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Energy and exergy analysis of thermoelectric heat pump system

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ABSTRACT

Exergy analysis has gained significance in analysing thermal energy systems as it locates and quantifies the irreversibilities in the system. This paper investigates the thermoelectric heat pump systems through exergy analysis. Four thermodynamic models of the thermoelectric heat pump considering the internal and external irreversibilities are developed and analysed in MATLAB Simulink environment with temperature dependent material properties for various operating temperatures. Moreover, analytical expressions for exergy efficiency and irreversibilities for the thermoelectric heat pump are derived. The results show that the exergy efficiency of the thermoelectric heat pump increases with increase in ΔTh. For a typical operating condition in an irreversible thermoelectric heat pump with 31 thermocouples and when T_H and T_C of 313 K and 303 K respectively, the maximum energy and exergy efficiency obtained are 4.01 and 12.81% at same optimum current of 5.55A. The results also show that the effect of internal irreversibilities is more pronounced than the external irreversibilities in the performance of the thermoelectric heat pump. The effects of irreversible heat transfer and contact resistance in the exergy efficiency are also studied. This study will be helpful in designing the actual thermoelectric heat pump systems.

1. Introduction

Thermoelectric devices are solid state direct energy conversion devices for converting heat into electricity and vice versa [1–4]. It operates on the combination of Seebeck, Peltier and Thomson effects. Thermoelectric devices have numerous advantages of being solid state device with no moving parts and require no maintenance. It provides noiseless operation and it offers light weight, compactness and hence, occupies small space [5]. The thermoelectric devices have better efficiencies at lower power levels compared with conventional thermodynamic devices used for power generation and space conditioning. Therefore, the thermoelectric devices are best suited for low power applications [6].

Thermoelectric heat pump (TEHP) works as a reversed heat engine operating between the two heat reservoirs as shown in Fig. 1 and its actual energy efficiency is lower than the ideal Carnot efficiency because of the irreversibilities induced by the electrical, thermal and the thermoelectric properties of the thermoelectric materials.

The thermoelectric heat pump systems can be used as cooler and/or heat pump by changing the direction of current flow through the thermoelectric couples. It does not require refrigerant to pump heat from the heat source to the heat sink since electrons serve this purpose. So there is no leakage of refrigerants and thus it does not contribute for ozone depletion and other environmental damages caused by the refrigerants as in conventional refrigerators. The thermal energy output of single thermoelectric couple is low and it can be increased to required level by adding several thermoelectric couples in series–parallel combination.

The efficiency of the thermoelectric devices depends on electrical conductivity (σ), thermal conductivity (k), and seebeck coefficient (α) of the thermoelectric material. The combination of these material properties of a thermoelectric material is defined as figure of merit (FOM) Rowe [3]. FOM is often defined as dimensionless figure of merit by multiplying it with mean operating temperature (T_m).

$$Z = \frac{\alpha^2 \sigma}{k} \quad (1)$$

$$ZT_m = \frac{\alpha^2}{\rho k} T_m \quad (2)$$

where, $T_m = \frac{T_H + T_C}{2}$ and symbols have their usual meanings.

To understand the reversible and irreversible effects in thermoelectric systems, one can classify them thermodynamically into four categories based on the irreversibilities in the system

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Nomenclature

A	area (m^2)
Ex	exergy (W)
I	current (A)
K	thermal conductance (W/K)
L	length (m)
P	electrical power (W)
Q	heat (W)
R	electrical resistance (Ω)
S	entropy (W/K)
T	temperature (K)
U	overall heat transfer coefficient (W/m^{2}K)
V	voltage (V)
Z	figure of merit (1/K)

Greek letters

α	Seebeck coefficient (V/K)
η	energy efficiency
k	thermal conductivity (W/mK)
ρ	electrical resistivity (Ω/m)
σ	electrical conductivity (S/m)
Δ	difference
Ψ	exergy efficiency

Subscripts

1	hot junction of TEHP
2	cold junction of TEHP
ce	ceramic layer
d	destroyed
en	endoreversible TEHP
ex	exoreversible TEHP
gen	generation
hp	heat pump
in	input
ir	irreversible TEHP
lost	lost
m	mean temperature
n	*n* type material
o	environment
p	*p* type material
t	total
C	cold side of TEHP
H	hot side of TEHP
I	ideal TEHP
Qh	heating power

such as ideal (or) reversible system, exoreversible system, endoreversible system, and irreversible system as shown in Fig. 2. Thermoelectric devices always have internal irreversibilities because of the intrinsic material properties. Super conductors have very low electrical resistivity but its electrical/thermal conductivity is high, and its seebeck coefficient is also very small so its figure of merit is very low and hence, these may not be the potential thermoelectric materials. Therefore, the term "ideal thermoelectric system" may not be thermodynamically possible.

Cvahtet and Strnad [7] thermodynamically analysed the ideal thermoelectric heat engine and heat pump and compared it with the actual systems. Nuwayhid et al. [8] and Wang et al. [9] have analysed the thermoelectric system based on entropy generation minimization method. Sharma et al. [10] have carried out simple exergy analysis of single and multistage exoreversible thermoelectric cooling system. Tipsaenporm et al. [11] have proposed thermodynamic analysis in thermoelectric cooler and found out second law efficiency is less than the first law (energy) efficiency.

Exergy analysis provides true measure of efficiency since it takes into considerations of first and second law of thermodynamics. With this technique the actual exergy destruction is in the system can be located so that the avoidable exergy losses can be reduced by taking corrective actions [12–16].

Based on the literature survey, it is found that exergy analysis in thermoelectric heat pump systems are not been carried out. The effect of contact resistance and irreversible heat transfer in the exergy efficiency is also not been studied. Therefore it becomes necessary to carryout exergy analysis in the thermoelectric heat pump system. In this study the authors have developed four thermodynamic models of thermoelectric heat pump to identify and quantify the internal and external irreversibilities in view of the exergy analysis.

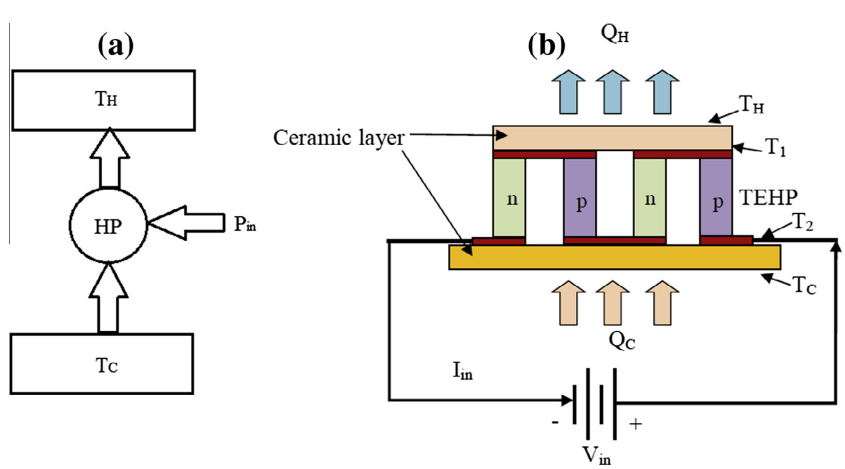

Fig. 1. (a) Reversed heat engine (Heat pump), (b) Thermoelectric heat pump.

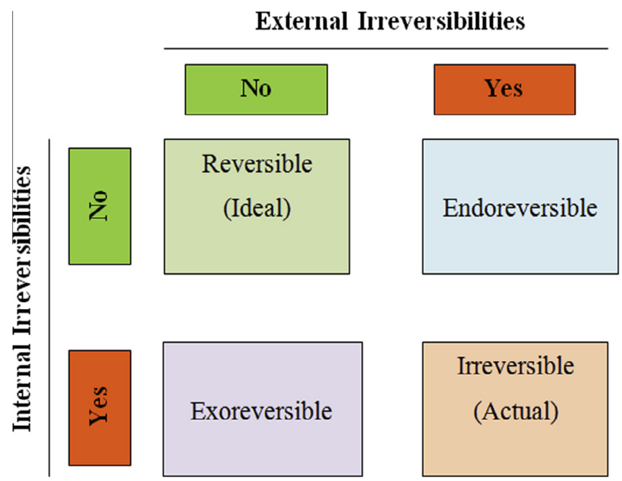

Fig. 2. Types of thermodynamic thermoelectric systems.

2. Thermodynamic modelling of thermoelectric heat pump systems

The thermodynamic models of the thermoelectric heat pump are developed and simulated in the MATLAB Simulink environment. The detailed thermodynamic modelling of thermoelectric heat pump is as follows.

2.1. Exergy analysis

The reversed heat engine shown in Fig. 1(a) then by applying the first law of thermodynamics, the energy balance can be given as follows [15]

$$Q_H - Q_C = P_{in} \tag{3}$$

By the second law of thermodynamics, the entropy generation is given by

$$S_{gen} = (\Delta S_{system} + \Delta S_{surrounding}) \geqslant 0 \tag{4}$$

or

$$S_{gen} = \frac{Q_H}{T_H} - \frac{Q_C}{T_C} \geqslant 0 \tag{5}$$

Combining first and second law of thermodynamics, the exergy balance in any thermodynamic system at steady state can be given as

$$Ex_{in} = Ex_{out} + Ex_{lost} + Ex_d \tag{6}$$

In the above equation, Ex_{in} is exergy input to the system, Ex_{out} is the exergy output of the system, Ex_{lost} is the exergy lost in the output stream (if present) and Ex_d are the exergy destroyed (Irreversibilities) in the process. In the thermoelectric devices the irreversibilities can be classified into two categories, as internal and external irreversibilities.

The exergy input to the system is the electrical power input (P_{in}) and it is 100% exergy. The exergy output is the thermal exergy (E_{Qh}) which is defined as follows

$$E_{Qh} = Q_H\left(1 - \frac{T_o}{T_H}\right) \tag{7}$$

where T_o is the environment temperature, then from Eq. (6) the exergy balance becomes

$$P_{in} = Q_C\left(\frac{T_o}{T_C} - 1\right) + Ex_d + Q_H\left(1 - \frac{T_o}{T_H}\right) \tag{8}$$

In the above equation the first term in the right hand side is the exergy contribution by the cold side of the heat pump (E_{Qc}). The second term is the exergy destruction and the third term is the exergy deposition into the hot side of the heat pump. In the ideal and exoreversible thermodynamic system $T_o = T_C$ therefore, the first term vanishes but in the endoreversible and irreversible thermodynamic system T_o differs from T_C therefore the exergy contribution by the cold side of heat pump will also be present. By rearranging the above equation we get,

$$Ex_d = P_{in} - Q_C\frac{T_o}{T_C} + Q_H\frac{T_o}{T_H} + Q_C - Q_H \tag{9}$$

Substituting P_{in} in Eq. (9) and solving for Ex_d, we get

$$Ex_d = T_o\left(\frac{Q_H}{T_H} - \frac{Q_C}{T_C}\right) \tag{10}$$

Comparing Eqs. (5) and (10), irreversibilities (Ex_d) becomes

$$Ex_d = T_oS_{gen} \tag{11}$$

Therefore, it is clear that the irreversibilities in the system are dependent on the environmental temperature. In the thermoelectric heat pump, cold side temperature T_C is the environment temperature (T_o). Then the exergy efficiency can be defined as

$$\Psi = \frac{Exergy\ Out}{Exergy\ in} = \frac{E_{Qh}}{P_{in}} = 1 - \frac{T_oS_{gen}}{P_{in}} \tag{12}$$

Therefore, by knowing the irreversibilities in the system one can find the exergy efficiency of a thermoelectric system. This procedure is followed by the authors in finding the irreversibilities in all four thermodynamic models of thermoelectric heat pump systems.

The temperature dependent properties of thermoelectric material (Bismuth Telluride – Bi_2Te_3) used in this study are given below as provided by Xuan et al. [17].

$$\alpha = [\alpha_p - (-\alpha_n)] = 2 \times (22224.0 + 930.6T_m - 0.9905T_m^2) \times 10^{-9} \tag{13}$$

$$\rho_n = \rho_p = (5112.0 + 163.4T_m + 0.6279T_m^2) \times 10^{-10} \tag{14}$$

$$k_n = k_p = (62605.0 - 277.7T_m + 0.4131T_m^2) \times 10^{-4} \tag{15}$$

Certain assumptions were made in the thermodynamic modelling of the thermoelectric heat pump systems to simplify the complexity of calculations, which are as follows:

- The study assumes temperature dependent thermoelectric properties for the analysis.
- One dimensional heat transfer along the length of the thermoelectric legs is considered.
- The temperatures of hot and cold side of the thermoelectric heat pump are maintained constant at T_H and T_C respectively.
- There is no other mode of heat transfer from the hot junction to cold junction of the thermoelectric heat pump other than the Fourier's heat conduction phenomena by the inherent thermal conductivity of thermoelectric materials.
- Thomson effect is not considered for the analysis.

2.2. Thermodynamic modelling of thermoelectric heat pump

The block diagram of irreversible thermoelectric heat pump is shown in Fig. 1(b). Number of thermocouples in a thermoelectric system (n) is 31 and all the thermocouples are connected electrically in series.

2.2.1. Ideal thermoelectric heat pump systems

The ideal thermoelectric heat pump system is one which does not possess internal and external irreversibilities.

Applying first law of thermodynamics to the ideal thermoelectric heat pump, the energy balance at hot and cold junction can be given as follows

$$Q_H = n(\alpha I T_1) \tag{16}$$

$$Q_C = n(\alpha I T_2) \tag{17}$$

In the ideal thermoelectric heat pump, the heat sink temperature and hot junction temperature are same ($T_H = T_1$) and the heat source temperature and cold junction temperature are same ($T_C = T_2$). For the ideal thermoelectric heat pump entropy generation S_{gen} is zero.

Substituting Eqs. (16) and (17) in Eq. (10), the irreversibilities in the ideal thermoelectric heat pump can be derived and written as

$$Ex_d = T_o\left(\frac{n\alpha I T_H}{T_H} - \frac{n\alpha I T_C}{T_C}\right) = 0 \tag{18}$$

The first law efficiency (energy efficiency) and the second law efficiency (exergy efficiency) of ideal thermoelectric heat pump can be written as

$$\eta_{I,hp} = \frac{Q_H}{P_{in}} = \frac{\alpha I T_H}{\alpha I (T_H - T_C)} = \frac{T_H}{T_H - T_C} \tag{19}$$

$$\Psi_{I,hp} = \frac{Ex_{out}}{Ex_{in}} = \frac{E_{Qh}}{P_{in}} = \frac{n\alpha I T_H\left(1 - \frac{T_o}{T_H}\right)}{n\alpha I (T_H - T_C)} = \frac{T_H\left(1 - \frac{T_o}{T_H}\right)}{T_H - T_C} \tag{20}$$

$$\Psi_{I,hp} = \frac{\eta_{I,hp}}{\left(1 - \frac{T_C}{T_H}\right)} = 100\% \tag{21}$$

It can be seen from Eq. (19) that, the Carnot efficiency is the limit of energy efficiency in the ideal thermoelectric heat pump.

The exergy efficiency of the ideal thermoelectric heat pump is also the function of the junction temperature and does not depend on the material properties. The exergy efficiency of thermoelectric heat pump in ideal thermodynamic conditions is 100% which is the maximum efficiency of any conceptual thermoelectric heat pump. It should be noted that the heat absorbed from the heat source and heat pumped to the heat sink of the thermoelectric heat pump is a function of material property (α) only. Peltier effect is a reversible phenomenon therefore, the irreversibilities in the ideal thermoelectric system is zero.

2.2.2. Endoreversible thermoelectric heat pump system

The endoreversible thermodynamic system is internally reversible and externally irreversible system. Therefore, in the endoreversible thermoelectric heat pump the internal irreversibilities are absent but the external irreversibilities due to the irreversible heat transfer at hot and cold sides are present.

By the first law of thermodynamics, the energy balance at hot and cold junction of endoreversible thermoelectric heat pump can be given as follows

$$Q_H = n(\alpha I T_1) \tag{22}$$

$$Q_C = n(\alpha I T_2) \tag{23}$$

In the endoreversible thermoelectric heat pump the irreversible heat transfer from the hot and the cold junction Q_H and Q_C can be written as follows:

$$Q_H = U_h A_h (T_1 - T_H) \tag{24}$$

$$Q_C = U_c A_c (T_C - T_2) \tag{25}$$

In the endoreversible thermoelectric heat pump, the heat sink temperature is T_H, the hot junction temperature is T_1, the heat source temperature is T_C and the cold junction temperature is T_2. The actual hot junction and cold junction temperature of the endoreversible thermoelectric heat pump can be analytically derived by substituting Eqs. (22) and (23) in Eqs. (24) and (25) and it can be written as follows.

$$T_1 = \frac{U_h A_h T_H}{(U_h A_h + n\alpha I)} \tag{26}$$

$$T_2 = \frac{U_c A_c T_C}{(U_c A_c - n\alpha I)} \tag{27}$$

By solving the above equation for constant T_H and T_C, we get $T_1 > T_H$ and $T_2 < T_C$. From Eq. (6) the exergy output of the endoreversible thermoelectric heat pump can be calculated. The irreversibilities in the endoreversible thermoelectric heat pump can be calculated from Eq. (10) and it is finite and non zero. The entropy generation S_{gen} can be calculated as per the entropy balance obtained from the second law of thermodynamics.

$$Ex_d = T_o\left(\frac{n\alpha I T_1}{T_H} - \frac{n\alpha I T_2}{T_C}\right) = n\left(\frac{\alpha I (T_1 T_C - T_H T_2)}{T_H}\right) > 0 \tag{28}$$

The first law efficiency and the second law efficiency of the endoreversible thermoelectric heat pump can be given as

$$\eta_{en,hp} = \frac{Q_H}{P_{in}} = \frac{n\alpha I T_1}{n\alpha I (T_1 - T_2)} = \frac{T_1}{T_1 - T_2} \tag{29}$$

In the endoreversible thermoelectric heat pump the power input P_{in} can be written as follows

$$P_{in} = Q_H - Q_C = n\alpha I (T_1 - T_2) \tag{30}$$

The exergy efficiency of endoreversible thermoelectric heat pump can be written as

$$\Psi_{en,hp} = \frac{Ex_{out}}{Ex_{in}} = \frac{E_{Qh}}{P_{in}} = \frac{P_{in} - I}{P_{in}} = 1 - \frac{T_C S_{gen}}{P_{in}} \tag{31}$$

$$\Psi_{en,hp} = 1 - \left(\frac{n\alpha I\left(\frac{T_1 T_C - T_2 T_H}{T_H}\right)}{P_{in}}\right) \tag{32}$$

$$\Psi_{en,hp} = \frac{\left(\frac{T_1}{T_1 - T_2}\right)}{\left(\frac{T_H}{T_H - T_C}\right)} = \frac{\eta_{en,hp}}{\eta_{I,hp}} \tag{33}$$

It can be seen from Eq. (29) that the energy efficiency of the endoreversible thermoelectric heat pump is lower than the ideal thermoelectric heat pump since $T_2 < T_C$ and $T_1 > T_H$. It is also observed that the irreversibilities in the thermoelectric heat pump are finite and caused because of the irreversible heat transfer in the system. The irreversibilities in the endoreversible thermoelectric heat pump can be termed as external irreversibilities. The exergy efficiency of endoreversible thermoelectric heat pump is less than the ideal case because of added irreversibilities in the system.

2.2.3. Exoreversible thermoelectric heat pump system

The exoreversible thermodynamic system is one which is externally reversible and internally irreversible. In the exoreversible thermoelectric heat pump the internal irreversibilities due to the material properties and the contact resistance between the metal and the thermocouple are present but the external irreversibilities due to irreversible heat transfer are absent.

By the first law of thermodynamics, the energy balance at hot and cold junction of exoreversible thermoelectric heat pump is given as follows

$$Q_H = n\left((\alpha I T_1) + \frac{I^2 R}{2} - K(T_1 - T_2)\right) \quad (34)$$

$$Q_C = n\left((\alpha I T_2) - \frac{I^2 R}{2} - K(T_1 - T_2)\right) \quad (35)$$

In the exoreversible thermoelectric heat pump, the heat sink temperature and the hot junction temperature are same ($T_H = T_1$), the heat source temperature and the cold junction temperature are same ($T_C = T_2$).

The thermal conductance (K) and the electrical resistance (R) of the thermocouple in exoreversible and irreversible thermoelectric heat pump is defined as

$$K = \left[\frac{k_n A_n}{L_n} + \frac{k_p A_p}{L_p}\right] + K_{conducting\ metal} \quad (36)$$

$$R = \left[\frac{\rho_n L_n}{A_n} + \frac{\rho_p L_p}{A_p}\right] + R_{contact} + R_{conducting\ metal} \quad (37)$$

The thermal conductance and electrical resistance of conducting metal is assumed negligible. In this case, the overall heat transfer coefficient between the heat source and cold junction and between the heat sink and hot junction of the thermoelectric heat pump is infinity as given by Eq. (38). Hence, the thermal resistance between the heat source and the cold junction and the thermal resistance between the hot junction and the heat sink of the thermoelectric heat pump is zero. The total heat transfer area (A_t) is the sum of heat transfer area at cold side (A_c) and the heat transfer area at the hot side (A_h).

$$U_c A_c = U_h A_h = \infty \quad (38)$$

From Eq. (6) the exergy output of the exoreversible thermoelectric heat pump can be calculated. The irreversibilities in the exoreversible thermoelectric heat pump can be calculated from Eq. (10) and it is finite and non zero because of the thermoelectric material properties and the contact resistance. The entropy generation S_{gen} can be calculated as per the entropy balance obtained by the second law of thermodynamics.

$$Ex_d = nT_o\left(\left(\frac{\alpha I T_H}{T_H} + \frac{I^2 R}{2T_H} - \frac{K(T_H - T_C)}{T_H}\right) - \left(\frac{\alpha I T_C}{T_C} - \frac{I^2 R}{2T_C} - \frac{K(T_H - T_C)}{T_C}\right)\right) > 0 \quad (39)$$

$$Ex_d = nT_o\left(\frac{I^2 R T_m}{T_H T_C} + \frac{K(T_H - T_C)^2}{T_H T_C}\right) > 0 \quad (40)$$

The first law efficiency and the second law efficiency of the exoreversible thermoelectric heat pump can be given as

$$\eta_{ex,hp} = \frac{Q_H}{P_{in}} = \frac{n((\alpha I T_H) + \frac{I^2 R}{2} - K(T_H - T_C))}{n[\alpha I(T_H - T_C) - I^2 R]} \quad (41)$$

$$\Psi_{ex,hp} = \frac{Ex_{out}}{Ex_{in}} = \frac{E_{Qh}}{P_{in}} = \frac{P_{in} - I}{P_{in}} = 1 - \frac{T_C S_{gen}}{P_{in}} \quad (42)$$

$$\Psi_{ex,hp} = 1 - \left(\frac{\left(\frac{I^2 R T_m}{T_H} + \frac{K(T_H - T_C)^2}{T_H}\right)}{\alpha I(T_H - T_C) - I^2 R}\right) \quad (43)$$

$$\Psi_{ex,hp} = \frac{\alpha I T_H\left(1 - \frac{T_C}{T_H}\right) + \frac{I^2 R}{2}\left(1 - \frac{T_C}{T_H}\right) - K(T_H - T_C)\left(1 - \frac{T_C}{T_H}\right)}{\alpha I(T_H - T_C) + I^2 R} \quad (44)$$

The optimum current at maximum exergy efficiency (I_{exergy}) can be obtained by differentiating the Eq. (44) with respect to current I and the optimum current at maximum energy efficiency (I_{energy}) can be obtained by differentiating the Eq. (41) with respect to current I [1–4]. The optimum current at maximum exergy efficiency is given below

$$I_{exergy} = \frac{\alpha(T_H - T_C)}{RT_m Z}(1 + \sqrt{1 + T_m Z}) = I_{energy} \quad (45)$$

The above equation is same as the equation for optimum current at maximum energy efficiency (I_{energy}). Therefore, it can be concluded that the energy and exergy efficiency will be maximum at same current (I_{energy}).

Substituting I_{exergy} in Eq. (44), the maximum exergy efficiency of the thermoelectric heat pump is derived by the authors as follows

$$\Psi_{ex,hp} = \left[\left(\frac{T_H}{T_H - T_C}\right)\left(\frac{\sqrt{1 + ZT_m} - \frac{T_C}{T_H}}{1 + \sqrt{1 + ZT_m}}\right)\left(1 - \frac{T_C}{T_H}\right)\right] = \left(\frac{\sqrt{1 + ZT_m} - \frac{T_C}{T_H}}{1 + \sqrt{1 + ZT_m}}\right) \quad (46)$$

It can be seen from Eq. (46) that the exergy efficiency of the thermoelectric heat pump is less than the endoreversible case and it is a function of material properties as well as the junction temperatures.

2.2.4. Irreversible thermoelectric heat pump system

The irreversible thermodynamic system is one which is internally and externally irreversible. In the irreversible thermoelectric heat pump the internal irreversibilities due to the material properties and the contact resistance between the metal and the thermocouple are present along with the external irreversibilities caused due to the irreversible heat transfer. Therefore, the irreversible thermoelectric heat pump is the closer approximation of the actual thermoelectric heat pump system.

By the first law of thermodynamics, the energy balance at hot and cold junction of irreversible thermoelectric heat pump is given as follows

$$Q_H = n\left((\alpha I T_1) + \frac{I^2 R}{2} - K(T_1 - T_2)\right) \quad (47)$$

$$Q_C = n\left((\alpha I T_2) - \frac{I^2 R}{2} - K(T_1 - T_2)\right) \quad (48)$$

The heat flow from the hot and the cold junctions (Q_H and Q_C) in irreversible thermoelectric heat pump can also be written as follows

$$Q_H = U_h A_h (T_1 - T_H) \quad (49)$$

$$Q_C = U_c A_c (T_C - T_2) \quad (50)$$

Solving Eqs. (47)–(50)for the hot and cold junction temperatures of irreversible thermoelectric heat pump, the expression for T_1 and T_2 is written as follows as derived by the authors

$$T_1 = \frac{[(U_h A_h T_H + 0.5nI^2 R)(A_c U_c + nK + n\alpha I) + (0.5n^2 KRI^2 + nKU_c A_c T_C)]}{[(nK + A_c U_c + n\alpha I)(A_h U_h + nK - n\alpha I) - (n^2 K^2)]} \quad (51)$$

$$T_2 = \frac{[(A_c U_c T_C + 0.5nI^2 R)(A_h U_h + nK - n\alpha I) + (0.5n^2 KRI^2 + nKU_h A_h T_H)]}{[(nK + A_c U_c + n\alpha I)(A_h U_h + nK - n\alpha I) - (n^2 K^2)]} \quad (52)$$

In the irreversible thermoelectric heat pump, the heat sink temperature and the hot junction temperature are different ($T_H < T_1$), the heat source temperature and the cold junction temperature are different ($T_C > T_2$).

In the irreversible thermoelectric heat pump, the heat transfer coefficient between the heat source and the cold junction is finite because of the finite thermal conductivity of the ceramic layer as given by Eq. (53). The value of U_c and U_h is assumed as 170 W/m^2 K as given by Chen et al. [18]. The heat transfer area is equally distributed to the hot and cold sides of the thermoelectric cooler as $A_c = A_h = 0.07$ m^2 per 31 thermocouples.

$$U_cA_c = \frac{k_{ce}A_c}{L_{ce}} \tag{53}$$

where, k_{ce} is the thermal conductivity of the ceramic layer. The heat transfer coefficient between the hot junction and the heat sink (U_hA) also follows the same notation of Eq. (53).

From Eq. (6) the exergy output of the irreversible thermoelectric heat pump can be calculated. The irreversibilities in the system can be calculated from Eq. (10) and it is finite and non zero. The entropy generation S_{gen} can be calculated as per the entropy balance obtained by the second law of thermodynamics.

$$Ex_d = nT_o\left(\left(\frac{\alpha IT_1}{T_H} + \frac{I^2R}{2T_H} - \frac{K(T_1-T_2)}{T_H}\right) - \left(\frac{\alpha IT_2}{T_C} - \frac{I^2R}{2T_C} - \frac{K(T_1-T_2)}{T_C}\right)\right) > 0 \tag{54}$$

$$Ex_d = nT_o\left(\frac{\alpha I(T_1T_C - T_2T_H)}{T_HT_C} + \frac{I^2RT_m}{T_HT_C} + \frac{K(T_H-T_C)(T_1-T_2)}{T_HT_C}\right) > 0 \tag{55}$$

The first law efficiency and the second law efficiency of the irreversible thermoelectric heat pump can be given as

$$\eta_{ir,hp} = \frac{Q_H}{P_{in}} = \frac{n((\alpha IT_1) + \frac{I^2R}{2} - K(T_1-T_2))}{n[\alpha I(T_1-T_2) - I^2R]} \tag{56}$$

$$\Psi_{ir,hp} = \frac{Ex_{out}}{Ex_{in}} = \frac{E_{Qh}}{P_{in}} = \frac{P_{in} - I}{P_{in}} = 1 - \frac{T_CS_{gen}}{P_{in}} \tag{57}$$

$$\Psi_{ir,hp} = 1 - \left(\frac{\left(\frac{\alpha I(T_1T_C - T_2T_H)}{T_H} + \frac{I^2RT_m}{T_H} + \frac{K(T_H-T_C)(T_1-T_2)}{T_H}\right)}{\alpha I(T_H-T_C) - I^2R}\right) \tag{58}$$

By simplifying the above equation we get the exergy efficiency of irreversible thermoelectric heat pump as follows

$$\Psi_{ir,hp} = \frac{\alpha IT_1\left(1-\frac{T_C}{T_H}\right) + \frac{I^2R}{2}\left(1-\frac{T_C}{T_H}\right) - K(T_1-T_2)\left(1-\frac{T_C}{T_H}\right)}{\alpha I(T_1-T_2) + I^2R} \tag{59}$$

The optimum current at maximum exergy efficiency of irreversible thermoelectric heat pump (I_{exergy}) can be obtained by differentiating the above equation with respect to current I, thus the optimum current in the irreversible thermoelectric heat pump at maximum exergy efficiency can be given below as derived by the authors

$$I_{exergy} = \frac{\alpha(T_1-T_2)}{RT_mZ}\left(1 + \sqrt{1+T_mZ}\right) = I_{energy} \tag{60}$$

The above equation is same as the equation for optimum current at maximum energy efficiency (I_{energy}) for irreversible thermoelectric heat pump but the mean temperature (T_m) is the mean of T_1 and T_2. Therefore, the exergy efficiency and energy efficiency of irreversible thermoelectric heat pump is maximum at same optimum current ($I_{energy} = I_{exergy}$).

Substituting I_{exergy} in Eq. (59), the maximum exergy efficiency of irreversible thermoelectric heat pump as derived by the authors is given as follows. Note that $\eta_{ir,hp}$ is the maximum energy efficiency of irreversible thermoelectric heat pump.

$$\Psi_{ir,hp} = \left[\left(\frac{T_1}{T_1-T_2}\right)\left(\frac{\sqrt{1+ZT_m} - \frac{T_2}{T_1}}{1+\sqrt{1+ZT_m}}\right)\left(1-\frac{T_C}{T_H}\right)\right] = \left(\frac{\eta_{ir,hp}}{\eta_{I,hp}}\right) \tag{61}$$

It can be seen from Eq. (61) that the exergy efficiency of the irreversible thermoelectric heat pump is less than the ideal, endoreversible and exoreversible thermodynamic case and it is a function of material properties, junction temperatures and the heat source and heat sink temperatures. This expression for the exergy efficiency of thermoelectric heat pump may closely approximate the actual exergy efficiency of the thermoelectric heat pump.

3. Results and discussion

The energy and exergy analysis of the thermoelectric heat pump for all thermodynamic models have been carried out for various operating conditions in MATLAB Simulink environment.

The thermodynamic models of thermoelectric heat pump explained in Section 2.2 are analysed with fixed hot and cold junction temperatures. The cold side temperature (T_C) is fixed at 303 K in all the cases and the hot side temperature (T_H) is fixed at 313 K, 323 K and 333 K to calculate the performance parameters such as energy efficiency, exergy efficiency, thermal energy output. ΔTh is the temperature difference between the heat sink and the heat source of the thermoelectric heat pump.

The heating power (Q_H) of the thermoelectric heat pump in ideal and endoreversible thermodynamic conditions increases linearly because of the absence of internal irreversibilities. These two cases will be impractical because the thermal and electrical resistance of thermoelectric material is finite and non zero. The heating power of the thermoelectric heat pump for the exoreversible and irreversible thermodynamic conditions with ΔTh of 10 K and 30 K is presented in Fig. 3. It shows that the heating power of irreversible case is lower than the exoreversible case because of external irreversibilities. The heating power of the exoreversible and irreversible thermoelectric heat pump for different operating temperatures at maximum energy/exergy efficiency condition is presented in Table 1.

The energy efficiency of the thermoelectric heat pump for the ideal thermodynamic case is dependent on the temperature only as stated by Carnot theorem and it is well known. In the ideal case of the thermoelectric heat pump the energy efficiency is constant for all currents. The energy efficiency of endoreversible thermoelectric heat pump is decreasing with the increase in

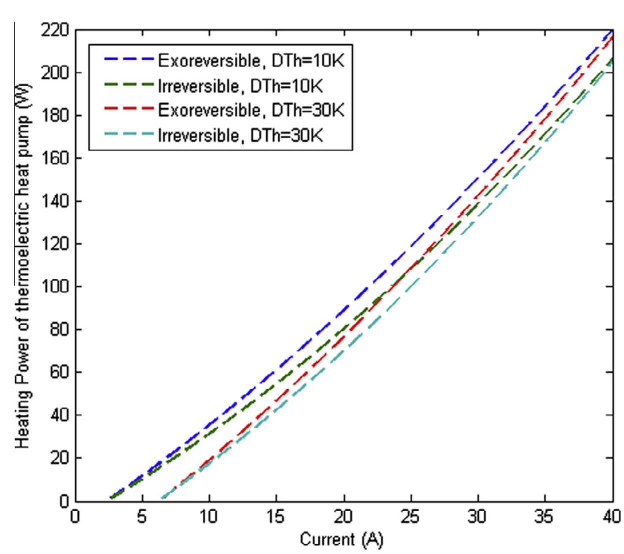

Fig. 3. Heating power of thermoelectric heat pump for different ΔTh.

Table 1
Heating power, maximum energy and exergy efficiency of exoreversible and irreversible thermoelectric heat pump with T_C of 303 K.

Performance parameters	Exoreversible TEHP			Irreversible TEHP		
$\Delta Th = (T_H - T_C)$	10 K	20 K	30 K	10 K	20 K	30 K
Q_H at maximum energy efficiency (W)	14.72	32.50	52.63	13.03	28.79	47.97
Maximum energy efficiency	4.52	2.56	1.91	4.01	2.30	1.73
Maximum exergy efficiency	0.14	0.15	0.17	0.12	0.14	0.15

current as shown in Fig. 4. This is because of the irreversibilities in the endoreversible thermoelectric heat pump increases with current (I) as given by Eq. (28) derived by the authors.

The energy efficiency of the exoreversible thermoelectric heat pump is shown in Fig. 5. It shows that there is an optimum current for maximum energy efficiency (I_{energy}) as given by Eq. (45). The energy efficiency of exoreversible thermoelectric heat pump is lower than the endoreversible case because of the internal irreversibilities induced by thermal and electrical properties of thermoelectric materials. The energy efficiency of exoreversible thermoelectric heat pump is maximum at I_{energy} of 5.65 A for ΔTh of 10 K.

The energy efficiency of the irreversible thermoelectric heat pump is presented in Fig. 6. It shows that the energy efficiency of irreversible thermoelectric heat pump is less than the exoreversible case. The optimum current for ΔTh of 10 K is 5.55 A, it is lower than the exoreversible case. This is because of the larger temperature differential between the hot and cold junction of irreversible thermoelectric heat pump compared with exoreversible thermoelectric heat pump ($T_1 - T_2 > T_H - T_C$). This can be analytically proven from Eqs. (45) and (60).

Exergy analysis gives the true efficiency of the thermal system since it consider second law of thermodynamics combined with first law of thermodynamics to analyse the thermal energy systems. The exergy analysis in all thermodynamic models of the thermoelectric heat pump can give the complete picture of external and internal irreversibilities occurring in the system.

The exergy efficiency of ideal thermoelectric heat pump is 100% since the internal and external irreversibilities are absent in the ideal thermodynamic system. The exergy efficiency of exoreversible thermoelectric heat pump is shown in Fig. 7. The energy and exergy efficiency has its maximum at I_{energy} as seen from Eq. (45). It also shows that the exergy efficiency of thermoelectric heat pump increases with increase in ΔTh. This is because the exergy output (thermal exergy) increases with increase in T_H as shown by Eqs. (7) and (34).

The exergy efficiency of the irreversible thermoelectric heat pump is presented in Fig. 8. It shows that the exergy efficiency of the irreversible thermoelectric heat pump is less than the exoreversible case. This is because of the added external irreversibilities with the internal irreversibilities in the thermoelectric heat pump. Figs. 6 and 8 shows that the maximum energy efficiency and maximum exergy efficiency occurs at same current.

The optimum current at the maximum energy/exergy efficiency conditions are same because the energy/exergy input (electrical energy is 100% exergy) is same in both energy and exergy analysis. Moreover, the exergy output (E_{Qh}) is only reduced by the Carnot factor from Q_H and not changing with any other parameters.

The irreversibilities in the thermoelectric heat pump system with constant ΔTh of 10 K for different thermodynamic cases are

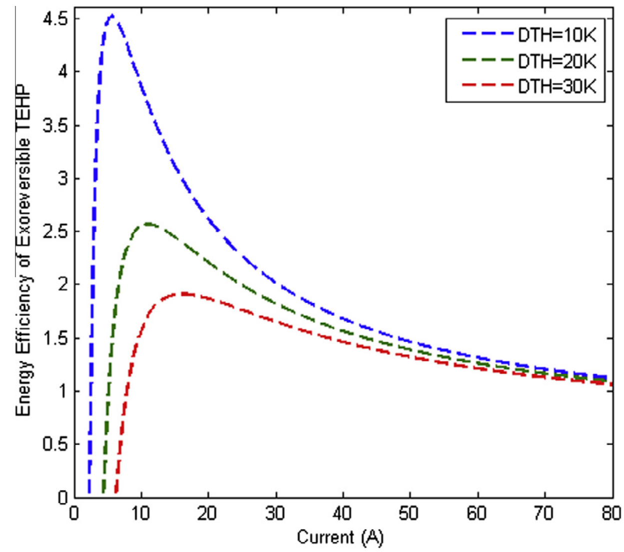

Fig. 5. Energy efficiency of exoreversible thermoelectric heat pump for different ΔTh.

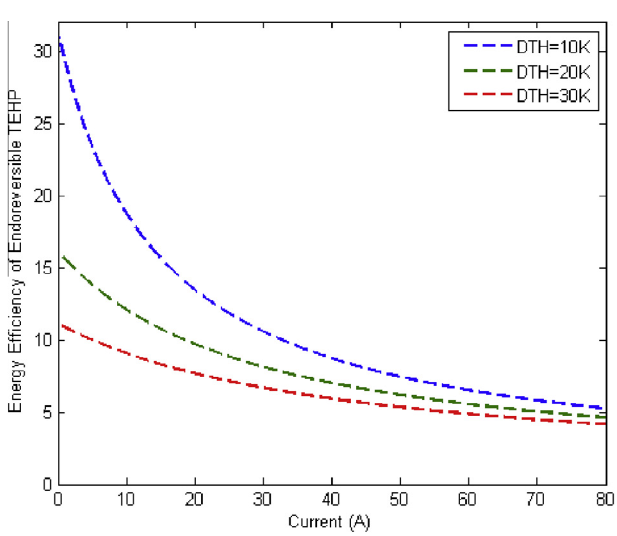

Fig. 4. Energy efficiency of endoreversible thermoelectric heat pump for different ΔTh.

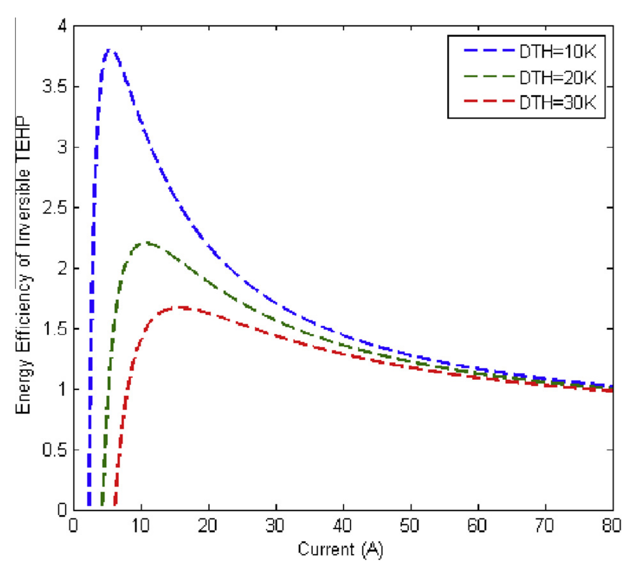

Fig. 6. Energy efficiency of irreversible thermoelectric heat pump for different ΔTh.

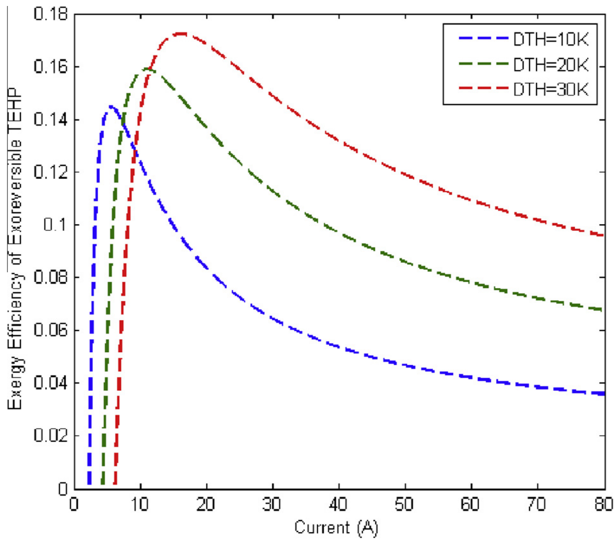

Fig. 7. Exergy efficiency of exoreversible thermoelectric heat pump for different ΔTh.

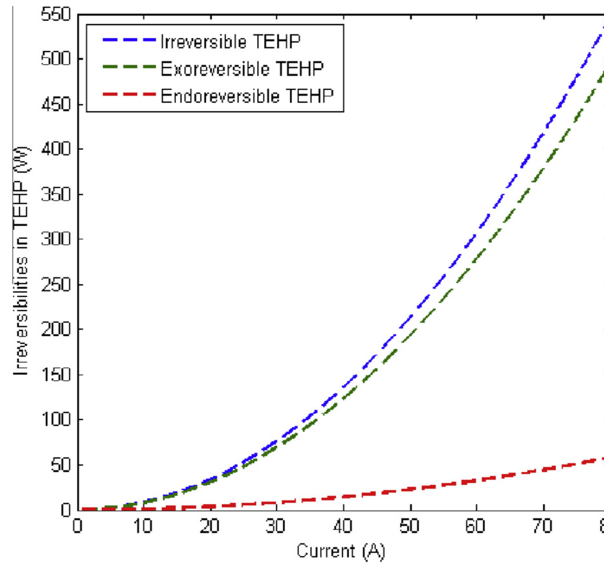

Fig. 9. Internal and external irreversibilities in different thermodynamic models of the thermoelectric heat pump.

presented in the Fig. 9. It shows that the irreversibilities in the irreversible thermoelectric heat pump are higher than the exoreversible case which is higher than the endoreversible case. It is also clear from the Fig. 9 that the internal irreversibilities are high when compared with the external irreversibilities. With the help of the exergy analysis presented in this paper, one can quantify the internal and external irreversibilities in the thermoelectric heat pump systems.

The external irreversibilities can be controlled by decreasing the overall thermal resistances at hot and cold sides of the thermoelectric heat pump and using the ceramic layer with higher thermal conductance. The internal irreversibilities are the function of electrical resistance and thermal conductance of the thermoelectric material as given by Eq. (40)(exoreversible case-only internal irreversibilities present). Therefore, the internal irreversibilities can only be reduced by increasing the FOM of the thermoelectric material. It is clear from Fig. 9 that the maximum exergy destruction is because of intrinsic thermoelectric material properties (exoreversible case-only internal irreversibilities present). Therefore, thermoelectric materials should be selected based on the minimum exergy destruction criterion.

The effect of irreversible heat transfer at the hot and cold side of the thermoelectric heat pump in the exergy efficiency is also studied and presented in Fig. 10. It shows that the decrease in total external heat transfer area decreases the maximum exergy efficiency since it increases the hot junction temperature and decreases the cold junction temperature thereby increases the effective temperature difference ($T_1 - T_2$) between the hot and cold junction of the thermoelectric heat pump, thus reduces the heating power and the exergy efficiency. It also tells that the exergy efficiency of exoreversible case is the maximum possible exergy efficiency of any actual thermoelectric heat pump systems since, the external irreversibilities are absent.

The irreversibilities in the thermoelectric heat pump for different heat transfer area are shown in Fig. 11. It shows that the decrease in the heat transfer area increases the irreversibilities in thermoelectric heat pump and the irreversibilities in the exoreversible thermoelectric heat pump are lower than the irreversible

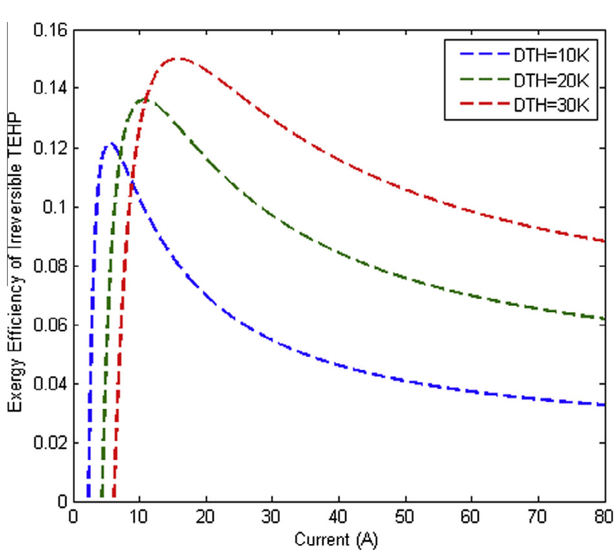

Fig. 8. Exergy efficiency of irreversible thermoelectric heat pump for different ΔTh.

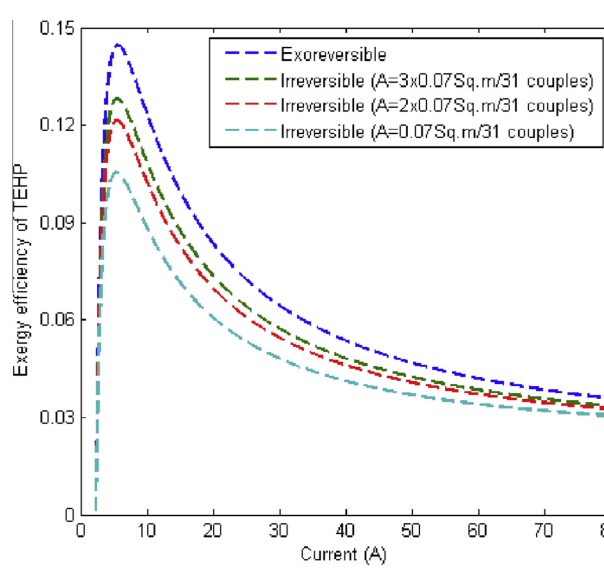

Fig. 10. Exergy efficiency of the thermoelectric heat pump for variable heat transfer area.

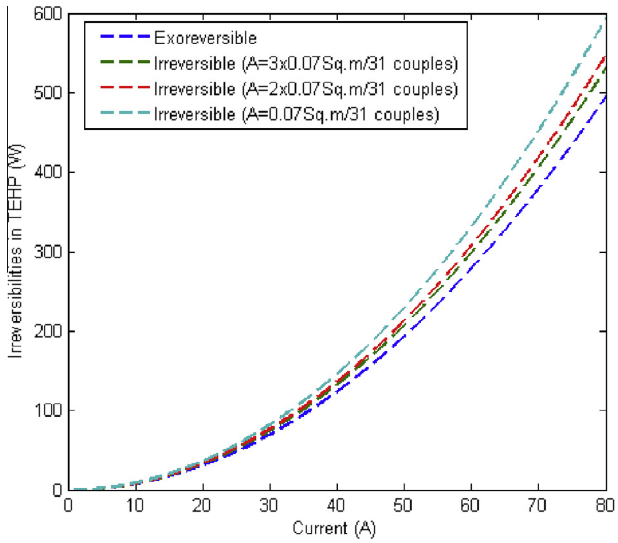

Fig. 11. Irreversibilities in thermoelectric heat pump for variable heat transfer area.

case. This makes the exergy efficiency of exoreversible thermoelectric heat pump higher than the irreversible cases. Therefore, the heat transfer area should be selected based on the minimum exergy destruction and economical consideration.

The effect of contact resistance $R_{contact}$ between the thermoelectric couples and metal contact in the exergy efficiency has been studied since it is one of the important parameter affecting the heating power and the efficiency of the thermoelectric devices. Fig. 12 shows the effect of contact resistance in the exergy efficiency. It shows that the increase in contact resistance decreases the exergy efficiency of the thermoelectric heat pump as expected. Therefore, better electrical contact should be maintained between the contacts. The electrical contact resistance is associated with manufacture quality factor (MQF) as stated by Rowe and Min [19]. Therefore, it is clear that the thermoelectric heat pump devices made of same material may have different exergy efficiency because of different manufacture process.

The irreversibilities in the thermoelectric heat pump for different contact resistance are shown in Fig. 13. It shows that the increase in contact resistance increases the irreversibilities in thermoelectric heat pump.

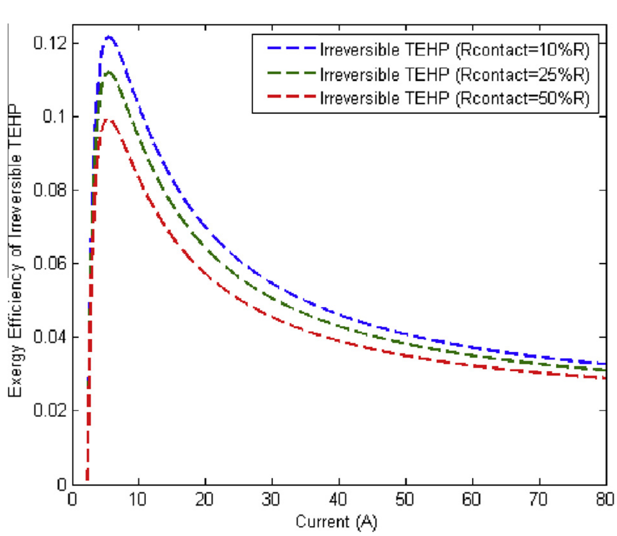

Fig. 12. Exergy efficiency of thermoelectric heat pump for variable contact resistance.

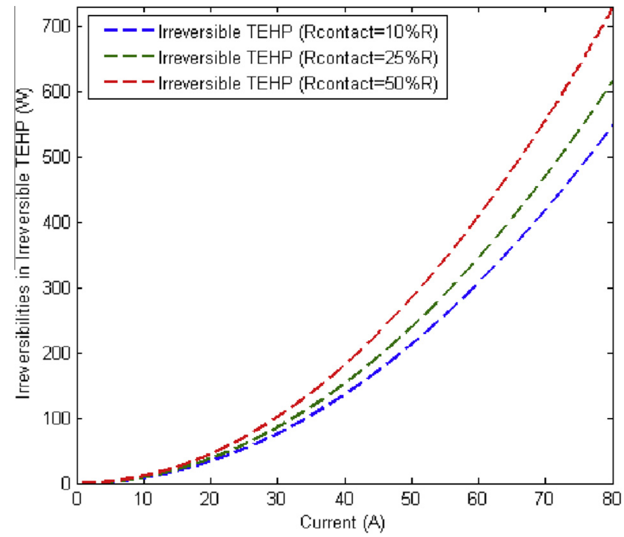

Fig. 13. Irreversibilities in thermoelectric heat pump for variable contact resistance.

Table 2
Comparison of performance parameters of the exoreversible and irreversible thermoelectric heat pump for ΔTh of 10 K.

Sl.No	Performance parameters	Exoreversible thermoelectric heat pump	Irreversible thermoelectric heat pump
1	Maximum energy efficiency	4.52	4.01
2	Maximum exergy efficiency	14.45%	12.81%
3	I_{energy}, I_{exergy}	5.65 A	5.55 A
4	Heating power at maximum energy efficiency	14.72 W	13.03 W
5	Irreversibilities at maximum energy efficiency	2.78 W	2.83 W

It can be seen from Figs. 11 and 13 that the decrease in the heat transfer area and increase in contact resistances increase the irreversibilities in the thermoelectric heat pump system. It is also proved that at higher currents the effect of contact resistance is more in the performance than the irreversible heat transfer because the irreversibilities are more with the increase in contact resistance at higher currents. Therefore, care must be taken during the assembling/designing of the thermoelectric heat pump systems.

The internal irreversibilities in the thermoelectric heat pump system is high compared with the external irreversibilities and once the thermoelectric material with higher FOM is found, then the exergy efficiency of the thermoelectric heat pump will be higher and this will be possible in near future. Table 2 aggregates the results obtained in this study for a typical operating condition.

4. Conclusions

Exergy analyses of all possible thermodynamic models of thermoelectric heat pump have been carried out. This study provides complete details about the exergy efficiency and the irreversibilities in the thermoelectric heat pump system.

- Analytical expressions for the exergy efficiency and irreversibilities in all thermodynamic models of thermoelectric heat pump system have been derived.

- The exergy efficiency of all the thermodynamic models of thermoelectric heat pump is lower than the energy efficiency. For example, in the irreversible thermoelectric heat pump the maximum energy efficiency obtained is 4.01 and the maximum exergy efficiency obtained is 12.81% for the same operating temperatures.
- The exergy efficiency of thermoelectric heat pump increases with increase in ΔTh.
- The energy and exergy efficiency of the exoreversible and irreversible thermoelectric heat pump are maximum at the same optimum current. For example, the energy and exergy efficiency of irreversible thermoelectric heat pump are maximum at same optimum current of 5.55 A.
- The exergy output of the thermoelectric heat pump increases with increase in ΔTh.
- The increase in total contact resistance decreases the exergy efficiency of the thermoelectric heat pump.
- The decrease in heat total external transfer area at hot and cold side of thermoelectric heat pump decreases the exergy efficiency.
- The effect of internal irreversibilities in the performance of the thermoelectric heat pump is more when compared with the external irreversibilities.

The above studies will be very helpful in designing of actual thermoelectric heat pump systems and give better understanding about the thermodynamic modelling, irreversibilities and entropy generation in the thermoelectric heat pump systems.

Conflict of interest

None declared.

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