Gm-C Filter – 1 Implementation

Fall 2005

Rev. 0.2

Outline

1- Introduction to Gm-C Filters

2- Filter Implementation Techniques

- SFG
- Element replacement
- Chain Method

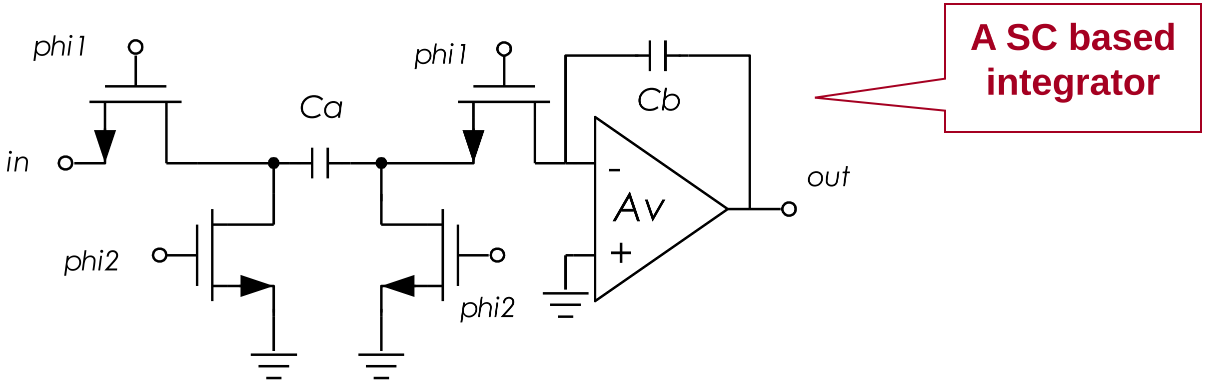
3- Differential Gm Cells

4- Double Input Gm Cells

Introduction: Comparison

Problems with SCF:

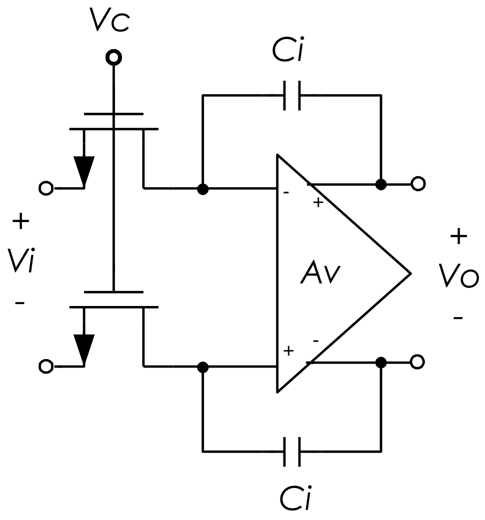
1- Need to smoothing post filtering
2- Need to anti-aliasing pre filtering
3- Need to high SR/BW amplifiers
4- Clock feed-through problem
5- Good for low-freq applications since needs to high frequency clock and hence high power dissipation (>1 MHz)



Introduction: Comparison

<u>Problems with Active-RC/MOSFET-C:</u>

1- Need of high SR/BW amplifiers
2- Proper for low-freq applications (>2-3 MHz)
3- Need very high **R** or **C** values with high precision to Implement filters



Introduction: Comparison

General Points for Integrated Filters

1- Designer requires deep knowledge on process:

- Rtotal < 40 kOhm (Accuracy 20%)
- Ctotal < 50 pF (Accuracy 10%)

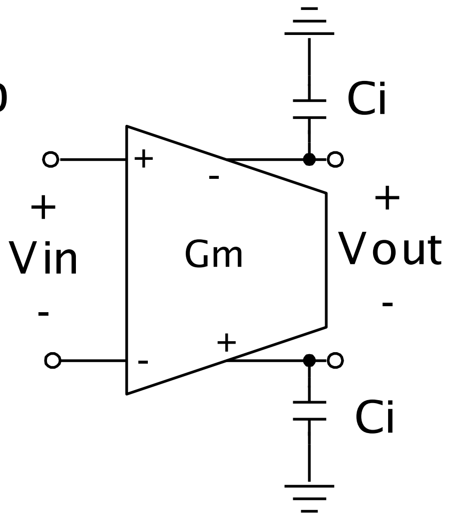
2- Tuning is unavoidable (else than SC filters)

3- Implementing very low or very high frequency filters are very difficult

4- Single ended configuration results in high non-linearity, distortion, and sensitivity to noise

Overall Specifications of Gm-C Filters:

1- Proper for High Freq Applications (~1 MHz and beyond) since there is no feedback
2- Uses just Gm Cell Plus Cap to Implement the Filter
3- Proper for Medium Accuracy Applications (DR~ 40-60 dB)
4- Tunability of filter by Gm variation
5- Less area of Gm cell respect to OpAmp



Ideal Gm-C Integrator:

$$\frac{V_o}{V_i} = \frac{G_m}{s \cdot C_i} \qquad \omega_0 = \frac{G_m}{C_i}$$

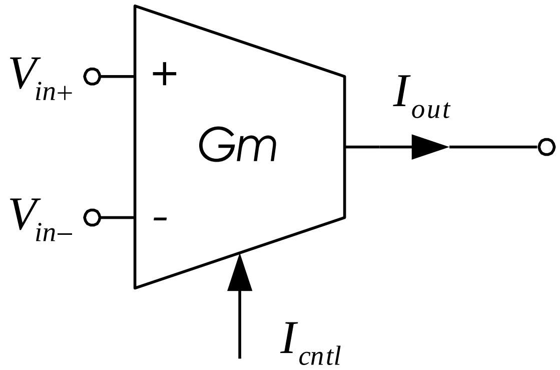
Overall Specifications of Gm-C Filters:

6-Gm cell must be linear for whole voltage swing (there is no feedback and no virtual ground)

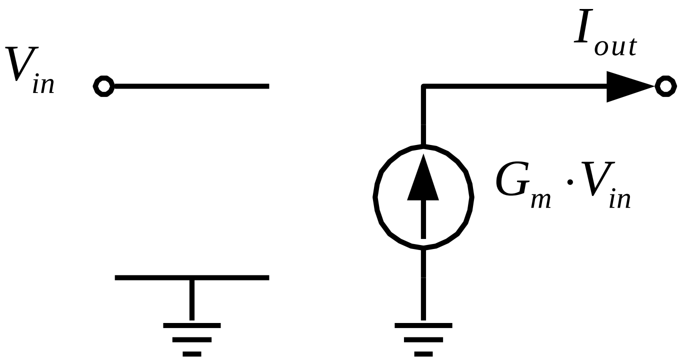
7- Needs to high output impedance

Introduction: Gm Cell

Gm symbol

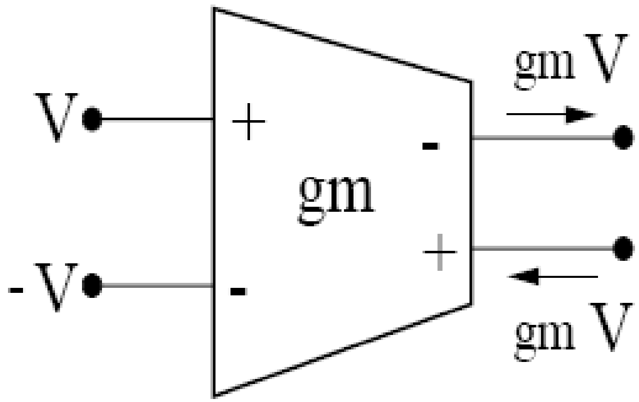


Ideal Gm Model: output current is proportional to the input voltage



Introduction: Gm Cell

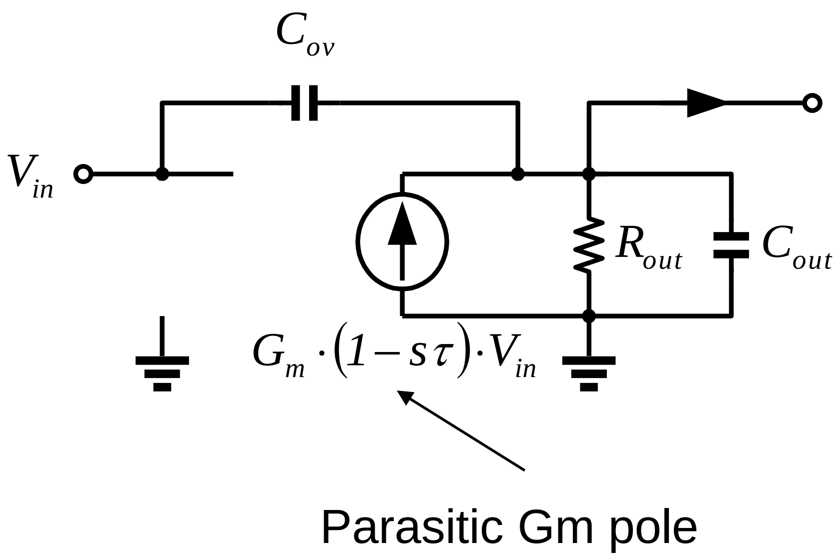
Symmetric Gm cell:

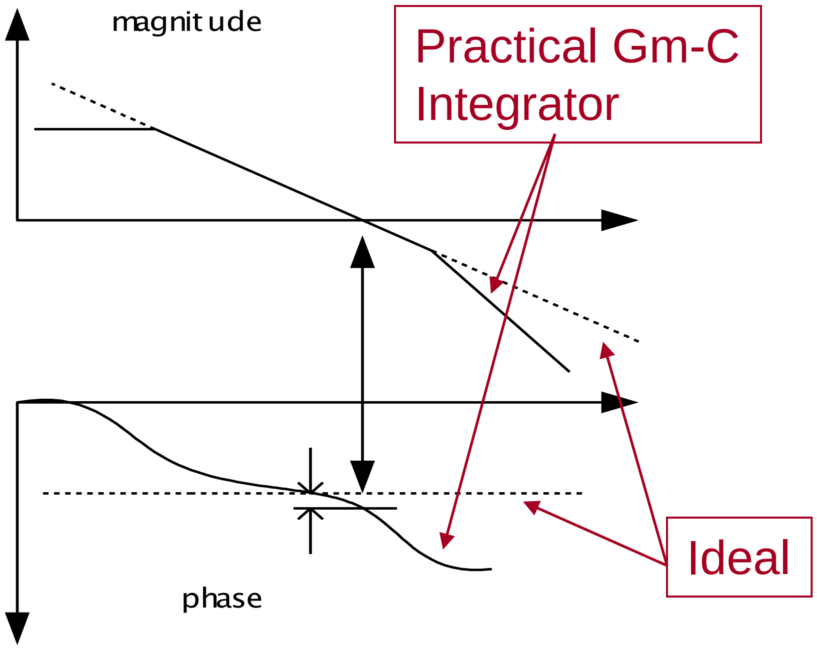


$$I_{out} = I_{plus} - I_{minus} = g_m(V_{plus} - V_{minus}) = g_m V_{in}$$

Introduction: Gm Cell

Practical Gm Model

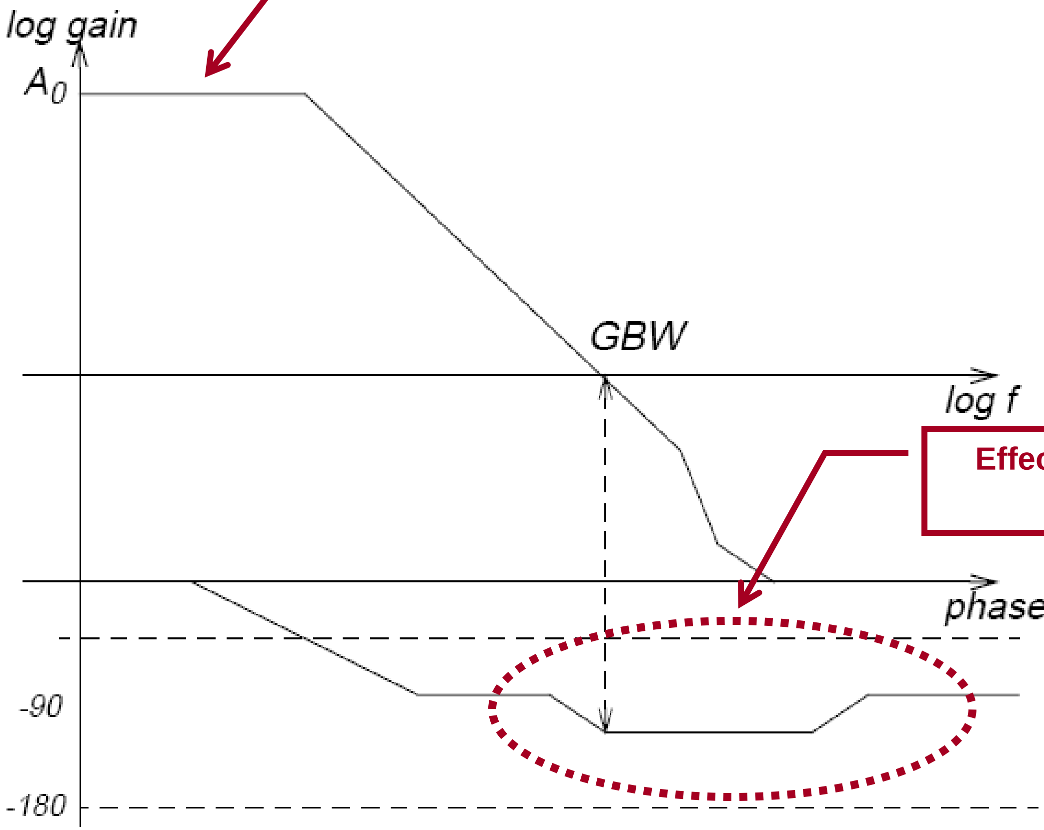




Introduction: Gm Cell

Practical integrator

Finite DC gain due to finite output resistance of Gm



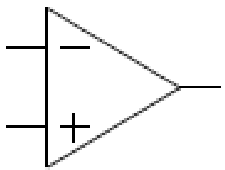
Effect of parasitic poles and zeros

Introduction: General Properties

Gm Cell VS OPAMP

Opamp

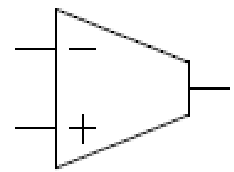
Voltage controlled voltage source



- Low output impedance
- Output in the form of voltage
- Can drive R-loads
- Good for RC filters, OK for SC filters
- Extra buffer adds complexity, power dissipation

OTA

Voltage controlled current source



- High output impedance
- In the context of filter design called gm-cells
- Output in the form of current
- Cannot drive R-loads
- Good for SC & gm-C filters
- Typically, less complex compared to opamp→ higher freq. potential
- Typically lower power
- no need to complex output stages

Filter Implementation

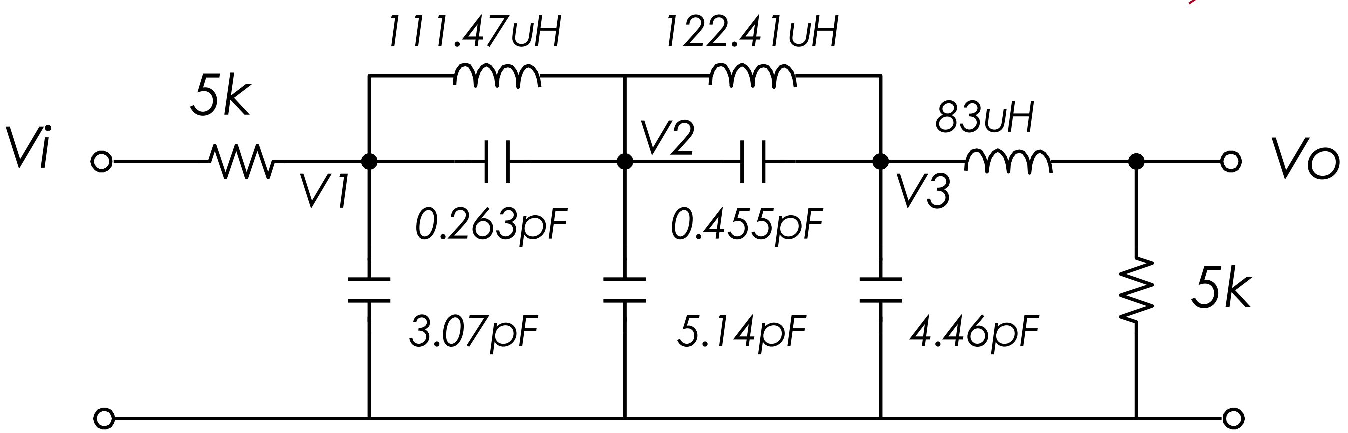
Implementation Techniques:

1- Element Substitution Technique
2- Signal Flow Graph Technique
3- Chain Method (Biquad)

Filter Implementation: Replacement Technique

Replacement Technique:

Replace Inductors/Resistors with Equivalent Gm-C Gyrator/Transconductor

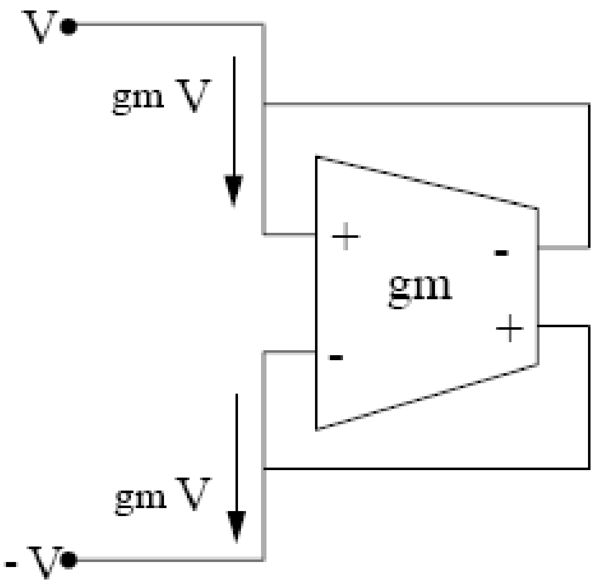


Example

Large inductors and capacitors So, could not be implemented on chip

Filter Implementation: Replacement Technique

Simulating the grounded R:

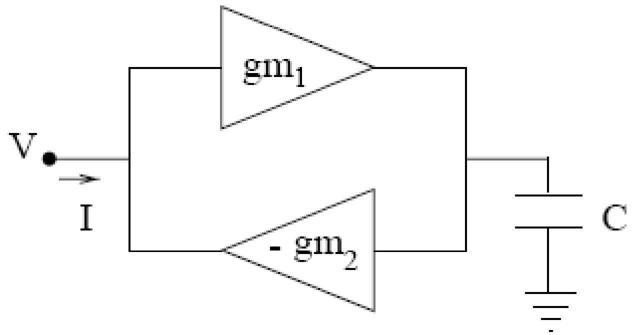


$$R = \frac{V}{I} = \frac{V}{g_m V} = \frac{1}{g_m}$$

Filter Implementation: Replacement Technique

Simulating the grounded L:

Active inductor = gyrator

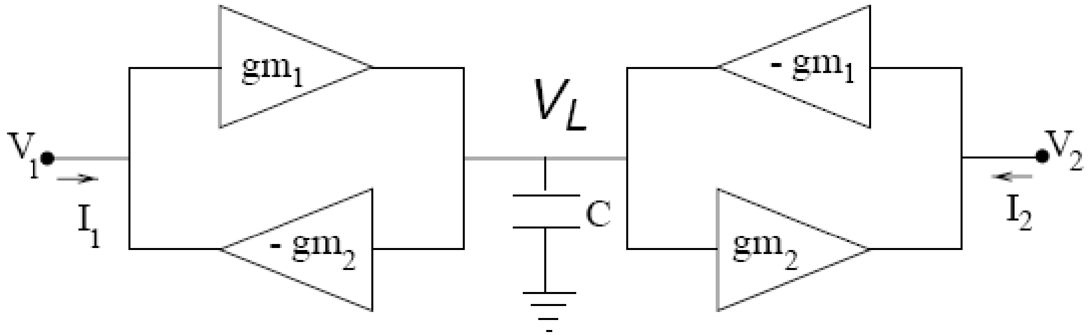


$$L = \frac{C}{g_{m1} g_{m2}}$$

$$Z = \frac{V}{I} = \frac{sC}{g_{m1} g_{m2}}$$

Filter Implementation: Replacement Technique

Simulating the floating L:



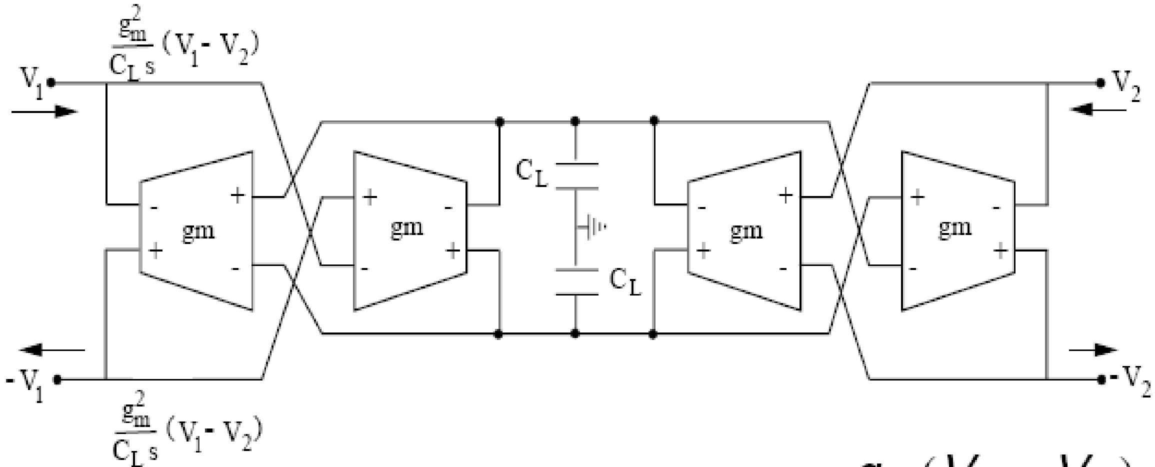
$$sC_LV_L + g_{m1}V_1 - g_{m1}V_2 = 0 \quad V_L = \frac{g_{m1}(V_2 - V_1)}{sC_L}$$

$$I_1 = -g_{m2}\frac{g_{m1}(V_2 - V_1)}{sC_L} \quad I_2 = g_{m2}\frac{g_{m1}(V_2 - V_1)}{sC_L}$$

$$L = \frac{V}{I} = \frac{C_L}{g_{m1}g_{m2}}$$

Filter Implementation: Replacement Technique

Simulating symmetrical floating gyrator:



$$sC_LV_L - g_mV_1 + g_mV_2 = 0 \qquad V_L = \frac{g_m(V_2 - V_1)}{sC_L}$$

$$I_1 = \frac{g_m^2(V_1 - V_2)}{sC_L} \qquad I_2 = \frac{g_m^2(V_2 - V_1)}{sC_L}$$

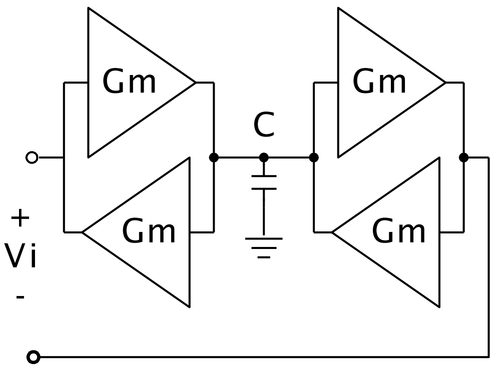
$$L = \frac{V}{I} = \frac{C_L}{g_m^2}$$

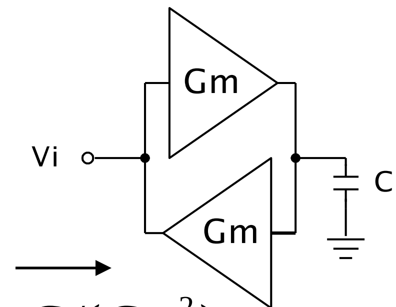
Filter Implementation: Replacement Technique

Simulating the passive devices:

Float L

$$L = C/(Gm^2)$$



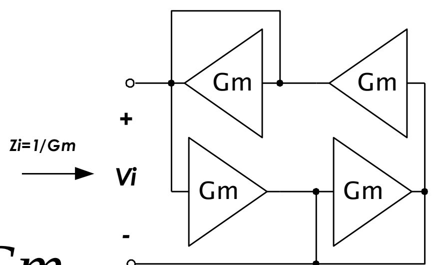


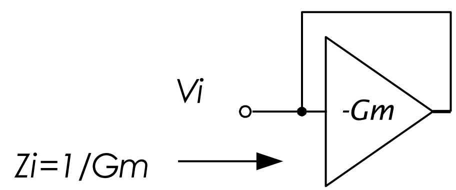
Grounded L

$$L = C/(Gm^2)$$

Float R

$$R = 1/Gm$$

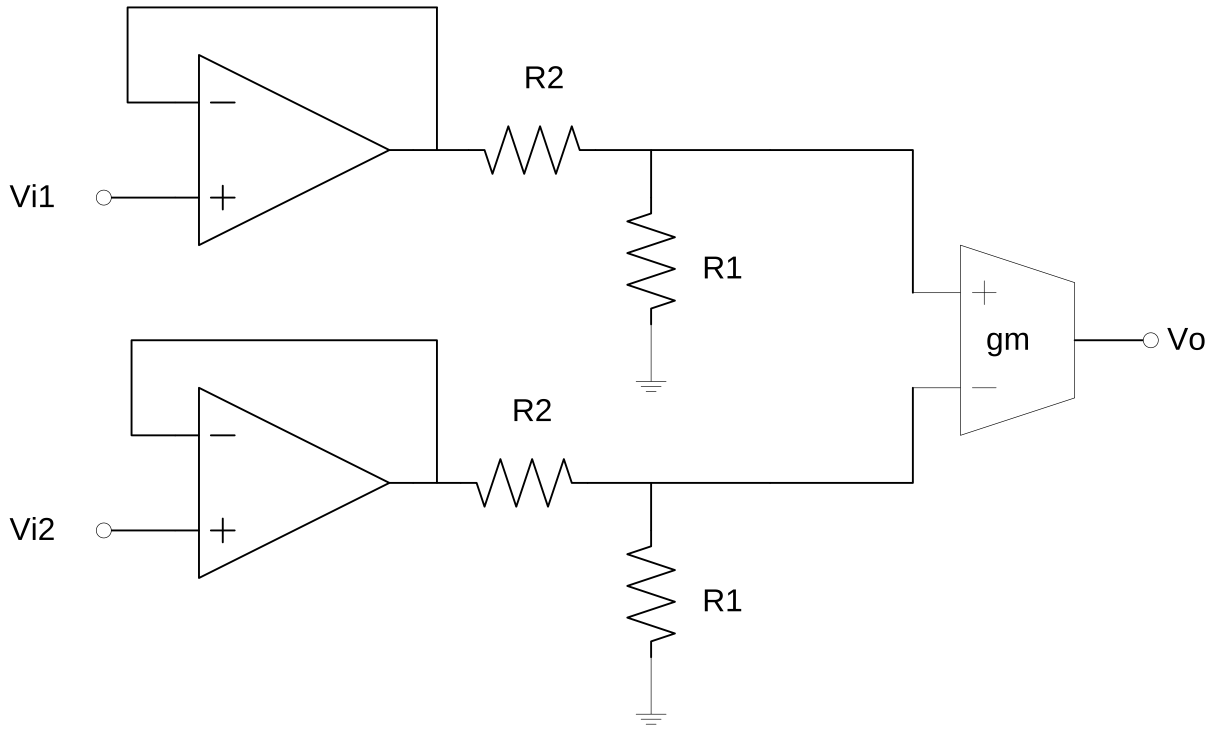




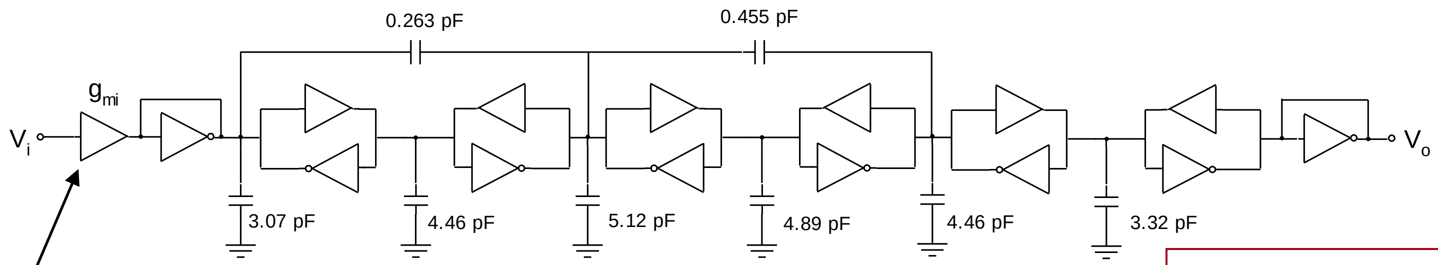
Grounded R

Filter Implementation: Replacement Technique

Example:
Implementing adder with desired weighting



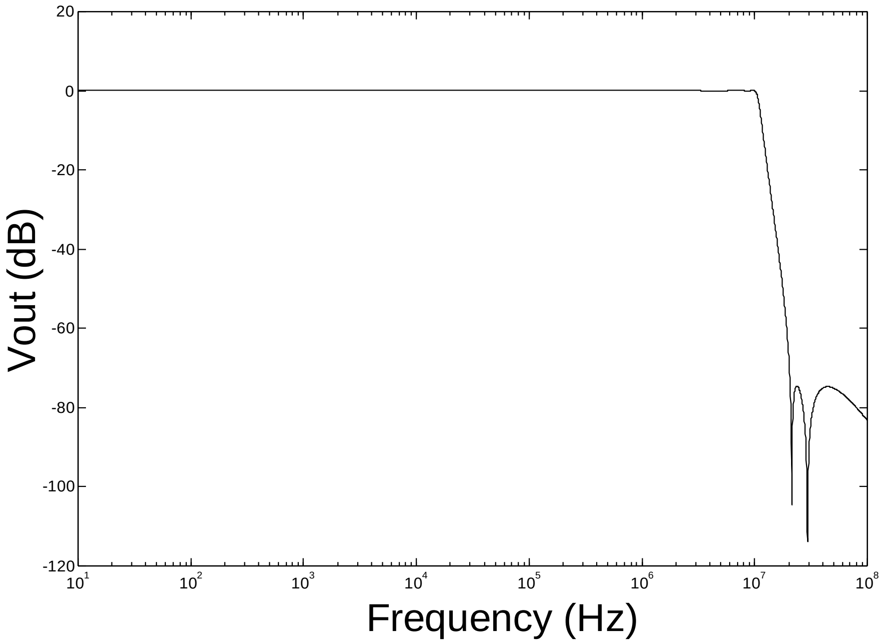
Filter Implementation: Replacement Technique



All Gm=200umho

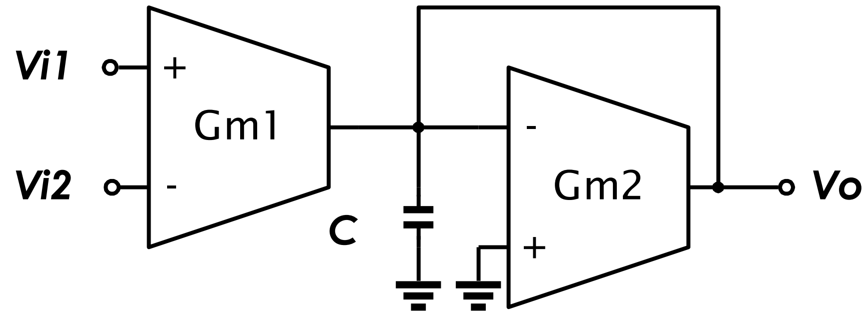
$$G_{mi} = 2 \cdot G_m$$

To Eliminate
the Input 6-dB Attenuation



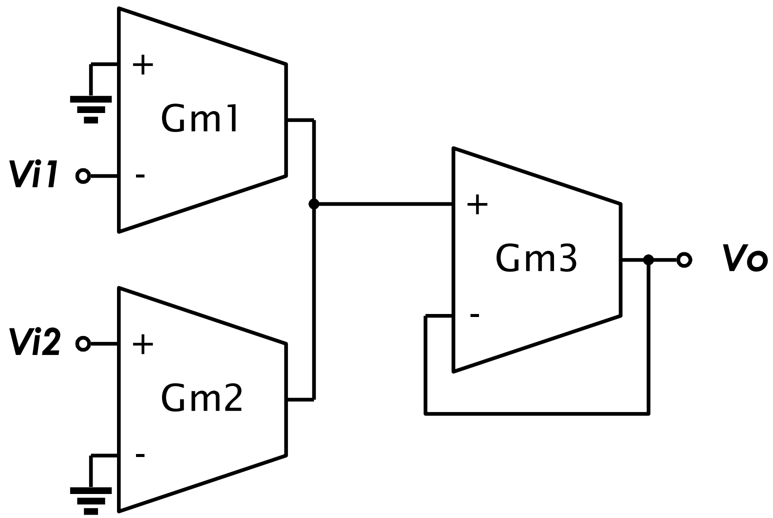
Filter Implementation: Replacement Technique

Some Other
Useful Building Blocks
For Replacement



$$Vo = \frac{Gm1}{sC + Gm2} \times (Vi1 - Vi2)$$

Lossy integrator



IMPORTANT NOTE:
Adding/subtracting of currents Is much simpler than voltage and hence could be helpful to Design high speed circuits

$$Vo = \frac{1}{Gm3} \times (-Gm1 \times Vi1 + Gm2 \times Vi2)$$

: Addition and division

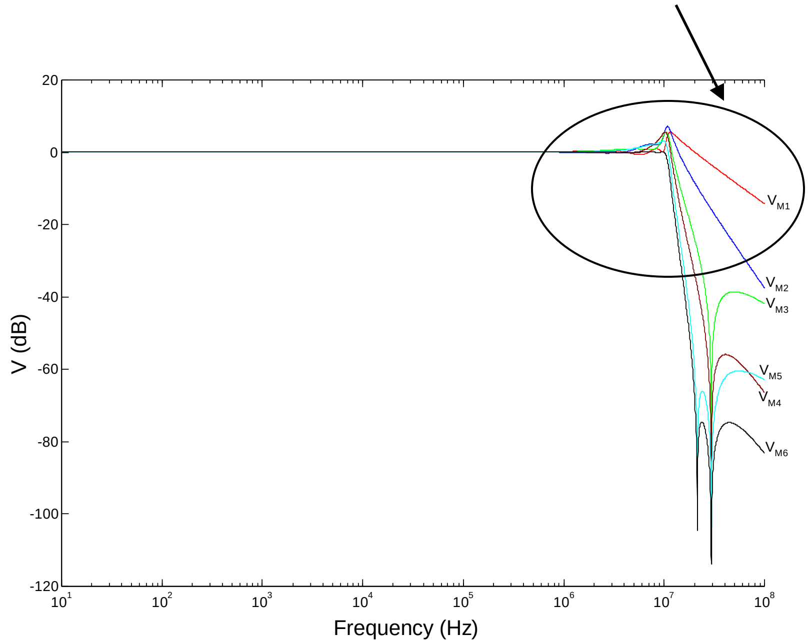
Filter Implementation: Replacement Technique Properties

Specifications:

1- Low sensitivity to device variations, since it is based on LC ladder topology

2- There is no general way for optimizing the DR (See the gain of intern=mediate nodes)

3- Hard to be tuned (difficult for tuning)



Filter Implementation: SFG

General Specifications of SFG method:

1- Low sensitivity to device variations
2- Filter implemented based on a LC ladder filter and using basic current/voltage equations in the filter
3- To convert finally all currents to voltage, a resistive coefficient of “R” will be used.
4- It is possible to optimize the DR (although it could be so much complex)
5- Number of transconductors is high

Filter Implementation: SFG

Method:

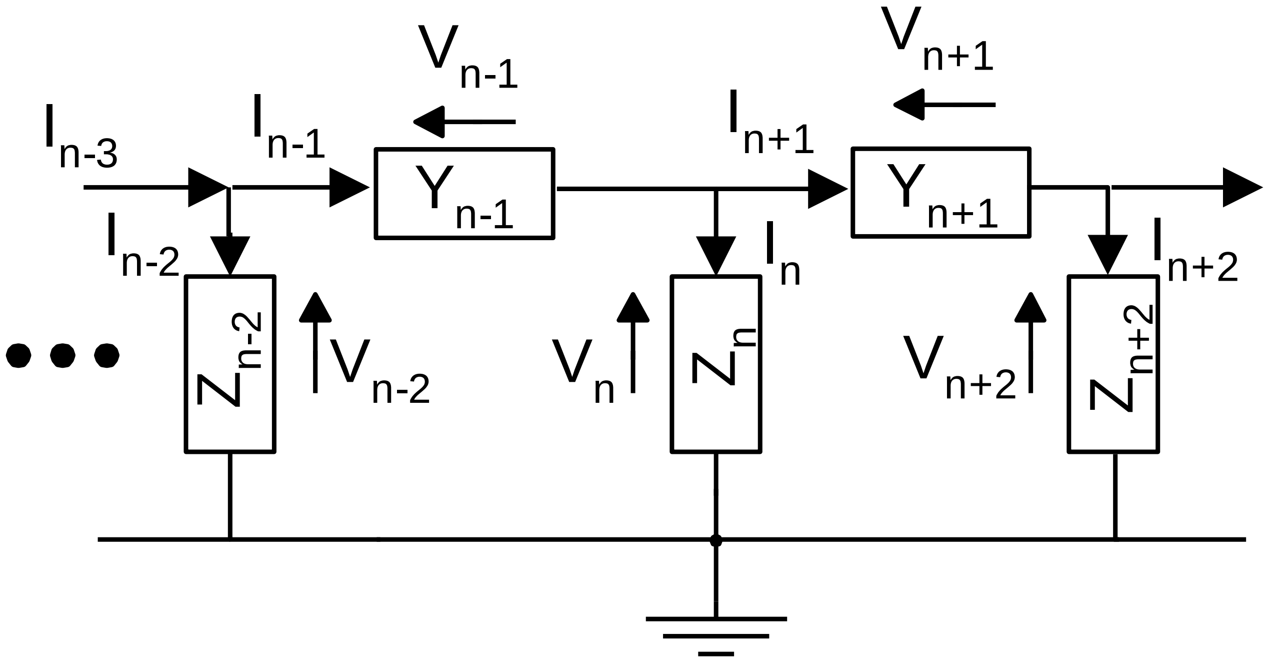
This method is based on common SFG method in filter implementation
And uses V/I equations to synthesis the filter:

0) Using approximations to extract filter transfer function
for example: Chebysheve, Butterworth, and so on.
1) Write the basic V/I equations
2) Select a nominal R* to convert all currents to voltages
3) Implement them based on active integrator (inverting or non-inverting) building blocks
4) Replacing active integrators by Gm-C counterparts
5) DR optimization

- It is preferred to use identical Gm cells for better matching and more simple tuning

Filter Implementation: SFG (Step-by-Step)

Step 1) Writing the basic equations
Step 2) Converting all currents to voltages



Filter Implementation: SFG

$$I_{n-2} = I_{n-3} - I_{n-1}$$

$$V_{n-2} = Z_{n-2} I_{n-2} = Z_{n-2}(I_{n-3} - I_{n-1})$$

$$V_{n-1} = V_{n-2} - V_n$$

$$I_{n-1} = Y_{n-1} V_{n-1} = Y_{n-1}(V_{n-2} - V_n)$$

$$I_n = I_{n-1} - I_{n+1}$$

$$V_n = Z_n I_n = Z_n(I_{n-1} - I_{n+1})$$

$$V_{n+1} = V_n - V_{n+2}$$

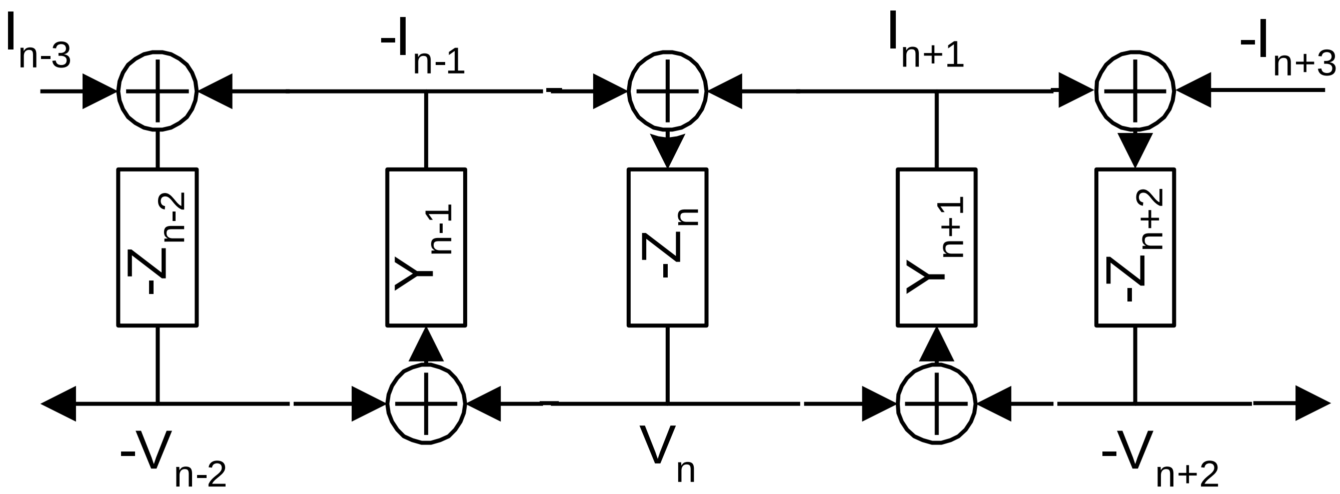
$$I_{n+1} = Y_{n+1} V_{n+1} = Y_{n+1}(V_n - V_{n+2})$$

$$I_{n+2} = I_{n+1} - I_{n+3}$$

$$V_{n+2} = Z_{n+2} I_{n+2} = Z_{n+2}(I_{n+1} - I_{n+3})$$

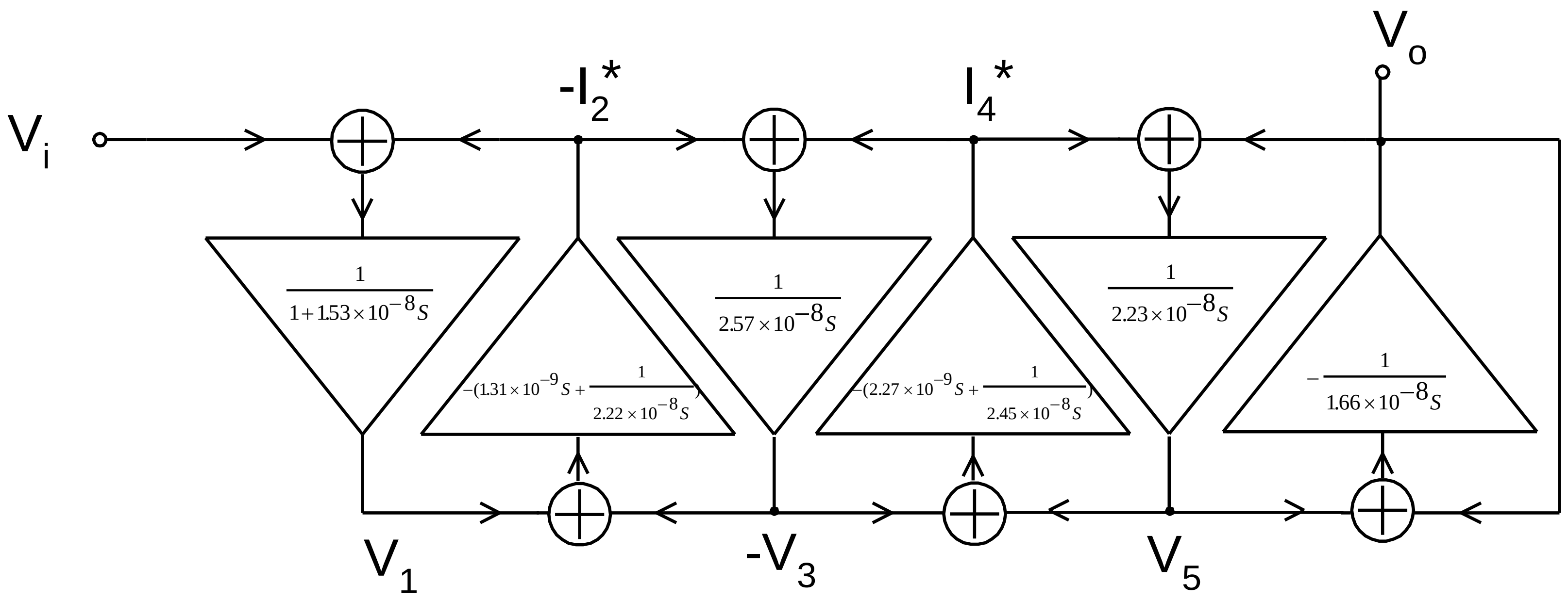
Filter Implementation: SFG

Implementing based on V/I equations:



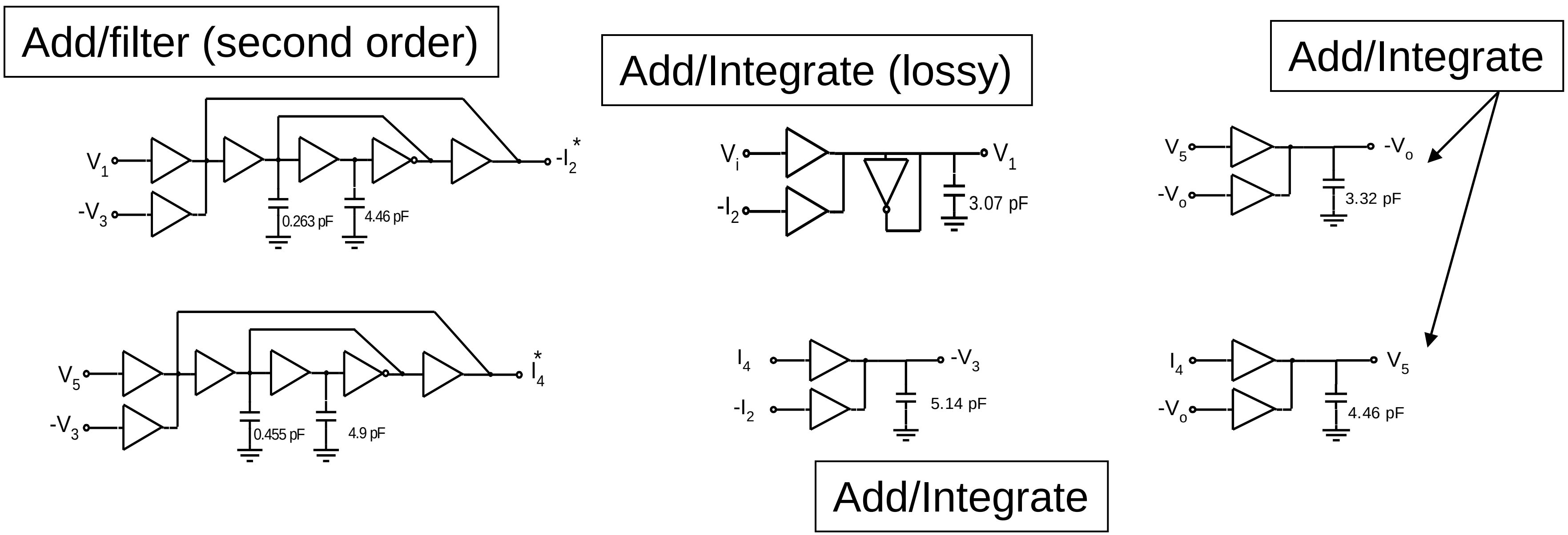
Filter Implementation: SFG

Step 3) Implementation based on active integrators



Filter Implementation: SFG

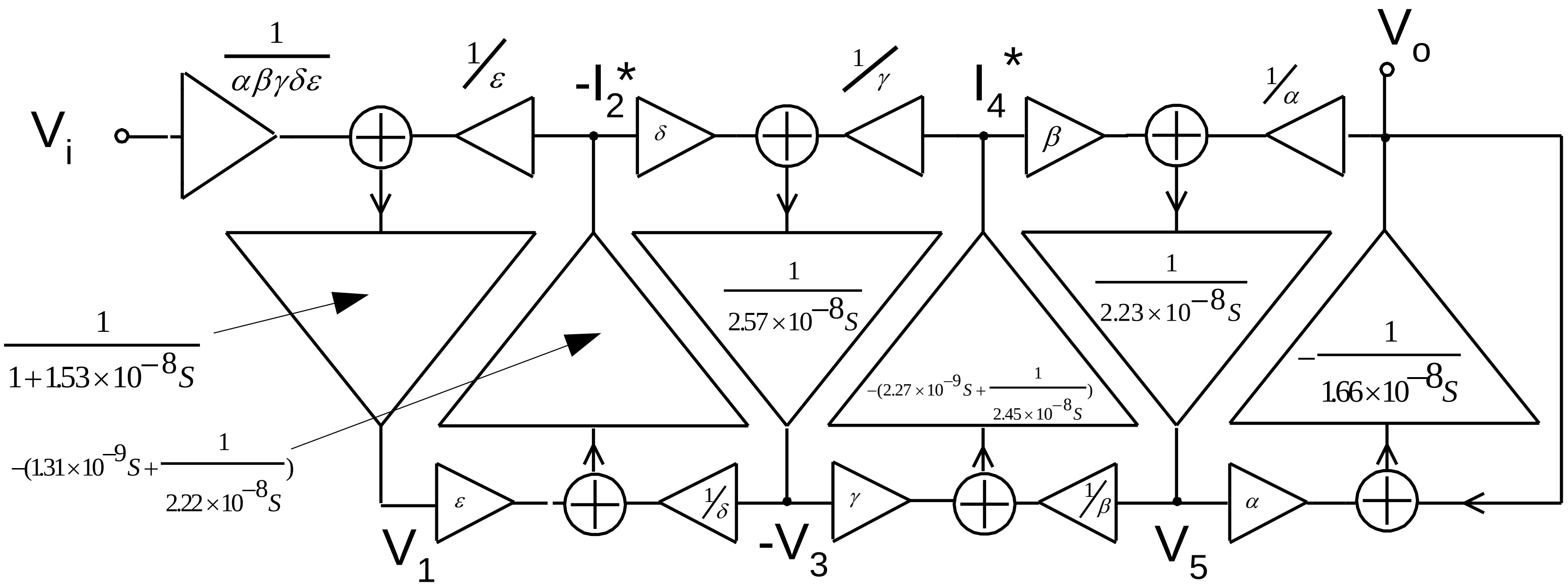
Step 4) Replacing by Gm-C integrators



Implementing the Basic Building Blocks Required in SFG Process

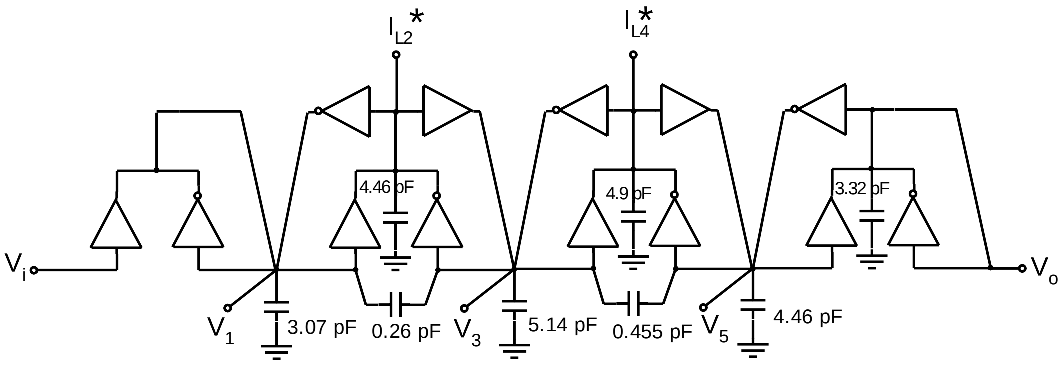
Filter Implementation: SFG

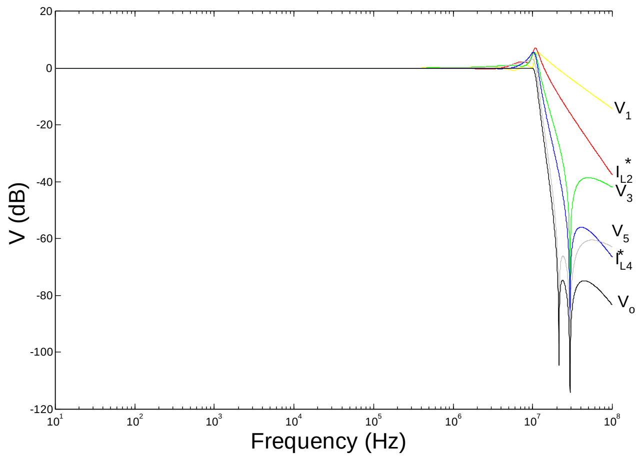
Step 5) DR Optimization



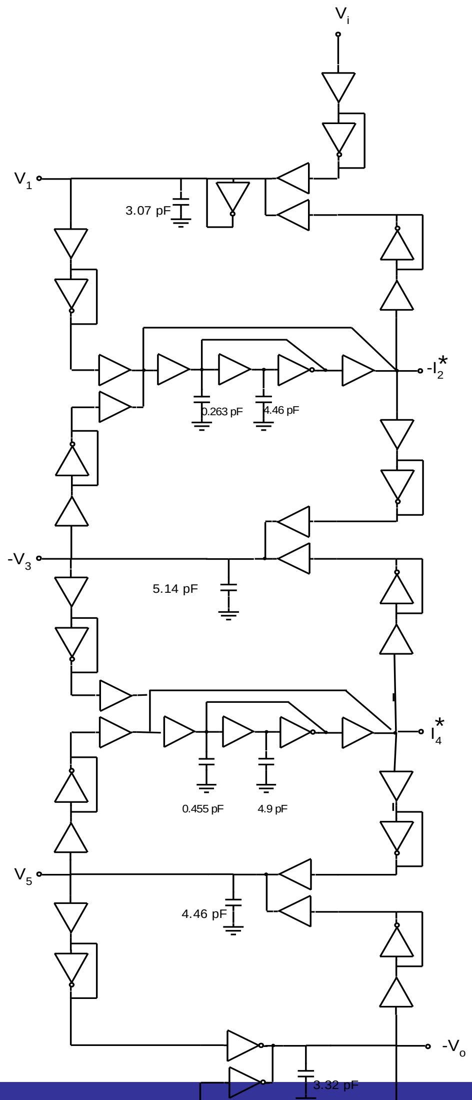
Filter Implementation: SFG

Final Implemented Filter (without DR optimization)



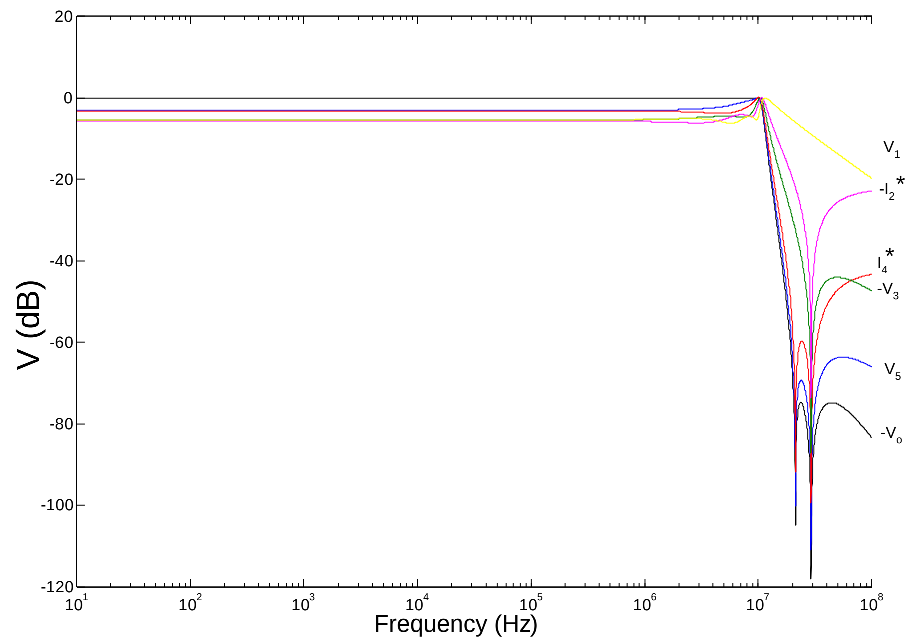


Filter Implementation: SFG



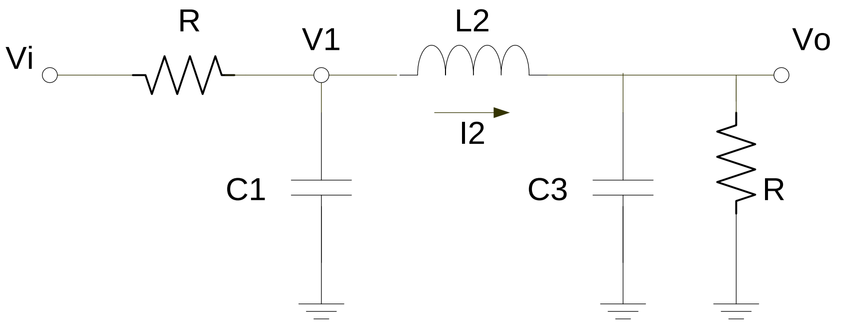
Final Implemented Filter (with optimized DR)

As can be seen, DR optimization need several extra Gm cells



Filter Implementation: SFG

Example 2: Implementing this filter



Step one :

Equation of voltage of Caps and current of Inductors.

$$V_1 = \frac{1}{C_1}\int\left(\frac{V_i - V_1}{R} - i_2\right)\cdot dt$$

$$i_2 = \frac{1}{L_2}\int\left(V_1 - V_o\right)\cdot dt$$

$$V_o = \frac{1}{C_3}\int\left(i_2 - \frac{V_o}{R}\right)\cdot dt$$

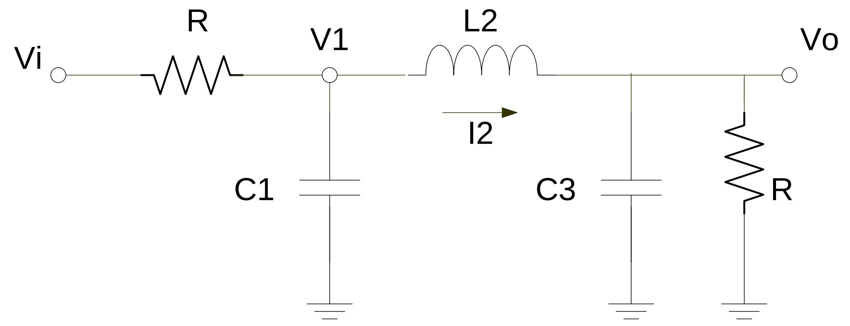
Step two :

Convert voltages to currents

$$i^* \rightarrow R^* \cdot i$$
$$R^* = R$$

Filter Implementation: SFG

Example 2: Implementing this filter



$$V_1 = \frac{1}{C_1}\int\left(\frac{V_i - V_1}{R} - i_2\right)\cdot dt$$

$$i_2 = \frac{1}{L_2}\int\left(V_1 - V_o\right)\cdot dt$$

$$V_o = \frac{1}{C_3}\int\left(i_2 - \frac{V_o}{R}\right)\cdot dt$$

➔

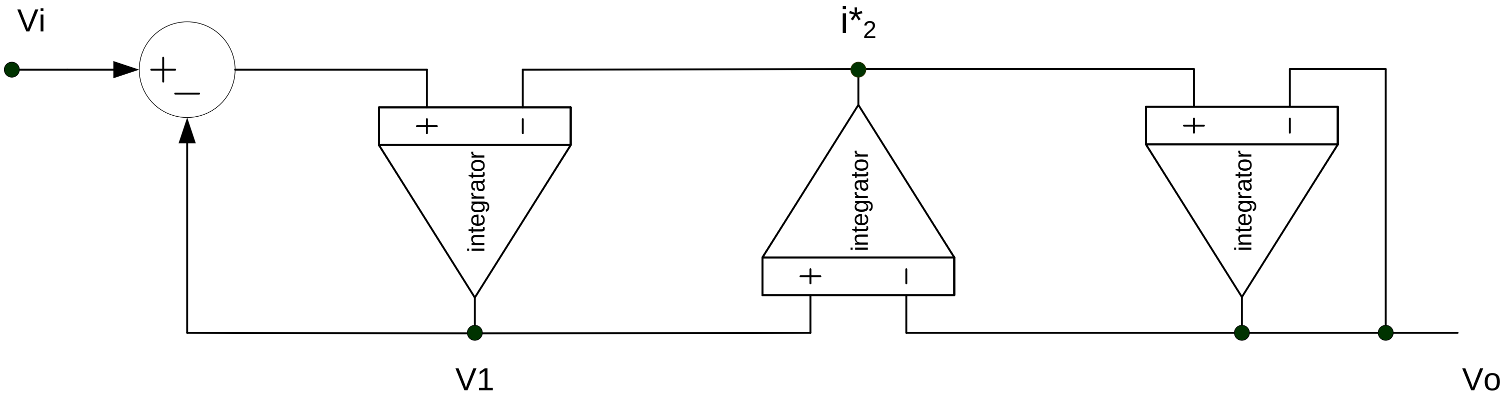
$$\begin{cases} V_1 = \dfrac{1}{R\cdot C_1}\int\left(\left(V_i - V_1\right) - i_2^*\right)\cdot dt \\ i_2^* = \dfrac{R}{L_2}\int\left(V_1 - V_o\right)\cdot dt \\ V_o = \dfrac{1}{R\cdot C_3}\int\left(i_2^* - V_o\right)\cdot dt \end{cases}$$

Filter Implementation: SFG

Example 2: Implementing this filter

Step three :

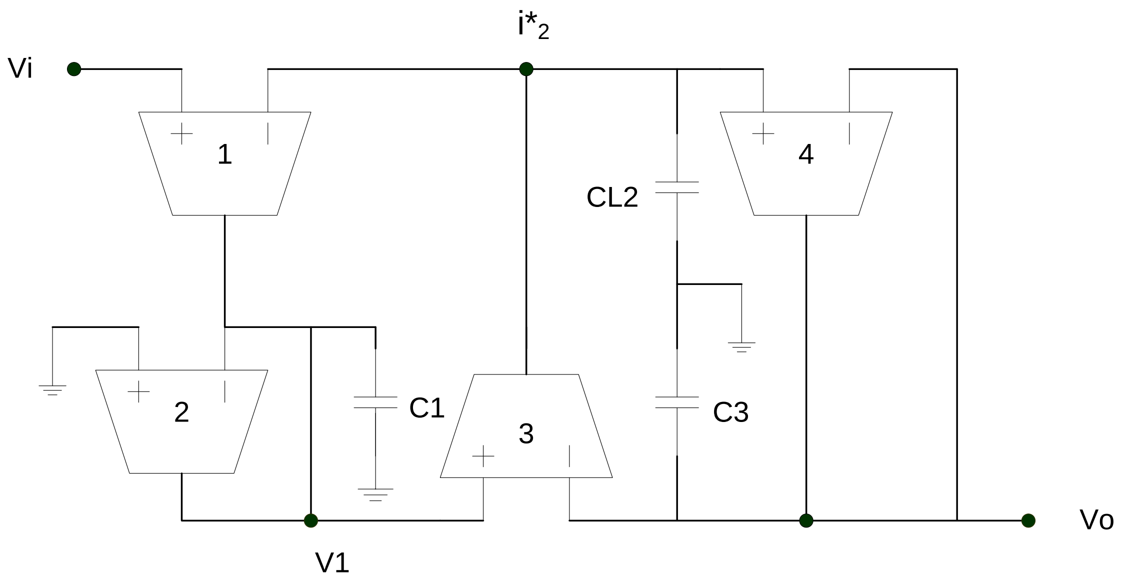
Implementation of above equations by using integrators in equivalent circuit



Filter Implementation: SFG

Step four :

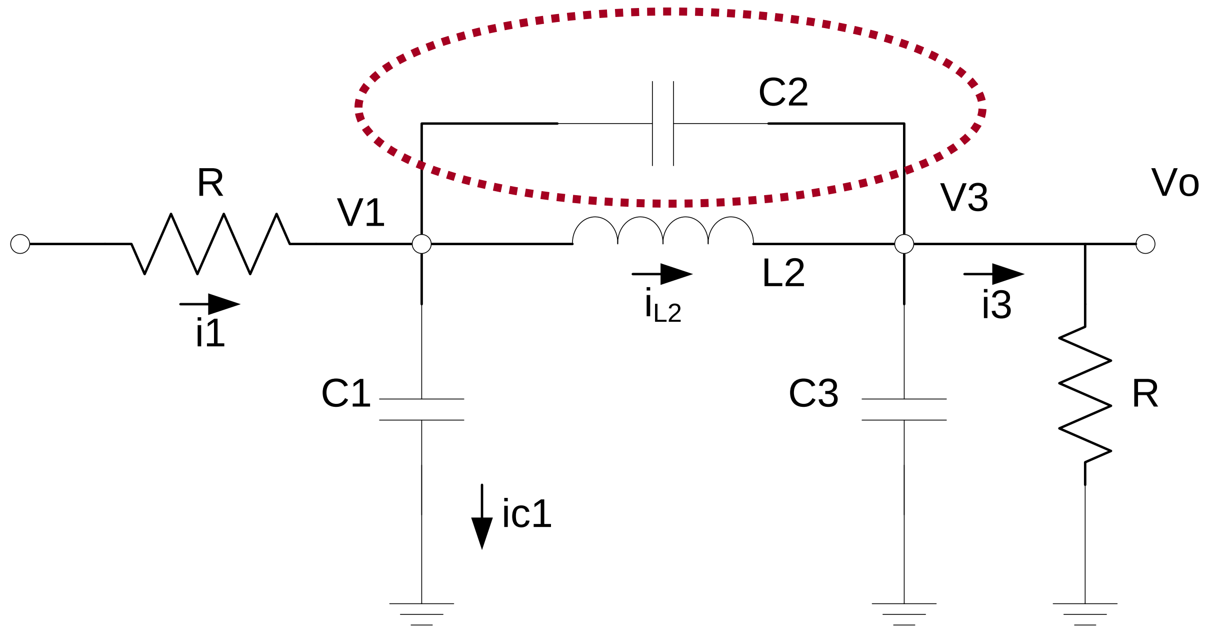
Implementation of above circuit by using Gm-C integrators and adders.



Where : $gm_1=gm_2=gm_3=gm_4=gm=1/R,\ CL_2=L_2.gm^2$

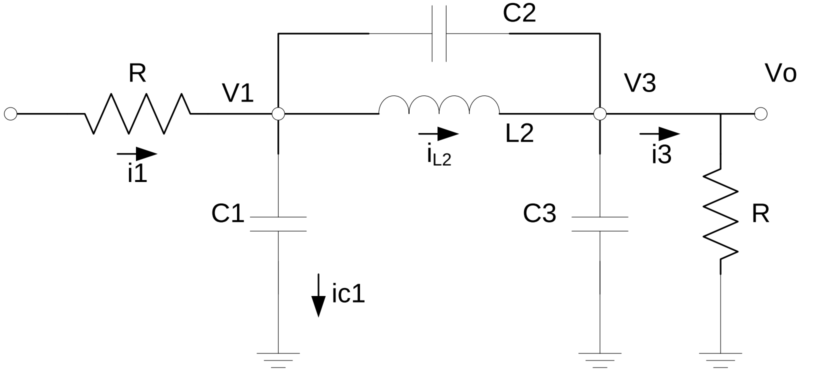
Filter Implementation: SFG

Implementing transmission zeros :



Filter Implementation: SFG

Implementing transmission zeros :



$$I_1 = \frac{V_i - V_1}{R}$$

$$V_1 = \frac{1}{S \cdot C_1}\left[I_1 - I_{L2} - S \cdot C_2\left(V_1 - V_3\right)\right]$$

$$I_{L2} = \frac{1}{S \cdot L_2}\left(V_1 - V_3\right)$$

$$V_3 = \frac{1}{S \cdot C_3}\left[I_{L2} - I_3 - S \cdot C_2\left(V_3 - V_1\right)\right]$$

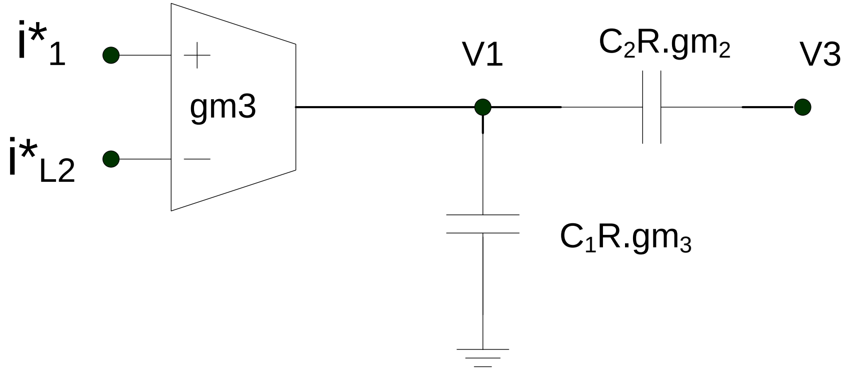
$$I_3 = \frac{V_3}{R}$$

$$I_i^* \rightarrow R^* \cdot I_i$$
$$R^* = R$$
➔➔

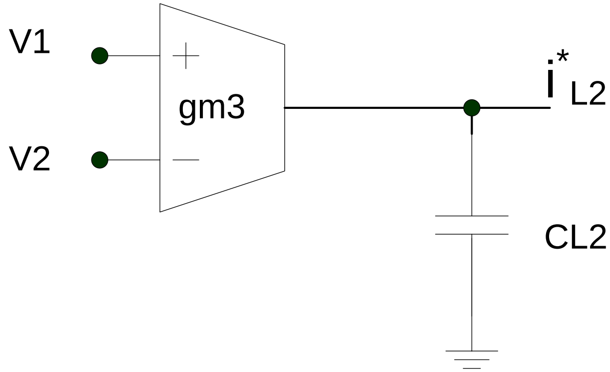
$$\begin{cases} I_1^* = V_i - V_1 \\ V_1 = \dfrac{1}{S \cdot C_1 \cdot R}\left[I_1^* - I_{L2}^* - S \cdot C_2\left(V_1 - V_3\right)\right] \\ I_{L2}^* = \dfrac{R}{S \cdot L_2}\left(V_1 - V_3\right) \\ V_3 = \dfrac{1}{S \cdot C_3 \cdot R}\left[I_{L2}^* - I_3^* - S \cdot C_2 \cdot R\left(V_3 - V_1\right)\right] \\ I_3^* = V_3 \end{cases}$$

Filter Implementation: SFG

Implementing transmission zeros :



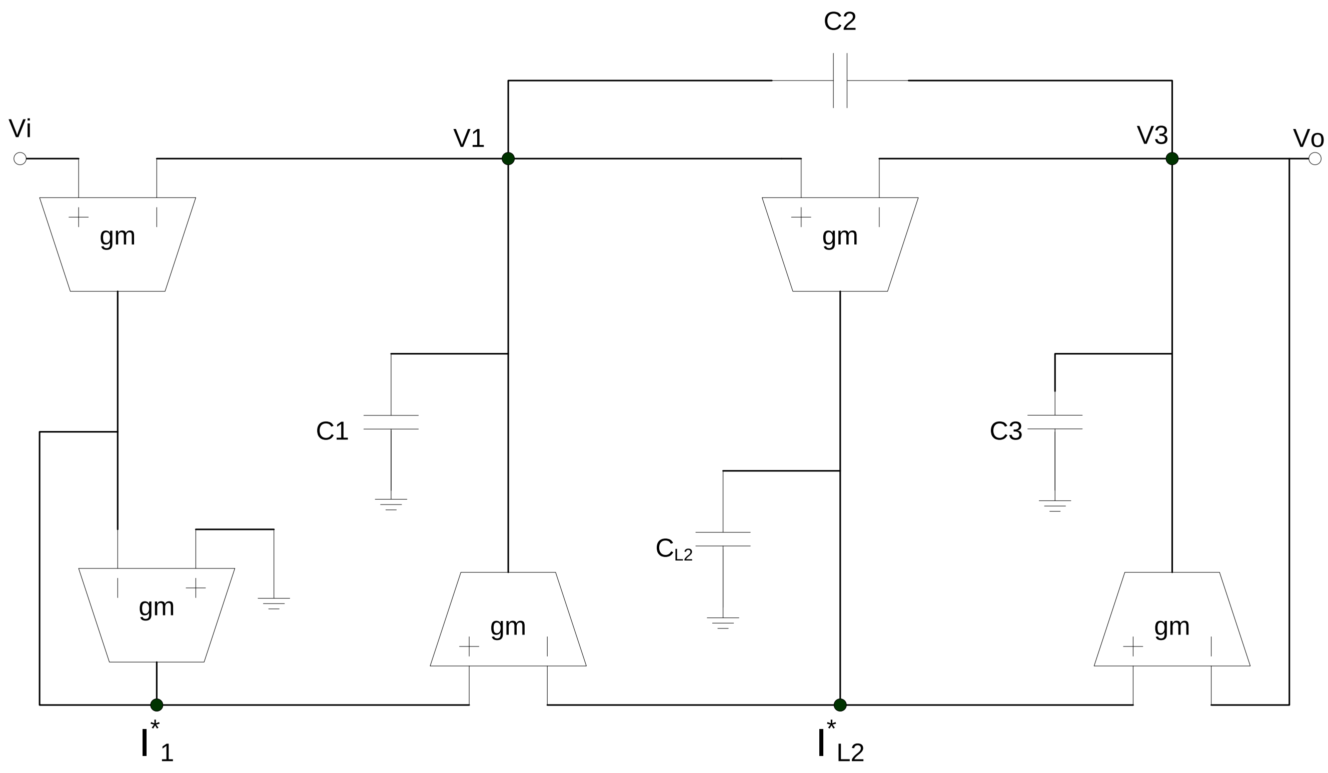
$$gm_3\left(I_1^* - I_{L2}^*\right) = S\left(C_1 \cdot R \cdot gm_3\right)V_1 + S\left(C_2 \cdot R \cdot gm_3\right)\left(V_1 - V_3\right)$$



$$C_{L2} = \frac{L_2 \cdot gm_4}{R}$$

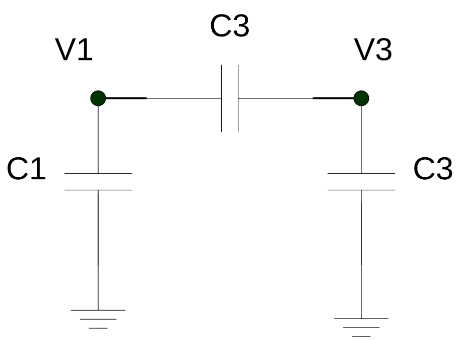
Filter Implementation: SFG

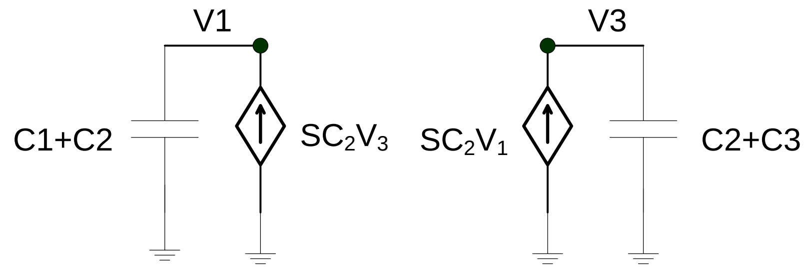
Implementing transmission zeros :



Filter Implementation: SFG

Implementing transmission zeros (second approach):





$$\begin{cases} S\cdot C_1\cdot V_1 + S\cdot C_2\left(V_1 - V_3\right) \\ S\cdot C_3\cdot V_3 + S\cdot C_2\left(V_3 - V_1\right) \end{cases} \equiv \begin{cases} S\cdot\left(C_1 + C_2\right)\cdot V_1 - S\cdot C_2 V_3 \\ S\cdot\left(C_2 + C_3\right)\cdot V_3 - S\cdot C_2 V_1 \end{cases}$$

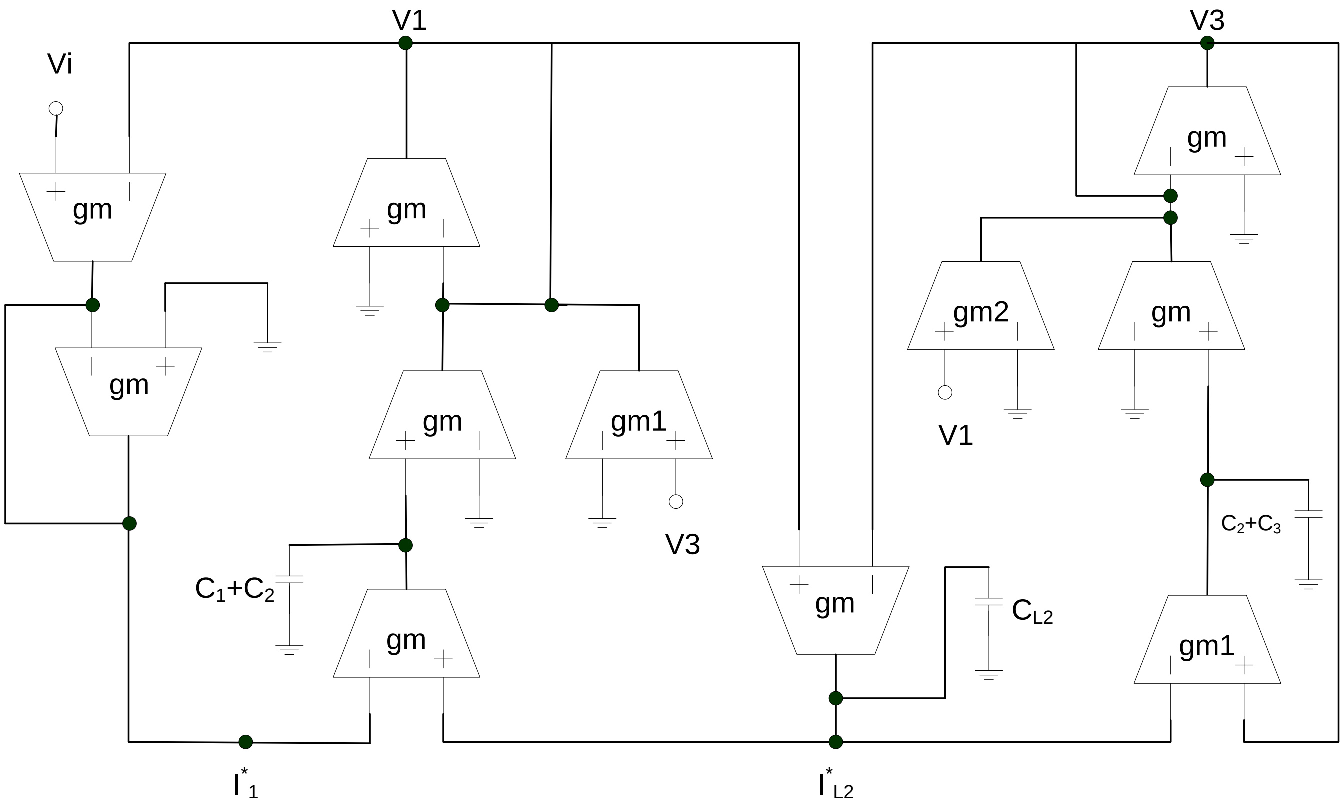
Filter Implementation: SFG

Implementing transmission zeros (second approach):

$$
\begin{cases}
I_1^* = V_i - V_1 \\
V_1 = \dfrac{1}{S(C_1 + C_2)\cdot R}\left[I_1^* - I_{L2}^* - S\cdot C_2 \cdot R \cdot V_3\right] \\
= \dfrac{1}{S(C_1 + C_2)\cdot R}\left(I_1^* - I_{L2}^*\right) + \dfrac{C_2}{C_1 + C_2}\cdot V_3 \\
I_{L2}^* = \dfrac{R}{S\cdot L_2}\left(V_1 - V_3\right) \\
V_3 = \dfrac{1}{S\cdot(C_2 + C_3)\cdot R}\left[I_{L2}^* - I_3^* - S\cdot C_2 \cdot R \cdot V_1\right] \\
= \dfrac{1}{S(C_2 + C_3)\cdot R}\left(I_{L2}^* - I_3^*\right) + \dfrac{C_2}{C_2 + C_3}\cdot V_1 \\
I_3^* = V_3
\end{cases}
$$

Filter Implementation: SFG

Implementing transmission zeros (second approach):



Filter Implementation: Chain

Specifications

1- Uses a chain of first/second order filters
2- Each stage could be implemented based on LC ladder
3- High sensitivity to device variation
4- Low number of Gm cells
5- DR could be simply optimized
6- Tuning is much easier than Replacement technique

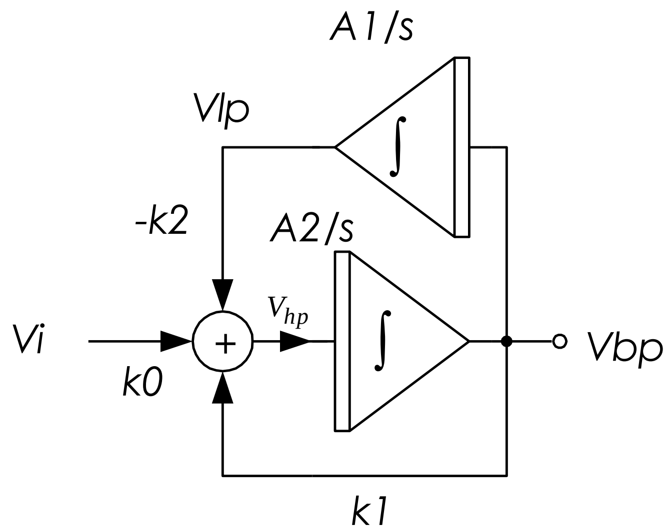
Filter Implementation: Chain

Two Integrator Filter Building Block (Base of many biquad stages)

$$V_{lp} = \frac{k_0 A_1 A_2}{s^2 + k_1 A_1 s + k_2 A_1 A_2}$$

$$V_{bp} = \frac{-k_0 A_1 s}{s^2 + k_1 A_1 s + k_2 A_1 A_2}$$

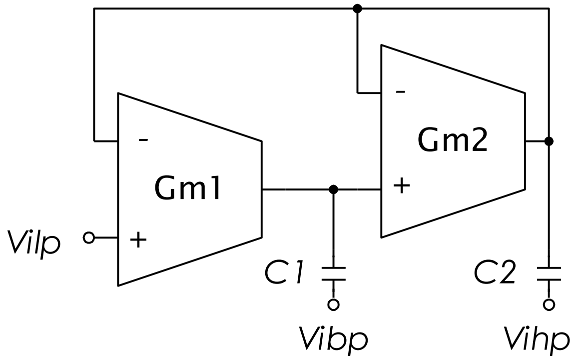
$$V_{hp} = \frac{k_0 s^2}{s^2 + k_1 A_1 s + k_2 A_1 A_2}$$



It is possible to implement all transfer functions of LP, BP, HP

Filter Implementation: Chain

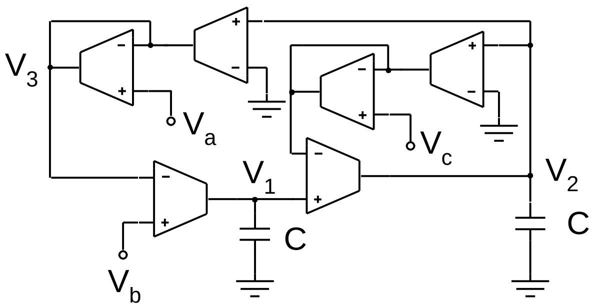
Examples of Biquad Stage

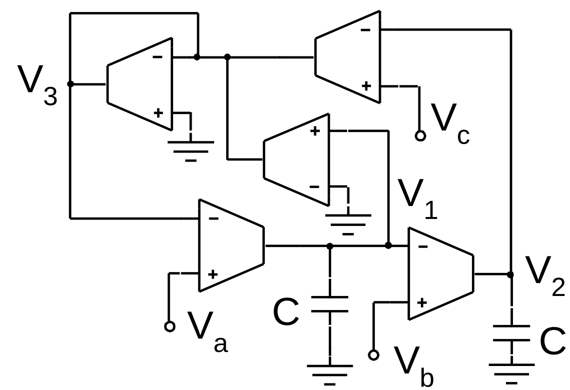


$$v_o = \frac{s^2(C_1C_2V_{ihp}) + s(C_1g_{m2}V_{ibp}) + (g_{m1}g_{m2}V_{ilp})}{s^2(C_1C_2) + s(C_1g_{m2}) + g_{m1}g_{m2}}$$

Other possible topologies:

Exercise: *Extract the transfer function of these biquad stages*





Filter Implementation: Chain

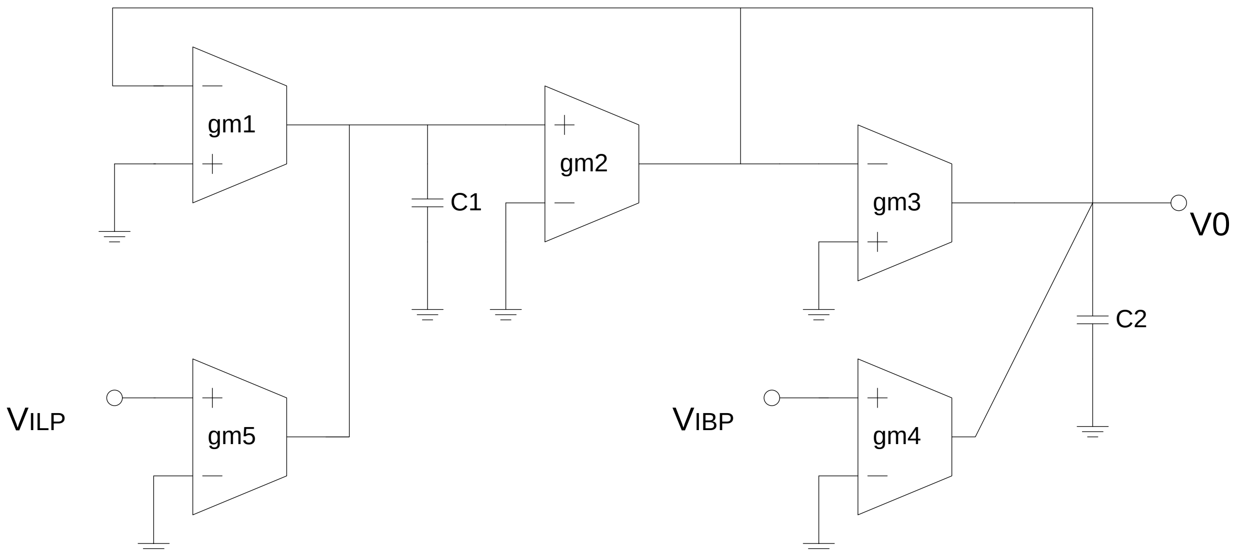
A complete Biquad Stage:

♦ Capability of tuning of Q or F seperately for poles and zeros

♦ High number of Gm cells

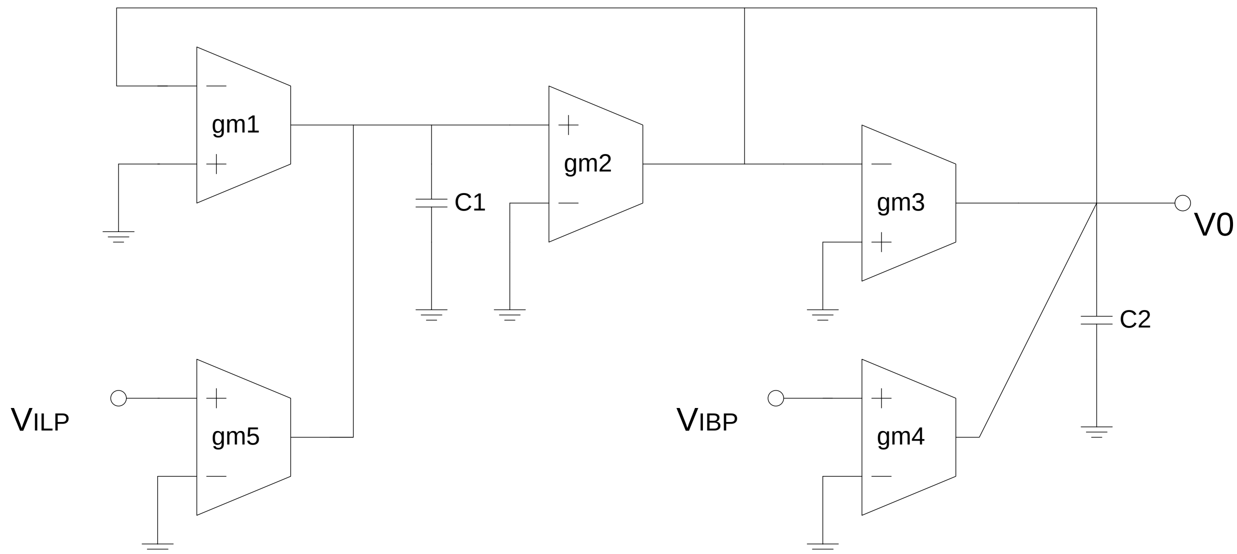
♦ All nodes have capacitor, so, parasitic capacitors could be absorbed. So, no extra parasitic pole/zero will be generated and just the existing poles/zeros of filter will be changed.

♦ Extra Gm cells could be removed if only a LPF is required



Filter Implementation: Chain

$$V_O = \frac{s^2 \cdot c_1 \cdot c_2 \cdot v_{IHP} + s \cdot c_1 \cdot gm_4 \cdot v_{IBP} + gm_2 \cdot gm_5 \cdot v_{ILP}}{s^2 \cdot c_1 \cdot c_2 + s \cdot c_1 \cdot gm_4 + gm_2 \cdot gm_1}$$

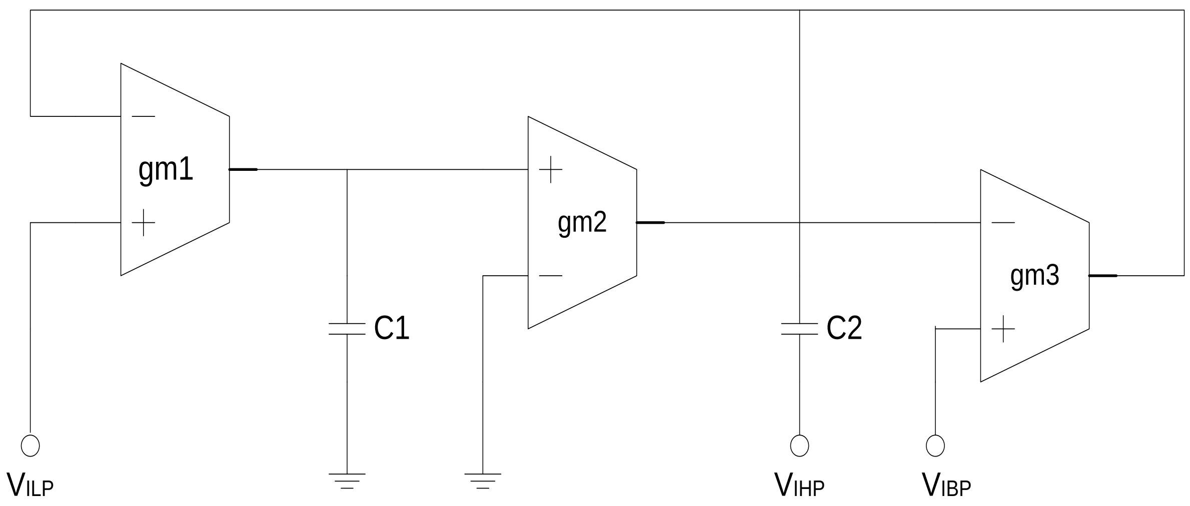


Filter Implementation: Chain

Input Condition	Transfer Function	Type of Circuit	ω_0	Q
$V_i=V_{ILP}$ $V_{IBP}=V_{ILP}=0$	$\frac{gm_1 \cdot gm_2}{s^2 \cdot c_1 \cdot c_2 + s \cdot c_1 \cdot gm_2 + gm_1 \cdot gm_2}$	LP with Tunable ω_0	$\frac{gm}{\sqrt{c_1 \cdot c_2}}$	$\sqrt{\frac{c_2}{c_1}}$
$V_i=V_{IBP}$ $V_{ILP}=V_{IHP}=0$	$\frac{s \cdot c_1 \cdot gm_2}{s^2 \cdot c_1 \cdot c_2 + s \cdot c_1 \cdot gm_2 + gm_1 \cdot gm_2}$	BP with Tunable ω_0	$\frac{gm}{\sqrt{c_1 \cdot c_2}}$	$\sqrt{\frac{c_2}{c_1}}$
$V_i=V_{IHP}$ $V_{ILP}=V_{IBP}=0$	$\frac{s^2 \cdot c_1 \cdot c_2}{s^2 \cdot c_1 \cdot c_2 + s \cdot c_1 \cdot gm_2 + gm_1 \cdot gm_2}$	HP with Tunable ω_0	$\frac{gm}{\sqrt{c_1 \cdot c_2}}$	$\sqrt{\frac{c_2}{c_1}}$
$V_i=V_{ILP}=V_{IHP}$ $V_{IBP}=0$	$\frac{s^2 \cdot c_1 \cdot c_2 + gm_1 \cdot gm_2}{s^2 \cdot c_1 \cdot c_2 + s \cdot c_1 \cdot gm_2 + gm_1 \cdot gm_2}$	Notch with Tunable ω_0	$\frac{gm}{\sqrt{c_1 \cdot c_2}}$	$\sqrt{\frac{c_2}{c_1}}$

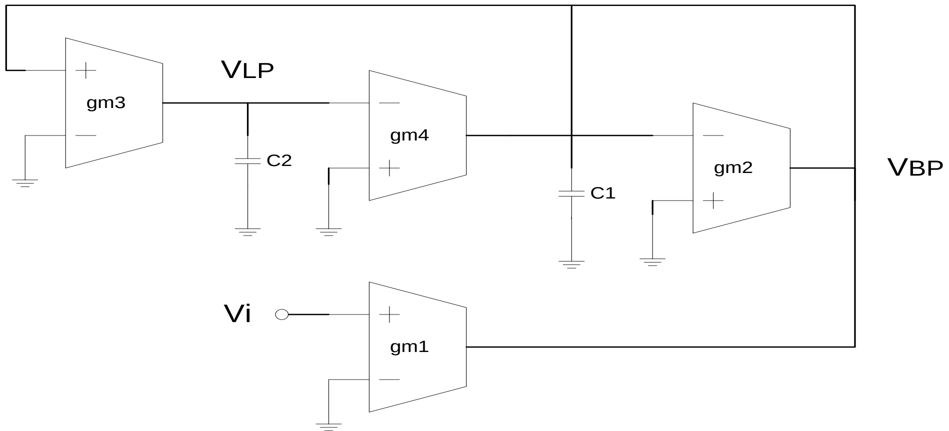
Filter Implementation: Chain

Equivalent Biquad stage uses two-input Gm cells



Filter Implementation: Chain

Example 1: Lowpass Biquad stage



KCLs

$$S \cdot V_{BP} \cdot C_1 = gm_1 \cdot V_i - gm_2 \cdot V_{BP} - gm_4 \cdot V_{LP}$$

$$S \cdot V_{LP} \cdot C_2 = gm_3 \cdot V_{BP}$$

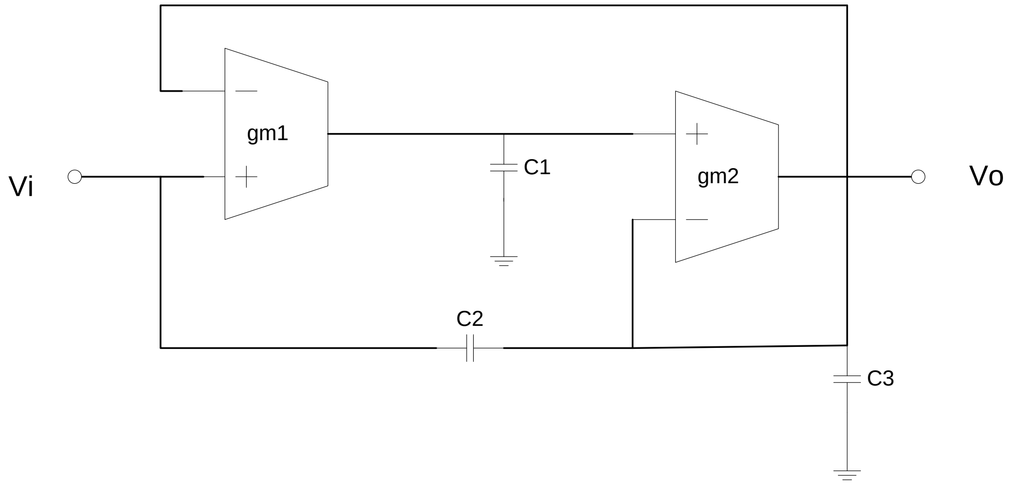
$\Rightarrow$

$$\frac{V_{BP}}{V_i} = \frac{S \cdot \frac{gm_1}{C_1}}{S^2 + S \cdot \frac{gm_2}{C_1} + \frac{gm_3 \cdot gm_4}{C_1 \cdot C_2}} = \frac{S \cdot \frac{gm_1}{C_1}}{S^2 + \frac{\omega_0}{Q} \cdot S + \omega_0^2}$$

$$\frac{V_{LP}}{V_i} = \frac{\frac{gm_3 \cdot gm_4}{C_1 \cdot C_2}}{S^2 + S \cdot \frac{gm_2}{C_1} + \frac{gm_3 \cdot gm_4}{C_1 \cdot C_2}}$$

Filter Implementation: Chain

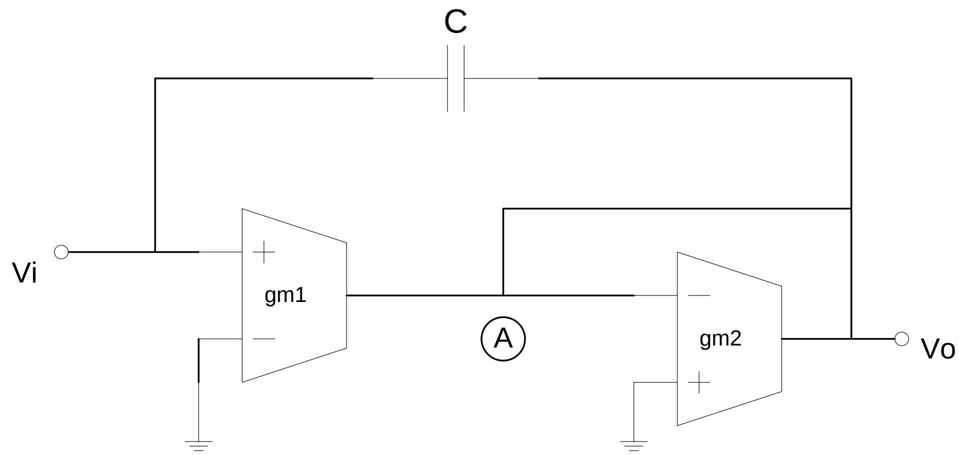
Example 2: Find the transfer function of this circuit



$$\frac{V_0}{V_i} = \frac{C_2}{C_1 + C_2} \cdot \frac{S^2 + \frac{gm_1 \cdot gm_2}{C_1 \cdot C_2}}{S^2 + S \cdot \frac{gm_2}{C_1 + C_2} + \frac{gm_1 \cdot gm_2}{C_1 \cdot (C_2 + C_3)}}$$

Filter Implementation: Chain

Example 3: Find the transfer function of this circuit



KCL @ node A ➔

$$gm_1 \cdot V_i - gm_2 \cdot V_o + S \cdot C(V_i - V_o) = 0$$

$$V_i(gm_1 + S \cdot C) = V_o(gm_2 + S \cdot C)$$

$$\Rightarrow$$

$$\frac{V_o}{V_i} = \frac{gm_1 + S \cdot C}{gm_2 + S \cdot C}$$

Filter Implementation: Chain

Main characteristics of Biquad Stage

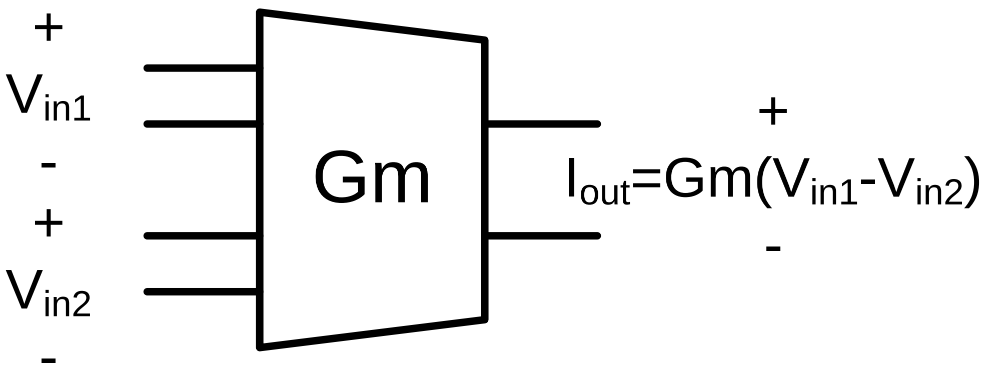
1- Nodes without Capacitor leads to high sensitivity to stray caps
2- Independent DR optimization of overall filter and biquad stage
3- Independent Freq and Q tuning capability
4- Q depends on ratio of capacitors and transconductors, so it could be accurate
5- Parasitic capacitors at the input or output of Gm cell, will not create new parasitic poles/zeros

Two Input Gm Cells

Using Two Input Gm Cells for Simplifying the Implementation:

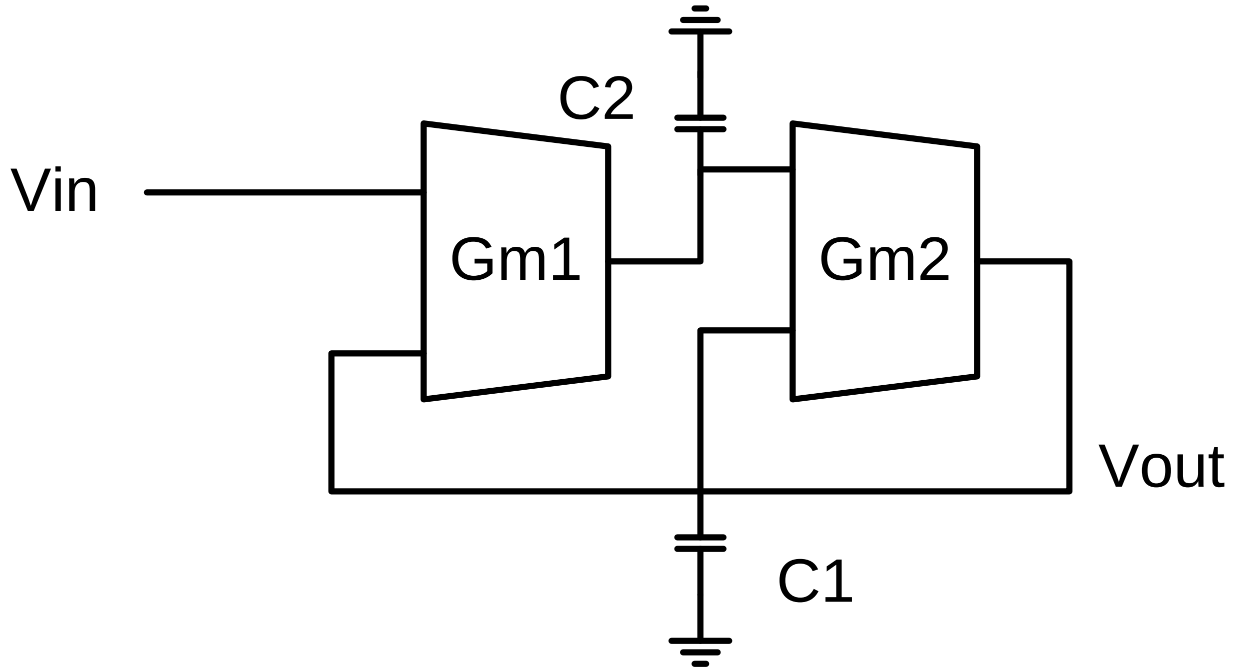
Two Input Gm Cell

- Reduces the circuit complexity
- Saves the number of output stages
- Saves the number of required CMFB circuits



Two Input Gm Cells

Example: Implementing Biquad Filter by two input Gm cells

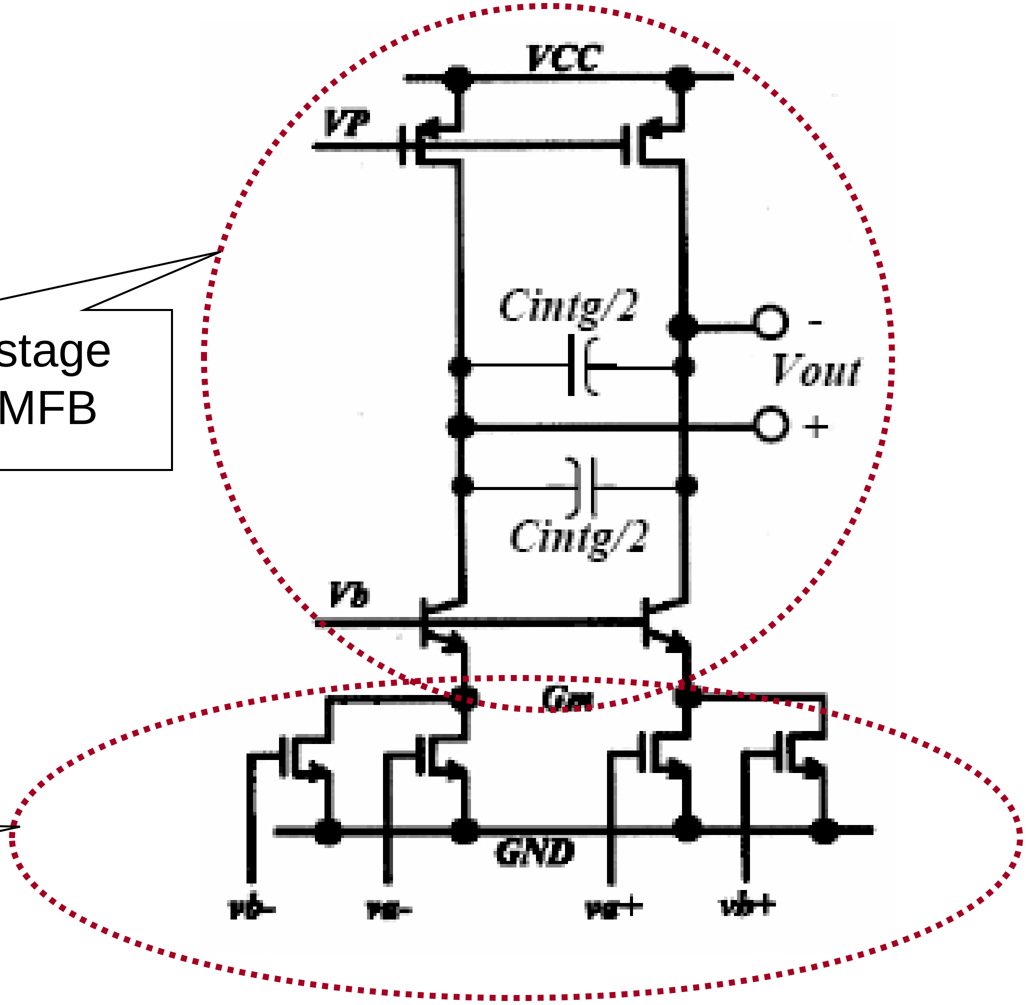


Two Input Gm Cells

Example: Two input Gm Cell

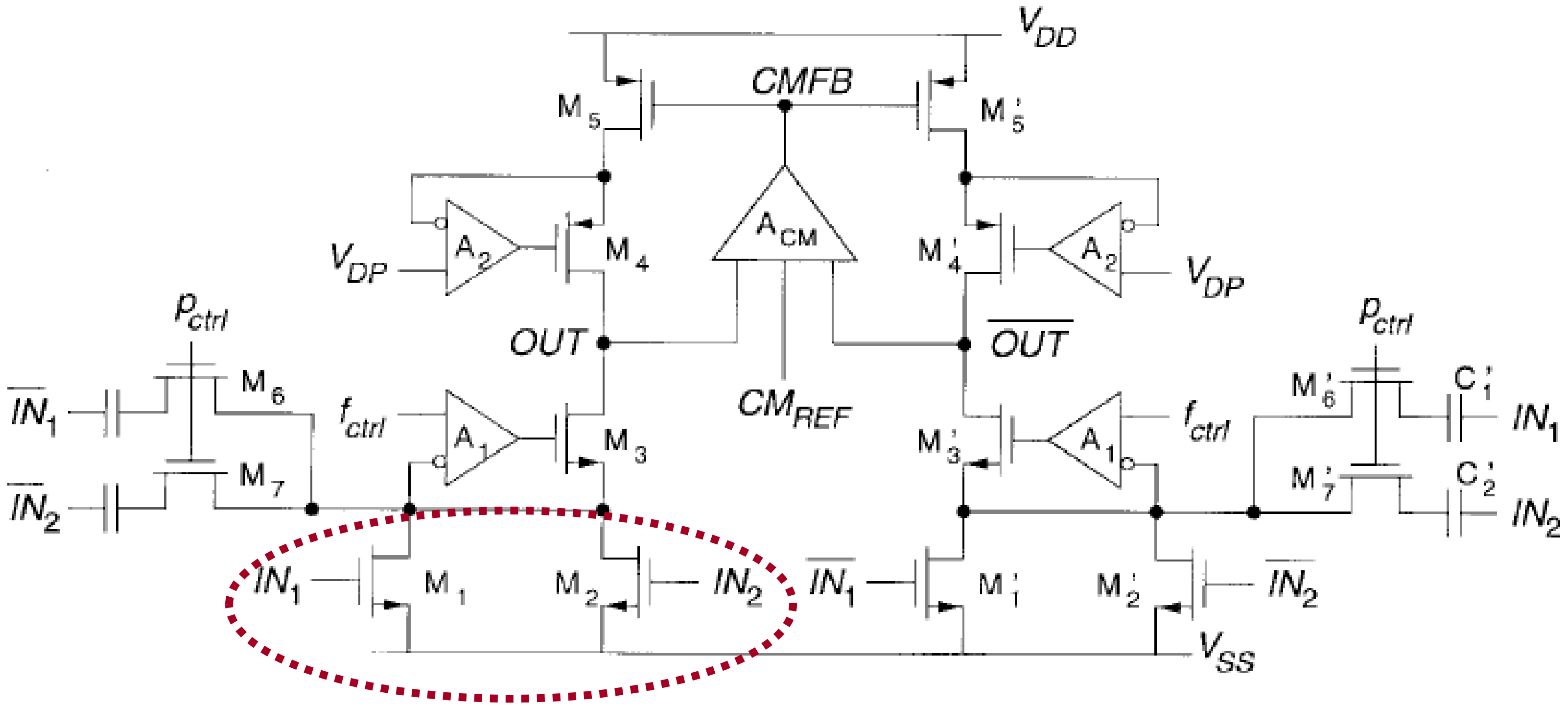
Share output stage
With share CMFB

Input stage
(could be even more than two inputs)



[JSSC, 1993, Laber]

Two Input Gm Cells

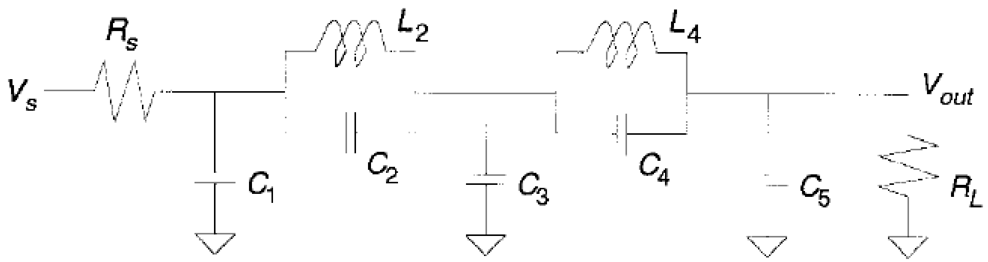


[JSSC, 1998, Yoo]

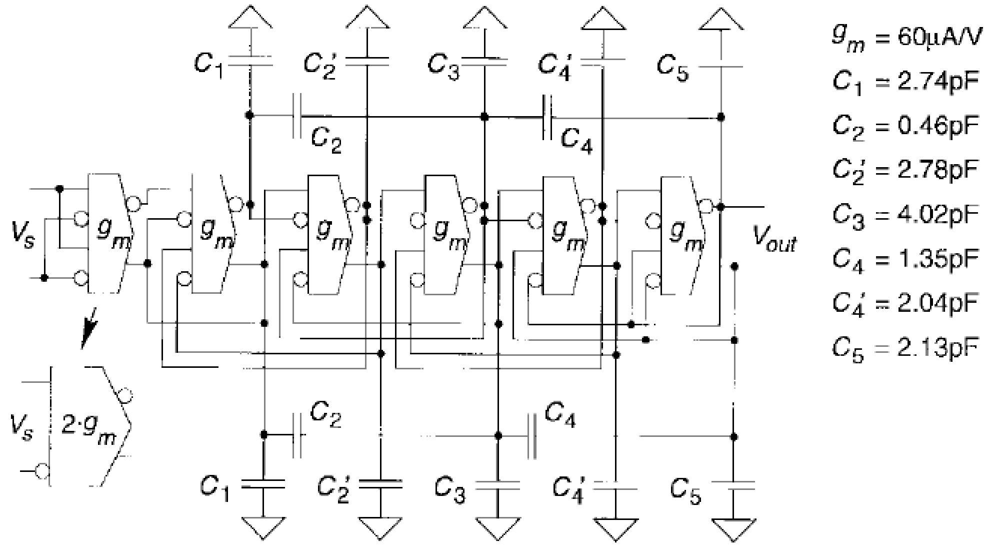
Two Input Gm Cells

Filter implementation using two input Gm cells

[JSSC, 1998, Yoo]



(a)



(b)

Single / Differential Ended

Converting S to D:

<u>Advantages:</u>

- Less sensitivity to supply/substrate noise
- Linearity (better HD2)
- Higher DR