Continuum Mechanics and Heat Transfer JEÐ503M 7.5 ECTS

Take home exam Fall 2022

Teachers:

Guðfinna Aðalgeirsdóttir,	gua@hi.is, Office: N-329 Askja
Angel Ruiz Angulo	angel@hi.is, office N-324 Askja
Sonja Heidi Maria Greiner,	shm19@hi.is, office N-372, Askja

This take-home exam is to be held from **8:00 on Monday 5.12.2022 until 12:00 on Monday 12.12.2022**. The exam consists **of 4 problems**. Write down clearly how you have found a solution to the problems, only answers or a single equation that you plug into will not be graded. It is useful to draw figures and explanations to accompany the solutions. You can return the solution to the exam in canvas (preferable) or in a closed envelope into the box with the course name on 1st floor in Askja (please let us know by email if you do that). You may consult notes, internet, and textbooks, but not each other. If you are stuck at some point assume an answer and carry on. If you feel like some required values/information are missing, try to look them up (for example in the appendix of the book) or assume a reasonable number/solution and continue (explain any assumption made). You should not work together on the solutions. Please sign the declaration at the end of the exam and hand in with your solutions.

Problem 1: Bridge over canyon on Reykjanes (25%)

It is planned to build a bridge over a 16 m wide canyon in the hiking area of Reykjanes peninsula. The idea is to build the bridge out of glass (density of glass is 2300 kg/m^3) so that the tourists can see into the canyon while walking over the bridge.

1) How thick does the bridge have to be so that it does not break? Assume that it breaks at 20 MPa tension. Assume that the ends of the bridge are fixed and cannot move (w=dw/dx=0 at both ends)
2) Is the answer different if the bridge is pinned and can rotate at its ends (dw/dx not equal to 0 but $d^2w/dx^2 = 0$)
3) The bridge is built and it is 25 cm thick. How much would the plate deflect in the middle from its own weight? How much does it deflect if 3 persons weighing 250 kg together stand on the middle of the bridge?
4) Assume that the bridge acts as Newtonian fluid with viscosity about 1×10^{19} Pa s. How much does the bridge deflect in the middle after 100 years?

Problem 2: Tectonics on the Reykjanes peninsula (25%)

The plate boundary between the North-American and the Eurasian plate runs across the Reykjanes peninsula (Southwest Iceland) and strikes approximately N76°E (76° East of North). Plate motion models predict that in the far-field the Eurasian plate moves in the direction of N103°E by 19mm/yr if the North-American plate is assumed to be stationary.

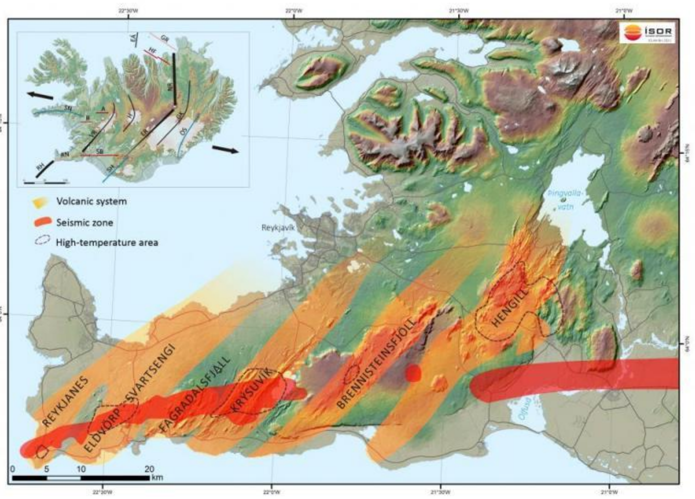

Figure 1: Reykjanes peninsula, volcanic systems and seismic zones are shown with orange and red colour, respectively (from https://en.isor.is/general-structure-and-volcanism-reykjanes-peninsula)

1) Calculate the velocity components in terms of E-W- & N-S displacement as well as the velocity components parallel (u_x) and perpendicular (u_y) to the direction of the plate boundary. Draw a sketch showing the velocity components.

Motion parallel and perpendicular to the plate boundary can be understood as shear and extension respectively.

2) Assume, that strain accumulates evenly within an 80 km wide deformation zone. Draw a figure that shows the horizontal deformation relative to the plate boundary in a simple, conceptual diagram (assume that the x-axis of your coordinate system runs along the plate boundary). Calculate the components of the strain rate tensor $\dot{\varepsilon}_{xx}$, $\dot{\varepsilon}_{xy}$ and $\dot{\varepsilon}_{yy}$.

3) Show, that the direction φ of the principal strain rates (relative to the strike of the plate boundary) can be calculated using only the angle ϑ between the plate boundary and the direction of motion through

$$\varphi = -\frac{1}{2}\arctan\left(\cot(\vartheta)\right)$$

4) Calculate the angle between the plate boundary and the direction of the first principal strain rate $\dot{\varepsilon}_1$, as well as the amplitude of the principal strain rates themselves ($\dot{\varepsilon}_1$ and $\dot{\varepsilon}_2$). Which direction experiences extension/compression? Please give the direction of the principal strain rate not only relative to the plate boundary, but also relative to North.

It is often assumed that dikes open parallel to the least compressive stress. If the dike opening is purely tensional, the dike's strike is expected to be parallel to σ_1

5) Have a look at the strike of the fissure swarms of volcanic systems on the peninsula (see elongated orange areas in figure 1) and at the strike of the Fagradalsfjall dike, which formed in March 2021 (e.g. https://en.vedur.is/media/uncategorized/Monitor-map-45x25-cm-unrest-en-20210309.png). Compare your findings to the direction of the principal strains and discuss. Could this be relevant e.g. in terms of hazard assessment on the Reykjanes peninsula?

Since the end of the last eruptive period 800 years ago until the eruption started in March 2021, it can be assumed that no new material has been added to the crust above the brittle-ductile boundary, which is thought to be located at ca. 7 km depth underneath Reykjanes. This means that the volume of the deformation zone is conserved and that the continuous stretching should result in very slight thinning of the brittle crust.

6) Assume that the vertical strain is distributed equally across the deformation zone. Calculate the vertical strain, which has accumulated during the last 800 years, based on the assumption of constant strain rates and conservation of volume of the deformation zone. Additionally, determine the strain that accumulated in the two horizontal directions.
Hint: Determine how much extension/compression occurred along the directions of the principal stress axes in total during the past 800 years. Make a sketch of the geometry with labels for all parameters you are using and clearly state your assumptions. Note that in the end you may need to re-name the principal stress directions to conform with the naming convention of the most compressive stress/strain as σ_1.
7) How much stress accumulated since the end of the last eruptive period 800 years ago (assuming purely elastic deformation)? Assume E = 80 GPa and ν = 0.25. Can you give two examples of how tensional stress and strain in the crust could be released apart from the emplacement of magmatic intrusions?
8) In extensional tectonic settings, σ_1 is usually oriented vertically. Discuss how you would expect the principal stresses to be oriented in a convergent tectonic setting, where two plates collide and why you expect them to be oriented in this specific way.

Problem 3: Eruption in Fagradalsfjall (25%)

During May 2021 Fagradalsfjall eruption, Iceland, evident periodic fountaining activity was observed, which was associated mainly to bubbles, many of them (Lamb et al., 2022). The findings showed an increase in bubble radii between early and late May, suggesting a widening of the upper conduit during the lava fountain sequence, indicating higher gas flux through the conduit culminating in shorter lava fountain events. For this problem we will explore bubbles rising in lava. We will simplify everything and assume only single bubble rising in a very wide conduit (this is to avoid any effects with the walls). The motivational sketch is in Figure 1.

Figure 1. Sketch showing bubbles rising within a lava conduit, and snapshots of the lava fountain sequence at Fagardalsfjall in May 2021 (Photo credit Talfan Barnie).

We will assume Stokes flow as we have equations for this that were derived in class, where we showed how to estimate the terminal velocity of a bubble rising in a liquid. This problem consists of 4 parts all equally weighted.

1. Re-derive the terminal velocity for a sphere of radius **a**, with density (ρ_s) in a fluid with density (ρ_f) and viscosity mu. Please begin with the velocity equations $\mathbf{u}_r$ and $\mathbf{u}_\theta$ that were provided in class, write every single assumption you take and be very clear on the steps you are taking. The equation for the terminal velocity you should get is:
$$U = \frac{2(\rho_f - \rho_s) g a^2}{9\mu}.$$
2. Now we will use the terminal velocity, we will push the limits of Stokes flow and move a bit out of the Re << 1, and we will allow 1<Re<10. A reasonable estimate for the viscosity of the basaltic magma is 10 Pa s. Also assuming (ρ_f - ρ_s) = 600 [kg m $^{-3}$] find what is the range of bubble **diameters** that are allowed for the range of Reynolds we are proposing, i.e., 1<Re<10. Hint: bubble diameter=2a; Re=ρ_f U 2a/ μ
3. Hint: Both for part 2 and 3, the equation in part 1 could also be expressed in terms of the drag coefficient (C_D) and this is a function of Reynolds number. For Stokes flow C_D = 24/Re. Here Re is based on the diameter, not the radius, so please be careful! Now, for the same limits I gave you (1<Re<10) what is the range of terminal velocities those bubbles could reach.

4. The paper by Lamb et al., 2022 mentions that bubbles could be of sizes 2-3.5m or even 6m in radius. For the parameters used in 2 and 3, what would be the terminal velocity and Reynolds number for those? It is easier to report this on a table. If the Re is too large, and you want to keep the dimensions right as the paper mentions, what could you change to keep the Re low? Notice that this solution is not unique; therefore, please justify all your assumptions.

   Just as a side note (#nerdalert), the reason why we would like to keep low Re is to remain spherical, at large Re the bubbles change shape and our equations do not longer apply! See Figure 2 below:

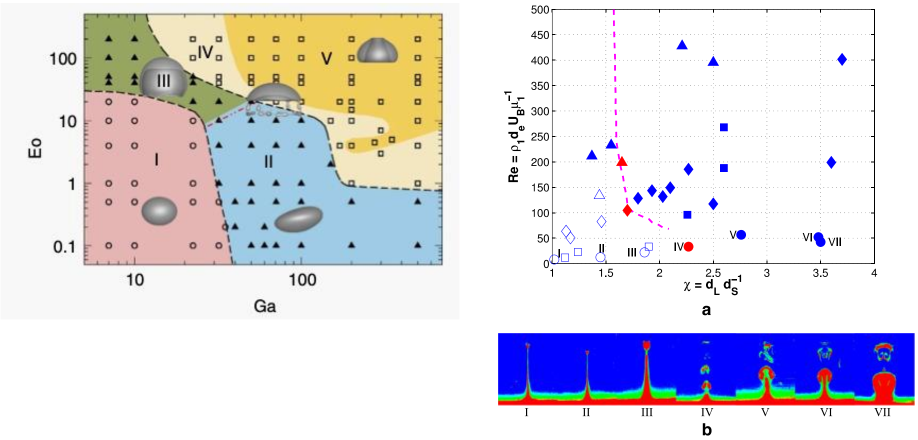

Figure 2. Left: Bubble shapes as a function of Galilei (Ga) – Eötvös (Eo) numbers (Tripathi et al., 2015); Right: bubbles Reynolds number as a function of the bubble aspect ratio, given by the ratio of the major and minor axis of an ellipse. Meaning that the closer to 1 the closer to be spherical bubbles, and Re is very small (Diaz-Damacillo et al., 2016).

References:

Lamb, O.D., Gestrich, J.E., Barnie, T.D., Jónsdóttir, K., Ducrocq, C., Shore, M.J., Lees, J.M. and Lee, S.J., 2022. Acoustic observations of lava fountain activity during the 2021 Fagradalsfjall eruption, Iceland. *Bulletin of Volcanology*, *84*(11), pp.1-18.

Tripathi, Manoj Kumar, Kirti Chandra Sahu, and Rama Govindarajan. "Dynamics of an initially spherical bubble rising in quiescent liquid." *Nature communications* 6.1 (2015): 1-9.

Diaz-Damacillo, L., A. Ruiz-Angulo, and R. Zenit. "Drift by air bubbles crossing an interface of a stratified medium at moderate Reynolds number." *International Journal of Multiphase Flow* 85 (2016): 258-266.

Problem 4: Heat transfer (25%)

a) Explain in words first and then set up the equation and explain how the linear stability analysis can be used to determine the onset of thermal convection in a layer of fluid heated from below. What drives the thermal convection of the fluid?

b) How is the dimensionless combination of parameters known as the Rayleigh number useful in the linear stability analysis of onset of thermal convection in a layer of fluid heated from below? what significance has the critical value of the Rayleigh number?

c) Now consider thermal convection in a layer of fluid saturated porous material contained between impermeable isothermal boundaries. How is the analysis and Rayleigh number different than in the previous convection situation? What drives the thermal convection in the porous layer? Compare the ratio between the wavelength of the convective cell and thickness of the layer for the smallest critical Rayleigh number when the fluid is freely moving (part a) and when the fluid is moving in a saturated porous material?

With my signature I confirm that I have worked on the solutions of the take-home exam by myself and I have not consulted other people while solving the problems.

Date Name