bound and its relation to the possible bias signals is given in Section 2. In Section 3, a lemma is proved which plays an important role in the robust analysis to be presented in Section 4. The robustness of the lead + bias controller is investigated by the Lyapunov method. The area of attraction and the tracking error bound is shown to be adjustable by a feedback gain. Simulation result is presented in Section 5 to demonstrate the robust tracking of a lead + bias controller applied to a simple two-link robot. A brief concluding remark is then presented in Section 6.

2. THE SYSTEM MODEL

The mathematic model of a n-link rigid-body robotic manipulator is given by

$$M(q)\ddot{q} + C(\dot{q}, q)\dot{q} + g(q) = \tau + \tau_d \tag{1}$$

where $q \in R^n$ denotes the generalized co-ordinates of the robot; $\tau \in R^n$ is the generalized control torque vector; τ_d represents the external disturbances generated by the environment; $M(q)$, $C(\dot{q}, q) \in R^{n\times n}$ and $g(q) \in R^n$ are nonlinear functions of q and $\dot{q}$. $M(q)$ is the system inertia matrix, $C(\dot{q}, q)\dot{q}$ represents the centripetal and Coriolis force while $g(q)$ denotes the gravitational force. For convenience, the system dynamic equation (1) is re-written in the form

$$M(q)\ddot{e} = \tau + \tau_* \tag{2}$$

where $e = q - q_d$, q_d is the desired trajectory vector and $\tau_* = \tau_d - M(q)\ddot{q}_d - C(\dot{q}, q)\dot{q} - g(q)$.

A lead + bias controller is synthesized by

$$\tau = \beta k \psi - \frac{1}{\rho} k e + \tau_b \tag{3}$$

where $\tau_b = \hat{M}\ddot{q}_d + \hat{C}\dot{q}_d + \hat{g}$ is the bias signal intended to compensate the nonlinear dynamics (it will be discussed later); ψ is a high-pass filtered version of e, synthesized by

$$\psi + \sigma k \int \psi \, dt = -ke \tag{4}$$

In general $k > 0$ can be replaced by a positive definite matrix. However, for clarity of derivations, k is chosen to be a positive scalar here. The other design constants ρ, β and σ are all positive. Their specific choice will be discussed in detail in Section 4.

Substituting (3) into (2), one can express the closed-loop system dynamics as

$$\ddot{e} = M^{-1}(q)k[\beta\psi - \frac{1}{\rho}e] + M^{-1}(q)\tau_0 \tag{5}$$

where $\tau_0 = \tau_b + \tau_{\#}$. According to the well-established results of the previous researchers,[1,11,12] $\| \tau_0 \|$ can be bounded by a second-order polynomial of $\| \dot{e} \|$ given as

$$\| \tau_0 \| = \| \tau_b + \tau_d - M(q)\ddot{q}_d - C(\dot{q}, q)\dot{q} - g(q) \| \leqslant c_0 + c_1 \| \dot{e} \|^2 \tag{6}$$

Denote as $\Delta M = \hat{M} - M(q)$, $\Delta C = \hat{C} - C(\dot{q}_d, q)$ and $\Delta g = \hat{g} - g(q)$, then a straightforward calculus will result in

$$c_0 = \sup \| \tau_d + \Delta M \ddot{q}_d + \Delta C \dot{q}_d + \Delta_g \|$$
$$c_1 = \sup_q \sup_{\|x\|=1} \| C(\dot{q}_d, q)x + C(x, q)\dot{q}_d \|$$
$$c_2 = \sup_q \sup_{\|x\|=1} \| C(x, q)x \|$$

Unlike c_1 and c_2, which only depend on the robot dynamics and the desired tracking speed, the constant bound c_0 is a function of the bias signal τ_b. It can be viewed as a measurement of the compensation error. In the ideal case where the system is free of external disturbances and the robot parameters are accurate, then $\tau_d = 0$, $\hat{M} = M(q)$, $\hat{C} = C(\dot{q}_d, q)$ and $\hat{g} = g(q)$. As a result, c_0 will be reduced to zero. In general, one cannot expect perfect compensation for various practical reasons. It is reasonable to set a conservative constant value for c_0.

Another class of bias signals are synthesized by neural networks. The connection weights of a neural net are adjusted by some learning rules. If the outputs of the neurons are bounded and the connection weights are constrained within a finite ball in the weight space, then (6) is valid and the robustness result based on such a polynomial bound also applies to a class of neural net controllers with a lead compensator as the feedback control part.

3. A LEMMA ON SOME POSITIVE DEFINITE FUNCTIONS

In this study, a Lyapunov-type stability analysis is conducted to investigate the robustness of the closed-loop system (5). Since the nonlinear compensation error τ_0 could grow with a magnitude of $\| \dot{e} \|^2$ as (6) suggests, it is very difficult to establish asymptotic stability for the closed-loop system. The main objective of this study is to establish uniformly ultimately boundedness to the tracking errors. The following Lemma plays an important part in the analysis.

Lemma 1

If a positive definite function $V(t)$ satisfies a differential inequality

$$\dot{V} \leqslant \gamma(t, V) \tag{7}$$

where $\gamma(t, V)$ is bounded for $V(t) \in [0, V_b)$, and

$$\gamma(t, V) < 0 \qquad \text{whenever } V(t) \in (V_a, V_b) \tag{8}$$

then $V(t)$ is uniformly bounded and $\lim_{t\to\infty} V(t) \leqslant V_a$ as long as $V(0) < V_b$.

Proof. Let $\{t_k = k\delta t\}$ denote a sequence of sampling strobes with $\delta t \to 0$. Then the knowledge, given by (7) and (8) can be combined to a simple expression

$$V(t_{k+1}) < V(t_k) \qquad \text{whenever } 0 \leqslant V_a < V(t_k) < V_b \tag{9}$$

The initial condition $V(0) < V_b$ includes two possibilities: $\{V_a < V(0) < V_b\} \cup \{V(0) \leqslant V_a\}$.

In the first case, $V_a < V(0) < V_b$. A sequence of samples of $V(t)$ can be obtained:

$$V_a = V(t_a) < V(t_{k+1}) < V(t_k) < \ldots < V(t_2) < V(t_1) < V(0) < V_b$$

The decreasing nature of such a sequence can be verified by the sufficient condition of (9). Since the above-sampled sequence is obtained with an infinitely small sampling interval δt, $V(t)$ must be monotonously decreasing for $0 \leqslant t \leqslant t_a$.

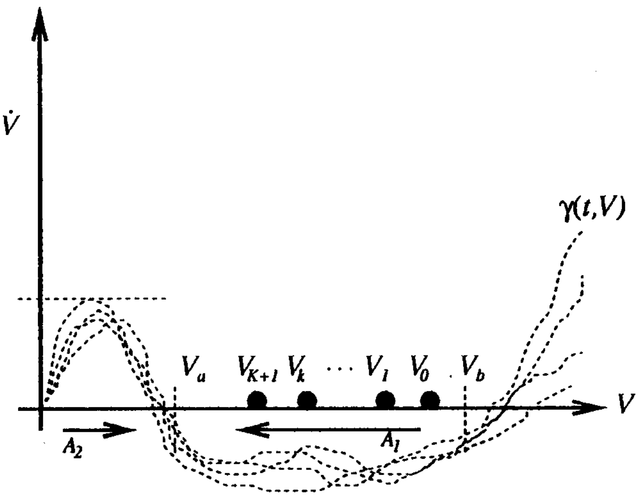

Figure 1. A visual illustration of Lemma 1

In the second case, $V(0) \leqslant V_a$, one must consider the worst possible case where $V(t)$ increases towards infinity. According to the given knowledge, $V(t)$ cannot jump from $V(t) \leqslant V_a$ to $V(t+\delta t) \geqslant V_b$ for an arbitrarily small δt, because $\dot{V}$ is bounded from above by $\gamma(t, V)$. However, $V(t)$ could reach V_a for the first time at some instant $t = t_a$. (if $V(0) = V_a$, then $t_a = 0$.)

Suppose somehow $V(t)$ manages to inch further towards V_b such that $V(t\xi) = V_a + \xi$ at $t_a < t_\xi$, where ξ is an arbitrarily small positive constant. Then (9) will force $V(t_\xi + \delta t) < V(t_\xi)$ because $V_a < V(t_\xi) < V_b$, making it impossible for $V(t)$ to exceed $V_a + \xi$ after $t = t_a$. By letting $\xi \to 0$, one obtains $\lim_{t\to\infty} V(t) \leqslant V_a$. Q.E.D.

An illustrative picture is provided in Figure 1 to give a visual explanation of the proof. In Figure 1, $\dot{V}$ is plotted versus V. One can see that $\dot{V}$ is bounded from above by $\gamma(t, V)$, which is represented by a family of functions because of another argument t. Figure 1 provides the following hints:

(1) $V(t)$ cannot jump from $V(t) \leqslant V_a$ to $V(t+\delta t) \geqslant V_b$ for an arbitrarily small δt, because $\dot{V} \leqslant \gamma(t, V) < \infty$ when $0 \leqslant V(t) < V_b$.
(2) Since $\dot{V} < 0$ whenever $V \in (V_a, V_b)$, the interval (V_a, V_b) creates a region of attraction as demonstrated by arrow A_1 in the figure. If initially $V_a < V(0) < V_b$, then $V(t)$ will be attracted towards V_a.
(3) Once $V(t) \leqslant V_a$, it will not be able to bounce back, though it is possible that $V(t)$ could be pushed towards V_a as arrow A_2 indicates.

The robustness of the lead + bias controller is to be established by finding a positive definite function which satisfies the condition of Lemma 1.

4. ROBUSTNESS OF THE LEAD COMPENSATOR

Although the high-passed error signal ψ is synthesized by (4) without physically involving the velocity error $\dot{e}$, mathematically, (4) is equivalent to

$$\dot{\psi} = -k\dot{e} - \sigma k\psi \tag{10}$$

if the initial value of ψ is set to zero. In the theoretical analysis, the closed-loop system will be described by (5) and (10). Their state-space representation is given by

$$\dot{\varepsilon} = -A\varepsilon + \tau_s \tag{11}$$

where $\varepsilon = [\dot{e}, \psi, e]^T$,

$$A = \begin{bmatrix} 0 & -M^{-1}\beta k & M^{-1}\frac{k}{\rho}I \\ kI & \sigma kI & 0 \\ -I & 0 & 0 \end{bmatrix} \quad \text{and} \quad \tau_s = \begin{bmatrix} M^{-1}(q)\tau_0 \\ 0 \\ 0 \end{bmatrix}$$

In the Lyapunov-type stability analysis of a closed-loop system like (11), it is customary to find positive definite matrices P and Q such that $PA + A^TP = Q$. The focal point is usually a positive function $L = \varepsilon^T P\varepsilon$. Its time derivative evaluated along (11) is written as

$$\dot{L} = -\varepsilon^T Q\varepsilon + 2\varepsilon^T P\tau_s + \varepsilon^T \dot{P}\varepsilon \tag{12}$$

where $\| 2\varepsilon^T P\tau_s + \varepsilon^T\dot{P}\varepsilon \|$ could be proportional to $\|\varepsilon\|^3$ because $\|\tau_0\| \propto \|\dot{e}\|^2$. In order to ensure stability to the closed-loop system, it is important to select P and Q such that $\lambda_{Q\min}$, the smallest eigenvalue of Q, can be set sufficiently large without affecting $\| 2\varepsilon^T P\tau_s + \varepsilon^T\dot{P}\varepsilon \|$.

4.1. The positive definite matrix pair P and *Q*

According to References 11 and 12, the inertia matrix $M(q)$ is uniformly bounded and positive definite. Therefore, one can write

$$\lambda_{\min} = \inf_{q} \inf_{\|x\|=1} \| M(q)x \| \quad \text{and} \quad \lambda_{\text{Max}} = \sup_{q} \sup_{\|x\|=1} \| M(q)x \| \tag{13}$$

Now, the three design parameters β, σ and ρ can be determined by

$$0 < \rho \leqslant \lambda_{\min}, \qquad \sigma \geqslant 3 \qquad \text{and } \beta = \rho(2+\sigma) \tag{14}$$

A possible P–Q matrix pair is given by

$$P = \begin{bmatrix} M & \rho I & \frac{1}{\beta}I \\ \rho I & 2\rho I & 0 \\ \frac{1}{\beta}I & 0 & \frac{k}{\rho}I \end{bmatrix} \quad \text{and } Q = \begin{bmatrix} \left(\rho k - \frac{1}{\beta}\right)I & 0 & 0 \\ 0 & \rho k(2\sigma I - \beta M^{-1}) & 0 \\ 0 & 0 & \frac{k}{\rho\beta}M^{-1} \end{bmatrix}$$

where (14) has been substituted to eliminate the off-diagonal sub-matrices of Q.

Denote as $y^T = [y_1^T, y_2^T, y_3^T]$ where $y_1, y_2, y_3 \in R^n$. Then $y^T P y$ can be expressed as

$$\begin{aligned} y^T P y &= y_1^T M y_1 + 2\rho \| y_2 \|^2 + \frac{k}{\rho} \| y_3 \|^2 + 2\rho y_1^T y_2 + \frac{2}{\beta} y_1^T y_3 \\ &\geqslant \rho \| y_1 + y_2 + \frac{1}{\beta\rho} y_3 \|^2 + \frac{1}{\rho}\left(k - \frac{1}{\beta^2}\right) \| y_3 \|^2 + \rho \| y_2 \|^2 \end{aligned} \tag{16}$$

where (13) and (14) have been substituted. When

$$\frac{1}{\rho}\left(k - \frac{1}{\beta^2}\right) > \lambda_{\text{Max}} \tag{17}$$

inequality (16) implies

$$\alpha^2_{\min} = \inf_q \inf_{\|y\|=1} y^{\mathrm{T}} P y \geqslant \rho \qquad \text{and } \alpha^2_{\max} = \sup_q \sup_{\|y\|=1} y^{\mathrm{T}} P y \leqslant \frac{k}{\rho} \tag{18}$$

The above inequality will be very helpful in the later stability analysis.

With the help of (14), it is not difficult to verify $x^{\mathrm{T}}(2\sigma I - \beta M^{-1})x \geqslant x^{\mathrm{T}}x$ for any $x \in R^n$. As indicated by (15), Q is positive definite if

$$\rho k > \frac{1}{\beta} \tag{19}$$

As long as (17) and (19) are satisfied, both P and Q are uniformly positive definite. The smallest eigenvalue of Q is given by $\lambda_{Q\min} \approx \rho k$.

It is also interesting to note that

$$\varepsilon^{\mathrm{T}} \dot{P} \varepsilon + 2\varepsilon^{\mathrm{T}} P \tau_s = \dot{e}^{\mathrm{T}} \dot{M} \dot{e} + 2(\rho\psi + \beta^{-1} e)^{\mathrm{T}} M^{-1} \tau_0 + 2\dot{e}^{\mathrm{T}} \tau_0 \tag{20}$$

which is independent of k. A substitution of (6) and (13) then results in

$$\|\varepsilon^{\mathrm{T}} \dot{P} \varepsilon + 2\varepsilon^{\mathrm{T}} P \tau_s\| \leqslant d_0 \|\varepsilon\| + d_1 \|\varepsilon\|^2 + d_2 \|\varepsilon\|^3 \tag{21}$$

where $d_0 > 0$, $d_1 > 0$ and $d_2 > 0$. Particularly $d_0 \propto c_0$. These constants are independent of k.

The whole discussion of this subsection can be summarized into one sentence: *by adjusting k, one can increase $\lambda_{Q\min}$ without affecting* $\|\varepsilon^{\mathrm{T}} \dot{P} \varepsilon + 2\varepsilon^{\mathrm{T}} P \tau_s\|$, which is essential to the following analysis.

4.2. *A polynomial bound for $\dot{L}$*

Substituting (21) into (12) results in

$$\begin{aligned} \dot{L} &\leqslant d_0 \|\varepsilon\| - (\lambda_{Q\min} - d_1)\|\varepsilon\|^2 + d_2 \|\varepsilon\|^3 \\ &= d_2 \|\varepsilon\| (\|\varepsilon\| - r_1)(\|\varepsilon\| - r_2) \end{aligned} \tag{22}$$

where $r_1 < r_2$ are given by

$$r_1 < r_2 = \frac{(\lambda_{Q\min} - d_1) \pm \sqrt{(\lambda_{Q\min} - d_1)^2 - 4 d_0 d_2}}{2 d_2} \tag{23}$$

They are positive constants if

$$(\lambda_{Q\min} - d_1)^2 > 4 d_0 d_2 \qquad \text{or } \lambda_{Q\min} > d_1 + 2\sqrt{d_0 d_2} \tag{24}$$

This condition can be easily satisfied by a sufficiently large k.

By substituting

$$\|\varepsilon\| = \frac{\|\varepsilon\|}{\sqrt{L}} \sqrt{L} = \frac{\sqrt{L}}{\alpha}$$

one can express inequality (22) in terms of L

$$\dot{L} \leqslant d_2 \frac{\sqrt{L}}{\alpha^3(t)} (\sqrt{L} - r_1 \alpha)(\sqrt{L} - r_2 \alpha) \tag{25}$$

where $\alpha(t)$ is defined as

$$\alpha^2 = \frac{L(t)}{\varepsilon^{\mathrm{T}} \varepsilon} = \frac{\varepsilon^{\mathrm{T}} P \varepsilon}{\varepsilon^{\mathrm{T}} \varepsilon}$$

When $\lambda_{Q\min} \approx \rho k$ is sufficiently large, one can write

$$r_1 = \frac{r_1 r_2}{r_2} = \frac{2d_0}{(\lambda_{Q\min} - d_1) + \sqrt{(\lambda_{Q\min} - d_1)^2 - 4d_0 d_2}} \propto \frac{d_0}{\rho k}$$

Similarly, it can be shown that

$$r_2 \propto \frac{\rho k}{d_2}$$

Taking (18) into account, it is not difficult to show

$$r_1 \alpha_{\max} \propto \frac{d_0}{\sqrt{\rho^3} k} \to 0 \qquad \text{and } r_2 \alpha_{\min} \propto \frac{k}{d_2} \sqrt{\rho^3} \to \infty \qquad \text{as } k \to \infty \tag{26}$$

According to (25), $\dot{L} < 0$ whenever $r_1 \alpha_{\max} < \sqrt{L} < r_2 \alpha_{\min}$. In other words, L satisfies the conditions of Lemma 1. As long as the initial error $\varepsilon(0)$ is small enough such that

$$L(0) < r_2^2 \alpha_{\min}^2 \propto \frac{k^2}{d_2^2} \rho^3 \tag{27}$$

then $L(t)$ is uniformly bounded. It will eventually converge into a final bound given by

$$\lim_{t \to \infty} L(t) \leqslant r_1^2 \alpha_{\max}^2 \propto \frac{d_0^2}{\rho^3 k} \tag{28}$$

It must be emphasized that a large k will reduce $r_1 \alpha_{\max}$ and enlarge $r_2 \alpha_{\min}$ as (26) indicates. This means that *the area of attraction* (27) *can be enlarged and the ultimate error bound* (28) *can be reduced by increasing k alone.* Even when k is not large enough to force a large $r_2 \alpha_{\min}$, the initial condition constraint (27) can be easily satisfied by specifying a smooth desired trajectory such that $\dot{q}_d = \dot{q}$ and $q_d = q$ at $t = 0$ (which implies $\| \varepsilon(0) \| = 0$).

4.3. Determine the design parameters

The design parameters are determined by several estimated parameters d_0, d_1 and d_2 given by (21), as well as $\lambda_{\min}$ and $\lambda_{\max}$ given by (13). If these parameters are not completely available, some conservative bounds on them can be substituted.

The first step is to fix β, σ and ρ according to (14). It must be emphasized that (14) is by no means a necessary condition. It is mainly intended to make Q, as given by (15), a block diagonal matrix. For analysis purposes, this make it easy to show the positive definiteness of Q. In fact, there are many other ways to determine these three parameters, such that P and Q are both uniformly positive definite.

The next step is to choose k. It is subject to four sufficient conditions (17), (19), (24) and (26). The four inequalities represent four open sets, all of them extending to positive infinity. Therefore their intersection must be an open set $k \in (k_{\min}, +\infty)$ where $k_{\min}$ is a finite positive constant. In practice, one can simply determine k by trial and error. The above four inequalities ensure a stable closed-loop when k is sufficiently large.

The lead + bias controller can be simplified until the bias signal τ_d is zero. Thus, d_0, d_1 and d_2 may be large. However, one can increase k to stabilize the system and make the tracking error arbitrarily small. This is suitable for industrial applications where another reason to avoid velocity feedback is to reduce cost, and where the tracking accuracy is not too strict.

Of course, the most effective way to improve tracking accuracy is to reduce d_0. According to (26), $r_1^2\alpha_{\max}^2 \propto d_0^2$. By substituting (6) and (13) into (20) to derive (21), it is not difficult to find $d_0 \propto c_0 = \sup_q \| \tau_d + \Delta M\ddot{q}_d + \Delta C\dot{q}_d + \Delta g \|$. Thus one can reduce the ultimate error bound (28) by improving the estimated $\hat{M}$, $\hat{C}$ and $\hat{g}$. In the ideal case, $\tau_d = 0$, $\hat{M} = M(q)$, $\hat{C} = C(\dot{q}_d, q)$ and $\hat{g} = g(q)$, c_0 will be reduced to zero. This means that the tracking errors of the closed-loop system will eventually converge into zero.

5. SIMULATION RESULTS

A simulation experiment is conducted to demonstrate the performance of a lead + bias compensator. A two-link planar robot described by

$$M(q)\begin{bmatrix}\ddot{q}_2\\ \ddot{q}_1\end{bmatrix} + C(\dot{q}, q)\begin{bmatrix}\dot{q}_1\\ \dot{q}_2\end{bmatrix} + g(q) = \begin{bmatrix}\tau_1\\ \tau_2\end{bmatrix}$$

is used as the control object, where

$$M(q) = \begin{bmatrix}(2l_1\cos(q_2) + l_2)l_2m_2 + l_1^2(m_1+m_2) & l_2^2m_2 + l_1l_2\cos(q_2)m_2\\ l_2^2m_2 + l_1l_2\cos(q_2)m_2 & l_2^2m_2\end{bmatrix}$$

$$C(\dot{q}, q) = \begin{bmatrix}-2l_1l_2m_2\sin(q_2)\dot{q}_2 & -l_1l_2m_2\sin(q_2)\dot{q}_2\\ l_1l_2m_2\sin(q_2)\dot{q}_1 & 0\end{bmatrix}$$

and

$$g(q) = \begin{bmatrix}g(m_2l_2\cos(q_1+q_2) + (m_1+m_2)l_1\cos(q_1))\\ m_2l_2g\cos(q_1+q_2)\end{bmatrix}$$

The parameters are chosen as $l_1 = 0{\cdot}7$, $l_2 = 0{\cdot}5$ (metre), $m_1 = 10$ and $m_2 = 5$(kg).

The bias signal τ_b is synthesized with inaccurate parameters $l_1 = 0{\cdot}7$, $l_2 = 0{\cdot}25$ (metres), $m_1 = 10$ and $m_2 = 1$ (kg). The inaccurate parameters of the second link simulate those cases where the robot carries an unknown payload. The desired trajectory is given by

$$q_{d1} = q_{d2} = 1 - \cos(2\pi t) \text{ (radians)}$$

which is a relatively fast movement. The lead compensator is synthesized by

$$\tau = 300\psi - 3000e + \tau_b \qquad \text{and} \qquad \psi + 300\int_0^t \psi = -300e$$

with tracking errors plotted in Figure 2. The relative tracking error is about 6 per cent measured peak versus peak. The control torques applied to the two joints are plotted in Figure 3.

In order to observe the performance of the lead + bias controller when the bias signal is zero, another simulation experiment is conducted where everything is the same as the previous experiment, except that τ_b is an all-zero vector. The tracking errors and control torques are plotted in Figures 4 and 5 respectively. It is interesting to note that the tracking error is not too bad, when the bias signal is missing even because of the large gains used in synthesizing the lead controller. However, the effect of the bias signal is still obvious: the tracking error of joint 1 becomes larger. Joint 2 keeps approximately the same performance because its parameters m_2 and l_2 were not correct in the first experiment.

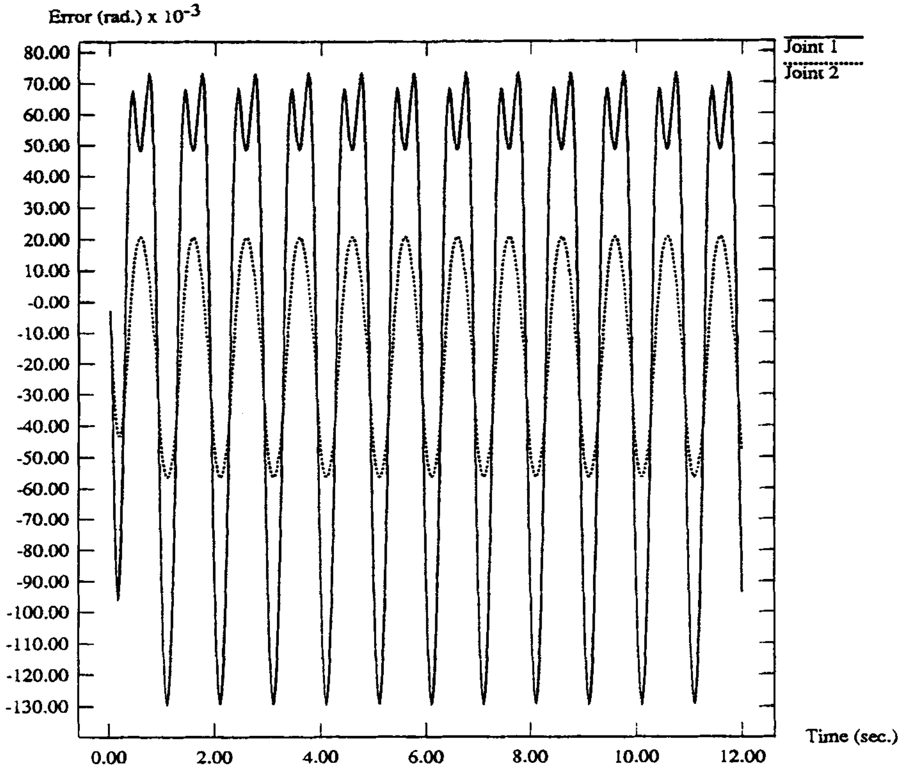

Figure 2. Tracking errors of the lead + bias compensator

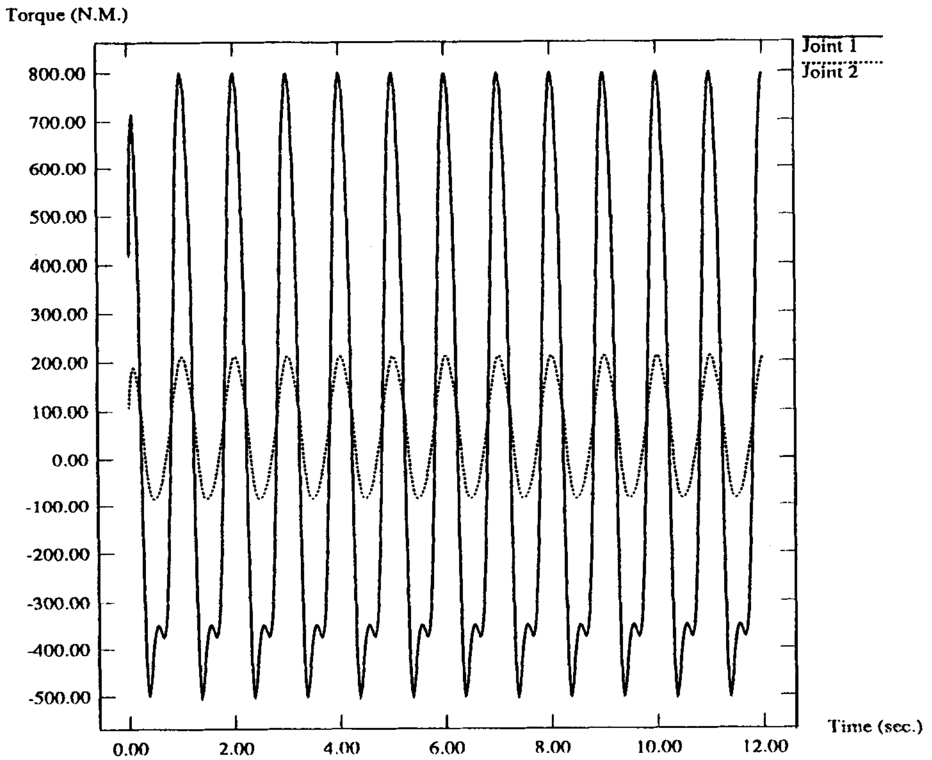

Figure 3. Control torques of the lead + bias compensator

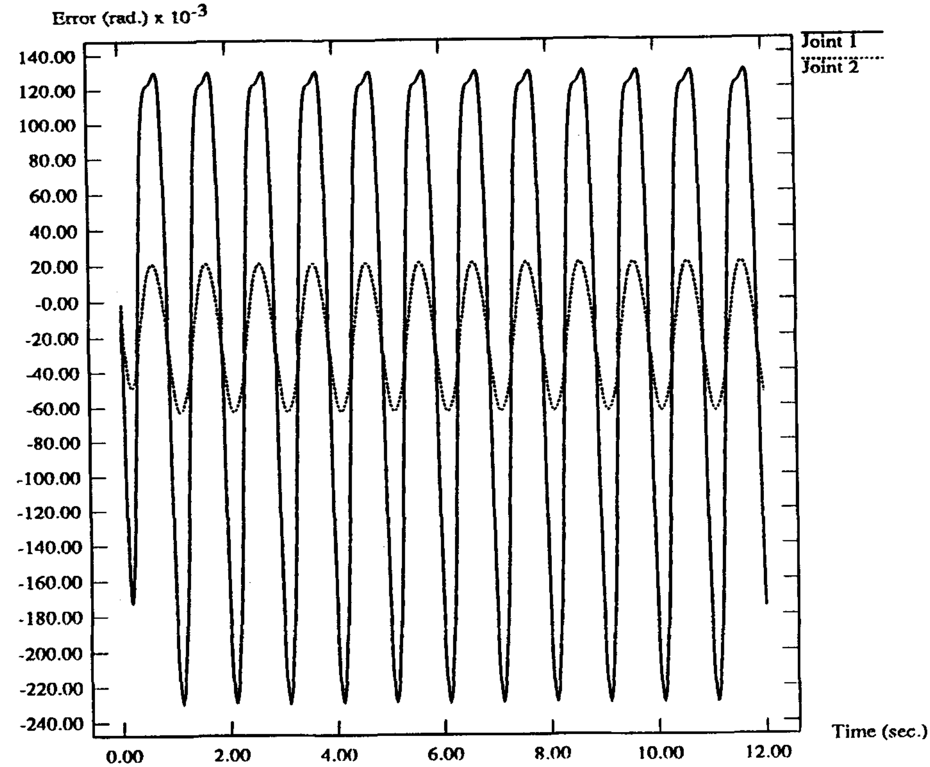

Figure 4. Tracking errors of the lead compensator

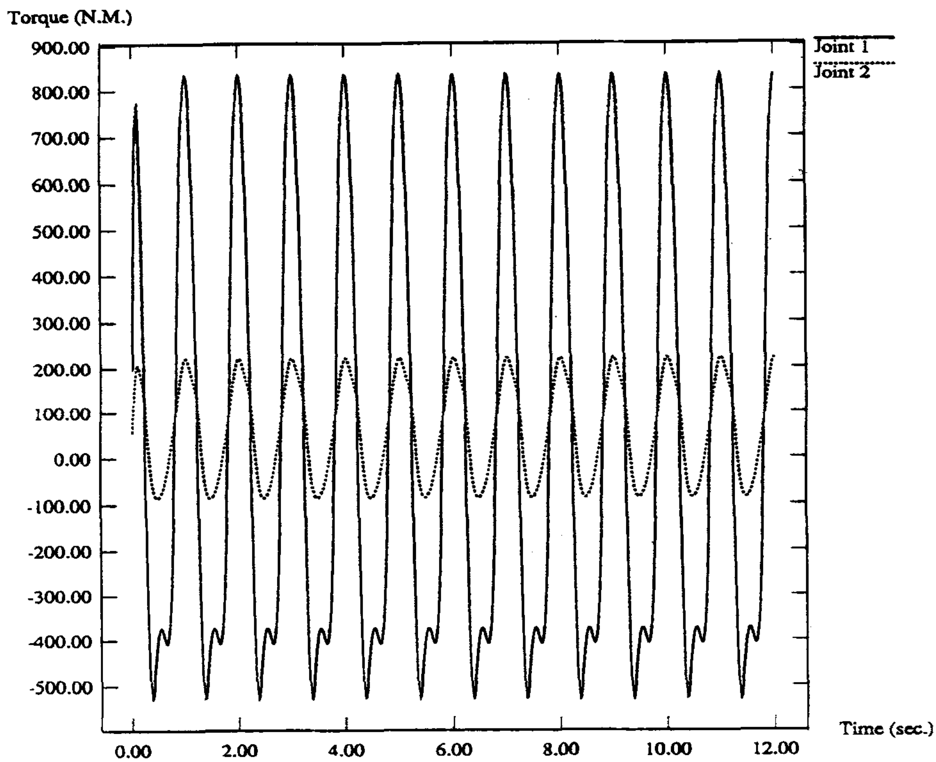

Figure 5. Control torques of the lead compensator

6. CONCLUSIONS

The robustness of a class of lead + bias compensators is established for robotic manipulators. The result also applies to adaptive controllers with lead compensation plus an adaptive bounded bias signal. In order to decouple the effect caused by adapting the bias signal, a conservative polynomial bound is introduced to describe the worst possible feedback effects. The closed-loop system is then proved to be uniformly bounded. The area of attraction can be enlarged and the tracking error can be reduced simultaneously by increasing the feedback gain. The simulation result is presented to demonstrate the performance of the lead + bias compensator applied to a two-link planar robot.

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REFERENCES

1. Abdallah, C., D. Dawson, P. Dorato and M. Jamshidi, 'Survey of robust control for rigid robots', *IEEE Control Systems,* **11**, 24–30 (1991).
2. Slotine, J. J., J. K. Hedrick, and E. A. Misawa, 'Sliding observer for nonlinear systems', *ASME J. Dyn. Sys., Meas. and Control,* **109**, 245–252, (1987).
3. Canudas de Wit, C., and J. J. Slotine, 'Sliding observers for robot manipulators', *IFAC Symposium on Nonlinear Control System Design*, Capri, Italy, August 1989. Also to appear in Automatica (1991).
4. Nicosia, S., P. Tomei, and A. Tornambe, 'An approximate observer for a class of nonlinear systems', *Syst. Contr. Lett.,* **12**, 43–51 (1989).
5. Nicosia, S., P. Tomei and A. Tornambe, 'A nonlinear observer for elastic robots', *IEEE J. Robot. Automat.,* **RA-4**, 45–52 (1988).
6. Nicosia, S., P. Tomei and A. Tornambe, 'Non-linear control and observation algorithms for a single-link flexible robot arm', *Int. J. Contr.,* **49**, 827–840 (1989).
7. Nicosia, S., and P. Tomei, 'Robot control by using only joint position measurements', *IEEE Trans. Automatic Control,* **AC-35**, 1058–1061 (1990).
8. Canudas de Wit, C., K. J. Astrom, and N. Fixot, 'Computed torque control via a nonlinear observer', *Int. J. Adap. Control,* **4**, 443–452 (1990).
9. Canudas de Wit, C., and N. Fixot, 'Robot control via robust state estimate feedback', to appear in *IEEE Trans. Automatic Control.*
10. Canudas De Wit, C., and N. Fixot, 'Adaptive control of robots via velocity estimated feedback', *IEEE Conference on Robotics and Automation,* Sacramento, CA, April, 1991, pp. 16–21.
11. Spong, M. W., and Vidyasagar, M., *Robot Dynamics and Control,* John Wiley, New York, 1989.
12. Craig, J. J., 'Adaptive Control of Mechanical Manipulators, Addison-Wesley, Reading, MA, 1988.

one has

$$K_N^{-1}\Phi_a\alpha_a = \Phi_a K_{Na}^{-1}\alpha_a = \Phi_a\alpha_{ak} \tag{5}$$

where $K_{Na} \triangleq \operatorname{diag}[k_{Ni}I_m]$ and $\alpha_{ak}^T \triangleq K_{Na}^{-1}\alpha_a$.

We suppose that in the right hand side of (5) only the parameter vector α_{ak} is *uncertain*.' The desired I_d is then synthesized by

$$I_d = \Phi_a(q, \dot{q}_d, \ddot{q}_d)\hat{\alpha}_{ak} - \gamma^2\Gamma(w + \kappa\tilde{q}) \tag{6}$$

$$w = \overline{w} + \gamma^2\tilde{q} \tag{7}$$

$$\dot{\overline{w}} = -2\gamma\overline{w} - 2\gamma^3\tilde{q} \tag{8}$$

$$\dot{\hat{\alpha}}_{ak} = \operatorname{Proj}(\hat{\alpha}_{ak}, -\sigma\Phi_a^T z), \quad \hat{\alpha}_{ak}(0) \in \Pi \tag{9}$$

$$z \triangleq \dot{\tilde{q}} - \frac{1}{\gamma}w + \frac{\kappa}{\gamma}\tilde{q} \tag{10}$$

where $\tilde{q} \triangleq q - q_d$ is the joint tracking error; $\hat{\alpha}_{ak}$ is the estimate of α_{ak}; Γ is an arbitrary positive definite constant diagonal matrix; γ, κ, and σ are positive constants; w and $\overline{w}$ are intermediate vectors; $\operatorname{Proj}(\cdot,\cdot)$ is a projection operator, which is constructed as follows.

Choose a set $\Pi = \{\alpha_{ak} | \theta_{i\min} < \alpha_{aki} < \theta_{i\max} \forall i \in \{1, n \times m\}\}$ with $\theta_{i\min}$ and $\theta_{i\max}$ some known real numbers. In this case, the projection operator defined by

$$\{\operatorname{Proj}(\hat{\alpha}_{ak}, -\sigma\Phi_a^T z)\}_i = \begin{cases} 0 & \text{if } \hat{\alpha}_{aki} = \theta_{i\max} \text{ and } \sigma(\Phi_a^T z)_i < 0 \\ -\sigma(\Phi_a^T z)_i & \text{if } [\theta_{i\min} < \hat{\alpha}_{aki} < \theta_{i\max}] \\ & \text{or } [\hat{\alpha}_{aki} = \theta_{i\max} \text{ and } \sigma(\Phi_a^T z)_i \geq 0] \\ & \text{or } [\hat{\alpha}_{aki} = \theta_{i\min} \text{ and } \sigma(\Phi_a^T z)_i \leq 0] \\ 0 & \text{if } \hat{\alpha}_{aki} = \theta_{i\min} \text{ and } \sigma(\Phi_a^T z)_i > 0 \end{cases} \tag{11}$$

satisfies

1) $\hat{\alpha}_{ak}(t) \in \Pi$ if $\hat{\alpha}_{ak}(0) \in \Pi$;
2) $\|\operatorname{Proj}(p, y)\| \leq \|y\|$;
3) $-(p - p^*)^T\Lambda\operatorname{Proj}(p, y) \geq -(p - p^*)^T\Lambda y$, where Λ is a positive definite symmetric matrix.

Remarks:

1) The choice of $\theta_{i\min}$ and $\theta_{i\max}$ is only related to the bound range of the projection operator and such a range in this paper is not restricted as long as the estimated parameters are bounded (required for the stability proof); hence one can always choose suitable $\theta_{i\min}$ and $\theta_{i\max}$, although such a choice may be conservative.
2) It can easily be shown that $\hat{\alpha}_{ak}$ does not involve link velocity measurements, though $\dot{\hat{\alpha}}_{ak}$ includes the signal $\dot{q}$. Therefore, I_d only needs link position measurements. This fact will be used later to prove that the controller for the overall system will depend only on the measurements of I and q.
3) The role of the projection operator is crucial. The boundedness of the estimated parameters $\hat{\alpha}_{ak}$ can be guaranteed; as will be clear from the theorem proof, it is this boundedness that makes it possible to prove the semi-global stability of the overall system.

C. Hybrid Adaptive Control for the Actuator Subsystem

We now turn to the development of a voltage input u, which forces $\tilde{I}$ to zero. However, as shown in [16], using the backstepping technique [19], we are required to calculate

$$\dot{I}_d = (d/dt)(\Phi_a(q, \dot{q}_d, \ddot{q}_d)\hat{\alpha}_{ak}) - \gamma^2\Gamma(\dot{w} + \kappa\dot{\tilde{q}})$$

where $(d/dt)(\Phi_a\hat{\alpha}_{ak}) = \dot{\Phi}_a\hat{\alpha}_{ak} + \Phi_a\dot{\hat{\alpha}}_{ak}$. Hence, the calculation of $\dot{\Phi}_a$ is involved. Also, the calculations of derivative $\dot{I}_d$ require measurements of the velocity $\dot{q}$. The challenge addressed here is to design the control input u without involving the computation of $\dot{\Phi}_a$ and the measurements of $\dot{q}$. In order to do so, we divide the embedded signals I_d as

$$I_d \triangleq I_p + I_c \tag{12}$$

$$I_p \triangleq \gamma^2\Gamma(\gamma^2\tilde{q} - w) \tag{13}$$

$$I_c \triangleq \Phi_a(q, \dot{q}_d, \ddot{q}_d)\hat{\alpha}_{ak} - \gamma^2\Gamma(\gamma^2 + \kappa)\tilde{q} \tag{14}$$

and simply substitute

$$\dot{I}_p = \gamma^2\Gamma(\gamma^2\dot{\tilde{q}} - \dot{w}) = 2\gamma^3\Gamma w \tag{15}$$

for $\dot{I}_d$. The effect of the signal I_c will be compensated in the actuator subsystem. We note that in (15) the relation $\dot{w} = -2\gamma w + \gamma^2\dot{\tilde{q}}$ has been used. So, no velocity $\dot{q}$ is involved in (15).

Following [16], it is assumed that the electrical parameters K_N, L, R, and K_e are all of uncertain values. However, there exist L_0, R_0, and K_{eo}, all known, such that

$$\|L - L_0\| \leq \delta_1; \|R - R_0\| \leq \delta_2; \|K_e - K_{eo}\| \leq \delta_3 \tag{16}$$

With the above in mind, the adaptive robust control law, forcing $\tilde{I} = 0$, is then synthesized by

$$u = L_0\dot{I}_p + R_0I_d + K_{eo}\dot{q}_d - (\hat{\delta}_1\|\dot{I}_p\| + \hat{\delta}_2\|I_d\| + \hat{\delta}_3\|\dot{q}_d\|)\operatorname{sgn}(\tilde{I}) \tag{17}$$

$$\dot{\hat{\delta}}_1 = \eta_1\|\dot{I}_p\|\|\tilde{I}\| \tag{18}$$

$$\dot{\hat{\delta}}_2 = \eta_2\|I_d\|\|\tilde{I}\| \tag{19}$$

$$\dot{\hat{\delta}}_3 = \eta_3\|\dot{q}_d\|\|\tilde{I}\| \tag{20}$$

where I_d, I_p, and $\dot{I}_p$ are defined in (6), (13), and (15), $\hat{\alpha}_{ak}$ is given by (9), η_i $(i = 1, 2, 3)$ are constants which determine the rates of adaptations.

Remarks:

1) Thanks to the definition of I_p, the time derivative of I_p does not involve the velocity measurements, which in turn implies no velocity measurements in the controller (17). Thus, the cascade control system only requires the measurements of I and q.
2) It is clear from (17), the time-derivative of the manipulator regressor matrix or upper bounds on the derivatives of the embedded controls are not involved. Therefore, the difficulty encountered in [15], [16] is removed.
3) Using the adaptive method, a scheme without using the velocity measurements was also proposed in [18]. The difference between adaptive and hybrid adaptive/robust schemes was discussed in the remark 3) of [16] and the reader may refer to it.
4) Similar to [16], the control law (17) involves the discontinuous function and may result in chattering behavior. For a discussion on how to remedy this the reader may refer to the remarks 4) and 5) of [16].

D. Stability Analysis

The stability of the closed-loop system described by (1), (2), (6), and (17) is established in the following theorem.

Theorem: If the robust control voltages u given by (6) and (17) are applied to the manipulator (1–2), then all closed-loop signals are bounded and $\lim_{t\to\infty} \tilde{q} = 0$, provided γ initially satisfies:

1) $\gamma \geq \max\{1, \kappa\}$;
2) the matrices P and Q in (24) being positive-definite;

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GLOBAL REGULATION OF UNCERTAIN MANIPULATORS USING BOUNDED CONTROLS

R. Colbaugh* E. Barany† K. Glass*

Departments of *Mechanical Engineering and †Mathematical Sciences
New Mexico State University, Las Cruces, NM 88003 USA

Abstract

This paper considers the position regulation problem for uncertain robot manipulators in the presence of constraints on the available actuator torques, and proposes two new controllers as solutions to this problem. The first controller is derived under the assumption that the manipulator state is measurable, while the second strategy is developed for those applications in which only position measurements are available. Each scheme consists of a nonadaptive component for gross position control and an adaptive component to ensure convergence to the desired position. The controllers are computationally simple, require very little information regarding the manipulator model or the payload, and ensure that the position error is globally convergent. The capabilities of the proposed control strategies are illustrated though both computer simulations and laboratory experiments with an IMI Zebra Zero manipulator.

1. Introduction

The control objective in position regulation of robot manipulators is to cause the manipulator to move from its initial state to any goal position specified by the user. This is also referred to as the position stabilization or point-to-point motion control problem, emphasizing the fact that only the final position is specified and the intermediate trajectory is not stipulated. It is well-known that simple proportional-derivative (PD) feedback controllers are capable of globally asymptotically stable regulation of rigid-link robots, provided that the effects of gravity on the manipulator are compensated. Ordinarily the requisite gravity compensation is achieved by including a gravity model in the control scheme, so that the resulting controller possesses a PD-plus-gravity structure [1].

While the PD-plus-gravity position control law is simple, elegant, and intuitively appealing, there are potential difficulties associated with this approach. First, providing gravity compensation by including a model of the manipulator gravity torques in the control law can be undesirable because this requires precise *a priori* knowledge of both the structure and the parameter values for this model, including the effects of any payload.

[13] for very recent progress on this problem).

This paper introduces two new position regulation schemes for uncertain manipulators with actuator constraints, one which utilizes state feedback and one which is implementable using only position measurements. Each controller is composed of a simple nonadaptive component which guarantees global convergence of the position to a (small) neighborhood of the desired set-point, together with a simple adaptive law which then ensures convergence to the goal position. We believe that these controllers represent the first solutions to the position regulation problem which provide global convergence without information regarding the manipulator model or the payload and in the presence of constraints on the available actuation. In fact, to the best of our knowledge there has not yet been proposed a globally convergent output feedback regulator for uncertain manipulators even in the case of unconstrained inputs (for example, the controllers [11,12] give semiglobal convergence). The efficacy of the proposed approach is illustrated though both computer simulations and laboratory experiments with an IMI Zebra Zero manipulator.

2. Preliminaries

Consider an n degree-of-freedom rigid-link manipulator with joint coordinates $\theta \in \Re^n$ and associated control torques $\mathrm{T} \in \Re^n$. The dynamic model for this system takes the form

$$\mathrm{T} = H(\theta)\ddot{\theta} + V_{cc}(\theta, \dot{\theta})\dot{\theta} + \mathrm{G}(\theta) \tag{1}$$

where $H \in \Re^{n \times n}$ is the manipulator inertia matrix, $V_{cc} \in \Re^{n \times n}$ quantifies Coriolis and centripetal acceleration effects, and $\mathrm{G} \in \Re^n$ is the vector of gravity forces. It is well-known that the dynamics (1) possesses considerable structure. For example, for any set of generalized coordinates θ, the matrix H is bounded, symmetric and positive-definite, the matrix V_{cc} is bounded in θ and depends linearly on $\dot{\theta}$, and the matrices H and V_{cc} are related according to $\dot{H} = V_{cc} + V_{cc}^T$. Additionally, the vector of gravity forces $\mathbf{G}$ is the gradient of a potential energy function and is bounded with bounded first partial derivatives [2].

The focus of this paper is the position regulation problem for uncertain manipulators with bounded controls. More specifically, we wish to develop a strategy for specifying the control input T, using only measurements of the system configuration θ and with no knowledge of the system dynamic model (1), so that (1) evolves from its initial state to the desired final state $\theta = \theta_d$, $\dot{\theta} = 0$ and the control input satisfies the constraint $\| \mathrm{T}(t) \| \leq T_{max}$ for some *a priori* specified bound T_{max}. Note that, for completeness, we will address the state feedback counterpart to this problem as well. It should also be mentioned that, while we restrict our attention to input constraints of the form $\| \mathrm{T}(t) \| \leq T_{max}$, other input constraints (such as constraints on the individual components of the input vector) can be addressed using methods precisely analogous to those employed in this paper [e.g., 14].

3. Global Regulation Schemes

We now turn to the development of two strategies for position regulation of uncertain manipulators with bounded controls. We first consider the case in which it is assumed that the entire manipulator state is available for feedback, and then show how the proposed approach can be extended for implementation in those applications in which only position measurements are available.

3.1 State Feedback Case

Consider the following simple (nonadaptive) position regulation scheme:

$$\mathrm{T} = k_1\gamma \mathrm{sat}(\dot{\mathrm{e}}) + k_2\gamma^2 \mathrm{sat}(\mathrm{e}) \tag{2}$$

where $\mathrm{e} = \theta_d - \theta$ is the position regulation error (recall that θ_d is constant), k_1, k_2, γ are positive scalar constants, and sat$(\cdot)$ is defined as follows: $\mathrm{sat}(\mathrm{x}) = \mathrm{x}$ if $\| \mathrm{x} \| \leq \epsilon$ and $\mathrm{sat}(\mathrm{x}) = \epsilon \mathrm{x} / \| \mathrm{x} \|$ otherwise, where ϵ is a positive scalar. Let G_{max} satisfy $\| \mathrm{G}(\theta) \| \leq G_{max}$ $\forall \theta$, and assume that $T_{max} \geq 3G_{max}$ (smaller lower bounds on T_{max} are possible, but this choice significantly simplifies some of the calculations needed later in the paper). Then choosing $k_1, k_2, \gamma, \epsilon$ so that $k_1\gamma\epsilon \leq G_{max}$ and $G_{max} < k_2\gamma^2\epsilon \leq 2G_{max}$ ensures that the control input generated by (2) satisfies the input constraint $\| \mathrm{T}(t) \| \leq T_{max}$. Moreover, in this case the scheme (2) possesses two desirable properties: the closed-loop system obtained by controlling (1) with (2) has a unique equilibrium θ^*, and this equilibrium is globally asymptotically stable. These properties are established in the following two lemmas.

Lemma 1: The closed-loop system obtained by applying the control law (2) to the manipulator dynamics (1) has a unique equilibrium θ^*, and this equilibrium can be made arbitrarily close to θ_d.

Proof: Any equilibrium point of the closed-loop system must satisfy the equation

$$\mathrm{G}(\theta^*) = k_2\gamma^2 \mathrm{sat}(\theta_d - \theta^*) \tag{3}$$

The condition $k_2\gamma^2\epsilon > G_{max}$ ensures that any equilibrium point must lie in the ball $\| \theta_d - \theta \| < \epsilon$. Inside this ball the condition (3) becomes

$$\mathrm{e} = \frac{1}{k_2\gamma^2}\mathrm{G}(\theta_d - \mathrm{e}) \tag{4}$$

Let M be a positive constant satisfying $M \parallel \theta_1 - \theta_2 \parallel \geq \parallel \mathbf{G}(\theta_1) - \mathbf{G}(\theta_2) \parallel \forall \theta_1, \theta_2$ (the boundedness of the partial derivatives of $\mathbf{G}$ ensures that such an M exists). Then choosing γ large enough so that $M/k_2\gamma^2 < 1$ ensures that $\mathbf{G}(\theta_d - \mathbf{e})/k_2\gamma^2$ defines a contraction mapping of the ϵ-ball around $\mathbf{e} = \mathbf{0}$ into itself. From this it can be concluded that (4) has a unique fixed point [15], which implies that the closed-loop system (1),(2) has a unique equilibrium. Inspection of (3) reveals that $\parallel \theta_d - \theta^* \parallel$ can be decreased arbitrarily by increasing γ (and decreasing ϵ correspondingly). ■

Lemma 2: The equilibrium point θ^* of the closed-loop system (1),(2) is globally asymptotically stable.

Proof: Defining $\mathbf{E} = \theta^* - \theta$ permits the closed-loop system (1),(2) to be written

$$H\ddot{\mathbf{E}} + V_{cc}\dot{\mathbf{E}} + k_1\gamma \text{sat}(\dot{\mathbf{E}}) + k_2\gamma^2 \text{sat}(\mathbf{E} + \theta_d - \theta^*) - \mathbf{G} = 0 \tag{5}$$

Observe that $\mathbf{E} = \mathbf{0}$ is the unique equilibrium point for this system. Let $U_g(\theta)$ denote the gravitational potential energy of the manipulator and define $U_E(\mathbf{E})$ as follows:

$$U_E = \begin{cases} \frac{1}{2} \parallel \mathbf{E} + \theta_d - \theta^* \parallel^2 & \parallel \mathbf{e} \parallel \leq \epsilon \\ \epsilon \parallel \mathbf{E} + \theta_d - \theta^* \parallel - \frac{1}{2}\epsilon^2 & \text{otherwise} \end{cases} \tag{6}$$

Consider the Lyapunov function candidate

$$V_1 = \frac{1}{2}\dot{\mathbf{E}}^T H \dot{\mathbf{E}} + k_2\gamma^2 U_E(\mathbf{E}) + U_g(\theta) - U_g(\theta^*) - \delta_1 \tag{7}$$

where δ_1 is a scalar constant chosen so that V_1 is nonnegative. It can be verified through straightforward (but tedious) calculation that V_1 is a positive-definite and proper function of the closed-loop system state if γ is chosen sufficiently large [14]. Differentiating (7) along (5) and simplifying gives

$$\dot{V}_1 = -k_1\gamma\dot{\mathbf{E}}^T \text{sat}(\dot{\mathbf{E}}) \leq 0 \tag{8}$$

Standard arguments can then be used to conclude that the equilibrium point $\mathbf{E} = \mathbf{0}$ is globally asymptotically stable [e.g., 16]. ■

Lemmas 1 and 2 show that the simple control strategy (2) provides a "practical" solution to the position regulation problem for uncertain manipulators with actuator constraints. That is, this scheme ensures that the position error will converge to an arbitrarily small neighborhood of the origin while using controls which satisfy the input constraint $\parallel \mathbf{T}(t) \parallel \leq T_{max}$. However, there is no guarantee that the position error will actually converge to the origin itself (in fact, this will not ordinarily happen). To provide convergence of the system (1) to the desired position θ_d, we propose to combine the control strategy (2) with an adaptive control law. Consider the following adaptive scheme:

$$\begin{aligned} \mathrm{T} &= \mathrm{f}(t) + k_1\gamma\dot{\mathrm{e}} + k_2\gamma^2\mathrm{e} \\ \dot{\mathrm{f}} &= \beta(1 - b\frac{\mathrm{ff}^T}{\| \mathrm{f} \|^2})(\dot{\mathrm{e}} + \frac{k_2}{k_1\gamma}\mathrm{e}) \end{aligned} \tag{9}$$

where $\mathrm{f}(t) \in \Re^n$ is the adaptive element, β is a positive scalar constant, and b is defined as follows: $b = 0$ if $\{\| \mathrm{f} \| < f_{max}\}$ or $\{\| \mathrm{f} \| = f_{max}$ and $\mathrm{f}^T(\dot{\mathrm{e}} + \frac{k_2}{k_1\gamma}\mathrm{e}) \leq 0\}$, and $b = 1$ otherwise, where $f_{max} = G_{max} + \delta_2$ for some (small) scalar δ_2. Including the term $b\mathrm{ff}^T/\| \mathrm{f} \|^2$ in the update law for f ensures that if $\| \mathrm{f} \| = f_{max}$ then the adaptation of f is directed away from the exterior of the ball $\{\mathrm{f} : \| \mathrm{f} \| \leq f_{max}\}$, so that f is prevented from escaping this ball. The use of such projection algorithms has been proposed to guarantee boundedness of adaptive elements [16], and in the present case it is easy to see that this boundedness is achieved: if f is initialized so that $\| \mathrm{f}(0) \| \leq f_{max}$ then $\| \mathrm{f}(t) \| \leq f_{max}\ \ \forall t$ (to see this, check that if $\| \mathrm{f} \| = f_{max}$ then $d(\| \mathrm{f} \|)/dt \leq 0$).

The stability properties of the adaptive regulation strategy (9) are summarized in the following lemma.

Lemma 3: If γ is chosen sufficiently large then the adaptive scheme (9) ensures that (1) evolves with all signals (semiglobally) uniformly bounded and so that e and $\dot{\mathrm{e}}$ converge asymptotically to zero.

Proof: Applying the control law (9) to the manipulator dynamics (1) yields the closed-loop error dynamics

$$\begin{aligned} H\ddot{\mathrm{e}} + V_{cc}\dot{\mathrm{e}} + k_1\gamma\dot{\mathrm{e}} + k_2\gamma^2\mathrm{e} + \Phi + \mathrm{G}(\theta_d) - \mathrm{G}(\theta) &= 0 \\ \dot{\Phi} = \beta(1 - b\frac{\mathrm{ff}^T}{\| \mathrm{f} \|^2})(\dot{\mathrm{e}} + \frac{k_2}{k_1\gamma}\mathrm{e}) & \end{aligned} \tag{10}$$

where $\Phi = \mathrm{f} - \mathrm{G}(\theta_d)$. Consider the Lyapunov function candidate

$$\begin{aligned} V_2 = & \frac{1}{2}\dot{\mathrm{e}}^T H\dot{\mathrm{e}} + \frac{1}{2}k_2\gamma^2\mathrm{e}^T\mathrm{e} + \frac{k_2}{k_1\gamma}\mathrm{e}^T H\dot{\mathrm{e}} + \frac{1}{2\beta}\Phi^T\Phi \\ & + U(\theta) - U(\theta_d) + \mathrm{G}^T(\theta_d)\mathrm{e} \end{aligned} \tag{11}$$

and note that V_2 is a positive-definite and proper function of e, $\dot{\mathrm{e}}$ and Φ if γ is chosen sufficiently large. Computing the derivative of (11) along (10) and simplifying yields the upper bound

$$\begin{aligned} \dot{V}_2 \leq & -(k_1\gamma - \frac{k_2}{k_1\gamma}\lambda_{max}(H)) \| \dot{\mathrm{e}} \|^2 + k_2\gamma \| \mathrm{e} \| \| \dot{\mathrm{e}} \| \\ & -(\frac{k_2^2\gamma}{k_1} - \frac{k_2 M}{k_1\gamma}) \| \mathrm{e} \|^2 + \frac{k_2 k_{cc}}{k_1\gamma} \| \mathrm{e} \| \| \dot{\mathrm{e}} \|^2 \end{aligned} \tag{12}$$

where $\lambda_{min}(\cdot), \lambda_{max}(\cdot)$ denote the minimum and maximum eigenvalue of the matrix argument, respectively, and k_{cc} is a positive constant satisfying $\| V_{cc} \| \leq k_{cc} \| \dot{\theta} \| \, \forall \theta$. Define $\mathrm{z}_1 = [\| \mathrm{e} \| \;\; \| \dot{\mathrm{e}} \|]^T$ and

$$Q_1 = \begin{bmatrix} \frac{k_2^2 \gamma}{k_1} - \frac{k_2 M}{k_1 \gamma} & -\frac{k_2 \gamma}{2} \\ -\frac{k_2 \gamma}{2} & k_1 \gamma - \frac{k_2}{k_1 \gamma} \lambda_{max}(H) \end{bmatrix}$$

and note that Q_1 is positive-definite if γ is chosen large enough. Then the following bound on $\dot{V}_2$ in (12) can be established:

$$\dot{V}_2 \leq -(\lambda_{min}(Q_1) - \frac{\alpha_1}{\gamma} {V_2}^{1/2}) \| \mathrm{z}_1 \|^2 \tag{13}$$

for some scalar constant α_1 which does not increase as γ is increased. Let $c_1(t) = \lambda_{min}(Q_1) - \alpha_1 {V_2}^{1/2}(t)/\gamma$ and choose γ large enough so that $c_1(0) > 0$ (this is always possible). Then (13) implies that $c_1(0) \leq c_1(t) \;\; \forall t$, so that $\dot{V}_2 \leq -c_1(0) \| \mathrm{z}_1 \|^2$. Standard arguments can then be used to show that all signals are bounded and that $\mathrm{e}, \dot{\mathrm{e}}$ converge to zero asymptotically [16]. ■

The preceding analysis shows that the adaptive regulation scheme (9) ensures that the manipulator (1) evolves to any user-specified final configuration and that the adaptive element f obeys the bound $\| \mathrm{f}(t) \| \leq G_{max} + \delta_2$. Note that the stability properties established in Lemma 3 are *semiglobal* in the sense that the basin of attraction can be increased arbitrarily by increasing the controller gain γ. Thus we have controller (2), which provides global convergence of the position error to a (small) neighborhood of the origin using bounded controls, and controller (9), which ensures convergence of the position error to zero. We will now show that these two schemes can be combined to yield a globally convergent position regulation strategy which is implementable using bounded controls and which does not require knowledge of the manipulator dynamic model.

Theorem 1: The position regulation schemes (2) and (9) can be combined to yield a control strategy which ensures global boundedness of all signals and global convergence of the position error to zero using controls which satisfy the constraint $\| \mathrm{T}(t) \| \leq T_{max}$.

Proof: Note first that increasing the controller parameter γ increases the basin of attraction of the closed-loop system (1),(9) and decreases the ultimate bound on the position error of the closed-loop system (1),(2). Thus there exists a value of γ, say γ^*, such that if $\gamma \geq \gamma^*$ then the region of ultimate convergence of (1),(2) is contained within the basin of attraction of (1),(9). Assume that $\gamma \geq \gamma^*$, and consider the following control strategy:

1. Use the regulation scheme (2) to drive the state error $(\mathrm{e}, \dot{\mathrm{e}})$ to the set $B_e = \{\mathrm{e}, \dot{\mathrm{e}} \mid \| \mathrm{e} \| \leq G_{max}/k_2\gamma^2 + \delta_3, \| \dot{\mathrm{e}} \| \leq \delta_3\}$, where δ_3 is a positive constant chosen small enough so that B_e is contained in the basin of attraction of (1),(9).

2. Once $(\mathrm{e}, \dot{\mathrm{e}}) \in B_\epsilon$, switch to the regulation scheme (9) to drive $(\mathbf{e}, \dot{\mathrm{e}})$ to the origin.

It is clear that this strategy will provide the desired stability and convergence properties, so that the only thing that remains is to check that the constraint on the control input is satisfied. Since it is obvious that the control input generated by the scheme (2) satisfies $\| \mathrm{T}(t) \| \leq T_{max}$, we turn our attention to the adaptive controller (9). Recall that the projection algorithm used in the update law for f in (9) ensures that $\| \mathrm{f} \| \leq G_{max} + \delta_2$, with δ_2 as small as desired. Given the proposed control law switching strategy, this bound on f together with the analysis presented in the proof of Lemma 3 permit the conclusion that V_2 in (11) can be upper bounded as follows:

$$V_2 \leq \frac{1}{2}(k_2\gamma^2 + \alpha_2)\frac{G_{max}^2}{k_2^2\gamma^4} + \frac{2}{3}G_{max} \tag{14}$$

where α_2 is a positive scalar constant which does not increase as γ increases. This upper bound can, in turn, be utilized to derive bounds on the possible magnitudes of e and $\dot{\mathrm{e}}$ under the control (9):

$$\| \mathrm{e}(t) \| \leq \frac{G_{max}}{\gamma^2}\left(\frac{1}{k_2} + \frac{1}{k_2^2}\right)^{1/2} \tag{15}$$
$$\| \dot{\mathrm{e}}(t) \| \leq \frac{G_{max}}{\gamma}\left(\frac{1}{\lambda_{min}(H)} + \frac{1}{\lambda_{min}(H)k_2}\right)^{1/2}$$

where, to simplify the calculations, we have assumed that γ is large and that β is chosen to be proportional to γ^2. The bounds (15) together with the bound on f then permit an upper bound on the control input to be established. While this bound depends on the values of k_1 and k_2 in general, routine calculation shows that for a large range of values for these gains we have $\| \mathbf{T}(t) \| \leq T_{max}$ [14]; for example, choosing $k_2 = 1$ and $k_1 \leq (\lambda_{min}(H))^{1/2}/2$ yields this result. ∎

3.2 Output Feedback Case

Implementing controllers which require full state measurement can be problematic in practice because velocity measurements are often either contaminated with noise or not available at all [2]. Thus in many applications it is desirable to obtain accurate position regulation using bounded controllers which do not require manipulator model information *or* velocity measurements. We show in this section that a straightforward modification of the state feedback controller detailed in Theorem 1 yields such an output feedback regulation scheme.

Consider the position regulation scheme obtained from (2) through the introduction of simple dynamic feedback in the control law:

$$\begin{aligned} \mathrm{T} &= k_1\gamma^2\mathrm{sat}(\mathrm{w}) + k_2\gamma^3\mathrm{sat}(\mathrm{e}) \\ \dot{\mathrm{w}} &= -2\gamma\mathrm{w} + \gamma^2\dot{\mathrm{e}} \end{aligned} \tag{16}$$

where w will be seen to provide a means of injecting damping into the closed-loop system. Observe that (16) is implementable without velocity information because, although $\dot{\mathrm{w}}$ depends on $\dot{\mathrm{e}}$, the control law requires only w and this term depends only on e (as can be verified through direct integration of the $\dot{\mathrm{w}}$ equation). It can be seen that the regulation strategy (16) retains the desirable properties of the controller (2) upon which it is based. For example, choosing the controller parameters $k_1, k_2, \gamma, \epsilon$ so that $k_1\gamma^2\epsilon \leq G_{max}$ and $G_{max} < k_2\gamma^3\epsilon \leq 2G_{max}$ ensures that the control input generated by (16) satisfies the input constraint $\| \mathrm{T}(t) \| \leq T_{max}$. The fact that the closed-loop system (1),(16) has a unique equilibrium θ^* follows directly from the proof of Lemma 1 (since the equilibrium condition is identical), and the global stability of this equilibrium is established in the following lemma.

Lemma 4: The equilibrium point θ^* of the closed-loop system (1),(16) is globally asymptotically stable.

Proof: Defining $\mathrm{E} = \theta^* - \theta$ permits the closed-loop system (1),(16) to be written

$$H\ddot{\mathrm{E}} + V_{cc}\dot{\mathrm{E}} + k_1\gamma^2\mathrm{sat}(\mathrm{w}) + k_2\gamma^3\mathrm{sat}(\mathrm{E} + \theta_d - \theta^*) - \mathrm{G} = 0 \tag{17}$$

with $\mathrm{E} = 0$ again the unique equilibrium point for the system. Define $U_g(\theta)$ and $U_E(\mathrm{E})$ as before, and let

$$U_w = \begin{cases} \frac{1}{2} \| \mathrm{w} \|^2 & \| \mathrm{w} \| \leq \epsilon \\ \epsilon \| \mathrm{w} \| - \frac{1}{2}\epsilon^2 & \text{otherwise} \end{cases}$$

Consider the Lyapunov function candidate

$$\begin{aligned} V_3 = {} & \frac{1}{2}\dot{\mathrm{E}}^T H \dot{\mathrm{E}} + k_2\gamma^3 U_E(\mathrm{E}) + k_1 U_w(\mathrm{w}) \\ & + U_g(\theta) - U_g(\theta^*) - \delta_4 \end{aligned} \tag{18}$$

where δ_4 is a scalar constant chosen so that V_3 is nonnegative. It can be verified through straightforward (but tedious) calculation that V_3 is a positive-definite and proper function of the closed-loop system state if γ is chosen sufficiently large [14]. Differentiating (18) along (17) and simplifying as in the proof of Lemma 2 gives

$$\dot{V}_3 = -2k_1\gamma\mathrm{w}^T\mathrm{sat}(\mathrm{w}) \leq 0 \tag{19}$$

Standard arguments can then be used to conclude that the equilibrium point $\mathbf{E} = \mathbf{0}$ is globally asymptotically stable [e.g., 16]. ∎

The preceding analysis shows that the simple control strategy (16) ensures that the position error will converge to an arbitrarily small neighborhood of the origin while using controls which satisfy the input constraint, and therefore provides a "practical" solution to our position regulation problem. However, there is no guarantee that the position error will actually converge to the origin when this control is employed, and this motivates the introduction of an adaptive component to the control strategy. Consider the following adaptive scheme:

$$\begin{aligned} \mathbf{T} &= \mathbf{f}(t) + k_1\gamma^2\mathbf{w} + k_2\gamma^2\mathbf{e} \\ \dot{\mathbf{w}} &= -2\gamma\mathbf{w} + \gamma^2\dot{\mathbf{e}} \\ \dot{\mathbf{f}} &= \mathcal{P}[\beta(\dot{\mathbf{e}} + \frac{k_2}{k_1\gamma}\mathbf{e} - \frac{1}{\gamma}\mathbf{w})] \end{aligned} \tag{20}$$

where $\mathcal{P}[\cdot]$ is the component-wise projection which ensures that $f_i \in [-f_{imax}, f_{imax}]$ $\forall i$ [16] (where the subscript i refers to the ith element of the vector and $f_{imax} = G_{imax} + \delta_5$ for some small δ_5). Observe that (20) can be implemented without velocity information because, although $\dot{\mathbf{w}}$ and $\dot{\mathbf{f}}$ depend on $\dot{\mathbf{e}}$, the control law terms $\mathbf{w}$ and $\mathbf{f}$ depend only on $\mathbf{e}$ (as can be verified through direct integration of the $\dot{\mathbf{w}}$ and $\dot{\mathbf{f}}$ equations).

The stability properties of the proposed adaptive regulation strategy (20) are summarized in the following lemma.

Lemma 5: If γ is chosen sufficiently large then the adaptive scheme (20) ensures that (1) evolves with all signals (semiglobally) uniformly bounded and so that $\mathbf{e}$ and $\dot{\mathbf{e}}$ converge asymptotically to zero.

Proof: Applying the control law (20) to the manipulator system dynamics (1) yields the closed-loop error dynamics

$$\begin{aligned} H\ddot{\mathbf{e}} + V_{cc}\dot{\mathbf{e}} + k_1\gamma^2\mathbf{w} + k_2\gamma^2\mathbf{e} + \mathbf{\Phi} + \mathbf{G}(\theta_d) - \mathbf{G}(\theta) &= \mathbf{0} \\ \dot{\mathbf{w}} &= -2\gamma\mathbf{w} + \gamma^2\dot{\mathbf{e}} \\ \dot{\mathbf{\Phi}} &= \mathcal{P}[\beta(\dot{\mathbf{e}} + \frac{k_2}{k_1\gamma}\mathbf{e} - \frac{1}{\gamma}\mathbf{w})] \end{aligned} \tag{21}$$

Consider the Lyapunov function candidate

$$V_4 = V_2 + \frac{1}{2}k_1\mathbf{w}^T\mathbf{w} - \frac{1}{\gamma}\mathbf{w}^T H\dot{\mathbf{e}} \tag{22}$$

and note that V_4 is a positive-definite and proper function of $\mathbf{e}$, $\dot{\mathbf{e}}$, $\mathbf{w}$ and $\mathbf{\Phi}$ if γ is chosen sufficiently large. Computing the derivative of (22) along (21) and simplifying as in the proof of Lemma 3 yields

$$\dot{V}_4 \leq -\mathbf{z}_2^T Q_2 \mathbf{z}_2 + \frac{k_{cc}}{\gamma} \parallel \dot{\mathbf{e}} \parallel^2 \left(\frac{k_2}{k_1} \parallel \mathbf{e} \parallel + \parallel \mathbf{w} \parallel\right) \tag{23}$$

where $\mathbf{z}_2 = [\parallel \mathbf{e} \parallel \;\; \parallel \dot{\mathbf{e}} \parallel \;\; \parallel \mathbf{w} \parallel]^T$, Q_2 is defined as

$$Q_2 = \begin{bmatrix} \frac{k_2^2\gamma}{k_1} - \frac{k_2 M}{k_1\gamma} & 0 & -\frac{M}{2\gamma} \\ 0 & (\gamma - \frac{k_2}{k_1\gamma})\lambda_{min}(H) & -\lambda_{max}(H) \\ -\frac{M}{2\gamma} & -\lambda_{max}(H) & k_1\gamma \end{bmatrix}$$

and it can be seen that Q_2 is positive-definite if γ is chosen large enough. Now arguments identical to those used in the proof of Lemma 3 can be used to show that all signals are uniformly bounded and that $\mathbf{e}$ and $\dot{\mathbf{e}}$ converge to zero asymptotically. ■

Observe that the output feedback regulation schemes (16) and (20) inherit the desirable properties of the state feedback strategies (2) and (9) upon which they are based: controller (16) provides global convergence of the position error to a (small) neighborhood of the origin using bounded controls, and controller (20) ensures convergence of the position error to zero. We will now show that these two schemes can be combined to yield a globally convergent position regulation strategy which is implementable using bounded controls and which does not require manipulator model information or velocity measurements.

Theorem 2: The position regulation schemes (16) and (20) can be combined to yield a control strategy which ensures global boundedness of all signals and global convergence of the position error to zero using controls which satisfy the constraint $\parallel \mathbf{T}(t) \parallel \leq T_{max}$.

Proof: Note that, just as was the case with state feedback, increasing the controller parameter γ increases the basin of attraction of the closed-loop system (1),(20) and decreases the ultimate bound on the position error of the closed-loop system (1),(16). Thus there exists a γ^* such that if $\gamma \geq \gamma^*$ then the region of ultimate convergence of (1),(16) is contained within the basin of attraction of (1),(20). Assume that $\gamma \geq \gamma^*$, and consider the following control strategy:

1. Use the regulation scheme (16) to drive $(\mathbf{e}, \dot{\mathbf{e}}, \mathbf{w})$ to the set $B_w = \{\mathbf{e}, \dot{\mathbf{e}}, \mathbf{w} \mid \; \parallel \mathbf{e} \parallel \leq G_{max}/k_2\gamma^2 + \delta_6, \parallel \dot{\mathbf{e}} \parallel \leq \delta_6, \parallel \mathbf{w} \parallel \leq \delta_6\}$, where δ_6 is a positive constant chosen small enough so that B_w is contained in the basin of attraction of (1),(20).
2. Once $(\mathbf{e}, \dot{\mathbf{e}}, \mathbf{w}) \in B_w$, switch to the regulation scheme (20) to drive $(\mathbf{e}, \dot{\mathbf{e}}, \mathbf{w})$ to the origin.

It is clear that this strategy will provide the desired stability and convergence properties, so that the only thing that needs to be checked is that the constraint on the control input is satisfied. The control input generated by the scheme (16) satisfies $\| \mathbf{T}(t) \| \leq T_{max}$ by design, so we need only check the adaptive controller (20). Recall that the projection algorithm used in the update law for $\mathbf{f}$ in (20) ensures that $\| \mathbf{f} \| \leq G_{max} + \delta_5$, with δ_5 as small as desired. Given the proposed control law switching strategy, this bound on $\mathbf{f}$ together with the analysis presented in the proof of Lemma 5 permit the conclusion that V_4 in (22) can be upper bounded as follows:

$$V_4 \leq \frac{1}{2}(k_2\gamma^2 + \alpha_3)\frac{G_{max}^2}{k_2^2\gamma^6} + \frac{2}{\beta}G_{max} \tag{24}$$

where α_3 is a positive scalar constant which does not increase as γ increases. This upper bound can be utilized to derive bounds for $\mathbf{e}$ and $\mathbf{w}$ which, together with the bound on $\mathbf{f}$, permit an upper bound on the control input to be established. The calculation of these bounds exactly parallels the calculations in the proof of Theorem 1 and thus is not given; the interested reader is referred to the report [14] for the details. Here it is simply noted that the bound $\| \mathbf{T}(t) \| \leq T_{max}$ is satisfied for a large range of values of the controller parameters; for example, choosing $k_2 \geq k_1$ yields this result. ∎

4. Simulation Results

The position regulation strategy proposed in Theorem 2 is now applied to an IMI Zebra Zero robot through computer simulation. The Zebra robot possesses a conventional design with six revolute joints configured in a "waist-shoulder-elbow-wrist" arrangement. The simulation environment incorporates models of all important dynamic subsystems and phenomena, such as the full nonlinear arm dynamics, joint stiction, sensor noise, and transmission effects, and therefore provides the basis for a realistic evaluation of controller performance. For all simulations the control laws are applied to the manipulator model with a sampling period of two milliseconds, and all integrations required by the controller are implemented using a simple trapezoidal integration rule with a time-step of two milliseconds.

In the first set of simulations, the position regulation scheme obtained by combining the controllers (16) and (20) as described in Theorem 2 is utilized to control the Zebra manipulator in joint-space. In the first simulation, the Zebra manipulator is initially at rest with joint-space position $\theta(0) = [0°, 0°, 0°, 0°, 0°, 0°]^T$, and is commanded to move to the desired final configuration $\theta_d = [0°, \theta_{2d}, \theta_{3d}, 0°, 0°, 0°]^T$, where two different goal positions are used: $\theta_{2d} = \theta_{3d} = 90°$ and $\theta_{2d} = \theta_{3d} = 360°$ This position control task is performed using the regulation scheme (16),(20) and, to provide a basis for comparison, the task is also executed using the (unprojected) adaptive strategy (20) alone; observe

Robust Control of Robots via Linear Estimated State Feedback

Harry Berghuis and Henk Nijmeijer

Abstract— **In this note we propose a robust tracking controller for robots that requires only position measurements. The controller consists of two parts: a linear observer part that generates an estimated error state from the error on the joint position and a linear feedback part that utilizes this estimated state. It is shown that this computationally efficient controller yields semi-global uniform ultimate boundedness of the tracking error. An interesting feature of the controller is that it straightforwardly extends recent results on robust control of robots by linear state feedback to linear estimated-state feedback.**

I. Introduction

Over the last decade, a lot of research effort has been put into the design of sophisticated control strategies for robots, see, for instance, [12]. In spite of these efforts, virtually all industrial robot systems today are still controlled by some kind of linear state feedback [1], [14]. The reasons for this are threefold. First, the linear state feedback (in literature frequently referred to as PD controller) is computationally simple and does not require any model knowledge, which makes it attractive from the viewpoint of implementation. Second, practice has proved that the PD controller is robust to disturbances like friction and load torques, which represents a prerequisite for the realm of applications. Third, since industrial robots are typically overdesigned in the sense that heavy and consequently rigid links are used together with high-gear transmission mechanisms [1], they can be described by linear and decoupled dynamics, for which linear state-feedback generally provides acceptable performance.

Owing to the increasing demands on productivity and efficiency of robots, there has been, in recent years, a tendency to develop fast lightweight robot constructions actuated by direct-drive motors. Contrary to the traditional manipulator, these newly developed robot systems are characterized by highly nonlinear and coupled dynamics. As was shown in [8], [10], [15], and more recently in [13], even for such systems the classical PD controller may work reasonably well. In particular, in [13] it is shown that in the presence of arbitrary but bounded nonlinearities in the system dynamics, the PD controller yields uniform ultimate boundedness (also known as practical stability) of the tracking errors; that is the error state tends in finite time to a bounded region around zero. An interesting feature of this stability result is that the authors can provide a relationship between the bound on the tracking errors and the PD feedback gains, for any initial condition. Moreover, it is demonstrated that the ultimate error bound can be made arbitrarily small by increasing the controller gains.

One characteristic feature of the state-feedback controller is that it requires both position and velocity measurements. In practice, however, this requirement is generally not fulfilled; although in robotic applications today high-precision sensors are used to obtain position information, velocity sensoring equipment is frequently omitted due to the savings in cost, volume, and weight that can be obtained [9]. For these reasons, a number of model-based robot control methods have been proposed recently that evade the velocity measurement problem by integrating a velocity observer in the control loop (e.g., [3], [4], [11]). These methods require exact knowledge of the nonlinear robot dynamics, which, in practice is generally not available. Motivated by this, Canudas de Wit and Fixot [5] have addressed the robust tracking control problem of robots using only position feedback. These authors combine a nonlinear switching type control method with a sliding mode velocity observer to face bounded uncertainties in the robot dynamics.

In this paper we present a novel approach to the robust control problem stated above. This approach originates from a strategy for combined controller-observer design for robots that we recently proposed in [3]; see also [2]. The rationale underlying this strategy is to extend in a natural way the passivity methodology to state-feedback robot control (cf. [12] and references therein) to the case that only joint position measurements are present. This allows us to develop a controller that consists of two parts:

1) a linear observer that generates an estimated error state from the position error
2) a linear feedback controller that employs the estimate of the error state.

By using stability analysis techniques that are similar to the ones in [13], it is proved that this linear estimated-state feedback controller provides uniform ultimate boundedness of the closed-loop error dynamics. As in [13], we given an explicit relation between the bound on the error state and the controller and observer feedback gains. This, together with the fact that the linear estimated-state feedback controller is easily implementable and needs only position information, makes the proposed controller particularly interesting from a practical point of view.

This paper is organized as follows. In Section II some mathematical preliminaries are given that support the stability analysis in the following sections. The proposed robot controller is introduced in Section III, together with its stability properties. Section IV contains a discussion of some characteristics of the novel control approach, and finally we give conclusions. Standard notation is used. In particular, vector norms are Euclidean, and for matrices the induced norm $\|A\| = \sqrt{\lambda_{\max}(A^T A)}$ is employed, with $\lambda_{\max}(\cdot)$ the maximum eigenvalue. Moreover, for any positive definite matrix $A(x)$ and for all x we denote by A_m and A_M the minimum and maximum eigenvalue of $A(x)$, respectively.

II. Mathematical Preliminaries

This section presents a stability result that plays a central role in the sequel. This result is a modified version of a theorem by Chen and Leitmann [6] (see also [13]), which basically states that a system is uniformly ultimately bounded if it has a Lyapunov function whose time-derivative is negative definite in an annulus of a certain width around the origin. For the sake of brevity, the proof is omitted. The following lemma is useful for the stability analysis.

Lemma 1: Consider the function $g(\cdot): \mathbb{R} \to \mathbb{R}$

$$g(y) = \alpha_0 + \alpha_1 y - \alpha_2 y^2, \qquad y \in \mathbb{R}^+ \tag{1}$$

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H. Berghuis was with the Department of Electrical Engineering, University of Twente, Netherlands and is now with Hollandse Signaalapparaten B.V., Afdeling DNCS-RS, PO Box 42, 7550 GD Hengelo, Netherlands.

H. Nijmeijer is with the Department of Applied Mathematics, University of Twente, PO Box 217, 7500 AE Enschede, Netherlands.

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where $\alpha_i > 0$, $i = 0, 1, 2$. Then $g(y) < 0$ if $y > \eta > 0$, where

$$\eta = \frac{\alpha_1 + \sqrt{\alpha_1^2 + 4\alpha_0\alpha_2}}{2\alpha_2}. \tag{2}$$

□

Proposition 1: Let $x(t) \in \mathbf{R}^m$ be the solution of the differential equation

$$\dot{x}(t) = f(x(t), t) \quad x(t_0) = x_0$$

and assume there exists a function $V(x(t), t)$ that satisfies

$$P_m\|x(t)\|^2 \le V(x(t), t) \le P_M\|x(t)\|^2 \tag{3a}$$

$$\dot{V}(x(t), t) \le g(\|x(t)\|) < 0 \quad \text{for all } \|x(t)\| > \eta > 0 \tag{3b}$$

with P_m and P_M positive constants, $g(\cdot)$ as in (1), and η as in (2). Define $\delta \equiv \sqrt{P_m^{-1}P_M}$ and $d > \delta\eta$. Then $x(t)$ is uniformly ultimately bounded, that is

$$\|x_0\| \le r \rightarrow \|x(t)\| \le d \quad \text{for all } t \ge t_0 + T(d, r) \tag{4}$$

where

$$T(d, r) = \begin{cases} 0 & r \le R \\ \frac{P_M r^2 - P_m R^2}{\alpha_2 R^2 - \alpha_1 R - \alpha_0} & r > R \end{cases} \tag{5}$$

and $R = \delta^{-1}d$. □

III. Linear Estimated State-Feedback Controller

The general equations describing the dynamics of an n degrees-of-freedom rigid robot manipulator are given by [14]

$$M(q)\ddot{q} + C(q, \dot{q})\dot{q} + G(q) + F(\dot{q}) + T = \tau \tag{6}$$

where q is the $[n \times 1]$ vector of generalized coordinates, $M(q) = M(q)^T > 0$ the $[n \times n]$ positive definite inertia matrix, $C(q, \dot{q})\dot{q}$ the Coriolis and centrifugal torques $[n \times 1]$, $G(q)$ the gravitational torques $[n \times 1]$, $F(\dot{q})$ the friction torques $[n \times 1]$, T an $[n \times 1]$ vector of load disturbances, and τ the $[n \times 1]$ vector of control torques. The matrix $C(q, \dot{q})$ is defined via the Christoffel symbols [12], which implies that $\dot{M}(q) - 2C(q, \dot{q})$ is skew symmetric. We use the following property.

Property 1: For revolute robots, $M(q)$, $C(q, \dot{q})$, and $G(q)$ are unbounded w.r.t. q, i.e., (cf. [7])

$$0 < M_m \le \|M(q)\| \le M_M \quad \text{for all } q \tag{7a}$$

$$\|C(q, x)\| \le C_M\|x\| \quad \text{for all } q, x \tag{7b}$$

$$\|G(q)\| \le G_M \quad \text{for all } q. \tag{7c}$$

In addition, the friction and load disturbance torques are bounded by (cf. [7])

$$\|F(\dot{q})\| \le F_{1,M} + F_{2,M}\|\dot{q}\| \quad \text{for all } \dot{q} \tag{7d}$$

$$\|T\| \le T_M. \tag{7e}$$

□

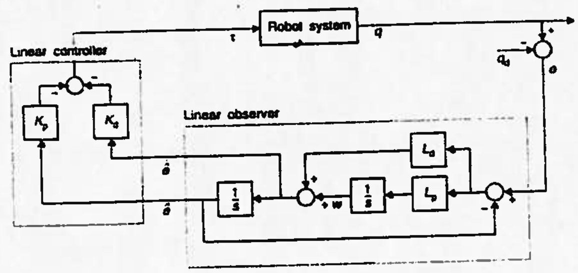

Fig. 1. Proposed linear controller-observer combination.

To solve the tracking control problem for (6) using position feedback only, consider the linear output-feedback robot control system (see Fig. 1)

$$\text{Controller } \{\tau = -K_d\dot{\hat{e}} - K_p\hat{e} \tag{8a}$$

$$\text{Observer } \begin{cases} \dot{\hat{e}} = w + L_d(e - \hat{e}) \\ \dot{w} = L_p(e - \hat{e}) \end{cases} \tag{8b}$$

where $q_d(t)$ represents the desired path to be tracked by the robot system, $\dot{e} \equiv \dot{q} - q_d$, $K_p = K_p^T > 0$ the controller proportional gain, $K_d = K_d^T > o$ the controller derivative gain, $L_p = L_p^T > 0$ the observer proportional gain, and $L_d = L_d^T > 0$ the observer derivative gain. This control system consists of two parts: a linear observer part (8b) that generates an estimated error state $[\dot{\hat{e}}^T, \hat{e}^T]^T$ from the tracking error e and a linear controller part (8a) that utilizes this estimated error state in the feedback loop.

Let us make the following assumption on the structure of K_p, K_d, L_p, and L_d.

Assumption 1: K_p, K_d, and L_p, L_d satisfy respectively

$$K_p = \lambda K_d, \quad K_d = (k_d + \gamma)I \tag{9a}$$

$$L_p = \lambda l_d, \quad L_d = (l_d + \lambda)I \tag{9b}$$

where $\lambda > 0$, $k_d > 0$, $\gamma > 0$ and $l_d > 0$ all scalar.

In addition, we require the following assumptions.

Assumption 2: The desired trajectory signals $\dot{q}_d(t)$ and $\ddot{q}_d(t)$ are bounded by V_M and A_M respectively, i.e.,

$$V_M = \sup_t\|\dot{q}_d(t)\|, \quad A_M = \sup_t\|\ddot{q}_d(t)\|. \tag{10}$$

Then our main result can be formulated as the following theorem.

Theorem 1: Consider the linear output-feedback robot controller (8) in closed loop with (6). Define $x(t)^T = [\dot{e}(t)^T\,(\lambda e(t))^T\,\dot{\tilde{q}}(t)^T\,(\lambda\tilde{q}(t))^T]$, where $e \equiv q - q_d$, $\tilde{q} \equiv q - \hat{q}$, and assume that $\|x_0\|$ represents an upper bound on the initial error state $x(0)$. Under the conditions

$$k_d > \lambda M_M \tag{11a}$$

$$l_d > 2M_m^{-1}\{k_d + \gamma\} \tag{11b}$$

$$\gamma > 2\epsilon^{-1}(\beta_0 + \beta_1(\delta\mu) + \beta_2(\delta\mu)^2) \tag{11c}$$

where

$$\beta_0 = M_M A_M + C_M V_M^2 + G_M + F_{1,M} + F_{2,M}V_M + T_M \tag{12a}$$

$$\beta_1 = 2(1 + \sqrt{2})C_M V_M + F_{2,M} \tag{12b}$$

$$\beta_2 = (1+\sqrt{2})C_M \tag{12c}$$

$$\delta = 3\sqrt{2(\lambda M_m)^{-1}k_d} \tag{12d}$$

$$\mu = \max\{\overline{\eta}, \|x_0\|\} \tag{12e}$$

and ϵ and $\overline{\eta}$ are positive constants that satisfy

$$\epsilon < \beta_2^{-1}(k_d - \lambda M_M) \tag{12f}$$

$$\overline{\eta} = \frac{\epsilon\beta_1 + \sqrt{(\epsilon\beta_1)^2 + 4\epsilon\beta_0(k_d - \lambda M_M - \epsilon\beta_2)}}{2(k_d - \lambda M_M - \epsilon\beta_2)} \tag{12g}$$

the closed-loop system is uniformly ultimately bounded, with

$$\|x(t)\| \le \delta\overline{\eta} \quad \text{for all } t \ge T(\delta\overline{\eta}, \|x_0\|) \tag{13}$$

where $T(\cdot)$ defined in (5). In the limiting case that $\epsilon \to 0$, and consequently $\gamma \to \infty$ and $l_d \to \infty$, the closed-loop system is asymptotically stable.

Proof: The closed-loop error dynamics (6), (8) are given by

$$\begin{aligned} M(q)\ddot{e} + C(q,\dot{q})s_1 + K_d s_1 \\ = K_d s_2 + C(q,\dot{q})\lambda e - C(q,s_2)\dot{q}_d - \Delta Y(\cdot) \end{aligned} \tag{14a}$$

$$\begin{aligned} M(q)\dot{s}_2 + C(q,\dot{q})s_2 + (l_d M(q) - K_d)s_2 \\ = -K_d s_1 + C(q, s_2 - \dot{q})\dot{e} - \Delta Y(\cdot) \end{aligned} \tag{14b}$$

where s_1 and s_2 are defined as

$$s_1 \equiv \dot{q} - \dot{q}_r \equiv \dot{e} + \lambda e \tag{15a}$$

$$s_2 \equiv \dot{q} - \dot{q}_0 \equiv \dot{\tilde{q}} + \lambda\tilde{q} \tag{15b}$$

and the perturbation term $\Delta Y(\cdot)$ satisfies

$$\begin{aligned} \Delta Y(q, \dot{q}_0, \dot{q}_d, \ddot{q}_d, t) \\ = M(q)\ddot{q}_d + C(q,\dot{q}_0)\dot{q}_d + G(q) + F(\dot{q}) + T. \end{aligned} \tag{16}$$

From Property 1 and Assumption 2 it follows that

$$\begin{aligned} \|\Delta Y(\cdot)\| &\le M_M A_M + C_M V_M^2 + G_M + F_{1,M} + F_{2,M}V_M + T_M \\ &\quad + \{F_{2,M} + (1+\sqrt{2})C_M V_M\}\|x\| \\ &\equiv \alpha_0 + \alpha_1\|x\|. \end{aligned} \tag{17}$$

Take as a candidate Lyapunov function the function

$$V(x,t) = \tfrac{1}{2}x^T P(x,t)x \tag{18}$$

where

$$P(x,t) = \begin{bmatrix} \begin{bmatrix} M(x,t) & M(x,t) \\ M(x,t) & 2\lambda^{-1}k_d \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} M(x,t) & M(x,t) \\ M(x,t) & 2\lambda^{-1}k_d + M(x,t) \end{bmatrix} \end{bmatrix}. \tag{19}$$

In the Appendix it is shown that condition (11a) implies that

$$\tfrac{1}{2}P_m\|x\|^2 \le V(x,t) \le \tfrac{1}{2}P_M\|x\|^2 \tag{20}$$

with P_m, P_M defined by

$$P_m = \tfrac{1}{3}M_m, \quad P_m = 6\lambda^{-1}k_d. \tag{21}$$

Along the error dynamics (14), the time-derivative of (18) becomes

$$\begin{aligned} \dot{V}(x,t) = &-x^T Q(x,t)x - s_2^T(l_d M(q) - 2k_d - 2\gamma)s_2 + \\ &-\gamma s_1^T s_1 + s_1^T\{-\Delta Y(\cdot) - C(q,s_2)\dot{q}_d\} \\ &+ \dot{e}^T C(q,\dot{q})(\lambda e) - \gamma s_2^T s_2 \\ &+ s_2^T\{-\Delta Y(\cdot) + C(q, s_2 - \dot{q})\dot{e}\} \end{aligned} \tag{22}$$

with

$$Q(x,t) = \begin{bmatrix} \begin{bmatrix} k_d - \lambda M(x,t) & 0 \\ 0 & k_d \end{bmatrix} & 0 \\ 0 & \begin{bmatrix} k_d & 0 \\ 0 & k_d \end{bmatrix} \end{bmatrix} \tag{23}$$

where Assumption 1 and the skew-symmetry of $\dot{M}(q) - 2C(q,\dot{q})$ has been used.

Using Property 1, Assumption 2, condition (11b) and (17), an upper bound on (22) is given by

$$\begin{aligned} \dot{V}(x,t) \le &-Q_m\|x\|^2 + \\ &-\gamma\|s_1\|^2 + \|s_1\|\{\beta_0 + \beta_1\|x\| + \beta_2\|x\|^2\} \\ &-\gamma\|s_2\|^2 + \|s_2\|\{\beta_0 + \beta_1\|x\| + \beta_2\|x\|^2\} \end{aligned} \tag{24}$$

where $Q_m = k_d - \lambda M_M > 0$, and β_i, $i = 0, 1, 2$ as defined in (12a)–(12c). Then the proof can be completed along the lines given in [13].

IV. Discussion

1) The basic improvement of the result in Theorem 1 in comparison with [13] is that the need of velocity feedback can be eliminated by a simple linear observer system, without affecting the stability properties of the closed loop. This is achieved despite the fact that the conditions on the controller gains remain essentially the same. An obvious additional constraint here is that l_d, the observer derivative gain, is required to be sufficiently large in order to guarantee uniform ultimate boundedness.

2) Like in [13], the uniform ultimate boundedness result is of local nature because condition (11) depends on the initial condition $x(0)$. Nonetheless, it is important to observe that these conditions can be met for arbitrary $x(0)$. In modern terminology this kind of stability is called semiglobal.

3) The following steps need to be taken to arrive at a stable implementation of the control law (8):

   - Determine the robot-specific quantities M_m, M_M, C_M, G_M, $F_{1,M}$, $F_{2,M}$, and T_M.
   - Specify upper bounds on the desired velocity and acceleration, V_M, respectively, A_M.
   - Select λ and k_d, taking into consideration (11a), and compute ϵ and $\overline{\eta}$ from (12f), (12g).
   - Fix an upper bound on the initial error state $x_0 = [\dot{e}(0)^T (\lambda e(0))^T \dot{\tilde{q}}(0)^T (\lambda\tilde{q}(0))^T]^T$.
   - Determine δ and μ in (12d) and (12e) respectively.
   - Choose γ and l_d, under the conditions (11b) and (11c) respectively.

These steps can be used as guidelines for the actual implementation of the control system (8). In this respect it should be emphasized that (13) provides a relation between the ultimate upper bound on the error state and the feedback gains. This relation can appropriately be used to guarantee a prespecified ultimate tracking performance. In practice, however, the tracking accuracy is likely to be better because the bound (13) is generally very conservative (cf. [2]).

State Observer-Based Robust Control Scheme for Electrically Driven Robot Manipulators

Masahiro Oya, Chun-Yi Su, and Toshihiro Kobayashi

***Abstract*—By using a state observer, a new robust trajectory tracking-control scheme is developed in this paper for electrically driven robot manipulators. The role of the observer is to estimate joint angular velocities. The proposed controller does not employ adaptation, but assures robust stability of tracking error between joint angles and desired trajectories. At sacrificing asymptotical stability of the tracking errors, the configuration of the proposed controller becomes very simple, compared with regressor-based adaptive controllers. It is shown in the closed-loop system using the proposed controller that the Euclidian norm of tracking errors arrives at any small closed region with any convergent rate by setting only one design parameter. Especially for the desired trajectories converging to constant ultimate values, it is assured that tracking errors converge to zero.**

***Index Terms*—Electrically driven robot, robot manipulators, robust control, state observer, tracking control.**

I. Introduction

As demonstrated in [1], the actuator dynamics constitute an important component of the complete robot dynamics. If the actuator dynamics is ignored, the designed controller may not yield good system overall performance. In recent years, controls for robot manipulators, including the actuator dynamics, have received considerable attention and several control schemes have been developed [2]–[14]. In the early works [2], [3], the controllers required full knowledge of system dynamics. If there are uncertainties in the dynamics, the controllers proposed in [2] and [3] may give a poor performance, and may even cause instability. To overcome the uncertainties in the dynamics, robust controllers have been proposed in [4]–[15]. However, these controllers normally require full state measurements. In general, full state measurements may not be available, due to cost of sensors, weight limitation, effects of noises, etc. Especially for the velocity measurement of joint angles, the required accuracy may not be achieved in practical applications, due to the existence of noises [16]. Most recently, control schemes without using velocity measurements were proposed in [17] and [18], where regressor-based adaptive controllers are employed. These controllers are effective for uncertainties in robot and actuator dynamics, and guarantee asymptotical stability of the tracking errors. However, the construction of the regressor is not trivial for a general robot, even when only desired values are involved.

In this paper, we will develop a new robust tracking-control scheme with the use of a state observer without involving adaptations, velocity measurements of joint angles, and the regressor. A precompensator is first introduced to obtain a new representation for the electrically driven robot dynamics. Then, a new robust controller is developed, which has the following features: 1) it is assured that the Euclidian norm of tracking errors can reach to any small closed region with any convergent rate by setting only one design parameter; 2) it is assured that the tracking error converges to zero when the desired trajectories $q_d(t)$ converge to ultimate constant values; and 3) the configuration of the developed robust controller is very simple, if compared with that of the regressor-based adaptive controllers.

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M. Oya and T. Kobayashi are with the Department of Control Engineering, Kyushu Institute of technology, Kitakyushu 804-8550, Japan (e-mail: oya@cntl.kyutech.ac.jp; koba@cntl.kyutech.ac.jp).

C.-Y. Su is with the Department of Mechanical and Industrial Engineering, Concordia University, Montreal, QC H3G 1M8, Canada (e-mail: cysu@me.concordia.ca).

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II. New Representation of Robot Manipulators With Integral Precompensator

Consider an n-link manipulator with revolute joints driven by armature-controlled dc motors with voltages being inputs to amplifiers. As in [3] and [4], the dynamics are described by

$$\left.\begin{array}{l} M_P(q)\ddot{q}(t) + B_P(q,\dot{q})\dot{q}(t) + g_P(q) = K_{PN}I_P(t) \\ M_P(q) = D_P(q) + J \end{array}\right\} \quad (1)$$

$$L\dot{I}_P(t) + RI_P(t) + K_e\dot{q}(t) = u_P(t) \quad (2)$$

where $q(t) \in R^n$ is joint angles, $I_P(t) \in R^n$ is the armature currents, and $u_P(t) \in R^n$ is armature voltages. $D_P(q) \in R^{n\times n}$ is a positive definite inertia matrix of the manipulator, $B_P(q,\dot{q}) \in R^{n\times n}$ is Coriolis and centrifugal torques, and $g_P(q) \in R^n$ is the gravitational torques. J, L, and R are the actuator inertia matrix, the actuator inductance matrix, and the actuator resistance matrix, respectively. K_e is the matrix that characterizes the voltage constant of the actuators, and K_{PN} is the matrix characterizing the electromechanical conversion between current and torque. J, L, R, K_e, and K_{PN} are positive definite constant diagonal matrices.

The control objective pursued here is as follows. For any given desired bounded trajectories $q_d(t)$, $\dot{q}_d(t)$, and $\ddot{q}_d(t)$, with some or all of the manipulator and actuator parameters unknown, derive a robust controller for the actuator u_P without using the measurement of joint velocities, such that the manipulator position vector $q(t)$ tracks $q_d(t)$.

In the following development, suppose that the armature current $I_P(t)$, joint angles $q(t)$, and armature voltage $u_P(t)$ are measurable, and the joint angles of the robot manipulator (1), (2) are held at some fixed angle by using some simple joint-angle feedback controller. Then, $u_P(t)$ can be represented as $u_P(t) = u(t) + \overline{u}$, where $\overline{u}$ is a constant voltage to hold the joint angle. In this case, the relations $\dot{q}(0) = 0$, $g_P(q(0)) = K_{PN}I_P(0)$, and $RI_P(0) = \overline{u}$ are satisfied. When the signals $I(t) = I_P(t) - I_P(0)$, $u(t) = u_P(t) - \overline{u}$ are used to initialize the manipulator (1), (2) in the case of the fixed-angle situation, the electrically driven robot manipulator can be described by

$$\left.\begin{array}{l} M(q)\ddot{q}(t) + B(q,\dot{q})\dot{q}(t) + g(q) = K_N I(t) \\ L\dot{I}(t) + RI(t) + K_e\dot{q}(t) = u(t) \\ M(q) = \dfrac{M_P(q)}{\underline{\rho}_{Pm}}, \quad B(q,\dot{q}) = \dfrac{B_P(q,\dot{q})}{\underline{\rho}_{Pm}} \\ g(q) = \dfrac{g_P(q) - g_P(q(0))}{\underline{\rho}_{Pm}}, \quad K_N = \dfrac{K_{PN}}{\underline{\rho}_{Pm}} \end{array}\right\} \quad (3)$$

where $\underline{\rho}_{Pm} = \lambda_{\min}[M_P(q)]$ is the lower limit of eigenvalues of $M_P(q)$, and the symbols $M(q)$, $B(q,\dot{q})$, $g(q)$, and K_N are introduced to normalize the lower-limit eigenvalue of the manipulator inertia matrix. It should be noted that the constant values $\overline{u}$ and $I_P(0)$ can be obtained from the measurable signals $I_P(t)$ and $u_P(t)$.

It is well known that manipulators and actuators are characterized by the following properties [19]–[21].

P1) The relation $B(q,x)y = B(q,y)x$ holds, and there exists a bounded positive constant $\overline{\rho}_b$ such that $\|B(q,x)y\| \leq \overline{\rho}_b\|x\|\|y\|$ for any two given vectors $x, y \in R^n$.

P2) The relation $\dot{M}(q)\dot{q}(t) - B(q,\dot{q})\dot{q}(t) = (1/2)[(\partial/\partial q)[\dot{q}(t)^T M(q)\dot{q}(t)]]^T$ is satisfied.

P3) The matrix $M(q)$ is symmetric positive definite and there exist bounded positive constants $\underline{\rho}_m (= 1)$ and $\overline{\rho}_m$ for any vector x such that $\underline{\rho}_m x^T x \leq x^T M(q) x \leq \overline{\rho}_m x^T x$.

P4) There exist bounded positive constant values $\underline{\rho}_{kn}, \overline{\rho}_{kn}$ for any vector x such that $\underline{\rho}_{kn} x^T x \le x^T K_N x \le \overline{\rho}_{kn} x^T x$.

P5) There exist bounded positive constants $\overline{\rho}_{m1}, \overline{\rho}_g, \overline{\rho}_{g1}, \overline{\rho}_r, \overline{\rho}_l, \overline{\rho}_{ke}$, such that $\|\dot{M}(q)\| \le \overline{\rho}_{m1}\|\dot{q}(t)\|$, $\|g(q)\| \le \overline{\rho}_g$, $\|\partial g(q)/\partial q\| \le \overline{\rho}_{g1}$, $\|R\| \le \overline{\rho}_r$, $\|L\| \le \overline{\rho}_l$, $\|K_e\| \le \overline{\rho}_{ke}$.

To make $q(t)$ track a desired trajectory $q_d(t)$, let us consider a pre-compensator of the following form:

$$u(t) = \mu(t) + \int_0^t \mu(\tau)d\tau = \frac{p+1}{p}[\mu(t)] \tag{4}$$

where the symbol p denotes the differential operator, and the signal $\mu(t)$ is the input to the compensator. Then, to develop a control scheme achieving the control objective, the robot model (3) is rewritten as follows (see Appendix I):

$$M(q)\ddot{q}(t) - \dot{M}(q)\dot{q}(t) + B(q,\dot{q})\dot{q}(t) + h_q(t) = K_N I_F(t) \tag{5}$$

$$L\dot{I}_F(t) + RI_F(t) + K_e\dot{q}(t) - h_I(t) = \mu(t) \tag{6}$$

and

$$\left.\begin{aligned} h_q(t) &= \int_0^t e^{-(t-\tau)}\left[\dot{M}(q)\dot{q}(\tau) + \frac{\partial g(q)}{\partial q}\dot{q}(\tau) + \frac{1}{2}f(\dot{q},\dot{q})\right]d\tau \\ f(\dot{q},\dot{q}) &= \left[\frac{\partial}{\partial q}\left[\dot{q}(t)^T M(q)\dot{q}(t)\right]\right]^T \\ h_I(t) &= \int_0^t e^{-(t-\tau)}K_e\dot{q}(\tau)d\tau \\ I_F(t) &= I(t) - \int_0^t e^{-(t-\tau)}I(\tau)d\tau = \frac{p}{p+1}[I(t)]. \end{aligned}\right\} \tag{7}$$

In the above expression, the function $f(a,b)$ is defined as $f(a,b) = [(\partial/\partial q)[a^T M(q)b]]^T$ for any vector a and $b \in R^n$, where the dependence on $q(t)$ is dropped for the simplicity in the later development.

It should be noted that property **P2** has been used to derive the new representation (5)–(7). Also, the gravitational torques $g(q)$ are represented by the term $(\partial_g(q)/\partial_q)\dot{q}(t)$.

III. Controller Development

In the following development, the design procedure will be described as a two-step process. Firstly, the signal $I_F(t)$ in (5) is regarded as the input signal. An embedded control input $I_{Fd}(t)$ is designed so that the desired tracking can be achieved. Secondly, $\mu(t)$ is designed so that $I_F(t)$ tracks $I_{Fd}(t)$ without using the signal $\dot{q}(t)$. In turn, this allows $q(t)$ to track the desired trajectory $q_d(t)$.

A. Synthesis of Embedded Signal $I_{Fd}(t)$

First, let us suppose that the signal $\dot{q}(t)$ is available and the signal $I_F(t)$ can be treated as an input signal. A robust controller will be synthesized, so that $q(t)$ tracks the desired trajectory $q_d(t)$ for the subsystem (5). To achieve such an objective, an additional property is exploited.

P6) There exists a bounded positive constant value $\overline{\rho}_f$ such that $\|f(a,b)\| \le \overline{\rho}_f\|a\|\|b\|$ for any $a, b \ne q(t)$.

For the development of robust controller, the following standard assumptions are required for the system (5)–(7).

A1) There exist known diagonal constant matrices $\widehat{K}_N, \widehat{R}, \widehat{K}_e$, and there exist bounded positive constant values $\overline{\rho}_{k\widehat{k}}, \underline{\rho}_{k\widehat{k}}, \overline{\rho}_{\widetilde{r}}, \overline{\rho}_{\widetilde{k}_e}$ such that $\underline{\rho}_{k\widehat{k}} x^T x \le x^T K_N \widehat{K}_N^{-1} x \le \overline{\rho}_{k\widehat{k}} x^T x$ for all x, $\|\widetilde{R}\| = \|R - \widehat{R}\| \le \overline{\rho}_{\widetilde{r}}$, $\|\widetilde{K}_e\| = \|K_e - \widehat{K}_e\| \le \overline{\rho}_{\widetilde{k}_e}$.

A2) There exists a known bounded function $\widehat{g}(q)$, and there exist bounded positive constant values $\overline{\rho}_{\widehat{g}1}, \overline{\rho}_{\widetilde{g}1}$ such that $\|(\partial\widehat{g}(q)/\partial q)\| \le \overline{\rho}_{\widehat{g}1}$, $\|(\partial\widetilde{g}(q)/\partial q)\| = \|(\partial g(q)/\partial q) - K_N\widehat{K}_N^{-1}(\partial\widehat{g}(q)/\partial q)\| \le \overline{\rho}_{\widetilde{g}1}$.

A3) For given desired trajectories $q_d(t)$, there exist bounded positive constant values $\overline{\rho}_d, \overline{\rho}_{di}$, $i = 1,2$, such that $\|q_d(t)\| \le \overline{\rho}_d$, $\|\dot{q}_d(t)\| \le \overline{\rho}_{d1}$, $\|\ddot{q}_d(t)\| \le \overline{\rho}_{d2}$.

A4) The initial value $q(0)$ is bounded.

It is noted that $\widehat{K}_N$, $\widehat{R}$, and $\widehat{K}_e$ are estimates for K_N, R, and K_e, respectively, $\widehat{g}(q)$ is an estimate of $g(q)$, and $\widetilde{\bullet}$ denotes estimated error.

To design an embedded control input so that $q(t)$ in the subsystems (5) tracks $q_d(t)$, in the following, we define:

$$s(t) = \beta^{-1}\dot{\widetilde{q}}(t) + \widetilde{q}(t), \quad \widetilde{q}(t) = q(t) - q_d(t) \tag{8}$$

where β is a positive design parameter introduced to improve tracking performance. It is noted that the norm of the initial tracking error $s(0)$ does not increase with respect to the design parameter β. Using the relation

$$\widehat{g}(q) - \int_0^t e^{-(t-\tau)}\widehat{g}(q)d\tau = \int_0^t e^{-(t-\tau)}\frac{\partial\widehat{g}}{\partial q}\dot{q}(\tau)d\tau + e^{-t}\widehat{g}(q(0)) \tag{9}$$

and the property **P1**), the error system can be obtained as

$$\begin{aligned} M(q)\dot{s}(t) = &-(\beta B(q, s-\widetilde{q}) - (\beta+1)M(q) + 2B(q,\dot{q}_d)) \\ &\times (s(t) - \widetilde{q}(t)) - \beta^{-1}\omega_{sd}(q,\dot{q}_d,\ddot{q}_d) - \beta^{-1}h(t) \\ &+ \beta^{-1}K_N I_F(t) - \beta^{-1}K_N\widehat{K}_N^{-1} \\ &\times \left(\widehat{g}(q) - \int_0^t e^{-(t-\tau)}\widehat{g}(q)d\tau\right) \end{aligned} \tag{10}$$

$$\begin{aligned} h(t) &= h_q(t) - \int_0^t e^{-(t-\tau)}K_N\widehat{K}_N^{-1}\frac{\partial\widehat{g}}{\partial q}\dot{q}(\tau)d\tau \\ &= \int_0^t e^{-(t-\tau)}\Big[\frac{\beta^2}{2}f(s-\widetilde{q}, s-\widetilde{q}) + \omega_{hd}(q,\dot{q}_d) \\ &\qquad + \beta(\dot{M}(q)(s(t)-\widetilde{q}(t)) \\ &\qquad + \frac{\partial\widetilde{g}}{\partial q}(s(t)-\widetilde{q}(t)) \\ &\qquad + f(\dot{q}_d, s-\widetilde{q}))\Big]d\tau \end{aligned} \tag{11}$$

where $\omega_{sd}(q,\dot{q},\ddot{q}_d)$, $\omega_{hsd}(q,\dot{q}_d)$ are given by

$$\left.\begin{aligned} \omega_{sd}(q,\dot{q}_d,\ddot{q}_d) &= M(q)(\ddot{q}_d(t) - \dot{q}_d(t)) + B(q,\dot{q}_d)\dot{q}_d(t) \\ &\quad - K_N\widehat{K}_N^{-1}e^{-t}\widehat{g}(q(0)) \\ \omega_{hd}(q,\dot{q}_d) &= \dot{M}(q)\dot{q}_d(t) + \frac{1}{2}f(\dot{q}_d,\dot{q}_d) + \frac{\partial\widetilde{g}}{\partial q}\dot{q}_d(t) \end{aligned}\right\}. \tag{12}$$

It should be emphasized that the notion for $f(s-\widetilde{q}, s-\widetilde{q})$ and $f(\dot{q}_d, s-\widetilde{q})$ should follow the definition of $f(a,b)$. For example, $f(\dot{q}_d, s-\widetilde{q}) = [(\partial/\partial q)[\dot{q}_d(t)^T M(q)(s-\widetilde{q})]]^T$.

Based on the error system (10), if the signal $I_F(t)$ can be treated as a control input signal, the signal $I_F(t)$ can be synthesized as

$$I_F(t) = -\beta\gamma_s\widehat{K}_N^{-1}s(t) + \widehat{K}_N^{-1}\left(\widehat{g}(q) - \int_0^t e^{-(t-\tau)}\widehat{g}(q)d\tau\right) \tag{13}$$

for any $\beta \ge 2$. Then, it can be proved that this robust control law can stabilize the error system (10) with proper selection of the scalar positive gain γ_s.

Remark: As a matter of fact, many robust control laws have been proposed in the literature and can be directly applied. However, the merit of this new control law is that only one design parameter β needs to be adjusted, and there is no regressor involved, which makes the controller design simple.

In the above control law, the measurement of joint velocities $\dot{q}(t)$ is required. However, as assumed in the paper, the signal $\dot{q}(t)$ is not available. To remove such a requirement, instead of using $I_F(t)$ in (13), an embedded signal $I_{Fd}(t)$ is defined as

$$I_{Fd}(t) = -\beta\gamma_s \widehat{K}_N^{-1}\widehat{s}(t) + \widehat{K}_N^{-1}\left(\widehat{g}(q) - \int_0^t e^{-(t-\tau)}\widehat{g}(q)d\tau\right) \tag{14}$$

where the signal $\widehat{s}(t)$ denotes the estimate of the tracking error signal $s(t)$ and is generated by

$$\left.\begin{aligned} \widehat{s}(t) &= (\gamma_e\beta^{-1}+1)\widetilde{q}(t) + \beta^{-1}\widetilde{q}(t) + s_e(t) \\ \dot{s}_e(t) &= \gamma_e\widetilde{q}(t) - \gamma_e\widehat{s}(t) \\ s_e(0) &= -(\gamma_e+1)\beta^{-1}\widetilde{q}(0) \end{aligned}\right\} \tag{15}$$

where the scalar constant γ_e is a positive design parameter and will be specified later.

It can be proved that the embedded signal $I_{Fd}(t)$ in (14) can still guarantee the tracking of the subsystem (5) if $I_{Fd}(t)$ is treated as an input signal $I_F(t)$. The proof is omitted, since the conclusion can easily be drawn for the proof of the main result (*Theorem 2*) in Section III-C.

B. Synthesis of Control Signal μ

From the dynamic (5), (6), it is clear that the signal $I_F(t)$ cannot be used as an input signal. It is the control signal μ that generates the $I_F(t)$. The design task turns to the development of the control signal μ, which forces $I_F(t)$ to track $I_{Fd}(t)$.

To develop such a controller, the error signals $\widetilde{I}_F(t)$, $\widetilde{s}(t)$ are, respectively, defined as

$$\widetilde{I}_F(t) = \beta^{-1}\gamma_s^{-1}\widehat{K}_N\left(I_F(t) - I_{Fd}(t)\right), \quad \widetilde{s}(t) = s(t) - \widehat{s}(t). \tag{16}$$

The error signal $\widetilde{I}_F(t)$ is defined so that $\|\widetilde{I}_F(0)\|$ does not increase with respect to β and γ_s. Then, it is seen from (10), (11), (15), (6), and (7) that the tracking error system can be described by

$$\begin{aligned} M(q)\dot{s}(t) = &-(\beta B(q, s-\widetilde{q}) - (\beta+1)M(q) + 2B(q,\dot{q}_d)) \\ &\times (s(t) - \widetilde{q}(t)) - \beta^{-1}h_{qg}(t) - \beta^{-1}h_{qd}(t) \\ &+ \gamma_s K_N\widehat{K}_N^{-1}\widetilde{I}_F(t) \\ &- \gamma_s K_N\widehat{K}_N^{-1}(s(t) - \widehat{s}(t)) \end{aligned} \tag{17}$$

$$\begin{aligned} M(q)\dot{\widetilde{s}}(t) = &-(\beta B(q, s-\widetilde{q}) + 2B(q,\dot{q}_d))(s(t)-\widetilde{q}(t)) \\ &- \beta^{-1}h_{qg}(t) - \beta^{-1}h_{qd}(t) + \gamma_s K_N\widehat{K}_N^{-1}\widetilde{I}_F(t) \\ &- \gamma_s K_N\widehat{K}_N^{-1}(s(t) - \widetilde{s}(t)) - \gamma_e M(q)\widetilde{s}(t) \end{aligned} \tag{18}$$

$$\left.\begin{aligned} h_{qg}(t) = &\int_0^t e^{-(t-\tau)}\left[\frac{\beta^2}{2}f(s-\widetilde{q}, s-\widetilde{q})\right. \\ &+ \beta(M(q)(s(t)-\widetilde{q}(t)) \\ &+ \frac{\partial\widetilde{g}}{\partial q}(s(t)-\widetilde{q}(t)) \\ &\left.+ f(\dot{q}_d, s-\widetilde{q}))\right]d\tau \\ h_{qd}(t) = &\,\omega_{sd}(q,\dot{q}_d,\ddot{q}_d) + \int_0^t e^{-(t-\tau)}\omega_{hd}(q,\dot{q}_d)d\tau \end{aligned}\right\} \tag{19}$$

$$\begin{aligned} L\dot{\widetilde{I}}_F(t) = &-R\widetilde{I}_F(t) - \frac{\widehat{K}_N}{\beta\gamma_s} \\ &\times \left(\widehat{R}I_{Fd}(t) + \widehat{K}_e\dot{q}_d(t) - \int_0^t e^{-(t-\tau)}\widehat{K}_e\dot{q}_d(\tau)d\tau\right) \\ &+ \frac{\widehat{K}_N}{\beta\gamma_s}\mu(t) + \omega_{Is}(t)s(t) + \omega_{I\widetilde{s}}(t)\widetilde{s}(t) \\ &+ \omega_{I\widetilde{q}}(t)\widetilde{q}(t) + \frac{1}{\gamma_s}h_{Ig}(t) + \frac{1}{\beta\gamma_s}h_{Id}(t) \end{aligned} \tag{20}$$

$$\left.\begin{aligned} \omega_{Is}(t) &= \widetilde{R} - \frac{1}{\gamma_s}\widehat{K}_N K_e + (\beta+1)L - \frac{L}{\gamma_s}\frac{\partial\widehat{g}}{\partial q} \\ \omega_{I\widetilde{s}}(t) &= -\widetilde{R} + \gamma_e L \\ \omega_{I\widetilde{q}}(t) &= \frac{1}{\gamma_s}\widehat{K}_N K_e - (\beta+1)L + \frac{L}{\gamma_s}\frac{\partial\widehat{g}}{\partial q} \end{aligned}\right\} \tag{21}$$

$$\left.\begin{aligned} h_{Ig}(t) = &(L-\widetilde{R})\int_0^t e^{-(t-\tau)}\frac{\partial\widehat{g}}{\partial q}(s(\tau)-\widetilde{q}(\tau))d\tau \\ &+ \widehat{K}_N K_e\int_0^t e^{-(t-\tau)}(s(\tau)-\widetilde{q}(\tau))d\tau \\ h_{Id}(t) = &-\widehat{K}_N\widetilde{K}_e\dot{q}_d(t) + \int_0^t e^{-(t-\tau)}\widehat{K}_N\widetilde{K}_e\dot{q}_d(\tau)d\tau \\ &- L\frac{\partial\widehat{g}}{\partial q}\dot{q}_d(t) + (L-\widetilde{R})\int_0^t e^{-(t-\tau)}\frac{\partial\widehat{g}}{\partial q}\dot{q}_d(\tau)d\tau \\ &+ (L-\widetilde{R})e^{-t}\widehat{g}(q(0)). \end{aligned}\right\} \tag{22}$$

In the above error system, it is obvious from the properties **P1**, **P3–P6**, and **A1–A4** that there exist bounded positive constants $\overline{\rho}_{qd}$, $\overline{\rho}_{Id}$ such that

$$\|h_{qd}(t)\| \le \overline{\rho}_{qd}, \quad \|h_{Id}(t)\| \le \overline{\rho}_{Id} \tag{23}$$

and there exist bounded positive constants $\overline{\rho}_{qi}$, $i = 1\ldots4$, $\overline{\rho}_{I1}$ and scalar positive signals $h_{qi}(t)$, $i = 1\ldots4$, $h_{I1}(t)$ such that

$$\begin{aligned} \beta^{-1}\|h_{qg}(t)\| \le &\frac{1}{2}\sqrt{\beta\gamma_0\overline{\rho}_{qd}}\overline{\rho}_{q1}h_{q1}(t) + \frac{1}{2}\sqrt{\beta\gamma_0\overline{\rho}_{qd}}\overline{\rho}_{q2}h_{q2}(t) \\ &+ \frac{1}{2}\sqrt{\beta\gamma_0\overline{\rho}_{qd}}\overline{\rho}_{q3}h_{q3}(t) + \overline{\rho}_{q4}h_{q4}(t) \end{aligned} \tag{24}$$

$$\|h_{Ig}(t)\| \le \overline{\rho}_{I1}h_{I1}(t) \tag{25}$$

$$\left.\begin{aligned} &\overline{\rho}_{q1} = \sqrt{\overline{\rho}_f}, \quad \overline{\rho}_{q2} = \sqrt{\frac{2\overline{\rho}_f}{\varepsilon_q}} \\ &\overline{\rho}_{q3} = \sqrt{\frac{\overline{\rho}_f}{\varepsilon_q}}, \quad \overline{\rho}_{q4} = \sqrt{\overline{\rho}_m + \overline{\rho}_{\widetilde{g}1} + \overline{\rho}_f\overline{\rho}_{d1}} \\ &\overline{\rho}_{I1} = (\overline{\rho}_\ell + \overline{\rho}_{\widetilde{r}})\overline{\rho}_{\widehat{g}1} + \|\widehat{K}_N\|\overline{\rho}_{ke} \end{aligned}\right\} \tag{26}$$

$$\left.\begin{aligned} \dot{h}_{q1}(t) &= -h_{q1}(t) + \sqrt{\beta\overline{\rho}_f}\sqrt{\frac{1}{\gamma_0\overline{\rho}_{qd}}}\|s(t)\|^2 \\ \dot{h}_{q2}(t) &= -h_{q2}(t) + \sqrt{2\beta\overline{\rho}_f}\sqrt{\frac{\varepsilon_q}{\gamma_0\overline{\rho}_{qd}}}\|s(t)\|\|\widetilde{q}(t)\| \\ \dot{h}_{q3}(t) &= -h_{q3}(t) + \sqrt{\beta\overline{\rho}_f}\sqrt{\frac{\varepsilon_q}{\gamma_0\overline{\rho}_{qd}}}\|\widetilde{q}(t)\|^2 \\ \dot{h}_{q4}(t) &= -h_{q4}(t) + \overline{\rho}_{q4}\|s(t) - \widetilde{q}(t)\| \\ \dot{h}_{I1}(t) &= -h_{I1}(t) + \|s(t)\| + \|\widetilde{q}(t)\| \\ h_{qi} &= 0, \quad i = 1,2,3, \quad h_{I1}(0) = 0 \end{aligned}\right\} \tag{27}$$

where γ_0 is a positive design parameter, ε_q is a positive constant, and these parameters are specified later.

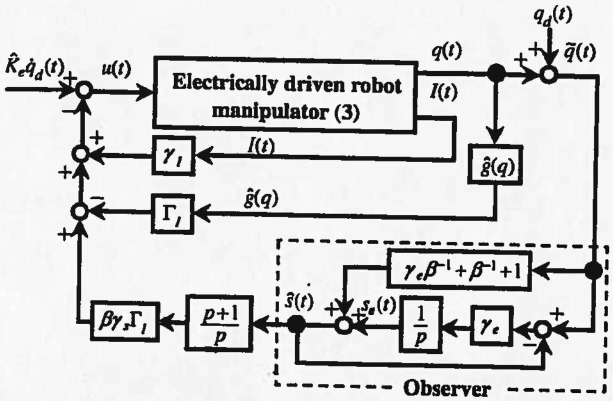

Fig. 1. Configuration of the closed-loop system using the proposed controller.

Based on the above error dynamics, the robust control law is then synthesized by

$$\mu(t)=-\beta\gamma_s\gamma_i\hat{K}_N^{-1}\tilde{I}_F+\hat{R}I_{Fd}(t)+\hat{K}_e\dot{q}_d(t)-\int_0^t e^{-(t-\tau)}\hat{K}_e\dot{q}_d(\tau)d\tau \tag{28}$$

and the design parameters are given by the following form:

$$\left.\begin{aligned} \gamma_s &= \gamma_{s1}\gamma_0\beta^2+\gamma_{s2}\gamma_0\beta+\gamma_{s3}\beta+\gamma_{s4}\\ \gamma_e &= \gamma_{e1}\gamma_s+\gamma_{e2}\gamma_0\beta^2+\gamma_{e3}\gamma_0\beta+\gamma_{e4}\beta+\gamma_{e5}\\ \gamma_I &= \gamma_{I1}\gamma_s+\gamma_{I2}\gamma_e+\gamma_{I3}\beta^2+\tfrac{1}{4}\bar{\rho}_\ell\beta+\gamma_{I4}+\gamma_{I5}\tfrac{1}{\gamma_s^2}\end{aligned}\right\} \tag{29}$$

where $\gamma_{si}, i=1\dots4$; $\gamma_{ei}, i=1\dots5$; $\gamma_{Ii}, i=1\dots5$ are positive design parameters.

Theorem 1: In (28), the control law is expressed as the input to the compensator. Based on the definition of $\mu(t)$ in (4), the control low (28) can be expressed as

$$\begin{aligned} u(t)&=-\beta\gamma_s\Gamma_I\left(\hat{s}(t)+\int_0^t\hat{s}(\tau)d\tau\right)-\gamma_I I(t)+\Gamma_I\hat{g}(q)+\hat{K}_e\dot{q}_d(t)\\ \Gamma_I&=\gamma_I\hat{K}_N^{-1}+\hat{R}\hat{K}_N^{-1}.\end{aligned} \tag{30}$$

Proof: See Appendix II.

The configuration of the closed-loop system using the proposed controller is shown in Fig. 1. The symbol p in Fig. 1 denotes the differential operator. In Fig. 1, the gains γ_s, γ_e, γ_I, and β are scalar, and matrices $\hat{K}_e$ and Γ_I are constant diagonal matrices. The configuration of the controller is very simple, as compared with the regressor-based adaptive controllers.

C. Stability Analysis

Before describing the stability analysis of the closed-loop system, the following lemma is required.

Lemma 1: Let us consider a nonnegative function $V(t)$. It is assumed that γ_0 is a fixed constant value, such that $\gamma_0\bar{\rho}_{qd}\geq 2V(0)$. If the derivative of $V(t)$ satisfies

$$\dot{V}(t)\leq -V(t)+\frac{1}{\gamma_0\bar{\rho}_{qd}}V(t)^2+\frac{5\bar{\rho}_{qd}}{4\beta^2} \tag{31}$$

then $V(t)$ is uniformly bounded for any $\beta>\sqrt{5\gamma_0^{-1}}$, and satisfies the relation

$$V(t)\leq\frac{\gamma_0\bar{\rho}_{qd}}{2}. \tag{32}$$

Proof: See Appendix III.

For stability analysis, we also need the following definitions:

$$\varepsilon_q=3\bar{\rho}_m+\bar{\rho}_f+4\bar{\rho}_b\bar{\rho}_{d1}+5+\bar{\rho}_{q4}^2. \tag{33}$$

$$\left.\begin{aligned} \bar{\rho}_{\omega I1}&=\|\hat{K}_N\|\bar{\rho}_{ke}+\bar{\rho}_{\hat{g}1}\bar{\rho}_\ell\\ \bar{\rho}_{vs1}&=\left(\frac{(\bar{\rho}_{m1}+2\bar{\rho}_b)^2+\bar{\rho}_{m1}^2+2\bar{\rho}_b^2}{4}\right.\\ &\quad\left.+\frac{(\bar{\rho}_{m1}+4\bar{\rho}_b)^2+32\bar{\rho}_b^2}{8\varepsilon_q}+\frac{2\bar{\rho}_b^2}{\varepsilon_q^2}\right)\bar{\rho}_{qd}\\ \bar{\rho}_{vs2}&=\sum_{i=1}^{3}\frac{\bar{\rho}_{qi}^2\bar{\rho}_{qd}}{2},\quad \bar{\rho}_{vs3}=3\bar{\rho}_m\\ \bar{\rho}_{vs4}&=\bar{\rho}_{m1}\bar{\rho}_{d1}+3\bar{\rho}_m+8\bar{\rho}_b\bar{\rho}_{d1}+4\bar{\rho}_{q4}^2+2\bar{\rho}_{qd}\\ \bar{\rho}_{vs5}&=\left(\frac{\bar{\rho}_{m1}^2}{4\varepsilon_q}+\frac{2\bar{\rho}_b^2}{\varepsilon_q^2}\right)\bar{\rho}_{qd}\\ \bar{\rho}_{vs6}&=\bar{\rho}_{m1}\bar{\rho}_{d1}+4\bar{\rho}_b\bar{\rho}_{d1}+4\bar{\rho}_{q4}^2+2\bar{\rho}_{qd}.\end{aligned}\right\} \tag{34}$$

The stability of the closed-loop system described by (5), (6), (14), (15), (28), and (29) is now stated by the following theorem.

Theorem 2: Let us consider the controller (15), (28), and (29) for the robot manipulator (3) with initial values $\dot{q}(0)=0$, $I(0)=0$, and $g(q(0))=0$. If the positive design parameters γ_0 and $\gamma_{si}, i=1\dots4$; $\gamma_{ei}, i=1\dots5$; $\gamma_{Ii}, i=1\dots5$ are fixed so that the following inequalities are satisfied:

$$\left.\begin{aligned} &\gamma_{s1}\geq\frac{1}{\underline{\rho}_{k\hat{k}}}\bar{\rho}_{vs1},\quad \gamma_{s2}\geq\frac{1}{\underline{\rho}_{k\hat{k}}}\bar{\rho}_{vs2}\\ &\gamma_{s3}\geq\frac{1}{\underline{\rho}_{k\hat{k}}}(\bar{\rho}_{vs3}+\bar{\rho}_f+\varepsilon_q+0.5\bar{\rho}_m)\\ &\gamma_{s4}\geq\frac{1}{\underline{\rho}_{k\hat{k}}}(5+\bar{\rho}_{vs4})\end{aligned}\right\} \tag{35}$$

$$\left.\begin{aligned} &\gamma_{e1}\geq 1+2\bar{\rho}_{k\hat{k}},\quad \gamma_{e2}\geq\bar{\rho}_{vs5},\quad \gamma_{e3}\geq\bar{\rho}_{vs2}\\ &\gamma_{e4}\geq 0.5\bar{\rho}_m,\quad \gamma_{e5}\geq\bar{\rho}_{vs6}+1\end{aligned}\right\} \tag{36}$$

$$\left.\begin{aligned} &\gamma_{I1}\geq\frac{\bar{\rho}_{k\hat{k}}^2}{2\underline{\rho}_{k\hat{k}}}+\frac{\bar{\rho}_{k\hat{k}}^2}{2},\quad \gamma_{I2}\geq\frac{\bar{\rho}_\ell^2}{2},\quad \gamma_{I3}\geq 3\bar{\rho}_\ell^2\\ &\gamma_{I4}\geq\frac{3}{2}(\bar{\rho}_{\tilde{r}}+\bar{\rho}_\ell)^2+\frac{3}{2}\bar{\rho}_\ell^2+\frac{\bar{\rho}_r^2}{2}\\ &\gamma_{I5}\geq 3\bar{\rho}_{\omega I1}^2+\bar{\rho}_{I1}^2+2\frac{\bar{\rho}_{Id}^2}{\bar{\rho}_{qd}}\end{aligned}\right\} \tag{37}$$

$$\begin{aligned}\gamma_0\geq\frac{2}{\bar{\rho}_{qd}}\Bigg[&\bar{\rho}_m\left(\|\tilde{q}(0)\|+\frac{1}{2}\left\|\dot{\tilde{q}}(0)\right\|\right)^2\\ &+\frac{\bar{\rho}_m}{4}\left\|\dot{\tilde{q}}(0)\right\|^2+\varepsilon_q\|\tilde{q}(0)\|^2\\ &+\bar{\rho}_\ell\left(\|\tilde{q}(0)\|+\frac{\underline{\rho}_{k\hat{k}}}{10}\|\hat{g}(q(0))\|\right)^2\Bigg].\end{aligned} \tag{38}$$

Then, the closed-loop system using the controller (15), (28), and (29) becomes stable for any β such that

$$\beta\geq 2 \text{ and } \beta>\sqrt{5\gamma_0^{-1}}. \tag{39}$$

Moreover, there exists a positive constant δ independent of the design parameter β such that

$$\|\tilde{q}(t)\|^2\leq\varepsilon_q^{-1}e^{-\frac{\beta}{2}t}V(0)+\frac{\delta}{\varepsilon_q\beta}. \tag{40}$$

Proof: Consider a positive definite function

$$V(t) = V_s(t) + V_{\tilde{s}}(t) + V_q(t) + V_I(t) + V_h(t) \tag{41}$$

$$\left.\begin{aligned} &V_s(t) = s(t)^T M(q) s(t), \;\; V_{\tilde{s}}(t) = \tilde{s}(t)^T M(q)\tilde{s}(t) \\ &V_q(t) = \varepsilon_q \tilde{q}(t)^T \tilde{q}(t), \;\; V_I(t) = \tilde{I}_F(t)^T L \tilde{I}_F(t) \\ &V_h(t) = \sum_{i=1}^{4} h_{qi}(t)^2 + h_{I1}(t)^2 \end{aligned}\right\} \tag{42}$$

where ε_q is defined in (33).

It can be proved that the derivatives of $V_s(t)$, $V_{\tilde{s}}(t)$, $V_q(t)$, $V_I(t)$, and $V_h(t)$ satisfy the following inequalities (see Appendix IV), where relation $\underline{\rho}_m = 1$ has been used in the derivation of the following inequalities (43) and (46), and the relation $\beta\varepsilon_q - 2\overline{\rho}_{q4}^2 \geq 0$ has been used in the derivation of the following inequality (45):

$$\begin{aligned} &\dot{V}_s(t) + \dot{V}_{\tilde{s}}(t) \\ &\quad\leq -\underline{\rho}_{k\hat{k}}\gamma_s \|s(t)\|^2 - (2\gamma_e - \gamma_s - 2\overline{\rho}_{k\hat{k}}\gamma_s)\|\tilde{s}(t)\|^2 \\ &\qquad + (\overline{\rho}_{vs1}\gamma_0\beta^2 + \overline{\rho}_{vs2}\gamma_0\beta + \overline{\rho}_{vs3}\beta + \overline{\rho}_{vs4})\|s(t)\|^2 \\ &\qquad + (\overline{\rho}_{vs5}\gamma_0\beta^2 + \overline{\rho}_{vs2}\gamma_0\beta + \overline{\rho}_{vs6})\|\tilde{s}(t)\|^2 \\ &\qquad + (\overline{\rho}_m\beta + \overline{\rho}_m + 4\overline{\rho}_b\overline{\rho}_{d1})\|\tilde{q}(t)\|^2 \\ &\qquad + \frac{1}{\gamma_0\overline{\rho}_{qd}}\left(V_s(t) + V_{\tilde{s}}(t) + V_q(t)\right)^2 + \sum_{i=1}^{3} h_{qi}(t)^2 \\ &\qquad + \frac{1}{2}h_{q4}(t)^2 + \left(\frac{\overline{\rho}_{k\hat{k}}^2}{\underline{\rho}_{k\hat{k}}} + \overline{\rho}_{k\hat{k}}^2\right)\gamma_s \left\|\tilde{I}_F(t)\right\|^2 \\ &\qquad + \frac{1}{\beta^2}\frac{\|h_{qd}(t)\|^2}{\overline{\rho}_{qd}} \end{aligned} \tag{43}$$

$$\begin{aligned} \dot{V}_I(t) \leq\; &-2\gamma_I \left\|\tilde{I}_F(t)\right\|^2 + \left[\left(6\overline{\rho}_{\omega I1}^2 + 2\overline{\rho}_{I1}^2 + 4\frac{\overline{\rho}_{Id}^2}{\overline{\rho}_{qd}}\right)\frac{1}{\gamma_s^2}\right. \\ &\qquad + 3(\overline{\rho}_{\tilde{r}} + \overline{\rho}_\ell)^2 + 3\overline{\rho}_\ell^2 + \overline{\rho}_{\tilde{r}}^2 \\ &\qquad \left. + 6\overline{\rho}_\ell^2\beta^2 + \overline{\rho}_\ell^2\gamma_e\right]\left\|\tilde{I}_F(t)\right\|^2 \\ &+ \|s(t)\|^2 + \|\tilde{q}(t)\|^2 + (\gamma_e + 1)\|\tilde{s}(t)\|^2 \\ &+ \frac{1}{2}h_{I1}(t)^2 + \frac{1}{4\beta^2}\frac{\|h_{Id}(t)\|^2\overline{\rho}_{qd}}{\overline{\rho}_{Id}^2} \end{aligned} \tag{44}$$

$$\begin{aligned} \dot{V}_q(t) \leq\; &-\left(\varepsilon_q\beta + 2\overline{\rho}_{q4}^2\right)\|\tilde{q}\|^2 + \left(\varepsilon_q\beta - 2\overline{\rho}_{q4}^2\right)\|s(t)\|^2 \\ &+ 4\overline{\rho}_{q4}^2\tilde{q}(t)^T s(t) \end{aligned} \tag{45}$$

$$\begin{aligned} \dot{V}_h(t) \leq\; &-V_h(t) - \sum_{i=1}^{3} h_{qi}(t)^2 - \frac{1}{2}h_{q4}(t)^2 - \frac{1}{2}h_{I1}(t)^2 \\ &+ \left(4 + 2\overline{\rho}_{q4}^2 + \overline{\rho}_f\beta\right)\|s(t)\|^2 \\ &+ \left(4 + 2\overline{\rho}_{q4}^2 + \frac{\overline{\rho}_f\beta}{2}\right)\|\tilde{q}(t)\|^2 - 4\overline{\rho}_{q4}^2\tilde{q}(t)^T s(t) \\ &+ \frac{2}{\gamma_0\overline{\rho}_{qd}}\left(V_s(t) + V_q(t)\right)V_h(t). \end{aligned} \tag{46}$$

Since $\beta \geq 2$, it follows from (33) that

$$\begin{aligned} -\beta\varepsilon_q &= -\frac{\beta}{2}\varepsilon_q - \frac{\beta}{2}\varepsilon_q \\ &\leq -\frac{\beta}{2}\varepsilon_q - \left(\overline{\rho}_m + \frac{\overline{\rho}_f}{2}\right)\beta - \overline{\rho}_m - 4\overline{\rho}_b\overline{\rho}_{d1} - 5 - \overline{\rho}_{q4}^2. \end{aligned} \tag{47}$$

Considering inequalities (35)–(37) are satisfied, it is seen from (43)–(47) that the following relation can be obtained:

$$\begin{aligned} \dot{V}(t) \leq\; &-\frac{\beta}{2}\left(V_s(t) + V_{\tilde{s}}(t) + V_q(t) + V_I(t)\right) - V_h(t) \\ &+ \frac{1}{\gamma_0\overline{\rho}_{qd}}V(t)^2 + \frac{1}{\beta^2}\frac{\|h_{qd}(t)\|^2}{\overline{\rho}_{qd}} + \frac{1}{4\beta^2}\frac{\|h_{Id}(t)\|^2\overline{\rho}_{qd}}{\overline{\rho}_{Id}^2} \\ \leq\; &-V(t) + \frac{1}{\gamma_0\overline{\rho}_{qd}}V(t)^2 + \frac{1}{\beta^2}\frac{\|h_{qd}(t)\|^2}{\overline{\rho}_{qd}} \\ &+ \frac{1}{4\beta^2}\frac{\|h_{Id}(t)\|^2\overline{\rho}_{qd}}{\overline{\rho}_{Id}^2} \\ \leq\; &-V(t) + \frac{1}{\gamma_0\overline{\rho}_{qd}}V(t)^2 + \frac{5\overline{\rho}_{qd}}{4\beta^2}. \end{aligned} \tag{48}$$

The initial value $V(0)$ does not increase with respect to β and γ_s. Considering this fact, it is seen from (38) and (41) that $2V(0) \leq \gamma_0\overline{\rho}_{qd}$ is assured for any $\beta \geq 2$ and $\gamma_s \geq (5/\underline{\rho}_{k\hat{k}})$. According to *Lemma 1*, it is concluded from (48) and (39) that the closed-loop system is stable, and $2V(t) \geq \gamma_0\overline{\rho}_{qd}$. Analyzing the derivative of the positive definite function $V_c(t) = V_s(t) + V_{\tilde{s}}(t) + V_q(t) + V_I(t)$ by using the fact $2V(t) \leq \gamma_0\overline{\rho}_{qd}$, it is easy to ascertain from (43)–(45) that there exists a positive constant δ_1 independent of the design parameter β

$$\dot{V}_c(t) \leq -\frac{\beta}{2}V_c(t) + \delta_1. \tag{49}$$

The relation (40) can be obtained from (49).

Remarks:

1) From *Theorem 2*, it can be concluded that the closed-loop system using the proposed controller is robust stable while the inequalities (35)–(39) are satisfied, and we can make $\|\tilde{q}(t)\|^2$ arrive at any small closed region with any convergent rate by setting the design parameter β.
2) Especially in the case of $V(0) = 0$, from (40), it is seen that the maximum value of $\|\tilde{q}(t)\|$ can be arbitrarily reduced by increasing the value of the design parameter β.
3) If a bounded disturbance $c(t)$ exists in robot dynamics (3) as $M(q)\ddot{q}(t) + B(q,\dot{q})\dot{q}(t) + g(q) = K_N I(t) + c(t)$. The changes appear only in $h_{qd}(t)$ (19) as

$$h_{qd}(t) = \omega_{sd}(q, \dot{q}_d, \ddot{q}_d) + \int_0^t e^{-(t-\tau)}\omega_{hd}(q, \dot{q}_d)d\tau + c(t) - \int_0^t e^{-(t-\tau)}c(\tau)d\tau.$$

In the closed-loop system using the proposed controller, it can also be shown that the tracking error $\tilde{q}(t)$ converges to zero if the desired trajectories $q_d(t)$ converge to constant ultimate values. To show this, let us consider the new signal

$$h_d(t) = \frac{\|h_{qd}(t)\|^2}{\overline{\rho}_{qd}} + \frac{\|h_{Id}(t)\|^2\overline{\rho}_{qd}}{4\overline{\rho}_{Id}^2}. \tag{50}$$

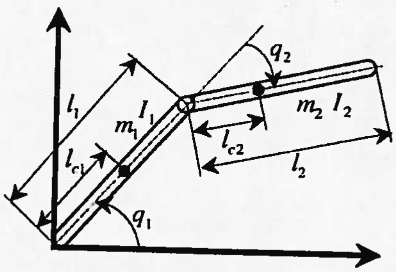

Fig. 2. Two-link manipulator.

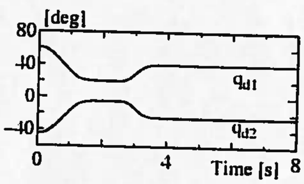

Fig. 3. Desired trajectory.

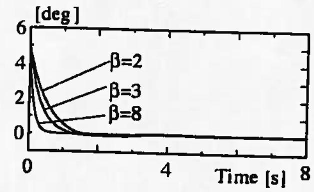

Fig. 4. Tracking error responses for $\beta = 2, 4, 10$ in the case of $\tilde{q}(0) = [5, -5]^T$ deg.

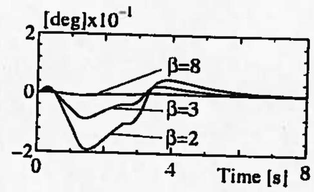

Fig. 5. Tracking error responses for $\beta = 2, 4, 10$ in the case of $\tilde{q}(0) = [0, 0]^T$ deg.

In the case of $q_d(t)$ converging to a constant vector, there exist bounded positive constants d_1, d_2 such that

$$h_d(t) \le d_2 e^{-d_1 t}. \tag{51}$$

Using the relation above, it can be seen from *Theorem 1* that the following corollary holds.

Corollary 1: Suppose $q_d(t)$ converges to a constant vector exponentially. If the fixed design parameters γ_0, γ_{si}, $i = 1 \ldots 4$; γ_{ei}, $i = 1 \ldots 5$; γ_{Ii}, $i = 1 \ldots 5$ satisfy (35)–(38), then the tracking error converges to zero for any β satisfying (39).

Proof: It is obvious that *Theorem 2* holds. Then, as stated in the proof of *Theorem 2*, $V(t)$ satisfies the relation $V(t) \le (1/2)\gamma_0 \overline{\rho}_{qd}$. According to the second inequality in (48), it can be seen that the following relation is satisfied:

$$\dot{V}(t) \le -\frac{1}{2}V(t) + \frac{h_d(t)}{\beta^2} \le -\frac{1}{2}V(t) + \frac{d_2}{\beta^2}e^{-d_1 t}. \tag{52}$$

It is concluded immediately from the equation above that $V(t)$ converges to zero and the tracking error $\tilde{q}(t)$ also converges to zero.

IV. Simulation Example

In this simulation, the controller is designed for a two-link robot manipulator, shown in Fig. 2. The nominal values of the manipulator and actuator parameters are given as [22] $l_1 = 0.6$ m, $m_1 = 18.3$ kg, $l_{c1} = 0.37$ m, $I_1 = 0.892$ kg·m^2, $l_2 = 1.02$ m, $m_2 = 28.5$ kg, $l_{c2} = 0.0.234$ m, $I_2 = 3.29$ kg·m^2, $m_e = 2$ kg, $J_1 = 7.91$ kg·m^2, $J_2 = 7.91$ kg·m^2, $L_1 = 5.2 \times 10^3$ V·s/A, $L_2 = 5.2 \times 10^{-3}$ V·s/A, $R_1 = 2\ \Omega$, $R_2 = 2\ \Omega$, $K_{e1} = 21$ V·s, $K_{e2} = 21$ V·s, $K_{N1} = 28.8$ V·s, $K_{N2} = 28.8$ V·s, where m_e denotes the weight of the end-effector.

Let the uncertainty in robot dynamics be originated by the weight of the end-effector varying in the range of 1–3 kg, and electrical parameters be assumed to have ±20% uncertainty. The controller shown in Fig. 1 is applied to the electrically driven robot manipulator with the true parameters that are given by $m_e = 3$ kg, $J_1 = 9.492$ kg· m^2, $J_2 = 9.492$ kg· m^2, $L_1 = 6.24 \times 10^{-3}$ V·s/A, $L_2 = 6.24 \times 10^{-3}$ V·s/A, $R_1 = 2.4\ \Omega$, $R_2 = 2.4\ \Omega$, $K_{e1} = 25.2$ V·s, $K_{e2} = 25.2$ V·s, $K_{N1} = 34.56$ V·s, $K_{N2} = 34.56$ V·s. The initial values of the manipulator are $\dot{q}_1(0) = \dot{q}_2(0) = 0$, $q_1(0) = 65$ deg, $q_2(0) = -40$ deg, and the initial value of the desired trajectories are set as $\dot{q}_{d1}(0) = \dot{q}_{d2} = 0$, $q_{d1}(0) = 60$ deg, $q_{d2}(0) = -45$ deg. The desired trajectories of joint 1 and joint 2 are shown in Fig. 3. It is noted that the desired trajectories converge to constant ultimate values. The design parameters are set so as to satisfy the inequalities in (35)–(38) and are given by $\gamma_0 = 1.3$, $\gamma_{s1} = 6.3 \times 10$, $\gamma_{s2} = 1.5 \times 10$, $\gamma_{s3} = 8.5 \times 10$, $\gamma_{s4} = 2.1 \times 10^2$, $\gamma_{e1} = 3.4$, $\gamma_{e2} = 1.1 \times 10^{-1}$, $\gamma_{e3} = 1.2 \times 10$, $\gamma_{e4} = 3.1$, $\gamma_{e5} = 1.5 \times 10^2$, $\gamma_{I1} = 1.7$, $\gamma_{I2} = 1.3 \times 10^{-3}$, $\gamma_{I3} = 7.6 \times 10^{-3}$, $\gamma_{I4} = 2.5 \times 10^{-2}$, $\gamma_{I5} = 3.9 \times 10^{-1}$, $\gamma_{I6} = 2.1 \times 10^4$.

Fig. 4 shows trajectory tracking errors for joint 1. As shown in Fig. 4, the tracking error converges to zero, and the convergent rate becomes more rapid as the design parameter β becomes larger. To show that $\|\tilde{q}(t)\|^2$ can be arbitrarily reduced in the case of $V(0) = 0$ in (40), tracking errors for joint 1 in the case of the initial tracking error $\tilde{q}(0) = 0$ are shown in Fig. 5. It can be seen that the maximum value of $\|\tilde{q}(t)\|^2$ decreases as the design parameter β increases. As shown in Figs. 4 and 5, it is concluded that the tracking performance can be easily improved by using design parameter β. We should mention that only joint 1 is illustrated in the above and that the joint 2 is omitted to save space.

V. Conclusions

In this paper, a novel robust tracking controller is developed for electrically driven robot manipulators. The main feature of the controller is that the measurements of joint velocities and calculation of the robot regressor are not required. Its configuration is very simple. Moreover, by theoretical analysis and numerical simulations, the proposed controller has the following properties. Tracking performance can be easily improved by setting only one design parameter. Especially in the case of $V(0) = 0$, the maximum value of $\|\tilde{q}(t)\|$ can be arbitrarily reduced by increasing the value of the design parameter β. Even if unmodeled bounded disturbances appear in robot dynamics, this property is still assured. If desired trajectories converge to constant ultimate values, the asymptotic stability of the tracking errors is assured.

APPENDIX I
DERIVATION OF NEW REPRESENTATION

Substituting the symbol τ for the symbol t in the first and second equation of (3), and multiplying both sides by $e^{-(t-\tau)}$, and then integrating from 0 to t with respect to τ, we obtain

$$\left.\begin{array}{l}\int_0^t e^{-(t-\tau)}\xi(\tau)d\tau + \int_0^t e^{-(t-\tau)}g(\tau)d\tau \\ \quad = K_N \int_0^t e^{-(t-\tau)}I(\tau)d\tau \\ \int_0^t e^{-(t-\tau)}\left(L\dot{I}(\tau) + RI(\tau) + K_e\dot{q}(\tau)\right)d\tau \\ \quad = \int_0^t e^{-(t-\tau)}u(\tau)d\tau \end{array}\right\} \tag{53}$$

where

$$\xi(t) = M(q)\ddot{q}(t) + B(q,\dot{q})\dot{q}(t). \tag{54}$$

Differentiating both sides of (53), we have the new representation of electrically driven robot manipulators as (5)–(7). Here, the relations $\dot{q}(0) = 0, I(0) = 0\ g(q(0)) = 0$, and the following relations are used to derive the new representation:

$$\begin{aligned}&\frac{d}{dt}\left[\int_0^t e^{-(t-\tau)}g(q(\tau))\,d\tau\right] \\ &= \frac{d}{dt}\left[g(q) - e^{-t}g(q(0)) - \int_0^t e^{-(t-\tau)}\frac{\partial g}{\partial q}\dot{q}(\tau)d\tau\right] \\ &= e^{-t}g(q(0)) + \int_0^t e^{-(t-\tau)}\frac{\partial g}{\partial q}\dot{q}(\tau)d\tau \end{aligned} \tag{55}$$

$$\begin{aligned}&\int_0^t e^{-(t-\tau)}M(q(\tau))\ddot{q}(\tau)d\tau \\ &= M(q)\dot{q}(t) - e^{-t}M(q(0))\dot{q}(0) \\ &\quad - \int_0^t e^{-(t-\tau)}\left(M(q(\tau))\dot{q}(\tau) + \dot{M}(q(\tau))\dot{q}(\tau)\right)d\tau \end{aligned} \tag{56}$$

$$\begin{aligned}&\frac{d}{dt}\left[\int_0^t e^{-(t-\tau)}I(\tau)d\tau\right] \\ &= I_F(t) \end{aligned} \tag{57}$$

$$\frac{d}{dt}\left[\int_0^t e^{-(t-\tau)}\dot{I}(\tau)d\tau\right] = \frac{d}{dt}\left[I_F(t) - e^{-t}I(0)\right] = \dot{I}_F(t) + e^{-t}I(0) \tag{58}$$

$$\begin{aligned}&\frac{d}{dt}\left[\int_0^t e^{-(t-\tau)}u(\tau)d\tau\right] \\ &= \frac{d}{dt}\left[\int_0^t e^{-(t-\tau)}\mu(\tau)d\tau + \int_0^t e^{-(t-\tau)}\int_0^\tau \mu(\sigma)d\sigma dt\right] \\ &= \frac{d}{dt}\left[\int_0^t e^{-(t-\tau)}\mu(\tau)d\tau + \int_0^t \mu(\tau)d\tau \right. \\ &\quad \left. - \int_0^t e^{-(t-\tau)}\mu(\tau)d\tau\right] = \mu(t). \end{aligned} \tag{59}$$

APPENDIX II
PROOF OF *THEOREM 1*

Let the signal $\xi(t)$ be given by

$$\xi(t) = \eta(t) - \int_0^t e^{-(t-\tau)}\eta(\tau)d\tau. \tag{60}$$

Using the integration by parts, it is easily ascertain that the following relation holds:

$$\int_0^t e^{-\tau}\int_0^\tau e^{\sigma}\eta(\sigma)d\sigma d\tau = -\int_0^t e^{-(t-\tau)}\eta(\tau)d\tau + \int_0^t \eta(\tau)d\tau. \tag{61}$$

From (60) and (61), it follows that:

$$\begin{aligned}&\xi(t) + \int_0^t \xi(\tau)d\tau \\ &= \eta(t) - \int_0^t e^{-(t-\tau)}\eta(\tau)d\tau + \int_0^t \eta(\tau)d - \int_0^t e^{-\tau}\int_0^\tau e^{\sigma}\eta(\sigma)d\sigma d\tau \\ &= \eta(t). \end{aligned} \tag{62}$$

Using the fact that the signal $\xi(t)$ given by (60) satisfies (62), it can be seen from (4), the third equation in (7), (14), (16), and (28) that the expression (30) is derived.

APPENDIX III
PROOF OF *LEMMA 1*

The roots of the equation

$$-V(t) + \frac{1}{\gamma_0\overline{\rho}_{qd}}V(t)^2 + \frac{5\overline{\rho}_{qd}}{4\beta^2} = 0 \tag{63}$$

are given by

$$\left.\begin{array}{l} D_- = \frac{\gamma_0\overline{\rho}_{qd}}{2}\left(1 - \sqrt{1 - 5\beta^{-2}\gamma_0^{-1}}\right) \\ D_+ = \frac{\gamma_0\overline{\rho}_{qd}}{2}\left(1 + \sqrt{1 - 5\beta^{-2}\gamma_0^{-1}}\right)\end{array}\right\}. \tag{64}$$

It can be seen that there are two different real roots for any $\beta > \sqrt{5\gamma_0^{-1}}$. From this fact, it follows that the following inequality holds:

$$-V(t) + \frac{1}{\gamma_0\overline{\rho}_{qd}}V(t)^2 + \frac{5\overline{\rho}_{qd}}{4\beta^2} \le 0, \quad \text{for } V(t) \in [D_-, D_+]. \tag{65}$$

Using the fact, it can be proved as stated below that the following properties hold:

1) in the case where $V(0) \le D_-$, $V(t)$ remains in the region $V(t) \le D_-$;
2) in the case where $D_- < V(0) < D_+$, $V(t)$ remains in the region $V(t) \le V(0)$.

It is assumed in *Lemma 1* that $V(0) \le (1/2)\gamma_0\overline{\rho}_{qd} < D_+$. If $D_- < V(0) < D_+$, from 2) it follows that $V(t) \le V(0) \le (1/2)\gamma_0\overline{\rho}_{qd}$. If $V(0) \le D_-$, from 1) it follows that $V(t) \le D_- < (1/2)\gamma_0\overline{\rho}_{qd}$. From the facts, it immediately follows that *Lemma 1* holds.

1) Now, let us suppose that there is $t_2 > 0$, such that $V(t_2) = \rho_D > D_-$. Since $V(0) \le D_-$, there is a $t_1 \in [0, t_2)$ such that

$$V(t_1) = D_-, D_- < V(t) \le \rho_D, \quad \text{for } t \in (t_1, t_2]. \tag{66}$$

However, integrating both sides of (31) from t_1 to t_2, it is seen from (65) that the following inequality is satisfied:

$$V(t_2) \le V(t_1) + \int_{t_1}^{t_2} \left(-V(t) + \frac{1}{\gamma_0 \overline{\rho}_{qd}} V(t)^2 + \frac{4\overline{\rho}_{qd}}{5\beta^2} \right) dt \le D_-. \tag{67}$$

The relation above contradicts supposition (66). Consequently, $V(t) \le D_-$ holds.

2) From (65) it can be seen that $V(t)$ does not increase while $V(t) \in [D_-, D_+]$. Additionally, from the fact that if property 1) holds, it is obvious that property 2) also holds.

Appendix IV
Derivation of (43)–(46)

Based on the definition of $\underline{\rho}_m$, we can simply take it as $\underline{\rho}_m = 1$. In this case, the relations $2\|a(t)\|\|s(t)\| \le \underline{\rho}_m \|a(t)\|^2 + \|s(t)\|^2 \underline{\rho}_m \le \|a(t)\|^2 + V_s(t)$, and $2\|a(t)\|\|\tilde{s}(t)\| \le \underline{\rho}_m \|a(t)\|^2 + \|\tilde{s}(t)\|^2 \underline{\rho}_m \le \|a(t)\|^2 + V_{\tilde{s}}(t)$ can be established for any signal $a(t)$. The following analysis is performed by using the above relations.

1) It can be seen from (42), (23), and (24) that the following inequalities hold:

$$\left.\begin{aligned}
&(\overline{\rho}_{m1} + 2\overline{\rho}_b)\beta \|s(t)\|^3\\
&\quad \le \frac{(\overline{\rho}_{m1}+2\overline{\rho}_b)^2\overline{\rho}_{qd}}{4}\gamma_0\beta^2 \|s(t)\|^2 + \frac{1}{\gamma_0\overline{\rho}_{qd}} V_s(t)^2\\
&(\overline{\rho}_{m1} + 4\overline{\rho}_b)\beta \|s(t)\|^2 \|\tilde{q}(t)\|\\
&\quad \le \frac{(\overline{\rho}_{m1}+4\overline{\rho}_b)^2\overline{\rho}_{qd}}{8\varepsilon_q}\gamma_0\beta^2 \|s(t)\|^2 + \frac{2}{\gamma_0\overline{\rho}_{qd}} V_s(t)V_q(t)\\
&2\overline{\rho}_b\beta \|s(t)\| \|\tilde{q}(t)\|^2\\
&\quad \le \frac{2\overline{\rho}_b^2\overline{\rho}_{qd}}{\varepsilon_q^2}\gamma_0\beta^2 \|s(t)\|^2 + \frac{1}{2\gamma_0\overline{\rho}_{qd}} V_q(t)^2\\
&2(\overline{\rho}_m\beta + \overline{\rho}_m + 2\overline{\rho}_b\overline{\rho}_{d1}) \|s(t)\| \|\tilde{q}(t)\|\\
&\quad \le (\overline{\rho}_m\beta + \overline{\rho}_m + 2\overline{\rho}_b\overline{\rho}_{d1}) \left(\|s(t)\|^2 + \|\tilde{q}(t)\|^2\right)\\
&2\beta^{-1} \|h_{qd}(t)\| \|s(t)\|\\
&\quad \le 2\overline{\rho}_{qd} \|s(t)\|^2 + \frac{1}{2\beta^2}\frac{\|h_{qd}(t)\|^2}{\overline{\rho}_{qd}}\\
&2\overline{\rho}_{k\hat{k}}\gamma_s \left\|\tilde{I}_F(t)\right\| \|s(t)\|\\
&\quad \le \underline{\rho}_{k\hat{k}}\gamma_s \|s(t)\|^2 + \frac{\overline{\rho}_{k\hat{k}}^2}{\underline{\rho}_{k\hat{k}}}\gamma_s \left\|\tilde{I}_F(t)\right\|^2\\
&2\beta^{-1} \|h_{qg}(t)\| \|s(t)\|\\
&\quad \le \left(\sum_{i=1}^{3} \frac{\overline{\rho}_{qi}^2\overline{\rho}_{qd}}{2}\gamma_0\beta + 4\overline{\rho}_{q4}^2\right) \|s(t)\|^2\\
&\qquad + \frac{1}{2}\sum_{i=1}^{3} h_{qi}(t)^2 + \frac{1}{4}h_{q4}(t)^2
\end{aligned}\right\} \tag{68}$$

$$\left.\begin{aligned}
&\overline{\rho}_{m1}\beta \|s(t)\| \|\tilde{s}(t)\|^2\\
&\quad \le \frac{\overline{\rho}_{m1}^2\overline{\rho}_{qd}}{4}\gamma_0\beta^2 \|s(t)\|^2 + \frac{1}{\gamma_0\overline{\rho}_{qd}} V_{\tilde{s}}(t)^2\\
&\overline{\rho}_{m1}\beta \|\tilde{q}(t)\| \|\tilde{s}(t)\|^2\\
&\quad \le \frac{\overline{\rho}_{m1}^2\overline{\rho}_{qd}}{4\varepsilon_q}\gamma_0\beta^2 \|\tilde{s}(t)\|^2 + \frac{1}{\gamma_0\overline{\rho}_{qd}} V_{\tilde{s}}(t)V_q(t)\\
&2\overline{\rho}_b\beta \|s(t)\|^2 \|\tilde{s}(t)\|\\
&\quad \le \frac{\overline{\rho}_b^2\overline{\rho}_{qd}}{2}\gamma_0\beta^2 \|s(t)\|^2 + \frac{2}{\gamma_0\overline{\rho}_{qd}} V_s(t)V_{\tilde{s}}(t)\\
&4\overline{\rho}_b\beta \|s(t)\| \|\tilde{s}(t)\| \|\tilde{q}(t)\|\\
&\quad \le \frac{4\overline{\rho}_b^2\overline{\rho}_{qd}}{\varepsilon_q}\gamma_0\beta^2 \|s(t)\|^2 + \frac{1}{\gamma_0\overline{\rho}_{qd}} V_{\tilde{s}}(t)V_q(t)\\
&2\overline{\rho}_b\beta \|\tilde{s}(t)\| \|\tilde{q}(t)\|^2\\
&\quad \le \frac{2\overline{\rho}_b^2\overline{\rho}_{qd}}{\varepsilon_q^2}\gamma_0\beta^2 \|\tilde{s}(t)\|^2 + \frac{1}{2\gamma_0\overline{\rho}_{qd}} V_q(t)^2\\
&4\overline{\rho}_b\overline{\rho}_{d1} \|\tilde{s}(t)\| \left(\|s(t)\| + \|\tilde{q}(t)\|\right)\\
&\quad \le 4\overline{\rho}_b\overline{\rho}_{d1} \|\tilde{s}(t)\|^2 + 2\overline{\rho}_b\overline{\rho}_{d1} \|s(t)\|^2\\
&\qquad + 2\overline{\rho}_b\overline{\rho}_{d1} \|\tilde{q}(t)\|^2\\
&2\overline{\rho}_{k\hat{k}}\gamma_s \left\|\tilde{I}_F(t)\right\| \|\tilde{s}(t)\|\\
&\quad \le \gamma_s \|\tilde{s}(t)\|^2 + \overline{\rho}_{k\hat{k}}^2\gamma_s \left\|\tilde{I}_F(t)\right\|^2\\
&2\beta^{-1} \|h_{qd}(t)\| \|\tilde{s}(t)\|\\
&\quad \le 2\overline{\rho}_{qd} \|\tilde{s}(t)\|^2 + \frac{1}{2\beta^2}\frac{\|h_{qd}(t)\|^2}{\overline{\rho}_{qd}}\\
&2\beta^{-1} \|h_{qg}(t)\| \|\tilde{s}(t)\|\\
&\quad \le \left(\sum_{i=1}^{3} \frac{\overline{\rho}_{qi}^2\overline{\rho}_{qd}}{2}\gamma_0\beta + 4\overline{\rho}_{q4}^2\right) \|\tilde{s}(t)\|^2\\
&\qquad + \frac{1}{2}\sum_{i=1}^{3} h_{qi}(t)^2 + \frac{1}{4}h_{q4}(t)^2
\end{aligned}\right\} \tag{69}$$

It can be seen from (68), (69), (17), and (18) that $V_s(t) + V_{\tilde{s}}(t)$ satisfies (43).

2) There exist upper bounds with respect to signals $\omega_I(t)$, $\omega_{I_{\tilde{s}}}(t)$, and $\omega_{I_{\tilde{q}}}(t)$ in (21) as follows:

$$\left.\begin{aligned}
&\|\omega_{Is}(t)\| \le \frac{\overline{\rho}_{\omega I1}}{\gamma_s} + \overline{\rho}_\varepsilon\beta + \overline{\rho}_{\tilde{r}} + \overline{\rho}_\varepsilon\\
&\|\omega_{I_{\tilde{s}}}(t)\| \le \gamma_s\overline{\rho}_\varepsilon + \overline{\rho}_{\tilde{r}}\\
&\left\|\omega_{I_{\tilde{q}}}(t)\right\| \le \frac{\overline{\rho}_{\omega I1}}{\gamma_s} + \overline{\rho}_\varepsilon\beta + \overline{\rho}_\varepsilon
\end{aligned}\right\} \tag{70}$$

where $\overline{\rho}\omega_{I1} = \|\widehat{K}_N\|\overline{\rho}_{ke} + \overline{\rho}_{\widehat{g}1}\overline{\rho}_\ell$. It is seen from (23), (25), and (70) that the following inequalities hold:

$$\left.\begin{aligned}
&2\left\|\widetilde{I}_F(t)\right\|\|\omega_{I_s}(t)\|\|s(t)\|\\
&\quad\le \|s(t)\|^2\\
&\qquad+\left(\frac{3}{\gamma_s^2}\overline{\rho}^2_{\omega I1}+3\overline{\rho}^2_\ell\beta^2+3(\overline{\rho}_{\widetilde{r}}+\overline{\rho}_\ell)^2\right)\left\|\widetilde{I}_F(t)\right\|^2\\
&2\left\|\widetilde{I}_F(t)\right\|\left\|\omega_{I_{\widetilde{s}}}(t)\right\|\|\widetilde{s}(t)\|\\
&\quad\le(\gamma_e+1)\|\widetilde{s}(t)\|^2+\left(\overline{\rho}^2_\ell\gamma_e+\overline{\rho}^2_{\widetilde{r}}\right)\left\|\widetilde{I}_F(t)\right\|^2\\
&2\left\|\widetilde{I}_F(t)\right\|\left\|\omega_{I\widetilde{q}}(t)\right\|\|\widetilde{q}(t)\|\\
&\quad\le\|\widetilde{q}(t)\|^2+\left(\frac{3}{\gamma_s^2}\overline{\rho}^2_{\omega I1}+3\overline{\rho}^2_\ell\beta^2+3\overline{\rho}^2_\ell\right)\left\|\widetilde{I}_F(t)\right\|^2\\
&\frac{2}{\gamma_s}\left\|h_{Iq}(t)\|\widetilde{I}_F(t)\right\|\\
&\quad\le\frac{1}{2}h_{I1}(t)^2+\frac{2}{\gamma_s^2}\overline{\rho}^2_{I1}\left\|\widetilde{I}_F(t)\right\|^2\\
&\frac{2}{\beta\gamma_s}\|h_{Id}(t)\|\left\|\widetilde{I}_F(t)\right\|\\
&\quad\le\frac{4\overline{\rho}^2_{Id}}{\overline{\rho}_{qd}\gamma_s^2}\left\|\widetilde{I}_F(t)\right\|^2+\frac{1}{4\beta^2}\frac{\|h_{Id}(t)\|^2\overline{\rho}_{qd}}{\overline{\rho}^2_{Id}}.
\end{aligned}\right\}\tag{71}$$

It can be seen from (20), (28), and (71) that $V_I(t)$ satisfies the inequality (44).

3) Since $\beta_{\varepsilon_q}-2\overline{\rho}^2_{q4}\ge 0$, the following relation holds:

$$2\left(\beta\varepsilon_q-2\overline{\rho}^2_{q4}\right)s(t)^T\widetilde{q}(t)\le\left(\beta\varepsilon_q-2\overline{\rho}^2_{q4}\right)\left(\|s(t)\|^2+\|\widetilde{q}(t)\|^2\right).\tag{72}$$

Using the relation above, the inequality (45) is obtained.

4) Using the relations

$$\left.\begin{aligned}
&2\sqrt{\beta\overline{\rho}_f}\sqrt{\frac{1}{\gamma_0\overline{\rho}_{qd}}}\|s(t)\|^2h_{q1}(t)\\
&\quad\le\frac{\overline{\rho}_f}{2}\beta\|s(t)\|^2+\frac{2}{\gamma_0\overline{\rho}_{qd}}V_s(t)h_{q1}(t)^2\\
&2\sqrt{\beta\overline{\rho}_f}\sqrt{\frac{\varepsilon_q}{\gamma_0\overline{\rho}_{qd}}}\|s(t)\|\|\widetilde{q}(t)\|h_{q2}(t)\\
&\quad\le\frac{\overline{\rho}_f}{2}\beta\|s(t)\|^2+\frac{2}{\gamma_0\overline{\rho}_{qd}}V_q(t)h_{q2}(t)^2\\
&2\sqrt{\beta\overline{\rho}_f}\sqrt{\frac{\varepsilon_q}{\gamma_0\overline{\rho}_{qd}}}\|\widetilde{q}(t)\|^2h_{q3}(t)\\
&\quad\le\frac{\overline{\rho}_f}{2}\beta\|\widetilde{q}(t)\|^2+\frac{2}{\gamma_0\overline{\rho}_{qd}}V_q(t)h_{q3}(t)^2\\
&2\overline{\rho}_{q4}\|s(t)-\widetilde{q}(t)\|h_{q4}(t)\\
&\quad\le\frac{1}{2}h_{q4}(t)^2+2\overline{\rho}^2_{q4}\left(\|s(t)\|^2-2s(t)^T\widetilde{q}(t)+\|\widetilde{q}(t)\|^2\right)\\
&2\left(\|s(t)\|+\|\widetilde{q}(t)\|\right)h_{I1}(t)\\
&\quad\le\frac{1}{2}h_{I1}(t)^2+4\|s(t)\|^2+4\|\widetilde{q}(t)\|^2
\end{aligned}\right\}\tag{73}$$

it can be seen from (27) that $V_h(t)$ satisfies the inequality (46).

References

[1] M. C. Good, L. M. Sweet, and K. L. Strobel, "Dynamic models for control system design of integrated robot and drive systems," *J. Dynam. Syst., Meas., Contr.*, vol. 107, pp. 53–59, 1985.

[2] D. G. Taylor, "Composite control of direct-drive robots," in *Proc. IEEE Conf. Decision and Control*, 1989, pp. 1670–1675.

[3] T.-J. Tarn, A. K. Bejczy, X. Yun, and Z. Li, "Effect of motor dynamics on nonlinear feedback robot arm control," *IEEE Trans. Robot. Automat.*, vol. 7, pp. 114–122, Feb. 1991.

[4] D. M. Dawson, Z. Qu, and J. J. Carroll, "Tracking control of rigid-link electrically driven robot manipulators," *Int. J. Contr.*, vol. 56-5, pp. 991–1006, 1992.

[5] M. S. Mahmoud, "Robust control of robot arms including motor dynamics," *Int. J. Contr.*, vol. 58-4, pp. 853–873, 1993.

[6] M. M. Bridges, D. M. Dawson, and X. Gao, "Adaptive control of rigid-link electrically-driven robot," in *Proc. IEEE Conf. Decision and Control*, 1993, pp. 159–165.

[7] R. Guenther and L. Hsu, "Variable structure adaptive cascade control of rigid-link electrically-driven robot manipulators," in *Proc. IEEE Conf. Decision and Control*, 1993, pp. 2137–2142.

[8] R. Colbaugh and K. Glass, "Adaptive regulation of rigid-link electrically-driven manipulators," in *Proc. IEEE Int. Conf. Robotics and Automation*, 1995, pp. 293–299.

[9] C.-Y. Su and Y. Stepanenko, "Hybrid adaptive/robust motion control of rigid-link electrically-driven robot manipulators," *IEEE Trans. Robot. Automat.*, vol. 11, pp. 426–432, June 1995.

[10] ——, "On the robust control of robot manipulators including actuator dynamics," *J. Robot. Syst.*, vol. 13, pp. 1–10, 1996.

[11] M. M. Bridges, D. M. Dawson, and J. Hu, "Adaptive control for a class of direct-drive robot manipulators," *Int. J. Adaptive Control, Signal Processing*, vol. 10, pp. 417–441, 1996.

[12] R. Colbaugh, K. Glass, and K. Wedeward, "Adaptive compliance control of electrically-driven manipulators," in *Proc. IEEE Conf. Decision and Control*, 1996, pp. 394–399.

[13] C.-Y. Su and Y. Stepanenko, "Backstepping-based hybrid adaptive control of robot manipulators incorporating actuator dynamics," *Int. J. Adaptive Control, Signal Processing*, vol. 11, pp. 141–153, 1997.

[14] C. Ishii, T. Shen, and Z. Qu, "Robust adaptive tracking control with L_2-gain disturbance attenuation for electrically-driven robot manipulators," in *Proc. IEEE Conf. Decision and Control*, 1999, pp. 3388–3393.

[15] C. Ishii, S. Shen, and Z. Qu, "Lyapunov recursive design of robust tracking control with L_2-gain performance for electrically-driven robot manipulators," in *Proc. IEEE Conf. Control Applications*, 1999, pp. 863–868.

[16] R. Klafterm, T. Chmielewski, and M. Negin, *Robotic Engineering: An Integrated Approach*. Englewood Cliffs, NJ: Prentice-Hall, 1989.

[17] T. Burg, D. Dawson, J. Hu, and M. de Queiroz, "An adaptive partial state-feedback controller for RLED robot manipulators," *IEEE Trans. Automat. Contr.*, vol. 41, pp. 1024–1030, July 1996.

[18] C.-Y. Su and Y. Stepanenko, "Redesign of hybrid adaptive/robust motion control of rigid-link electrically-driven robot manipulators," *IEEE Trans. Robot. Automat.*, vol. 14, pp. 651–655, Aug. 1998.

[19] P. Tomei, "Adaptive PD controller for robot manipulators," *IEEE Trans. Robot. Automat.*, vol. 7, pp. 565–570, June 1991.

[20] C. Canudas de Wit, B. Siciliano, and G. Bastin, *Theory of Robot Control*. New York: Springer-Verlag, 1996.

[21] R. Kelly, "Global positioning of robot manipulators via PD control plus a class of nonlinear integral action," *IEEE Trans. Automat. Contr.*, vol. 43, pp. 934–938, Aug. 1998.

[22] Y. Stepanenko, C.-Y. Su, and S. Tang, "Robust controller design and implementation for industrial robots: Electrically driven rigid body robots," in *Proc. American Control Conf.*, 1998, pp. 2206–2208.

[10] M. James, "Asymptotic analysis of nonlinear stochastic risk-sensitive control and differential games," *Math. Contr., Signals Syst.*, vol. 5, pp. 401–417, 1992.

[11] W. H. Fleming and W. M. McEneaney, "Risk-sensitive control and differential games," in *Stochastic Theory and Adaptive Control*, T. E. Duncan and B. Pasik-Duncan, Eds. New York: Springer-Verlag, 1992, pp. 185–197.

[12] M. James, J. Baras, and R. Elliott, "Risk-sensitive control and dynamic games for partially observed discrete-time nonlinear systems," *IEEE Trans. Automat. Contr.*, vol. 39, pp. 780–792, 1994.

[13] C. Charalambous, "The role of information state and adjoint in relating nonlinear output feedback risk-sensitive control and dynamic games," this issue, pp. 1163–1170.

[14] W. M. Wonham, "On the separation theorem of stochastic control," *SIAM J. Contr.*, vol. 6, pp. 312–326, 1968.

[15] A. Bensoussan, *Stochastic Control of Partially Observable Systems*. Oxford, U.K.: Cambridge Univ. Press, 1992.

[16] C. Charalambous, D. Naidu, and K. Moore, "Solvable risk-sensitive control problems with output feedback," in *Proc. 33rd IEEE Conf. Decision Contr.*, Lake Buena Vista, FL, 1994, pp. 1433–1434.

[17] A. Bensoussan and R. Elliott, "General finite dimensional risk sensitive problems and small noise limits," *IEEE Trans. Automat. Contr.*, vol. 41, pp. 210–215, 1996.

[18] R. S. Liptser and A. N. Shiryayev, *Statistics of Random Processes*. New York: Springer-Verlag, 1977, vol. 1.

[19] A. Bensoussan and R. Elliott, "A finite dimensional risk sensitive control problem," *SIAM J. Contr. Optim.*, vol. 33, pp. 1834–1864, 1996.

[20] V. Benes, "Exact finite-dimensional filters for certain diffusions with nonlinear drift," *Stochastics*, vol. 5, pp. 65–92, 1981.

[21] U. Haussmann and E. Pardoux, "A conditionally almost linear filtering problem with non-Gaussian initial condition," *Stochastics*, vol. 23, pp. 241–275, 1988.

[22] V. Benes and L. Shepp, "Wiener integral associated with diffusion processes," *Teoriya Veroiatnostei i Prem.*, vol. 3, pp. 498–501, 1968.

On Global Output Feedback Regulation of Euler–Lagrange Systems with Bounded Inputs

Antonio Loría, Rafael Kelly, Romeo Ortega, and Victor Santibáñez

***Abstract*—In this paper we identify a class of Euler–Lagrange systems with bounded inputs that can be globally asymptotically stabilized via dynamic output feedback. In particular, we prove that if the system is fully actuated and the forces due to the potential field can be "dominated" by the constrained control signals, then global output feedback regulation is still possible, incorporating some suitable saturation functions in the controller.**

***Index Terms*—Euler–Lagrange systems, passivity, saturations.**

I. Introduction

It is well known that in Euler–Lagrange (EL) systems the number and stability properties of the equilibria are univocally defined by its potential energy and dissipation functions: see, e.g., [1]. This fundamental feature is at the core of passivity-based control of EL systems, which hinges upon (physically appealing principles of) shaping the system's energy and injection of the required damping. These ideas—though well known in mechanics and first introduced in control in the seminal paper [2], then rigorously articulated in [3]—have been successfully used to control various practical EL systems, e.g., robots [2], [4], motors [5], and power converters [6].

In [1] we observed that in feedback interconnections of EL systems (which preserve the EL structure), the energy and dissipation functions of the closed loop are the *sum* of the corresponding functions of each subsystem. Motivated by this fact, we proposed in that paper to consider EL controllers. In this way we have a direct, systematic way to solve the energy shaping and damping injection tasks, i.e., by adding up the plant and controller total energies and dissipation functions. This procedure was used in [1] to identify a class of EL systems—which includes robot manipulators—that can be globally asymptotically stabilized via nonlinear dynamic *output* feedback. To inject the damping without velocity measurement, we added a dynamic extension and invoked a dissipation propagation condition. This condition, also known as pervasive damping [7], is connected with the detectability property of the system. An interesting particular case of the controllers derived in [1] is the one reported in [8], where it is shown that in set-point control tasks, velocity can be replaced by its dirty derivative-preserving global asymptotic stability.

The main motivation of this paper is to extend the methodology of [1] to the practically important situation when the system is subject to *input constraints*. Our contribution is the proof that if the system is fully actuated and the forces due to the potential field can be "dominated" by the constrained control signals, then global output feedback regulation is still possible, incorporating some suitable saturation functions in the controller. Instrumental for the solution of the problem is the result of [9], where global *state* feedback regulation of robots with saturated inputs is established. A corollary of this paper is, therefore, the extension—to the *output feedback* case—of the result in [9] (see also [10]).

Besides the obvious interest of ensuring stability with bounded controls, there is another motivation to introduce saturations in the loop. This stems from the fact that the action of the approximate differentiation filter can also be regarded as a linear *high-gain* observer (see [11]), which may introduce very large values of the state estimate over a short period of time. Introducing saturated controls, as first proposed in [12], is a way to overcome this difficulty. It should also be mentioned that dirty derivative filters with saturations are used as well (among other various tools) in [13] for semiglobal output feedback stabilization of general nonlinear systems.

II. Problem Formulation and Background

A. Problem Formulation

We consider in this paper plants described EL systems of the form

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}_p(q_p, \dot{q}_p)}{\partial \dot{q}_p}\right) - \frac{\partial \mathcal{L}_p(q_p, \dot{q}_p)}{\partial q_p} = u_p - \frac{\partial \mathcal{F}_p(\dot{q}_p)}{\partial \dot{q}_p} \qquad (1)$$

where $q_p \in \mathbb{R}^n$ are the generalized coordinates assumed *measurable*, $u_p \in \mathbb{R}^n$ are the control inputs

$$\mathcal{L}_p(q_p, \dot{q}_p) \triangleq T_p(q_p, \dot{q}_p) - V_p(q_p)$$

is the Lagrangian function

$$T_p(q_p, \dot{q}_p) = \tfrac{1}{2}\dot{q}_p^\top D_p(q_p)\dot{q}_p, \qquad D_p(q_p) = D_p^\top(q_p) > 0$$

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A. Loría is with the Department of Engineering Cybernetics, Norwegian University of Science and Technology, N-7034 Trondheim, Norway.

R. Kelly and V. Santibáñez are with the Applied Physics Division at CICESE, AP 2615, Ensenada, BC, 22800, México.

R. Ortega is with the LSS, Supélec, Gif sur Yvette, France.

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is the kinetic energy, and $V_p(q_p)$ is the potential energy which we assume is twice differentiable and bounded from below. Further, we assume that there exist some constants $0 < k_g < \infty$ and $0 < k_v < \infty$ such that

$$k_v \triangleq \sup_{q_p \in \Re^n} \left\| \frac{\partial V_p}{\partial q_p} \right\| \tag{2}$$

$$k_g \triangleq \sup_{q_p \in \Re^n} \left\| \frac{\partial^2 V_p(q_p)}{\partial q_{p_i} \partial q_{p_j}} \right\| \tag{3}$$

with $\|\cdot\|$ the Euclidean norm. $\mathcal{F}_p(\dot{q}_p)$ is the Rayleigh dissipation function which satisfies

$$\dot{q}_p^\top \frac{\partial \mathcal{F}_p(\dot{q}_p)}{\partial \dot{q}_p} \geq \alpha_p \|\dot{q}_p\|^2 \tag{4}$$

for all $\dot{q}_p \in \Re^n$ with some $\alpha_p \geq 0$, and $(\partial \mathcal{F}_p/\partial \dot{q}_p)(0) = 0$. It is important to remark that EL systems are fully characterized by their *EL parameters*[1]

$$\Sigma_p \colon \{T_p, V_p, F_p\}.$$

In this paper we are interested in the problem of global asymptotic output feedback stabilization of EL systems (1) subject to *input constraints*

$$|u_{p_i}| \leq u_{p_i}^{\max}, \qquad i = 1, \cdots, n. \tag{5}$$

In particular, we want to extend the design methodology proposed in [1] to the important bounded input case.

B. Background

The following facts, further elaborated in [1], are in order.

Fact 1 [1, Proposition 2.1]: EL systems define *passive operators* [14] from the inputs u_p to the generalized velocities $\dot{q}_p$. This follows from integration of the key energy balance equation:

$$\dot{H}_p(t) = \dot{q}_p^\top u_p - \dot{q}_p^\top \frac{\partial \mathcal{F}_p(\dot{q}_p)}{\partial \dot{q}_p}$$

and (4), where

$$H_p(q_p, \dot{q}_p) \triangleq T_p + V_p$$

is the system's total energy. □

Fact 2: Let the *controller* also be an EL system with EL parameters

$$\Sigma_c \colon \{T_c, V_c, F_c\}$$

with generalized coordinates $q_c \in \Re^n$. For simplicity we consider here

$$T_c = \frac{1}{2}\dot{q}_c^\top D_c(q_c)\dot{q}_c, \qquad D_c(q_c) = D_c^\top(q_c) > 0$$

although as explained in [1], with an obvious abuse of notation, we can also take $T_c \equiv 0$. We assume V_c is bounded from below and $(\partial \mathcal{F}_c/\partial \dot{q}_c)(0) = 0$. Notice the dependence of V_c on the plant coordinates q_p.

The controller dynamics is given by

$$\frac{d}{dt}\left(\frac{\partial T_c(q_c, \dot{q}_c)}{\partial \dot{q}_c}\right) - \frac{\partial T_c(q_c, \dot{q}_c)}{\partial q_c} + \frac{\partial V_c(q_c, q_p)}{\partial q_c} + \frac{\partial \mathcal{F}_c(\dot{q}_c)}{\partial \dot{q}_c} = 0 \tag{6}$$

while the feedback interconnection between plant and controller is established by

$$u_p = -\frac{\partial V_c(q_c, q_p)}{\partial q_p}. \tag{7}$$

[1] Notice that, in contrast to [1], we consider here only *fully actuated* EL systems (i.e., number of degrees of freedom equal to the number of control actions). Therefore, only three EL parameters are needed.

Then, the closed loop is also an EL system with generalized coordinates $q = [q_p^\top, q_c^\top]^\top$ and EL parameters $\{T(q, \dot{q}), V(q), \mathcal{F}(\dot{q})\}$ defined as

$$T(q, \dot{q}) \triangleq T_p(q_p, \dot{q}_p) + T_c(q_c, \dot{q}_c)$$

$$V(q) \triangleq V_p(q_p) + V_c(q_c, q_p)$$

$$\mathcal{F}(\dot{q}) \triangleq \mathcal{F}_p(\dot{q}_p) + \mathcal{F}_c(\dot{q}_c).$$

□

Fact 3 [1, Proposition 2.3]: The equilibria of the closed-loop system $(q, \dot{q}) = (\overline{q}, 0)$ are uniquely determined by the potential energy as the solutions $\overline{q}$ of $\partial V(q)/\partial q = 0$. The equilibrium is *unique and stable* if $\overline{q}$ is an *unique* and *global* minimum of $V(q)$ (see Definitions 3) and 4) of Appendix B). Further, it is globally asymptotically stable (GAS) if

$$\dot{q}_c^\top \frac{\partial \mathcal{F}_c(\dot{q}_c)}{\partial \dot{q}_c} \geq \alpha_c \|\dot{q}_c\|^2 \tag{8}$$

for some $\alpha_c > 0$, and the function $\partial V(q)/\partial q_c = 0$ has only *isolated zeros* in q_p for each given q_c.

□

Our motivation to consider EL controllers for regulation of EL systems stems from the facts above and may be summarized as follows: since the feedback interconnection of two EL systems is an EL system, and the dynamic behavior of an EL system is fully characterized by the EL parameters, we propose to choose some *desired* closed-loop EL parameters and obtain from them the EL controller. In particular, for *regulation* tasks we choose $V(q)$ to have a unique and global minimum at the desired equilibrium and pick $\mathcal{F}_c(\dot{q}_c)$ to inject the damping required for asymptotic stability.

Remark 4: From the discussion above it is clear that the success of our approach hinges upon our ability to "dominate" the plant's potential energy $V_p(q_p)$ with $V_c(q_c, q_p)$. Now, since u_p is restricted by (5), this implies—via (7)—a bound on $\partial V_c(q_c, q_p)/\partial q_p$, which in turn imposes a bound on the growth rate of $V_p(q_p)$. This restriction on $V_p(q_p)$ is inherent to the energy shaping approach taken here.

Remark 5: Since we are dealing with fully actuated systems, the simplest way to "dominate" the plant potential energy is to cancel $V_p(q_p)$ and impose a desired shape to the closed-loop function. This, however, might entail some potential robustness problems, hence we favor a solution that does not rely on this cancellation. Interestingly enough, if we use a controller that does not cancel the vector of potential forces, the growth rate restriction on $V_p(q_p)$ mentioned above is imposed only at the desired position. The price paid, however, is that in this case we need to use high gains in $V_c(q_c, q_p)$ to dominate $V_p(q_p)$, and this translates into stiffer requirements on the input saturation bound.

Instrumental for the solution of the stabilization problem is the utilization of *saturation functions*, which we define as follows.

Definition 6: A saturation function $\mathrm{sat}(x) \colon \Re \to \Re$ is a C^2 strictly increasing function[2] that satisfies:

1) $\mathrm{sat}(0) = 0$;
2) $|\mathrm{sat}(x)| < 1$;
3) $\partial^2 \mathrm{sat}(x)/x = 0 \ \forall x \neq 0 \in \Re$.

For instance, we can take $\mathrm{sat}(x) \triangleq \tanh(\omega x), \omega > 0$, as proposed in [9] and [15].

It can be proven that saturation functions defined as in (6) satisfy the following properties, required for our further developments.

P1) $\int_0^{\tilde{q}_{p_i}} \mathrm{sat}(x)\,dx \geq \frac{1}{2}\mathrm{sat}(\tilde{q}_{p_i})\tilde{q}_{p_i}, \ \forall \tilde{q}_{p_i} \in \Re$.

[2] As it will become clear later, the "strict" qualifier for $\mathrm{sat}(x)$ is required to ensure $\partial V(q)/\partial q_c = 0$ has only *isolated zeros* in q_p, as required by Fact 3.

P2) For all $\varepsilon > 0$ we have that

$$\mathrm{sat}(\tilde{q}_{p_i})\tilde{q}_{p_i} \geq \frac{\mathrm{sat}(\varepsilon)}{\varepsilon}\tilde{q}_{p_i}^2, \qquad \forall |\tilde{q}_{p_i}| < \varepsilon \tag{9}$$

$$\mathrm{sat}(\tilde{q}_{p_i})\tilde{q}_{p_i} \geq \mathrm{sat}(\varepsilon)|\tilde{q}_{p_i}|, \qquad \forall |\tilde{q}_{p_i}| \geq \varepsilon. \tag{10}$$

III. Main Result

Proposition 7

Consider the EL system (1) with saturated inputs (5), *measurable* output q_p, and a *constant* desired reference value $q_{pd} \in \mathbb{R}^n$.

1) Controllers with Cancellation of Potential Forces: Assume that the system's potential energy verifies the strict inequality

$$\sup_{q_p \in \mathbb{R}^n} \left| \left(\frac{\partial V_p}{\partial q_p}\right)_i \right| < u_{p_i}^{\max}, \qquad \forall i \in [1, \cdots, n] \tag{11}$$

with $(\cdot)_i$ the ith component of the vector. Under these conditions, any EL controller (6), (7) with dissipation function $\mathcal{F}_c(\dot{q}_c)$ satisfying (8) and potential energy

$$V_c(q_c, q_p) = V_{c_2}(q_c, q_p) - V_p(q_p)$$

where

$$V_{c_2}(q_c, q_p) \triangleq \sum_{i=1}^{n} \left(\frac{k_{2_i}}{b_i} \int_0^{(q_{c_i} + b_i q_{p_i})} \mathrm{sat}(x_i)\, dx_i + k_{3_i} \int_0^{\tilde{q}_{p_i}} \mathrm{sat}(x_i)\, dx_i \right) \tag{12}$$

where $\tilde{q}_p \triangleq q_p - q_{p_d}, b_i > 0$, and $k_{2_i}, k_{3_i} > 0$ sufficiently small, makes

$$(\dot{q}_p, q_p, \dot{q}_c, q_c) = (0, q_{pd}, 0, q_{cd}) \tag{13}$$

with q_{cd} some constant, a GAS equilibrium point of the closed loop.

2) Controllers Without Cancellation of Potential Forces: Choose now the potential energy of the EL controller as

$$V_c = V_{c_2}(q_c, q_p) - q_p^\top \frac{\partial V_p}{\partial q_p}(q_{pd}) \tag{14}$$

where $V_{c_2}(q_c, q_p)$ is given by (12), we take k_{2_i} sufficiently small, and $\min_i \{k_{3_i}\} > k_{3_i}^{\min}$ with $k_{3_i}^{\min}$ as some suitably defined positive constant. Then (13) is a GAS equilibrium point of the closed loop (1), (5)–(7) provided[3]

$$u_{p_i}^{\max} > \left| \left(\frac{\partial V_p}{\partial q_p}(q_{pd})\right)_i \right| + k_{3_i}, \qquad i \in [1, \cdots, n]. \tag{15}$$

In particular, if we take $\mathrm{sat}(x) = \tanh(x)$, then

$$k_{3_i}^{\min} \triangleq \frac{4k_v}{\tanh\left(\frac{4k_v}{k_g}\right)} \tag{16}$$

where k_v and k_g are given by (2) and (3), respectively. □

Remark 8: The proposition above characterizes—in terms of the EL parameters $\mathcal{T}_c(q_c, \dot{q}_c)$, $\mathcal{F}_c(\dot{q}_c)$, and V_c—a *class* of output feedback GAS controllers for EL systems with *saturated inputs*, thus providing an extension to the constrained input case of the result reported in [1]. Also, as a corollary of our proposition we obtain an extension to the *output* feedback case of the result in [9], where a *full state* feedback solution to the problem of global regulation of rigid robots with saturated inputs was presented.

[3] Notice that the gradient of the system's potential energy is evaluated here at the *desired* reference.

Remark 9: A key feature of the controller given in Part 2 of Proposition 7 is that, to enhance its robustness, we *avoid* explicit cancellations of the plant dynamics. In this respect our result supersedes that of [10], where the following GAS controller, which relies on exact cancellation of potential forces, was proposed:

$$\dot{q}_{c_i} = -k_{1_i} q_{c_i} - k_{2_i}\, \mathrm{sat}(q_{c_i} - \tilde{q}_{p_i})$$

$$u_{p_i} = k_{2_i}\, \mathrm{sat}(q_{c_i} - \tilde{q}_{p_i}) + \left(\frac{\partial V_p(q_p)}{\partial q_p}\right)_i$$

where $k_{2_i} > 0$ and k_{2_i} is taken sufficiently small. Remark that this is an EL controller with EL parameters

$$\mathcal{T}_c(q_c, \dot{q}_c) = 0, \qquad \mathcal{F}_c(\dot{q}_c) = \tfrac{1}{2}\|\dot{q}_c\|^2$$

$$V_c = \sum_{i=1}^{n} \tfrac{1}{2} k_{1_i} q_{c_i}^2 - V_p(q_p) + \sum_{i=1}^{n} k_{2_i} \int_0^{(q_{c_i} - \tilde{q}_{p_i})} \mathrm{sat}(x_i)\, dx_i.$$

Remark 10: The price paid for the robustness enhancement in Part 2 is that higher gains have to be injected into the loop through k_{3_i}. As seen from the proposition, this imposes an additional requirement of sufficiently large input constraints for stability. The condition on k_{3_i} stems from the fact that to impose a desired minimum point to the closed-loop potential energy, now we have to dominate (and not to cancel) the system's potential energy.

IV. Proof of Main Results

For ease of presentation we will consider for both parts of the proposition the EL controller parameters

$$\mathcal{T}_c(q_c, \dot{q}_c) = 0, \qquad \mathcal{F}_c = \tfrac{1}{2}\dot{q}_c^\top K_2 B^{-1} A^{-1} \dot{q}_c$$

with

$$A \triangleq \mathrm{diag}\{a_i\} > 0, \qquad B \triangleq \mathrm{diag}\{b_i\} > 0$$

$$K_2 \triangleq \mathrm{diag}\{k_{2_i}\} > 0.$$

This choice, together with (6) and (7), yields the EL controller

$$\dot{q}_{c_i} = -a_i\, \mathrm{sat}(q_{c_i} + b_i q_{p_i})$$

$$u_{p_i} = -\frac{k_{2_i}}{a_i}\dot{q}_{c_i} - k_{3_i}\, \mathrm{sat}(\tilde{q}_{p_i}) + \left(\frac{\partial V_p(q_p)}{\partial q_p}\right)_i \tag{17}$$

for the first part of the proposition, while for the second controller we only change u_{p_i} to

$$u_{p_i} = -\frac{k_{2_i}}{a_i}\dot{q}_{c_i} - k_{3_i}\, \mathrm{sat}(\tilde{q}_{p_i}) + \left(\frac{\partial V_p}{\partial q_p}(q_{pd})\right)_i. \tag{18}$$

Notice the difference in the third right-hand terms of the control signals.

It is interesting to remark that if in the controllers above we write $(\cdot)$ instead of $\mathrm{sat}(\cdot)$, we *exactly recover* the (approximate differentiation) output feedback GAS controllers of [8]; see also [16]. The proposition then shows that by simply including the saturations we can preserve GAS even under input constraints.

As pointed out in Fact 2 of Section II, the closed-loop system is an EL system with potential energy $V(q) = V_c(q_c, q_p) + V_p(q_p)$, hence the proof of both results is carried out by proving the conditions of Fact 3. Notice that the difficulty lies in proving that $V(q)$ has a global minimum at (13) with $q_{cd} \triangleq -B^{-1}q_{pd}$. For this we use the following fact.[4]

[4] It is worth mentioning that this fact is a generalization of the ideas exposed in [17] and [18].

Fact 11: Let $f(x)\colon \mathbb{R}^n \to \mathbb{R}$ be a C^1 function. Assume

1) $f(x) > 0$, for all $x \neq 0 \in \mathbb{R}^n$ and $f(0) = 0$;
2) $\|(\partial f/\partial x)(x)\| > 0$, for all $x \neq 0 \in \mathbb{R}^n$;
3) $f(x) \to \infty$ if $\|x\| \to \infty$.

Then the function $f(x)$ is globally positive definite and radially unbounded with a unique and global minimum at $x = 0$, according to definitions of Appendix B. □

Remark 12: Condition 1 implies that $f(x)$ is positive definite with zero, a strict global minimum. Nevertheless, it is important to remark that this condition alone does not imply the uniqueness of the minimum. Condition 2 implies that zero is the only critical point, hence that zero is also a unique minimum of $f(x)$. Finally, the third condition corresponds to the definition of radially unboundedness.

A. Proof of Part 1

Notice first that for this simpler case we have that $V(q) = V_{c2}(q_c, q_p)$. Using properties P1) and P2), it is easy to show that this function satisfies all conditions of Fact 11 above.[5] To establish that the equilibrium is GAS, we use again Fact 3 and notice that for each q_c, the function

$$\frac{\partial V(q)}{\partial q_c} = K_2 B^{-1} \operatorname{sat}(q_c + Bq_p)$$

has only *isolated zeros* in q_p (since B is full rank).

Now, applying the triangle inequality to (17) and using the fact that $|\operatorname{sat}(x)| < 1$, we have the bound

$$|u_{p_i}| < k_{2_i} + k_{3_i} + \left|\left(\frac{\partial V_p}{\partial q_p}\right)_i\right|.$$

Thus, under (11), we can always choose sufficiently small $k_{2_i}, k_{3_i} > 0$ such that (5) holds. □

B. Proof of Part 2

The closed-loop potential energy is now

$$V(q) = V_{c2}(q_c, q_p) + V_p - q_p^\top \frac{\partial V_p}{\partial q_p}(q_{pd}). \tag{19}$$

In Appendix I we show that if there exists a k_v such that for (2) there exists $k_{3_i}^{\min} > 0$ such that $V(q)$ has a global minimum at the desired equilibrium for all $k_{3_i} \geq k_{3_i}^{\min}$, the proof is completed observing that GAS follows along the lines of Part 1 and using (18) to get the bound

$$|u_{p_i}| < k_{2_i} + k_{1_i} + \left|\left(\frac{\partial V_p}{\partial q_p}(q_{pd})\right)_i\right|.$$

From (15) it is easy to see that for a sufficiently small k_{2_i}, (5) is satisfied. Finally, the value of $k_{3_i}^{\min}$ for the case when $\operatorname{sat}(x) = \tanh(x)$ is obtained in Appendix I. □

V. Simulation Results

Using SIMULINK™ of MATLAB™, we tested our algorithm in the two-link robot arm of [19] with a desired reference $q_d = \operatorname{col}[\pi/2, \pi/2]$. We have imposed the input constraint $u_{p_i}^{\max} = 320$ [Nm] in (5). To meet the conditions of Proposition 7 we chose $A = \operatorname{diag}\{100, 100\}$, $B = \operatorname{diag}\{130, 130\}$, while the controller gains were set to $K_3 = \operatorname{diag}\{180, 180\}$, $K_2 = \operatorname{diag}\{125, 125\}$ according to (15).

Then, in order to evaluate the performance of our controller, we tested as well the one proposed in [8] with exactly the same gain values and starting from initial conditions $q_{p0} = \operatorname{col}[\pi/4, \pi/4]$; in accordance with the previous discussions we set $q_{c0} = \operatorname{col}[-32.5\pi, -32.5\pi]$ in order to make $\vartheta_0 = 0$.

[5] The proof of this claim is strictly contained in the proof of Part 2. See Appendix A.

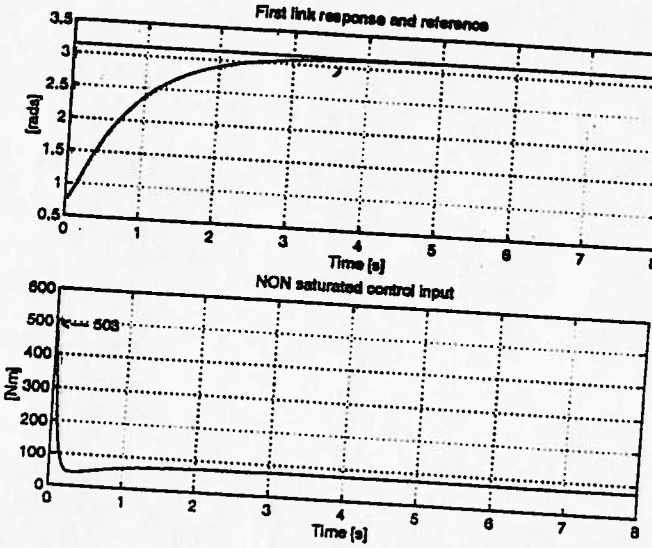

Fig. 1. EL controller of Kelly [8].

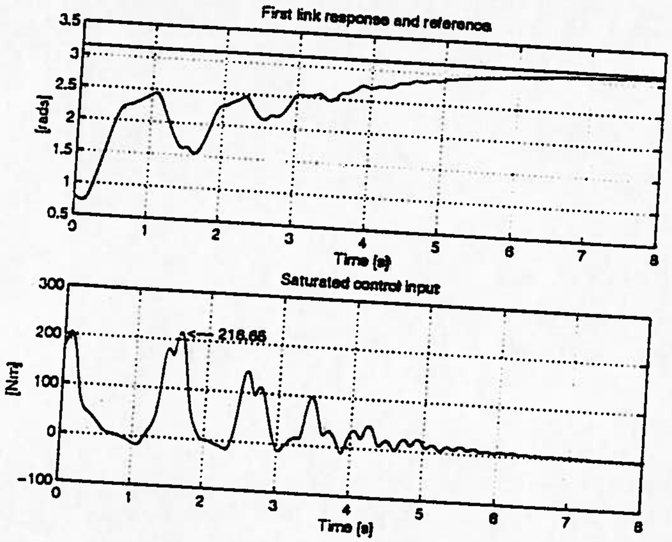

Fig. 2. Saturated EL controller.

In Fig. 1 we show the transient of the first link position using the algorithm of [8], i.e., the *nonsaturated* controller and the control input signal yielded by this controller. In Fig. 2 we show the response of the same link driven by the saturated controller of Proposition 7 and its control input.

On one hand, notice that the transient produced by the nonsaturated controller is much faster than the response using saturated controls. On the other hand, it must be remarked that the control input yielded by the linear controller fails to satisfy the input constraint: in particular, the maximum absolute value of u_p is 503 [Nm] for the first link. In contrast to this, the saturated controller yields a control input with $|u_{p_1}|^{\max} = 211$ [Nm].

Thus we verify what is not surprising—that there is a compromise between a fast transient and small control inputs.

VI. Concluding Remarks

We have extended in this paper our results on output feedback stabilization of EL systems to the practically important case of bounded inputs. As a corollary of our work we have improved in several directions the result of [9] on state feedback global stabilization of robot manipulators with saturated inputs. First, we have removed the requirement of measurement of generalized velocities. In particular, we have shown that a suitably saturated approximate differentiator can be used to estimate these signals. Second, we have developed this theory for a wider class of EL systems—that contains as a particular case robot manipulators—and to general saturation functions. Finally, we have identified a class of EL controllers, characterized in terms of their EL parameters, that achieve the global output feedback stabilization objective. A subject of current research is to devise a technique to select from this class one that optimizes a performance criterion.

It might be argued that limiting ourselves to EL controllers is unnecessarily restrictive. Our motivation for this choice is twofold: first by preserving the EL structure of the closed loop the behavior of the system is fully characterized by its energy and dissipation functions. From a Lyapunov perspective, this procedure naturally leads to the choice of an energy-based Lyapunov function candidate. Second, we have shown in [1] that the class of EL controllers contains several well-known schemes which were derived from apparently unrelated perspectives, e.g., [11], [20], and [4]. Henceforth, we provide a unified framework to compare them.

Finally, we have illustrated in simulations the practical advantages of using bounded controls. In particular, we compared the performance of the controller of [8] and its "saturated" equivalent. It was shown that for a particular case with nonzero initial conditions, the linear controller of [8] failed to satisfy the input constraints.

Appendix A
Proof of Positive Definiteness of $V(q)$

First, notice that the first right-hand term of (19) is a nonnegative function of q_c, q_p which is zero at $q_c = B^{-1}q_p$. Hence, to prove that (19) has a unique global minimum at (13), it suffices to show that the three remaining terms [i.e., the second term of V_{c2} and the last two terms of (19)] have a unique global minimum at $q_p = q_{pd}$, or equivalently, that the function $f(\tilde{q}_p): \mathbb{R}^n \to \mathbb{R}$

$$f(\tilde{q}_p) \triangleq \sum_{i=1}^{n}\left\{k_{3_i}\int_0^{\tilde{q}_{p_i}} \mathrm{sat}(x_i)\,dx_i\right\} + V_p(\tilde{q}_p + q_{pd}) - V_p(q_{pd}) - \tilde{q}_p^\top \frac{\partial V_p}{\partial q_p}(q_{pd}) \tag{20}$$

has a global and unique minimum at zero. We will establish the proof by verifying the conditions of Fact 11 in order to define a $k_{3_i}^{\min}$ which ensures this to be the case.

Condition 1

To prove that $f(x) > 0$ for all $x \neq 0 \in \mathbb{R}^n$, we shall prove first that for all $\varepsilon > 0$

$$\sum_{i=1}^{n} k_{3_i}\int_0^{\tilde{q}_{p_i}} \mathrm{sat}(x)\,dx \geq \min_i\{k_{3_i}\}\frac{\mathrm{sat}(\varepsilon)}{2\varepsilon}\|\tilde{q}_p\|^2, \quad \forall \|\tilde{q}_p\| < \varepsilon \tag{21}$$

$$\sum_{i=1}^{n} k_{3_i}\int_0^{\tilde{q}_{p_i}} \mathrm{sat}(x)\,dx \geq \min_i\{k_{3_i}\}\frac{\mathrm{sat}(\varepsilon)}{2}\|\tilde{q}_p\|, \quad \forall \|\tilde{q}_p\| \geq \varepsilon \tag{22}$$

where for the sake of clarity we consider two cases separately.

Case 1 ($\|\tilde{q}_p\| < \varepsilon$): Notice that in this case we have that $|\tilde{q}_{p_i}| < \varepsilon\ \forall i \leq n$; then using P1) and (9) we get

$$\sum_{i=1}^{n} k_{3_i}\int_0^{\tilde{q}_{p_i}} \mathrm{sat}(x)\,dx \geq \sum_{i=1}^{n} k_{3_i}\frac{\mathrm{sat}(\varepsilon)}{2\varepsilon}\tilde{q}_{p_i}^2 \geq \min_i\{k_{3_i}\}\frac{\mathrm{sat}(\varepsilon)}{2\varepsilon}\|\tilde{q}_p\|^2.$$

Case 2 ($\|\tilde{q}_p\| \geq \varepsilon$): Within this case we shall consider three different cases.

Case a ($|\tilde{q}_{p_i}| < \varepsilon\ \forall i \leq n$): Again, using P1) and (9) we get

$$\sum_{i=1}^{n} k_{3_i}\int_0^{\tilde{q}_{p_i}} \mathrm{sat}(x)\,dx \geq \sum_{i=1}^{n} k_{3_i}\frac{\mathrm{sat}(\varepsilon)}{2\varepsilon}\tilde{q}_{p_i}^2 \geq \min_i\{k_{3_i}\}\frac{\mathrm{sat}(\varepsilon)}{2\varepsilon}\|\tilde{q}_p\|^2$$

and considering that $\|\tilde{q}_p\| \geq \varepsilon$, we obtain (22).

Case b ($|\tilde{q}_{p_i}| \geq \varepsilon\ \forall i \leq n$): From P1) and (10) notice that

$$\sum_{i=1}^{n} k_{3_i}\int_0^{\tilde{q}_{p_i}} \mathrm{sat}(x)\,dx \geq \sum_{i=1}^{n} k_{3_i}\frac{\mathrm{sat}(\varepsilon)}{2}|\tilde{q}_{p_i}| \geq \min_i\{k_{3_i}\}\frac{\mathrm{sat}(\varepsilon)}{2}\sum_{i=1}^{n}|\tilde{q}_{p_i}|.$$

Then (22) easily follows, observing that $\|\tilde{q}_p\| \leq \Sigma_{i=1}^n |\tilde{q}_{p_i}|$.

Case c ($|\tilde{q}_{p_i}| \geq \varepsilon, |\tilde{q}_{p_j}| < \varepsilon\ \forall i, j \leq n, i \neq j$): Without loss of generality, we can take $i \leq n/2$ and $1 \leq j < n/2$; then a simple analysis along the lines of cases a and b shows that (22) holds as well in this case.

Now we prove that for all $\varepsilon > 0$ there exist $\beta_1(\varepsilon), \beta_2(\varepsilon) \in \mathbb{R}$ such that

$$V_p - V_p(q_{p_d}) - \tilde{q}_p^\top \frac{\partial V_p(q_p)}{\partial q_p}(q_{p_d}) \geq \begin{cases}\beta_1\|\tilde{q}_p\|^2, & \forall \|\tilde{q}_p\| < \varepsilon\\ \beta_2\|\tilde{q}_p\|, & \forall \|\tilde{q}_p\| \geq \varepsilon.\end{cases} \tag{23}$$

On one hand, using (3) it can be proven that

$$V_p(q_p) - V_p(q_{p_d}) - \tilde{q}_p^\top \frac{\partial V_p(q_p)}{\partial q_p}(q_{p_d}) \geq -\frac{k_g}{2}\|\tilde{q}_p\|^2.$$

On the other hand, invoking the Mean Value Theorem we have that $\exists \xi \in \mathbb{R}^n$ such that

$$V_p(q_{p_d}) - V_p = \left(\frac{\partial V_p(q_p)}{\partial q_p}(\xi)\right)(q_{p_d} - q_p) \leq k_v\|q_{p_d} - q_p\|.$$

Then using (2) we can write

$$V_p(q_p) - V_p(q_{p_d}) - \frac{\partial V_p(q_p)}{\partial q_p}(q_{p_d})^\top \tilde{q}_p \geq -2k_v\|\tilde{q}_p\|.$$

We finally conclude from (21)–(23) that

$$f(\tilde{q}_p) \geq \begin{cases}\left(\min_i\{k_{3_i}\}\dfrac{\mathrm{sat}(\varepsilon)}{2\varepsilon} - \dfrac{k_g}{2}\right)\|\tilde{q}_p\|^2, & \forall \|\tilde{q}_p\| < \varepsilon\\ \left(\min_i\{k_{3_i}\}\dfrac{\mathrm{sat}(\varepsilon)}{2} - 2k_v\right)\|\tilde{q}_p\|, & \forall \|\tilde{q}_p\| \geq \varepsilon.\end{cases} \tag{24}$$

The proof of Condition 1 is completed, observing that (24) happens to hold, provided

$$\min_i\{k_{3_i}\} \geq k_{3_i}^{\min} > \max\left\{\frac{\varepsilon k_g}{\mathrm{sat}(\varepsilon)}, \frac{4k_v}{\mathrm{sat}(\varepsilon)}\right\} \tag{25}$$

is satisfied.

Condition 2

Taking the partial derivative of $f(\tilde{q})$ we get

$$\frac{\partial f}{\partial \tilde{q}_p}(\tilde{q}_p) = K_3 \begin{bmatrix} \mathrm{sat}(\tilde{q}_{p_1}) \\ \mathrm{sat}(\tilde{q}_{p_2}) \\ \vdots \\ \mathrm{sat}(\tilde{q}_{p_n}) \end{bmatrix} + \frac{\partial V_p}{\partial \tilde{q}_p}(\tilde{q}_p) - \frac{\partial V_p}{\partial q_p}(q_{pd}). \quad (26)$$

Now, taking the norm and using the triangle inequality we obtain

$$\left\|\frac{\partial f}{\partial \tilde{q}_p}(\tilde{q}_p)\right\| \geq \left\| K_3 \begin{bmatrix} \mathrm{sat}(\tilde{q}_{p_1}) \\ \mathrm{sat}(\tilde{q}_{p_2}) \\ \vdots \\ \mathrm{sat}(\tilde{q}_{p_n}) \end{bmatrix} \right\| - \left\| \frac{\partial V_p}{\partial \tilde{q}_p}(\tilde{q}_p) - \frac{\partial V_p}{\partial q_p}(q_{pd}) \right\|. \quad (27)$$

On one hand, from (3), (2) and using the Mean Value Theorem we have that for all $\varepsilon > 0$

$$-\left\| \frac{\partial V_p}{\partial \tilde{q}_p}(\tilde{q}_p) - \frac{\partial V_p}{\partial q_p}(q_{pd}) \right\| \geq \begin{cases} -k_g\|\tilde{q}_p\|, & \text{if } \|\tilde{q}_p\| < \varepsilon \\ -2k_v, & \text{if } \|\tilde{q}_p\| \geq \varepsilon. \end{cases}$$

On the other hand, since K_3 is diagonal and using P2), we get

$$\left\| K_3 \begin{bmatrix} \mathrm{sat}(\tilde{q}_{p_1}) \\ \mathrm{sat}(\tilde{q}_{p_2}) \\ \vdots \\ \mathrm{sat}(\tilde{q}_{p_n}) \end{bmatrix} \right\| \geq \begin{cases} \min_i \{k_{3_i}\} \dfrac{\mathrm{sat}(\varepsilon)}{\varepsilon}\|\tilde{q}_p\|, & \text{if } \|\tilde{q}_p\| < \varepsilon \\ \min_i \{k_{3_i}\} \mathrm{sat}(\varepsilon), & \text{if } \|\tilde{q}_p\| \geq \varepsilon. \end{cases}$$

Thus, we are able to write

$$\left\|\frac{\partial f}{\partial \tilde{q}_p}(\tilde{q}_p)\right\| \geq \begin{cases} \left[\min_i \{k_{3_i}\} \dfrac{\mathrm{sat}(\varepsilon)}{\varepsilon} - k_g\right]\|\tilde{q}_p\|, & \text{if } \|\tilde{q}_p\| < \varepsilon \\ \left[\min_i \{k_{3_i}\} \mathrm{sat}(\varepsilon) - 2k_v\right], & \text{if } \|\tilde{q}_p\| \geq \varepsilon. \end{cases}$$

From here it is easy to see that Condition 2 is satisfied provided (25) holds with ε as in P2), k_g, and k_v defined by (2) and (3), respectively.

The proof is completed observing from (23) that $f(\tilde{q}_p)$ is radially unbounded.

Notice that in the case of $\mathrm{sat}(x) = \tanh(x)$ we have that P2) is true with[6] $\varepsilon \triangleq 2k_v/k_g$, then (16) immediately follows. □

Remark: It is worth remarking that the proof for the simpler case when $V(q_p, q_e) = V_{e_2}(q_e, q_p)$ is strictly contained in the proof above: see (21) and (22).

Appendix B
Definitions

For the sake of clarity we recall below some definitions borrowed from [21] and [22].

Let $f: \mathbb{R}^n \to \mathbb{R}$ be a smooth function, then we define the following.

1) *Critical point*: A point $x^* \in \mathbb{R}^n$ is called *critical point* of $f(x)$ if and only if $(\partial f/x)(x^*) = 0$.
2) *Local minimum*: A point $x^* \in \mathbb{R}^n$ is a *local minimum* of $f(x)$ if $f(x) \geq f(x^*)$ in a neighborhood B_δ of x^* with $0 < \delta < \infty$.
3) *Global or absolute minimum*: A point $x^* \in \mathbb{R}^n$ is an *absolute minimum* of $f(x)$ if $f(x) \geq f(x^*)$ for all $x \in \mathbb{R}^n$.
4) *Unique minimum*: A point $x^* \in \mathbb{R}^n$ is a *unique minimum* of $f(x)$ if there are no other local minima of $f(x)$ in $\mathbb{R}^n$.
5) *Strict local minimum*: A point $x^* \in \mathbb{R}^n$ is a *strict local minimum* of $f(x)$ if there exists a neighborhood B_δ of x^* with $0 < \delta < \infty$ such that $f(x) > f(x^*)$ for all $x \in B_\delta$.

[6] See [9].

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References

[1] R. Ortega, A. Loria, R. Kelly, and L. Praly, "On passivity-based output feedback global stabilization of Euler-Lagrange systems," *Int. J. Robust Nonlinear Contr.—Special issue on contr. nonlinear mechanical syst.*, H. Nijmeijer and J. van der Schaft, Eds., vol. 5, no. 4, pp. 313–325, 1995.
[2] M. Takegaki and S. Arimoto, "A new feedback method for dynamic control of manipulators," *ASME J. Dyn. Syst. Meas. Contr.*, vol. 103, pp. 119–125, 1981.
[3] R. Ortega and M. Spong, "Adaptive motion control of rigid robots: A tutorial," *Automatica*, vol. 25/26, pp. 877–888, 1989.
[4] A. Ailon and R. Ortega, "An observer-based set-point controller for robot manipulators with flexible joints," *Syst. Contr. Lett.*, vol. 21, pp. 329–335, 1993.
[5] G. Espinosa and R. Ortega, "State observers are unnecessary for induction motor control," *Syst. Contr. Lett.*, vol. 23, no. 5, pp. 315–323, 1994.
[6] H. Sira-Ramirez and R. Ortega, "Passivity-based control of DC to DC converters," *Automatica*, to be published.
[7] C. I. Byrnes and C. F. Martin, "An integral-invariance principle for nonlinear systems," *IEEE Trans. Automat. Contr.*, vol. 40, pp. 983–994, June 1995.
[8] R. Kelly, "A simple set-point robot controller by using only position measurements," in *Proc. 12th IFAC World Congr.*, Sydney, Australia, 1993, vol. 6, pp. 173–176.
[9] R. Kelly, V. Santibáñez, and H. Berghuis, "Global regulation for robot manipulators under actuator constraints," CICESE, Ensenada, BC, Mexico., Tech. Rep., 1994.
[10] I. V. Burkov, "Stabilization of mechanical systems via bounded control and without velocity measurement," in *Proc. 2nd Russian–Swedish Contr. Conf.*, St. Petersburg, Russia, 1995, pp. 37–41.
[11] H. Berghuis and H. Nijmeijer, "Global regulation of robots using only position measurements," *Syst. Contr. Lett.*, vol. 21, pp. 289–293, 1993.
[12] F. Esfandiari and H. K. Khalil, "Output feedback stabilization of fully linearizable systems," *Int. J. Contr.*, vol. 56, pp. 107–1037, 1992.
[13] A. Teel and L. Praly, "Tools for semiglobal stabilization by partial state and output feedback," *SIAM J. Contr. Opt.*, vol. 33, no. 5, pp. 1443–1488, 1995.
[14] C. Desoer and M. Vidyasagar, *Feedback Systems: Input–Output Properties*. New York: Academic, 1975.
[15] L. Cai and G. Song, "A smooth robust nonlinear controller for robot manipulators with joint stick-slip friction," in *Proc. IEEE Conf. Robotics Automat.*, Atlanta, GA, 1993, pp. 449–454.
[16] R. Kelly, R. Ortega, A. Ailon, and A. Loria, "Global regulation of flexible joints robots using approximate differentiation," *IEEE Trans. Automat. Contr.*, vol. 39, pp. 1222–1224, June 1994.
[17] V. Santibáñez and R. Kelly, "Set-point control in task space of robot manipulators with torque constraints: State and output feedback case," in *Proc. 13th IFAC World Congr.*, San Francisco, CA, July 1996.
[18] ——, "Global regulation for robot manipulators under SP-SD feedback," in *Proc. IEEE Conf. Robotics Automat.*, Minneapolis MN, 1996.
[19] H. Berghuis, "Model-based robot control: From theory to practice," Ph.D. dissertation, Univ. Twente, The Netherlands, 1993.
[20] P. Tomei, "A simple PD controller for robots with elastic joints," *IEEE Trans. Automat. Contr.*, vol. 36, pp. 1208–1213, Oct. 1991.
[21] E. Mardsen and A. J. Tromba, *Vector Calculus*. New York: W. H. Freeman, 1988.
[22] R. A. Wismer and R. Chattergy, *Introduction to Nonlinear Optimization*. New York: North Holland, 1979.