Adaptive and Variable Structure Control of Robot Manipulators

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Adaptive Control of Manipulators

References

Approximation Methods (1979-1985)

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Linear-parameterization Methods (1986-present)

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Difficulty of Feedback Linearization (Computed Torque) Control

Dynamic equation

$$M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + G(\mathbf{q}) = \tau \qquad (1)$$

where $\mathbf{q} \in R^n$ is the joint positions and $\tau \in R^n$ is the applied joint torques.

The feedback linearization (computed torque) control law is defined as

$$\tau = M(\mathbf{q})\mathbf{v} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + G(\mathbf{q}) \qquad (2)$$

where vector $\mathbf{v}$ is the outer loop control signals.

Since there are considerable uncertainties in all robot dynamic models, for example, model parameters such as link length, mass and inertia, variable payloads, etc, which are either impossible to know precisely or varying unpredictably, in this case, the feedback linearization method or the computed torque method cannot be directly used.

Adaptive Feedback Linearization (Computed Torque) Control of Manipulators

Objectives

Apply adaptive control methods in which controllers are designed to be adjustable so as to automatically compensate for these uncertainties.

Properties of Robotic Dynamics

- Positive definite
  The inertia matrix $M(\mathbf{q})$ is symmetric, positive definite, and both $M(\mathbf{q})$ and $M(\mathbf{q})^{-1}$ are uniformly bounded as function of $\mathbf{q}$.
- Skew-symmetric
  A suitable definition of $C(\mathbf{q}, \dot{\mathbf{q}})$ makes matrix $(\dot{M} - 2C)$ skew-symmetric. In particular, this is true if the elements of $B(\mathbf{q}, \dot{\mathbf{q}})$ are defined as

$$C_{ij} = \frac{1}{2}[\dot{\mathbf{q}}^T \frac{\partial M_{ij}}{\partial \mathbf{q}} + \sum_{k=1}^{n} (\frac{\partial M_{ik}}{\partial q_j} - \frac{\partial M_{jk}}{\partial q_i})\dot{q}_k] \qquad (3)$$

- Linear parameterization

$$M(\mathbf{q})\ddot{\mathbf{q}} + C(\mathbf{q}, \dot{\mathbf{q}})\dot{\mathbf{q}} + G(\mathbf{q}) = Y(\mathbf{q}, \dot{\mathbf{q}}, \ddot{\mathbf{q}})\theta \qquad (4)$$

where $Y \in R^{n \times m}$ is a matrix of known functions, known as the regressor; $\theta \in R^m$ is a vector of the manipulator inertia parameters (masses, moments of inertia, etc.).

Control Objectives

For any given desired trajectory $\mathbf{q}_d \in R^n$, with some or all the manipulator parameters unknown, find τ so that

$$\lim_{t \to \infty} \|\mathbf{q}(t) - \mathbf{q}_d(t)\| = 0.$$

3. Adaptive Case

The adaptive implementation of the inverse dynamics control law is obtained by replacing M, C, and G by their estimates, i.e.,

$$\tau = \hat{M}(q)(\ddot{q}^d - K_v \dot{e} - K_p e) + \hat{C}(q, \dot{q})\dot{q} + \hat{G}(q). \tag{6}$$

We assume that $\hat{M}$, $\hat{C}$, and $\hat{G}$ have the same functional form as M, C, g with estimated parameters $\hat{\theta}_1, \cdots, \hat{\theta}_r$. Thus,

$$\hat{M}\ddot{q} + \hat{C}\dot{q} + \hat{G} = Y(q, \dot{q}, \ddot{q})\hat{\theta} \tag{7}$$

where $\hat{\theta}$ is the vector of estimated parameters.

Substituting (6) into (1) gives

$$M\ddot{q} + C\dot{q} + G = \hat{M}(\ddot{q}^d - K_v \dot{e} - K_p e) + \hat{C} + \hat{G} \tag{8}$$

Adding and subtracting $\hat{M}\ddot{q}$ on the left-hand side of (8) and using (7) we can write

$$\begin{aligned} \hat{M}(\ddot{e} + K_v \dot{e} + K_p e) &= \tilde{M}\ddot{q} + \tilde{C}\dot{q} + \tilde{G} \\ &= Y(q, \dot{q}, \ddot{q})\tilde{\theta} \end{aligned} \tag{9}$$

where $\tilde{(\cdot)} := \hat{(\cdot)} - (\cdot)$. Finally, the error dynamics may be written as

$$\ddot{e} + K_v \dot{e} + K_p e = \hat{M}^{-1} Y \tilde{\theta} := \Phi\tilde{\theta}. \tag{10}$$

We may write the system (10) in state space as

$$\dot{x} = Ax + B\Phi\tilde{\theta} \tag{11}$$

where A is the Hurwitz matrix

$$A = \begin{bmatrix} 0 & I \\ -K_p & -K_v \end{bmatrix}; \text{ and } B = \begin{bmatrix} 0 \\ I \end{bmatrix}; x = \begin{bmatrix} e \\ \dot{e} \end{bmatrix}. \tag{12}$$

Assumptions:
1) $\ddot{q}$ = acceleration is measurable
2) $\hat{M}^{-1}$ is bounded.

Theorem 1: Choose the update law

$$\dot{\hat{\theta}} = \dot{\tilde{\theta}} = -\Gamma^{-1}\Phi^T B^T P x \tag{13}$$

where $\Gamma = \Gamma^T > 0$ and P is the unique symmetric positive definite solution to the Lyapunov equation

$$A^T P + PA + Q = 0 \tag{14}$$

for a given symmetric, positive definite Q. Under these conditions then, the solution x of (11) satisfies

$$x \to 0 \qquad \text{as } T \to \infty \tag{15}$$

with all signals remaining bounded.

Proof: Choose the Lyapunov function candidate

$$V = x^T P x + \tilde{\theta}^T \Gamma \tilde{\theta}. \tag{16}$$

The time derivative of V along trajectories of (11) is computed to be

$$\dot{V} = -x^T Q x + 2\tilde{\theta}^T [\Phi^T B^T P x + \Gamma \dot{\tilde{\theta}}]. \tag{17}$$

Using the parameter update law (11), this reduces to

$$\dot{V} = -x^T Q x \leq 0. \tag{18}$$

. *A New Adaptive Inverse Dynamics Scheme*

The main drawback of the result in the previous section is the requirement that $\hat{M}$ remain uniformly positive definite.
we choose an inverse dynamics control law of the form

$$\tau = M_0(q)(V + \delta V) + C_0(q, \dot{q})\dot{q} + G_0 \tag{20}$$

where $M_0 = M_0^T > 0$, C_0, G_0 are *a priori* estimates of M, C, G, respectively, with **fixed parameters**, V is given by (2), and δV is an additional outer loop control that compensates for the deviations ΔM, ΔC, ΔG, where $\Delta(\cdot) = (\cdot)_0 - (\cdot)$. In the present setup we will choose δV adaptively. It is important to note that the terms M_0, C_0, G_0 in (20) are not updated on-line, and hence the invertibility of M_0 is not an issue.

$V = \ddot{q}_d - K_p e - K_v \dot{e}$

If we now combine (20) with (1) we have an equation similar to (9)

$$M_0(\ddot{e} + K_v \dot{e} + K_p e - \delta V) = \Delta M \ddot{q} + \Delta C \dot{q} + \Delta G$$
$$= Y(q, \dot{q}, \ddot{q})\Delta\theta \tag{21}$$

where the last equality is obtained using Property 3 of linearity in the parameters. Note, in (21), that $\Delta\theta = \theta_0 - \theta$ is a fixed vector in $\mathbb{R}^r$ and not a function of time, since the terms in (20) are fixed estimates. Finally, we write

$$\ddot{e} + K_v \dot{e} + K_p e = M_0^{-1} Y \Delta\theta + \delta V := \Phi_0 \Delta\theta + \delta V. \tag{22}$$

Choosing the control δV as

$$\delta V = -\Phi_0 \Delta\hat{\theta} \tag{23}$$

yields an equation identical to (11) with Φ replaced by Φ_0. Note that $\Delta\tilde{\theta} = \tilde{\theta}$ and that now Φ_0 is not a function of the estimated parameters since M_0 is fixed. Choosing an update law for $\Delta\hat{\theta}$ according to

$$\Delta\dot{\hat{\theta}} = \Gamma^{-1} \Phi_0^T B^T P x \tag{24}$$

where P satisfies (14), and choosing a Lyapunov function candidate

$$V = x^T P x + \Delta\tilde{\theta}^T \Gamma \Delta\tilde{\theta} = x^T P x + \tilde{\theta}^T \Gamma \tilde{\theta} \tag{25}$$

a proof identical to that of Theorem 1 shows that $x \to 0$ as $t \to \infty$, with all signals remaining bounded.

The Algorithm of Slotine and Li

Advantages:

* Without measurement of the acceleration.
* Without inversion of estimated inertia matrix
* The controller is globally stable.

Controller

$$\tau = \hat{M}\ddot{q}_r + \hat{C}(q, \dot{q}_r)\dot{q}_r + \hat{G}(q) - \underbrace{K_D s}_{\text{PD Feed back control part.}}$$

where

$$\dot{q}_r \triangleq \dot{q}_d - \Lambda\tilde{q} \;,$$

Λ is a positive definite matrix

$\tilde{q} \triangleq q - q_d$; K_D is a uniformly positive matrix ; s is defined as

$$s = \dot{q} - \dot{q}_r = \dot{\tilde{q}} + \Lambda\tilde{q}$$

q_d = desired trajectory.

Unknown parameter.

$$\tau = Y(q, \dot{q}, \dot{q}_r, \ddot{q}_r)\hat{\theta} - K_D s$$

Using linearization property we can write this.

This suggests that choosing the adaptive law such that

$$Y^T(q, \dot{q}, \dot{q}_r, \ddot{q}_r)s + \Gamma\dot{\tilde{\theta}} = 0$$

that is

$$\dot{\hat{\theta}} = -\Gamma^{-1}Y^T s$$

$$\left(\dot{\hat{\theta}} = \dot{\tilde{\theta}} \text{ since } \theta \text{ is constant}\right)$$

The resulting expression of $\dot{V}$ is

$$\dot{V} = -s^T K_D s < 0 \quad (16)$$

~~the~~ Lemma:

Let $Y = H(s)X$, with $H(s)$ an $n\times m$ transfer matrix which is strictly proper and exponential stable.

Then if $x \in L_2^m$, we have

i) $y \in L_2^n \cap L_\infty^n$ and $\dot{y} \in L_2^n$

ii) y is continuous, and $y \to 0$ as $t \to \infty$

iii) if $x \to 0$, as $t \to \infty$, then $\dot{y} \to 0$

$L_\infty^n \triangleq \sup|x|$

Expression (16) imply that

$s \in L_2^n \cap L_\infty^n$!

~~Hence~~

Since $\tilde{q} = (s^* I - K_0)^{-1} s$

By using Lemma, we ~~can~~ can conclude ~~that~~

$$\tilde{q} \in L_2^n \cap L_\infty^n$$

$$\tilde{q} \to 0 \text{ as } t \to \infty$$

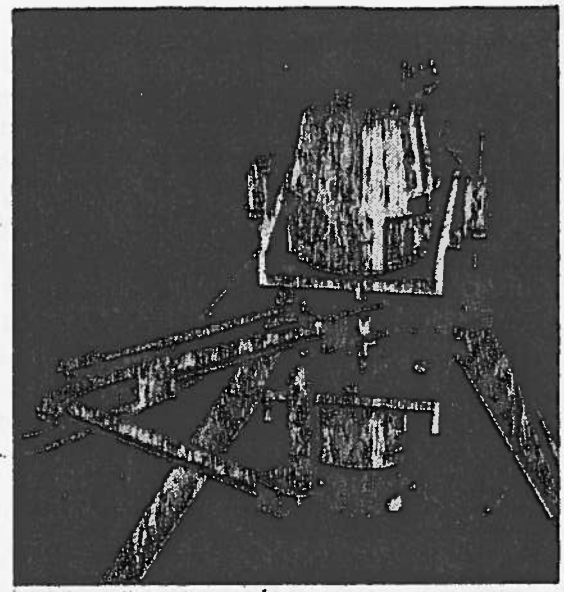

Fig. 2. The experimental two degree-of-freedom manipulator.

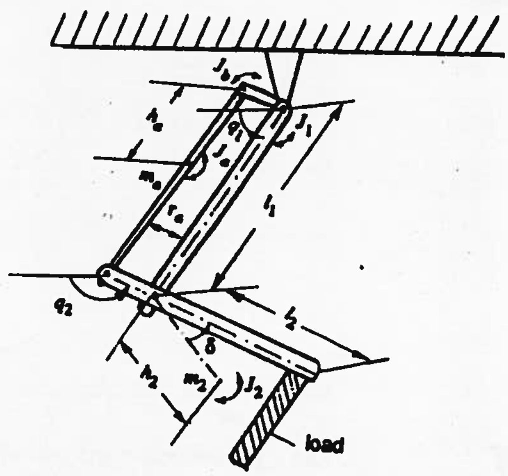

Fig. 3. The schematic structure of the manipulator.

B. Dynamic Model and Adaptive Controller Design

The dynamic model of the manipulator can be derived from Lagrange's equations to be

$$a_1\ddot{q}_1+(a_3c_{21}+a_4s_{21})\ddot{q}_2-a_3s_{21}\dot{q}_2^2+a_4c_{21}\dot{q}_2^2=\tau_1 \tag{9-a}$$

$$(a_3c_{21}+a_4s_{21})\ddot{q}_1+a_2\ddot{q}_2+a_3s_{21}\dot{q}_1^2-a_4c_{21}\dot{q}_1^2=\tau_2 \tag{9-b}$$

where $c_{21} = \cos(q_2 - q_1)$, $s_{21} = \sin(q_2 - q_1)$. It is clearly linear in terms of the four parameters a_1, a_2, a_3, a_4, which are related to the physical parameters of the links in Fig. 3 through

$$a_1=J_1+J_a+m_2l_1^2+m_ah_a^2 \qquad a_2=J_2+J_b+m(2)h_2^2+m_ar_a^2$$

$$a_3=m_2h_2l_1\cos\delta-m_ah_ar_a \qquad a_4=m_2h_2l_1\sin\delta \tag{10}$$

with the load treated as part of the second link. Defining the components of the matrix C as

$$C(1,1)=C(2,2)=0$$

$$C(1,2)=(a_4c_{21}-a_3s_{21})\dot{q}_2 \qquad C(2,1)=(a_3s_{21}-a_4c_{21})\dot{q}_1$$

the skew-symmetry of $\dot{H} - 2C$ can also be confirmed easily.

For simplicity, the feedback gain matrix K_D and the adaptation gain matrix Γ in the controller design are chosen to be diagonal

$$K_D=\operatorname{diag}(k_{d1}, k_{d2}) \qquad \Gamma=\operatorname{diag}(\gamma_1, \gamma_2, \gamma_3, \gamma_4).$$

The explicit form of the control law $\tau = \hat{H}\ddot{q}_r + \hat{C}\dot{q}_r - K_Ds$ in terms of the parameter vector a is

$$\tau_1=Y_{11}a_1+Y_{13}a_3+Y_{14}a_4-k_{d1}s_1 \tag{11-a}$$

$$\tau_2=Y_{22}a_2+Y_{23}a_3+Y_{24}a_4-k_{d2}s_2 \tag{11-b}$$

where

$$Y_{11}=\ddot{q}_{r1} \qquad Y_{13}=c_{21}\ddot{q}_{r2}-s_{21}\dot{q}_2\dot{q}_{r2} \qquad Y_{14}=s_{21}\ddot{q}_{r2}+c_{21}\dot{q}_2\dot{q}_{r2}$$

$$Y_{22}=\ddot{q}_{r2} \qquad Y_{23}=c_{21}\ddot{q}_{r1}+s_{21}\dot{q}_{r1}\dot{q}_1 \qquad Y_{24}=s_{21}\ddot{q}_{r1}-c_{21}\dot{q}_{r1}\dot{q}_1.$$

The adaptation law can be explicitly written as

$$\dot{\hat{a}}_1=-\gamma_1Y_{11}s_1 \tag{12-a}$$

$$\dot{\hat{a}}_2=-\gamma_2Y_{22}s_2 \tag{12-b}$$

$$\dot{\hat{a}}_3=-\gamma_3(Y_{13}s_1+Y_{23}s_2) \tag{12-c}$$

$$\dot{\hat{a}}_4=-\gamma_4(Y_{14}s_1+Y_{24}s_2). \tag{12-d}$$

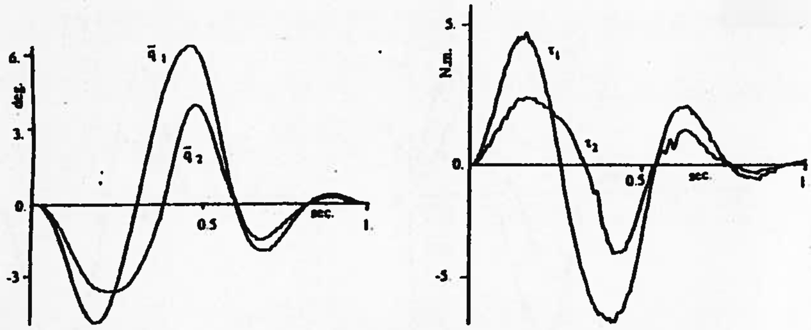

Fig. 5. PD control without load.

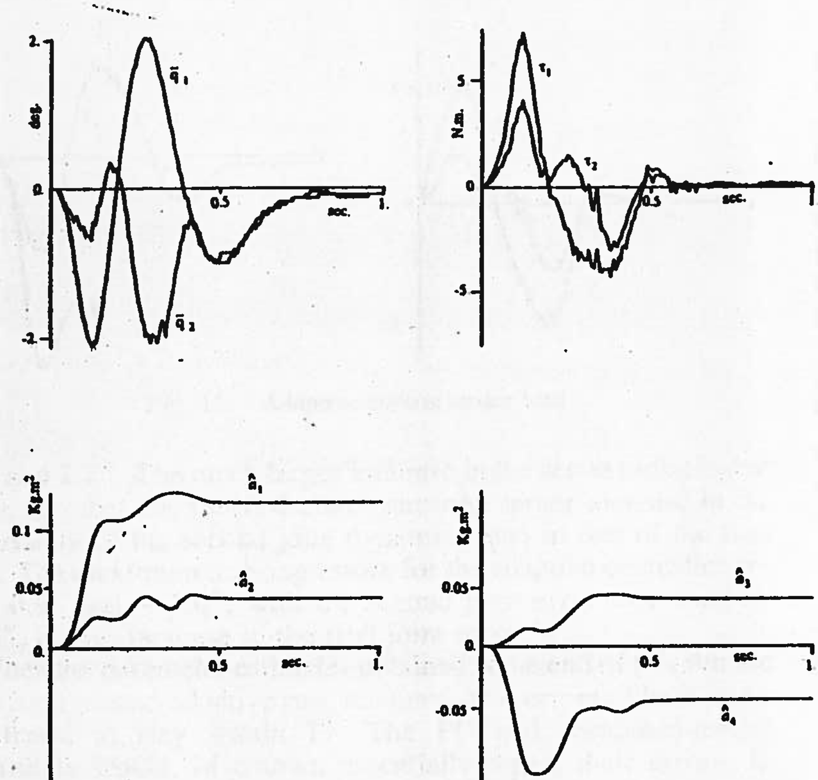

Fig. 6. Adaptive control starting from zero estimates.

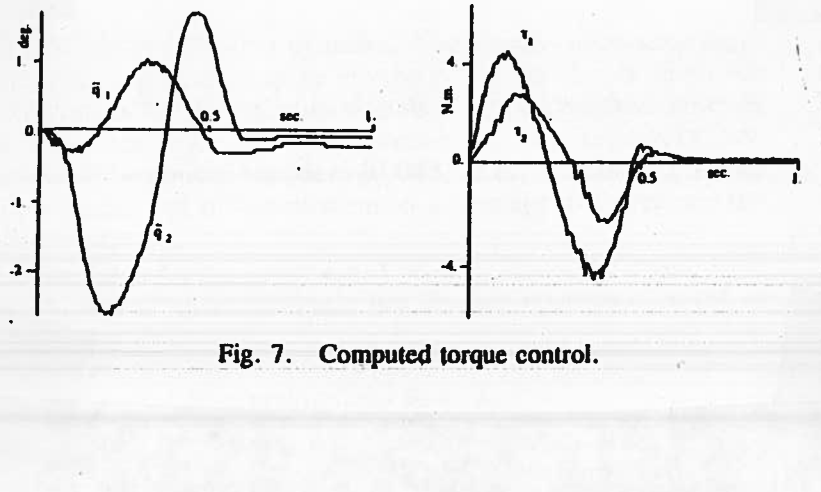

Fig. 7. Computed torque control.

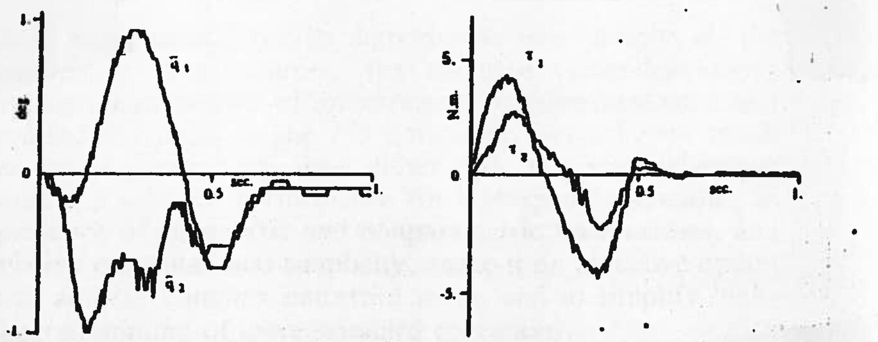

Fig. 8. Adaptive control starting with nonzero estimates.

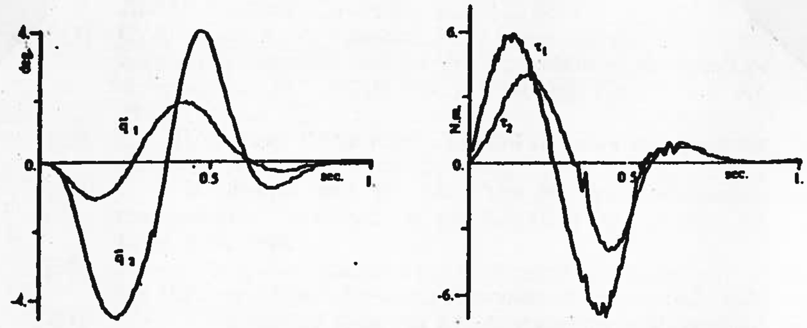

Fig. 10. Computed torque control under large load.

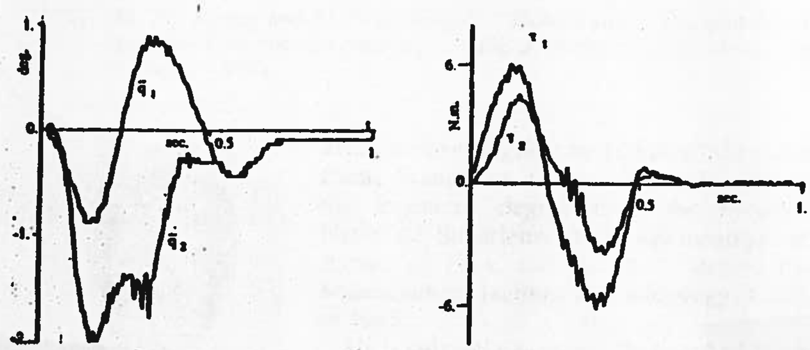

Fig. 11. Adaptive control under load.

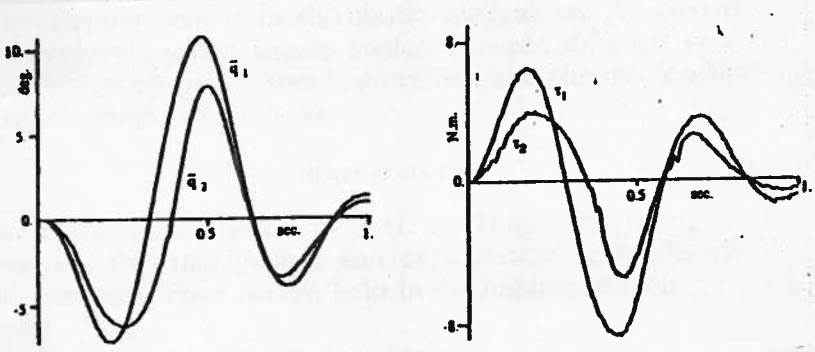

Fig. 9. PD control under load.

Algorithm of Slotine and Li

$$\tau = Y(q,\dot q,\dot q_r,\ddot q_r)\hat{\alpha} - K_D s \quad \text{-- } (*)$$

$$\dot{\hat{\alpha}} = -\Gamma Y^T s$$ → The only parameter we can chose here.

Where $Y(q,\dot q,\dot q_r,\ddot q_r)$ is defined as

General dynamic equation $M\ddot q + C(q,\dot q)\dot q + G = \tau$ (1)

$$M\ddot q_r + C\dot q_r + G = Y(q,\dot q,\dot q_r,\ddot q_r)\alpha$$

$$s \triangleq \dot q - \dot q_r = \dot{\tilde q} + \Lambda\tilde q$$

$$\dot q_r = \dot q_d - \Lambda\tilde q$$

Objective:

Design a variable structure control by using the controller structure of (*) without estimating the parameters.

$$\tau = Y(q,\dot q,\dot q_r,\ddot q_r)\Psi - K_D s \qquad (2)$$

where Ψ is switching function designed according to the variable structure theory.

Since
$$M\dot s = M\ddot q - M\ddot q_r$$
$$= \tau - C(q,\dot q)\dot q - G - M\ddot q_r$$
$$= \tau - C(q,\dot q)\dot q_r - G - M\ddot q_r - C(q,\dot q)s$$

$$= \tau - Y(q, \dot{q}, q_r, \dot{q}_r)\alpha - C(q, \dot{q})S$$

$$= Y(q, \dot{q}, q_r, \dot{q}_r)\Psi - Y(q, \dot{q}, q_r, \dot{q}_r)\alpha$$

$$- C(q, \dot{q})S - K_D S \quad (3)$$

The generalized Lyapunov function

$$V = \frac{1}{2} S^T M S$$

{original Lyapunov function was

$V = \frac{1}{2} S^T M S + \tilde{\alpha} \Gamma \tilde{\alpha}$; since we cannot estimate $\tilde{\alpha}$ we just take the first part}

Differentiating with respect to time yields,

$$\dot{V} = S^T M \dot{S} + \frac{1}{2} S^T \dot{M} S$$

$$= S^T (Y\Psi - Y\alpha - K_D S) + \frac{1}{2} S^T \underbrace{(\dot{M} - 2C)}_{0} S$$

(Using skew symmetric property)

$$= S^T (Y\Psi - Y\alpha - K_D S) \quad (4)$$

Now choosing

$$\psi_i = -\hat{\alpha}_i \, \mathrm{sgn}\left(\sum_{i=1}^{n} s_j Y_{ji}\right)$$

where $\hat{\alpha}_i \geq |\alpha_i|, \ \forall i$

then,

$$\dot{V} = -s^T K_D s - \sum_{i=1}^{m} \hat{\alpha}_i \left| \sum_{j=1}^{n} s_j Y_{ji} \right| - \sum_{i=1}^{m} \alpha \sum_{j=1}^{n} s_j Y_{ji} \leq -s^T K_D s \quad (5)$$

Therefore,

$\|S\| \to 0$ at least exponentially

converges to zero which implies

that g $\tilde{q} \to 0$ as $t \to \infty$

Remarks:

(1) In sliding mode, the resulting system equation

Λ = a positive constant matrix.

$$\dot{\tilde{q}} = -\Lambda \tilde{q} \quad \text{------} \quad (6)$$

$s = \dot{\tilde{q}} + \Lambda \tilde{q} = 0$

Equation (6) represents n uncoupled first order linear system and the system only depends on the parameter

1.

Clearly the robustness to the uncertainty is guaranteed.

(2) It differs from adaptive control in that no learning mechanism is used, and has the advantage of prescribed transient response in sliding mode.

Estimating $\hat{\alpha}$ parameter

$$\mathring{\hat{\alpha}}_i = \eta_i \left| \sum_{j=1}^{n} S_j Y_{ji} \right|, \quad i = 1, \ldots n$$

where, η_i are arbitrary constants.
Consider a Lyapunov function

$$V = \frac{1}{2} S^T M S + \frac{1}{2} \sum_{i=1}^{m} (|\alpha_i| - \hat{\alpha}_i)^2 / \eta_i$$

$$\Downarrow$$

$$Y\alpha$$

~~$M\ddot{q} + a\dot{q} +$~~

The time derivative is given by

$$\dot{V} = S^T M \dot{S} + \frac{1}{2} S^T \dot{M} S + \sum_{i=1}^{m} (|\alpha_i| - \hat{\alpha}_i) \frac{(-\dot{\hat{\alpha}}_i)}{\eta_i}$$

$$= S^T (Y\psi - Y\alpha - K_D S) - \cancel{K_D S}$$

$$- \sum_{i=1}^{m} |\alpha_i| \left| \sum_{j=1}^{n} S_j Y_{ji} \right| + \sum_{i=1}^{m} \hat{\alpha}_i \cancel{|}$$

$$\left| \sum_{j=1}^{n} S_j Y_{ji} \right| \leq -S^T K_D S$$