**Evaluating Congestion in Two-Stage Supply Chain Network Structures: Link Control by Previous and Next Stages**

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Abstract

Congestion in the input and intermediate products of the supply chain is a critical issue that reduces efficiency and performance in any organization. Eliminating congestion can enhance supply chain performance. Therefore, addressing congestion and finding methods to eliminate the resulting inefficiencies is highly important for managers and researchers. The supply chain of any organization has a network structure, and each network consists of multiple sub-sections. Therefore, congestion and its evaluation must be considered in the entire network and also in the sub-sections to avoid inadequate or incorrect performance assessments. This paper presents the congestion concept for two-stage structures and proposes a new method for evaluating congestion in a two-stage serial supply chain in line with the development of the Cooper method. The suggested method introduces two scenarios based on the expansion of two production possibility sets (PPSs). And incorporates the concept of a 'link' to identify potential congestion between stages and the entire network. Finally, to demonstrate the applicability of the proposed method, a practical example is provided to analyze and compare the results of the two scenarios.

Keywords: Supply chain, congestion, data envelopment analysis, intermediate products

1. **Introduction**

Data Envelopment Analysis (DEA), introduced by Charnes, Cooper, and Rhodes [1], is a widely used management tool for assessing the relative efficiency of comparable decision-making units (DMUs) that utilize multiple inputs to produce multiple outputs**.** One of the applications of this mathematical technique is the examination of supply chain performance as a production process. Considering that in DEA, if a unit is deemed inefficient, this lack of success, in addition to managerial weakness, can be due to the presence of congestion in the unit's inputs, and since the performance of units is influenced by the resources used, output also increases with an increase in input resources. However, this may not always be the case, such that an increase in input leads to a decrease in output, in which case congestion has occurred, and given that the economic concept of congestion is widely observable in most phenomena, its presence in the production process of the supply chain cannot be ignored and considering that supply chain sustainability refers to achieving the social, environmental, and economic goals of organizations with effective coordination of internal processes and structures. so, in addition to initial inputs and final outputs, understanding internal structures can provide a broader perspective on supply chain performance and management.

Among the most recent studies on supply chain management, Nadime et al. [2 ] conducted a literature review on supply chain management, and the results of this study guide businesses and researchers. Liagkouras et al. [3 ] presented a literature review on supply chain management, examining the various challenges faced by modern supply chain networks in an integrated manner, to cover the gaps.

 Congestion in the input and internal structures (intermediate products) is also an important issue in evaluating supply chain performance because the presence of congestion in the input and intermediate products is a key issue whose identification, evaluation, and elimination can contribute to practical benefits, including reduced costs and increased productivity. Research related to congestion was first initiated by Färe and Svensson [4], while before that, this topic was unknown in economics. After that, their study was developed by Färe and Grosskopf [5], and they proposed a model based on the content of DEA in calculating congestion, which until then was the only available method for evaluating congestion in problems. This led to further research in this area until the single-model method was presented by Cooper [6] in 1996, following congestion studies. Cooper's research was expanded by Brockett et al., [7], and finally the BCSW method was introduced. In further research, Jahanshahloo and Khodabakhshi [8], based on the output improvement model, presented a method for calculating congestion, which was presented in the form of two models and required solving three linear programming problems. Asgharian et al.,[9], to improve the presented method with his colleagues, reduced it to solving two problems within one model, which is of great computational importance. Nora and colleagues [10] proposed a new method for calculating the congestion of each DMU, which significantly reduces calculations. Wei and Yan [11] examined the necessary and sufficient conditions for the occurrence of congestion with respect to the type of return to scale due to DEA efficiency. Tone and Sahoo [12] introduced a new method for calculating scale among congestion by introducing the concepts of strong and weak congestion. In recent years, numerous studies have explored congestion in the DEA. For instance, Karimi et al. [13] introduced a novel approach for assessing congestion in DEA models with integer-valued data. Meanwhile, Khoveyni et al., [14] examined congestion identification in contexts involving negative data, further distinguishing between the least and most congested DMUs.

Meng et al., [15] introduced a two-stage model for evaluating congestion in mixed energy systems. This method was applied to analyze inefficiency and congestion in 16 OPEC countries, showing that fossil energy plays a significant role in congestion in these countries. Mehdiloozad et al., [16] showed that all points in a given region exhibit similar congestion characteristics, even when dealing with negative data. Chen et al. [17] categorized energy congestion into two distinct types in their research: desirable and undesirable energy congestion. Chen et al. [18] proposed a new congestion measurement approach and identified three types of congestion with specific objectives. Saati and Shadab [19] examine supply chain congestion related to inputs or intermediate products and explore various scenarios that can lead to congestion in intermediate products to optimize supply chain efficiency. Their study focused on a two-stage serial supply chain, identifying its internal units and the set of production possibilities that show strong or weak congestion in intermediate products through comparative analysis.

Shabanpour et al. [20] presented a new DEA model that showed that increasing congested inputs may lead to more outputs. Adimi et al. [21] were able to present concepts related to the phenomenon of congestion regardless of the efficiency value. Salehi et al. [22] also proposed a new method for identifying and assessing congestion based on the definitions of congestion. Ren et al. [23] also showed in an innovative way that congestion can be eliminated by increasing the inputs related to research activities in Chinese universities. They also addressed the relationship between congestion and overinvestment and were able to analyze the results. Navidi et al. [24] present a method for measuring congestion without solving the model.

 GholamAzad and Pourmahmoud [25] were able to present a new method for measuring congestion at network stages, but congestion is not examined in the entire structure of a system. Also, Shadab et al. [26] presented a new algorithm through the relationship between the s-shape and the shape of the production function and the geometric characteristics of the support points to identify weak and strong DMUs.

 Shadab et al. [27] considered a two-stage sustainable supply chain structure and then determined the congestion status with respect to the role of intermediate products and identified different scenarios where congestion might occur in intermediate products. Bazyar et al. [28] have presented new processes for evaluating congestion and poor congestion specifically in the context of sustainable supply chains. These processes are based on empirical analysis and provide practical insights for the water and wastewater industry in Iran. Kasaei and colleagues. [29] proposed a two-stage series network structure that defines the concept of congestion for systems with a general two-stage structure and stages. Jabbari and colleagues [30] also evaluate congestion in units with increases and decreases in inputs.

In this article, we study congestion in a two-stage system. We analyze two scenarios involving intermediate products. This approach evaluates potential congestion in different sections and the entire system. It helps organizations with supply chain structures assess congestion both system-wide and in specific sections. The goal is to identify and resolve congestion issues effectively.

The remainder of the paper is structured as follows: Section 2 reviews the basic concepts and presents the single-model Cooper method and its extended version. In section 3, a new definition of congestion in two-stage structures is provided. Then, considering the extended model of the Cooper method, a new method for evaluating congestion is introduced. The proposed method will be presented under two distinct scenarios in Section 3. In section 4, a practical example will be provided and conclusions will be drawn.

**2. Preliminaries:**

In this article, it is assumed that there are n DMUs, each of which converts m distinct inputs into n distinct outputs.

 Input and output of the j-th decision unit  is displayed byand. It is also assumed that  and  .

2.1. Single-stage structure

2.1.1: Decision-making unit in a single-stage structure

Definition 2.1: The decision-making unit in a single-stage structure is the unit in which the output vector Y is generated using the input vector X. The structure of a single-stage DMU is displayed below.



Fig. 1: One-step structure.

**2-1-2 -Production Possibility Set in a One-Stage Structure:**

In a single-stage structure, the production possibility set is as follows:



Under variable returns to scale (VRS), the production possibility set transforms to (1):

 (1)

**2.1.3- Congestion in a single-stage structure:**

**Definition 2.2:** Evidence of congestion is present in the performance of when reductions in one or more inputs are associated with increases in one or more outputs—without worsening any other input or outputs. More precisely, congestion is evidenced when the attainment of maximal output requires a reduction in one or more of the input amounts used [4].

**2.1.4. Cooper's method for assessing congestion:**

Cooper et al. [4], in 2002, proposed model (2) to identify and evaluate congestion.

 (2)

congestion in this model is measured using the following theorem:

**Theorem 2.1**: Congestion is present if and only if in an optimal solution  of model (2) at least one of the following two conditions is satisfied [4]:

1)  and there is at least one 

2) There exists at least one  and at least one 

**2.2. Two-stage structure**

**2.2.1. Decision-making unit in a two-stage structure:**

**Definition 2.3:** The decision-making unit with a series two-stage network structure refers to a unit that itself consists of two stages, according to Fig. 2. In such a unit that produces the output vector Y using the input vector X , there are some intermediate productions called the vector Z that is the output vector of the first stage. In other words, in such units, to generate the final output Y , the main input vector X is first provided to the first stage to generate the output Z . Then, the output vector of the first stage is given to the second stage as the input vector to generate the final output vector Y [29].



Fig.2: DMU with two-stage series network structure.

**2-2-2- Production possibility set in a two-stage structure:**

The PPS set for n observed DMUs  with a two-stage network structure is defined as relation (3). Note that the PPS in relation (3) is constructed by considering the principles of input and output feasibility.

 (3)



**2-2-3- Congestion in a two-stage structure:**

**Definition 2.4:** A DMU with the series two-stage network structure exhibits overall congestion when at least one of the following conditions is met [29].

Case1. A decrease (increase) in one /more components of the primary input X leads to an increase (decrease) in one / more components of the final output Y, of course, without worsening (improving) other input and output components and assuming that the intermediate production Z remains constant.

Case2. A decrease (increase) in one /more components of the middle input Z from the second stage leads to an increase (decrease) in one / more final out put components of Y, of course, without worsening (improving) other input and output components and assuming that the primary input X does not worsen.

**2-2-4: Cooper's developed model for measuring Congestion in a two-stage structure:**

Sadat Kassaei et al. [29], in 2023 extended model (2) and proposed model (4) for congestion evaluation:



(4)

It should be noted that although model (4) is formulated as a single linear model, its estimation involves a four-step optimization process. Specifically, this includes maximizing the variable  first, followed by the sequential optimization of the variables , respectively. And then minimizing the variables and , respectively.

According to the definitions of congestion, Theorem 2.2 is presented to identify and evaluate congestion in a two-stage serial network structure:

Theorem 2.2: exhibits overall congestion if for the optimal solution of model (4), , at least one of the following conditions holds [29]:



**3-Proposed method**

In this part, to investigate congestion, a two-stage system with a series structure, as shown in Fig.2, is considered. The congestion assessment is presented in the form of two scenarios: link control by the previous and next stage, and using the extended Cooper model method.

For this evaluation, we consider n DMUs. In each , input vector  enters the first stage, and the output vector from the first stage, enters the second stage as input. Also, the vector  is the output of the second stage.

The definition of congestion considered in this section will be 2.4.

Table 1**:** The symbols (variables) used in the models presented in the third section

|  |  |
| --- | --- |
|  | The ith input to DMUj in the first stage. |
|  | The rth output to DMUj in the second stage. |
|  | The lth intermediate products to DMUj from the first stage to the second stage. |
|  | Reduction (surplus variable) of input in the first stage. |
|  | Increase (shortage variable) output in the second stage. |
|  | Reduction (surplus variable) of links (intermediate products) in the first stage. |
|  | Axis of DMUj coefficients in Previous section |
|  | Axis of DMUj coefficients in Next section |
|  | The ith input to DMUo in the first stage. |
|  | The rth output to the DMUo in the second stage. |
|  | The lth intermediate products to DMUo from the first stage to the second stage. |

**3-1- Scenario 1: Link control by the previous section**

In this scenario, the control value of the intermediate products is determined by the previous section, and the next section has no role in determining the value of the link. Therefore, the link plays the role of an output under the control of the previous section (Assuming continuity between sending and receiving the value of the link). In this scenario, PPS and congestion are obtained as follows:

 (5)

By removing the input possibility principle, the production possibility set (5) changes to equation (6):



 (6)

In this scenario, the intermediate product z plays the role of the output of the first step. And the second stage has no decision to receive the amount of intermediary products, and only the first stage decides on the products sent. And since intermediate products are outputs in this scenario, they do not play a role in calculating the congestion. The model presented for assessing the congestion in scenario 1 is model (7).



 (7)

Although model (7) is expressed in a linear model, it necessitates a three-step optimization process. These steps consist of maximizing the variable φ and then the variables and minimizing the variables is. To obtain congestion in this scenario, model (7) is solved. The first constraint in model (7) represents the input constraint, the second constraint represents the output constraint, and the third constraint represents the intermediate product constraint (output role).

**Theorem 3.1:** Under scenario 1, Congestion is present in with the series two-stage network structure, if and only if an optimal solution of model (7) is true in at least one of the following conditions (1) and (2):



Proof: Suppose an optimal solution obtained from model (7) satisfies the condition  and at least one of the conditions or. In this case, considering the constraints of model (7), the optimal solution of model (7) satisfies the Relation (8):



 (8)

From equation (8), it follows that is a member of the PPS (6) that, using an initial input less than  (at least in one input component) together with an intermediate output , and produce an output more than the final output  (at least in one output component). This means that  according to case 1 of definition 2.4, it has congestion.

Conversely, suppose that displays congestion according to Definition 2.4. We want to show that at least one of the conditions (1) and (2) holds. Suppose be the optimal solution of model (7). We prove that if , then  or  occurs.

Assume, by contradiction, thatand . Now consider , , ). is a feasible solution for model (7).

For this feasible solution, by calculating the objective function value of model (7), we have:

. This contradicts the optimality condition of the solution.

**3-2-Scenario 2: Link control by the next section**

In this scenario, optimal values of intermediate products are determined in the second stage, while the first stage lacks control over them. Thus, intermediate products solely function as inputs fully governed by the second stage. In this scenario, PPS and the assessment of congestion are as follows:

 (9)

By removing the input possibility principle, PPS (9) is changed into (10).



 (10)
To assess the congestion for scenario 2, Model (11) is proposed.

 (11)

Although model (7) is expressed in a single linear model, it necessitates a four-step optimization process. These steps include maximizing the variable φ, and then the variables. Also, minimizing variables , , respectively. In model (11), the first constraint is the input constraint, the second constraint is the output constraint, and the third constraint is related to the intermediate products constraint (input role).

**Theorem 3.2:** Under scenario 2,  with the series two-stage network structure will have congestion, if and only if, an optimal solution of model (11) satisfies at least one of the following conditions (1) and (2):



Proof: Suppose that model (11) achieves an optimal solution such that it meets at least one of the conditions or. In this case, two situations may occur.

Situation1: . In this case, according to the constraints of model (11), the relationship (12)

holds for the optimal solution.



 (12)

From relation (12), it can be seen that is a member of the PPS (10) that can produce an output greater than the final output , (at least in one input component) along with the intermediate output less than  (at least in one component), without worsening the primary input . This means that  exhibits congestion according to case 2 of Definition 2.4.

Situation2: .

In this case, similar to case 1, according to the constraints of model (11) in the optimal solution, relation (13) is obtained.



 (13)

And from the relation (13), it is concluded that is a member of the PPS (10) that uses a primary input less than (at least in one of the input components) along with the intermediate outputs  leads to the production of the final output greater than  (at least in one of the input components). This means that exhibits congestion according to case 1 of Definition 2.4.

Conversely, suppose that displays congestion according to Definition 2.4. We want to show that at least one of the conditions (1) and (2) holds. Suppose be the optimal solution of model (11). We prove that if , then  or  occurs.

Assume, by contradiction, thatand . Now consider ,,. is a feasible solution for model (11).

For this feasible solution, by calculating the objective function value of model (11), we have:

 This contradicts the optimality condition of the solution.

**4-Applied example**

We consider 20 branches of one of Iran's commercial banks which have a two-stage series structure. Using the proposed method, we will detect congestion in them for the first time. By reviewing past documents and conducting interviews with bank experts, less important or unavailable performance indicators were eliminated. The final input, intermediate, and output indicators were then compiled for 20 bank branches in Tehran in 2020. In this two-stage model, the first stage uses inputs (human resources and operational costs) to generate intermediate outputs (deposits), which then serve as inputs for the second stage. The bank uses these deposits to issue loans and generate profits, ultimately improving branch efficiency and performance. However, inefficiencies can lead to congestion, reducing productivity.

The proposed method helps identify congested branches, allowing managers to take corrective measures and enhance overall system performance. In this model, the bank operates as atwo-stage DMU (Decision-Making Unit)**.** Thefirst stage consumes all initial inputs to generate intermediate outputs. These intermediate outputs then serve as theinputs for the second stage**,** where they are further processed to produce the final outputs. This structure allows for a detailed efficiency analysis of each stage, highlighting how resources are transformed sequentially within the organization. The two-stage approach helps identify bottlenecks, optimize processes, and improve overall performance by evaluating both phases independently and collectively. In this example, we examine a two-stage banking model. In thefirst stage**,** the bank utilizes key inputs **such as**human **resources** andoperational costs to optimize **its** internal processes and service delivery.

These inputs enable the bank to function efficiently and maintain its daily operations. In the second stage, the outputs generated from these processes—primarily deposits—serve as measurable indicators of the bank's performance. This model demonstrates how investments in workforce and operational expenditures contribute to increased deposit mobilization, ultimately enhancing the bank's financial outcomes. The bank uses deposits (its output) to invest in loans. This generates profit. Higher profit improves the branch’s performance and efficiency. Some branches face inefficiency. This can lead to congestion. Congestion is a sign of inefficiency. By detecting congestion, managers can improve struggling branches. This article proposes a method. It identifies congested branches. Managers can then take action. Their goal is to resolve inefficiencies. This enhances overall system performance.

**Indicators of Interest:**

**• Number of Employees**: This input is the average number of line employees in each branch over a one-year period.

**• Operating expenses**: According to the documentation available in the bank, this entry consists of three sub-criteria: personnel expenses, administrative expenses, and depreciation expenses of movable property, which are created from the sum of these operating expenses.

**• Amount of deposits (current-short-term-long-term):** These intermediate products include the average total of current, long-term and short-term deposits over a one-year period. These indicators are the most important source of funds for branches that they can use for investment.

• **Facility Amount:** This output includes the types of loans provided by the bank to customers.

Table 1 introduces the inputs, outputs, and intermediate products for 20 bank branches.

|  |
| --- |
| Table 2: Inputs, outputs, and intermediate products  |
| Steps and links |  Symbols | Definitions  |
| First stage entrance |    | Number of employees |
| First stage entrance |  | Expenses |
| Intermediate products (output of the first stage and input of the second stage) |  | Current deposits |
| Intermediate products (output of the first stage and input of the second stage) |  | Short-term deposits |
| Intermediate products (output of the first stage and input of the second stage) |  | Long-term deposits  |
| Output of the second stage |  | Facilities granted – loans |

Fig.3 shows the structure of a two-stage series system in a bank.



Figure 1 Fig.3: Structure of a two-stage series system in a bank

Data related to the twenty branches of the bank, whose column titles are introduced in Table 1, are shown in Table 2.

|  |
| --- |
| Table 3. Input, output, and intermediate product values ​​of 20 branches of bank. |
|  |  |  |    |   |   |   |
|  | 10.1 | 1581530065 | 17438750 | 45566146458 | 1442225266 | 1442225266 |
|  | 29.76 | 3131924929 | 151127977 | 99186787500 | 5754250480 | 5754250480 |
|  | 18.11 | 2238573784 | 62119505 | 55521833817 | 3901860850 | 3901860850 |
|  | 16.72 | 3245132567 | 408980744 | 88040074650 | 5062045362 | 5062045362 |
|  | 6.59 | 1175879227 | 3925000 | 32181411150 | 2120095503 | 2120095503 |
|  | 13.3 | 2481630750 | 44715177 | 51267691874 | 2393377035 | 2393377035 |
|  | 25.72 | 4414234890 | 221707542 | 2.27575E+1 | 4739523507 | 4739523507 |
|  | 14.47 | 1567046715 | 285330075 | 38745612254 | 1032535423 | 1032535423 |
|  | 8.25 | 2115093658 | 142274000 | 30502903269 | 749476169 | 749476169 |
|  | 10.1 | 2481630750 | 35000000 | 24201394769 | 871615396 | 871615396 |
|  | 14.99 | 1033342335 | 46795937 | 45185645297 | 1436547076 | 1436547076 |
|  | 8.32 | 1787297050 | 22444388 | 40208164863 | 1200175994 | 1200175994 |
|  | 12.26 | 1033342335 | 58306997 | 36015245593 | 2468566570 | 2468566570 |
|  | 14.3 | 1787297050 | 243848400 | 34896651543 | 1019676241 | 1019676241 |
|  | 5.1 | 1033342335 | 175106795 | 21564061127 | 812115513 | 812115513 |
|  | 11.29 | 2717397505 | 161159500 | 53333333398 | 1288144991 | 1288144991 |
|  | 11.53 | 1955112558 | 476526310 | 29448960511 | 837547026 | 837547026 |
|  | 31.02 | 99999999999 | 129540330 | 1.06245E+1 | 3433656869 | 3433656869 |
|  | 27.72 | 2200053376 | 109268323 | 83888688459 | 2628130463 | 2628130463 |
|  | 8.57 | 301578529 | 14147000 | 27766460455 | 487716957 | 487716957 |

To detect congestion in twenty bank branches, we first solve model (7) and assume that scenario one holds. The results from model (7) are shown in Table 3.

|  |
| --- |
| Table 4: shows the results of model (7) for scenario 1 |
|  |  |  |  |  |
| 0 | 0 | 0.72 | 5.91 |  |
| 0 | 0 | 0 | 245.96 |  |
| 0 | 0 | 0 | 46.56 |  |
| 0 | 0 | 0 | 22.27 |  |
| 0 | 0 | 0 | 78.87 |  |
| 0 | 0 | 0 | 3.13 |  |
| 0 | 0 | 0 | 1 |  |
| 0 | 0 | 0 | 4.64 |  |
| 0 | 0 | 0 | 14.34 |  |
| 0 | 0 | 0 | 12.23 |  |
| 0 | 0 | 0 | 2.1 |  |
| 0 | 0 | 0 | 21.75 |  |
| 0 | 0 | 0 | 26.35 |  |
| 0 | 0 | 0 | 1.16 |  |
| 0 | 0 | 0 | 6.26 |  |
| 0 | 0 | 0 | 275.53 |  |
| 0 | 0 | 0 | 20.12 |  |
| 0 | 0 | 0 | 1.39 |  |
| 0 | 0 | 0 | 1.72 |  |
| 0 | 0 | 0 | 5.08 |  |

To obtain congestion in scenario 1, from the three optimization steps of model (7), the values ​​of the variables are shown in the four columns of Table 3 for 20 DMUs. The results of these steps include a variable column, whose value is greater than one for each DMU in this scenario. **The third column displays the first input slack of the first stage () and shows a value greater than zero only in the first DMU, and a zero value for all other units. The fourth column represents the second input slack of the first stage (****) and shows a value of zero across all units.** The last column  is related to the output slack of the second stage, whose value is zero for every twenty DMUs. According to the first condition of Theorem 3.1, the input slack value is greater than zero only in the first unit, and the value of its variable φ is also greater than one in that unit, so only has congestion.

To detect congestion in scenario two for 20 bank branches, we solve model (11). The results

of model (11) are shown in Table 4.

|  |
| --- |
| Table 5: Results from model (11) in scenario 2. |
|  |  |  |  |  |  |  |  |
| 0 | 34830175.45 | 16043438 879 | 0 | 0 | 5.48 | 1.33 |  |
| 0 | 106894862 | 80346627396 | 0 | 0 | 19.12 | 193.27 |  |
| 0 | 0 | 20617652865 | 0 | 0 | 6.04 | 14.98 |  |
| 0 | 0 | 66505394355 | 289069217.6 | 0 | 5.9 | 11.09 |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 |  |
| 0 | 45914716.25 | 7192194299 | 0 | 0 | 0 | 1.44 |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 |  |
| 0 | 58422983.54 | 19669232518 | 51519153.11 | 0 | 2.02 | 4.28 |  |
| 0 | 0 | 7629120473 | 0 | 0 | 1.86 | 8.71 |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 9.25 |  |
| 0 | 4362268.95 | 10372371778 | 6580952.33 | 0 | 5.64 | 1.05 |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 |  |
| 0 | 119322087.3 | 33071557911 | 0 | 0 | 1.52 | 219.16 |  |
| 0 | 54068274.72 | 5425279172 | 239576476.7 | 0 | 2.53 | 18.18 |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 |  |
| 0 | 52270196.9 | 60595672534 | 862782074.5 | 0 | 18.21 | 1.56 |  |
| 0 | 0 | 0 | 0 | 0 | 0 | 1 |  |

The results of Table 4 indicate that the variable equals zero for all 20 DMUs. The third and fourth columns, which are the two input slacks of the first step, as and, have a value of zero for all 20 units. **The next three columns in Table 4 show the slack values of the second-stage intermediate variables (, ,).** The last column is related to the output slack of the second stage in the form of a variable. For all 20 DMUs, the value ofis greater than zero. The findings in Table 5 reveal that in scenario 2, units 1, 2, 3, 4, 6, 8, 9 ,14 ,16 ,17 show congestion. Units 5, 7,10,11,12,13,15,18,20 have no congestion.

**Conclusion:**

Since congestion may result in organizational inefficiencies and resource depletion, its detection is critical. In this study, we presented a new model, formulated for the first time through an enhanced Cooper approach, designed for two-stage serial-structured networks in two different scenarios. This new method proposed a solution for identifying congestion in stages and the entire system of organizations that can be used in organizations such as educational systems, universities, urban traffic, banks, post offices, power plants, refineries, training centers, universities, industrial and manufacturing units, and environmental units, and the like. Finally, to demonstrate the applicability of the proposed models, 20 supply chains consisting of 20 branches of Iran's commercial banks were used. Each bank was considered a two-stage supply chain. With the proposed method, the presence of congestion in the stages and in the entire system of these twenty banks was detected.

**Suggestions for the future:**

1. Design and propose a method for the congestion of stages and the whole that requires fewer calculations. (To reduce the complexity of calculations)

2. Development of the proposed method for extended network structures such as multi-stage structures

3. Detection of congestion in network structures that have undesirable final output.

**CONFLICTS OF INTEREST STATEMENT**

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**Research Involving Human and /or Animals:** The research does not involve humans and/or animals.

**Informed Consent:** Since the research does not involve humans and/or animals, there is no informed consent statement.

**Refrences**

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